

# Towards Explaining Distribution Shifts

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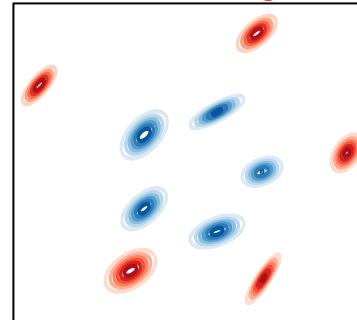


Sean Kulinski

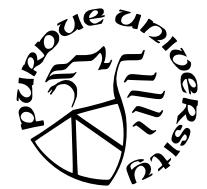
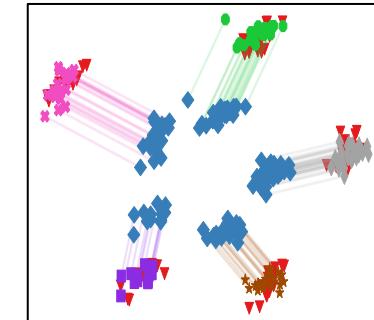


David Inouye

Oracle Shift from  
 $P_{src}$  to  $P_{tgt}$



Proposed Distribution  
Shift Explanation

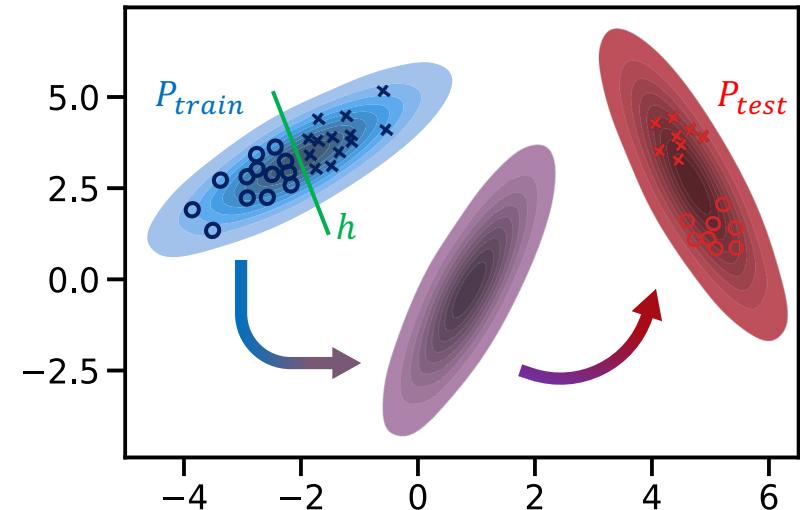


# A distribution shift is when a data distribution changes from what is expected

- In machine learning, a distribution shift is when a **testing distribution** no longer matches the **training distribution**

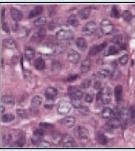
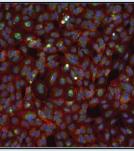
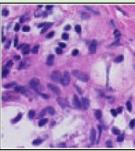
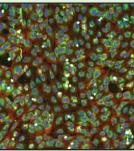
$$P_{test}(x) \neq P_{train}(x)$$

- Under distribution shift, the patterns learned by **a model** might not be present in  $P_{test}$



# Distribution shifts are ubiquitous

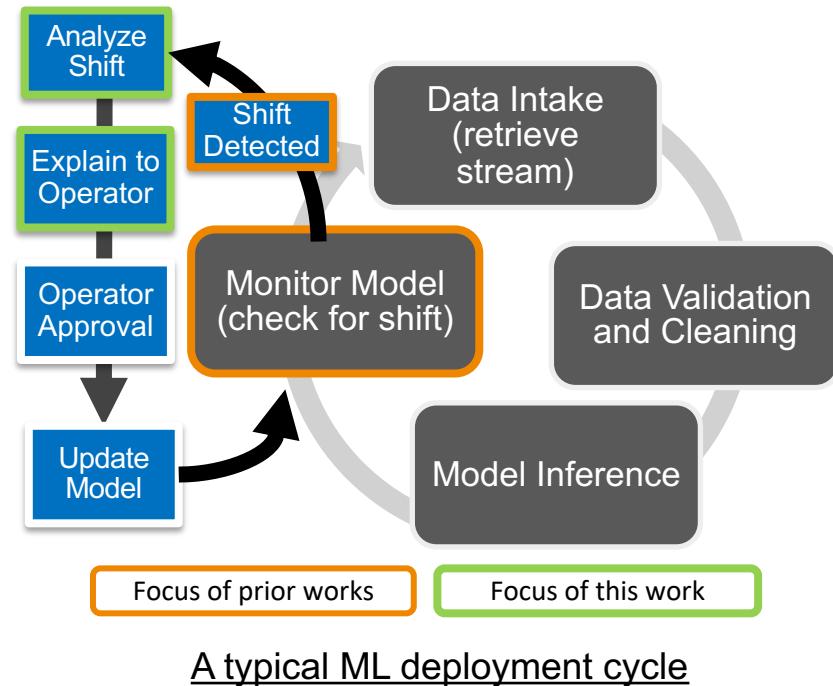
- Any changes in a current data generating environment can cause shifts
- Applying a model to a new domain is almost always a shift

Dataset	iWildCam	Camelyon17	RxRx1	FMoW	PovertyMap	GlobalWheat	OGB-MolPCBA	CivilComments	Amazon	Py150
Input (x)	camera trap photo	tissue slide	cell image	satellite image	satellite image	wheat image	molecular graph	online comment	product review	code
Prediction (y)	animal species	tumor	perturbed gene	land use	asset wealth	wheat head bbox	bioassays	toxicity	sentiment	autocomplete
Domain (d)	camera	hospital	batch	time, region	country, ru/ur	location, time	scaffold	demographic	user	git repo
Source example							<chem>O=C1NC(=O)c2ccccc2N1C</chem>	What do Black and LGBT people have to do with bicycle licensing?	Overall a solid package that has a good quality of construction for the price.	<pre>import numpy as np  ...  norm=np.____</pre>
Target example							<chem>Oc1ccc(cc1)N2Cc3ccsc3N(C(=O)c4ccccc4)C2=O</chem>	As a Christian, I will not be patronizing any of those businesses.	I *loved* my French press, it's so perfect and came with all this fun stuff!	<pre>import subprocess as sp  p=sp.Popen() stdout=p.____</pre>

Exemplar Real-World Distribution Shift datasets from Stanford WILDS benchmarks overview

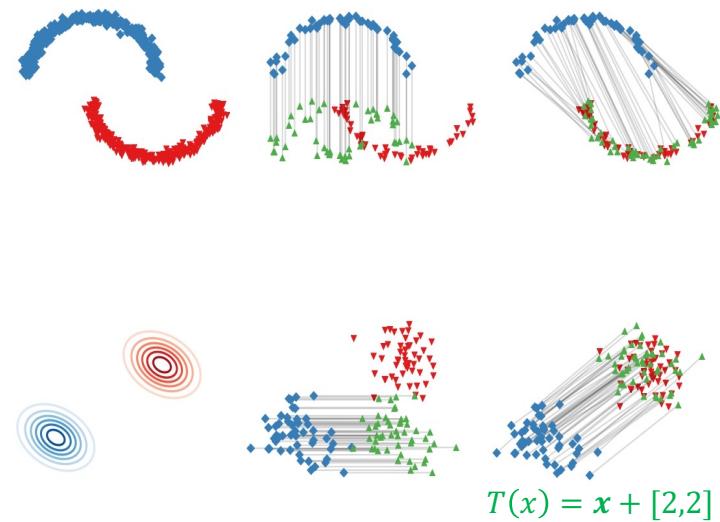
# Knowing what has changed under a shift allows us to more **effectively** respond to mitigate the shift

- **Problem:** Most prior works focus on only *detecting* a shift and do not help with “How should I respond?”
- To most effectively mitigate the shift, an operator needs to know what changed
  - E.g., “Preferences of 18-25 year-olds changed” or “X feature of the data intake pipeline is broken”
- **Our goal:** Aid the operator by **explaining** how  $P_{src}$  shifted to  $P_{tgt}$



# Distribution shifts can be explained by hypothesizing how to map $P_{src}$ to $P_{tgt}$

- Given two distributions  $P_{src}$ ,  $P_{tgt}$ :
  - a transport map  $T(\cdot)$ , is a function which moves a point from  $P_{src}$  to  $P_{tgt}$ , such that  $P_{T(P_{src})} \approx P_{tgt}$
- If  $T$  is interpretable, it can explain how  $P_{src}$  shifted to  $P_{tgt}$



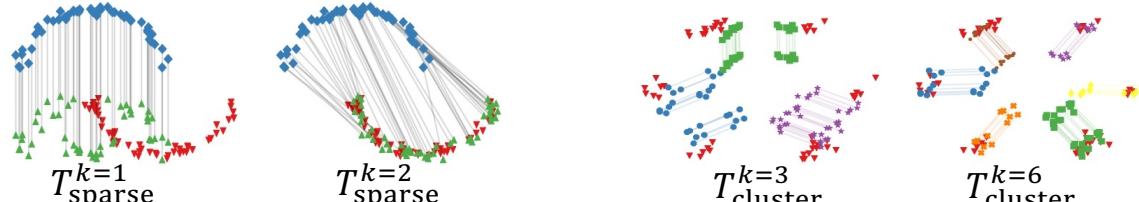
$$T(x) = x + [2,2]$$

# We can leverage prior Optimal Transport work to find **good** interpretable mappings

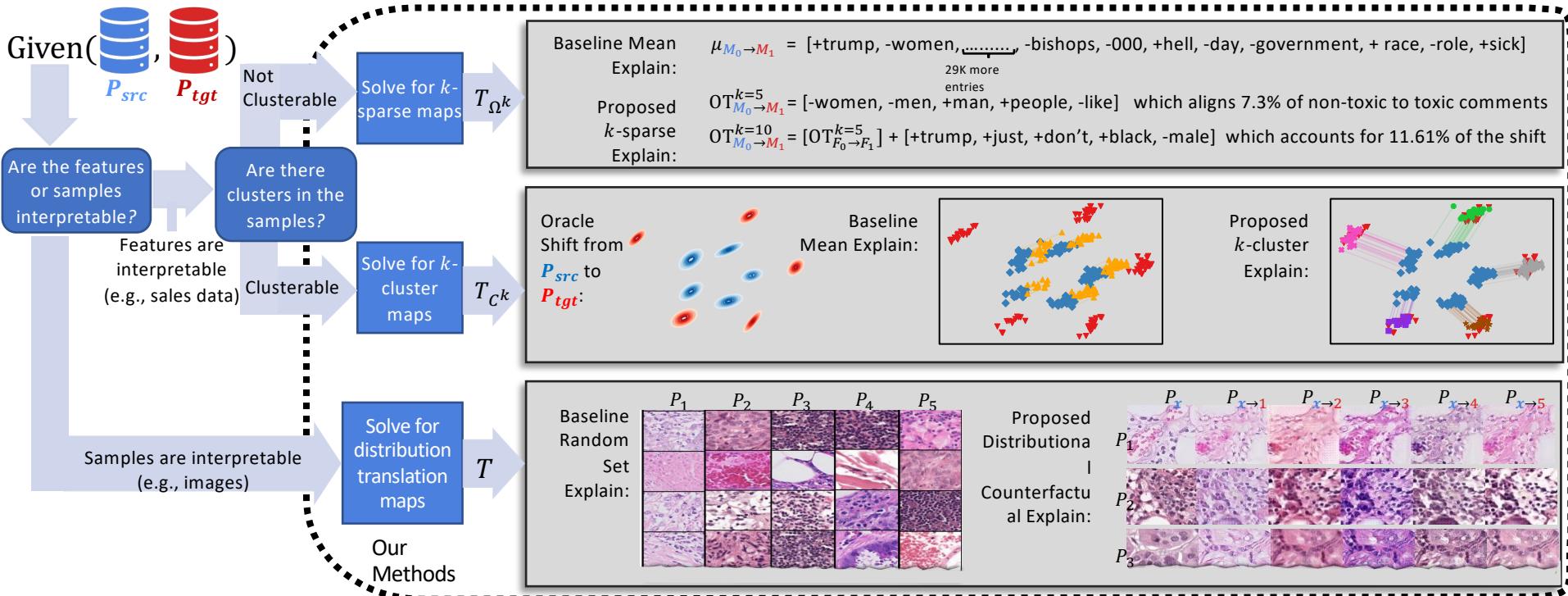
- By relaxing alignment in Optimal Transport and restricting our possible mappings to be interpretable we get *Intrinsically Interpretable Transport*:

$$T_{IIT} := \operatorname{argmin}_{\underbrace{T \in \Omega_{int}}_{\Omega_{int}: \text{A set of interpretable mappings}}} \mathbb{E}_{P_{train}} [\underbrace{c(x, T(x))}_{\text{Cost function: } T \text{ should retain as much of the original point as possible}}] + \lambda \phi(\underbrace{P_{T(X)}}_{\text{Divergence: } T \text{ should align } P_{T(X)} \text{ and } P_{test}}, \underbrace{P_{test}}_{})$$

- $\Omega_{int}$  can be defined based on context, or one can use our pre-defined mappings:  $k$ -sparse feature mappings or  $k$ -cluster mappings



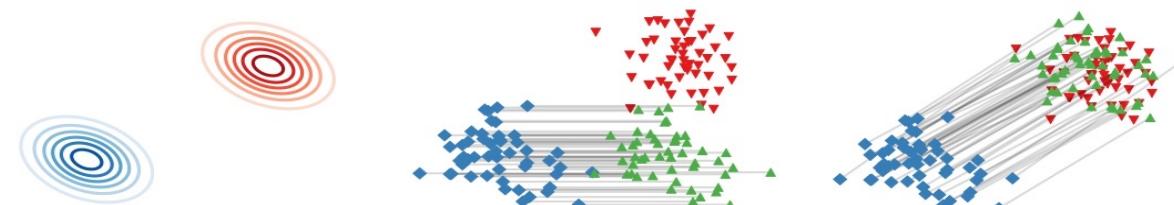
# Methodology for solving for a shift explanation



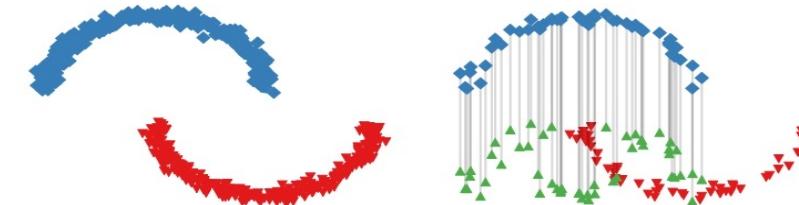
# $k$ -Sparse Feature Mappings can show how features moved along defined axes

- $\Omega_{sparse}^k$ : Find a  $T$  which yields the best alignment, while only moving points along  $k$  dimensions

Simple  
mean shift:



Complex  
conditional shift:



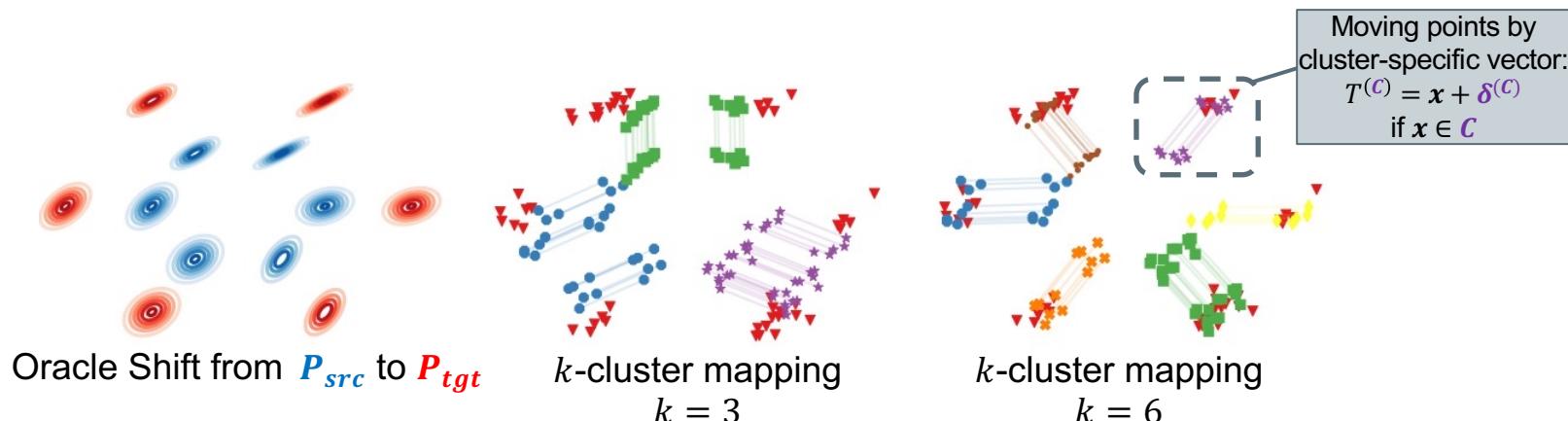
Oracle Shifts from  $P_{src}$  to  $P_{tgt}$

$T_{sparse}^{k=1}$

$T_{sparse}^{k=d}$

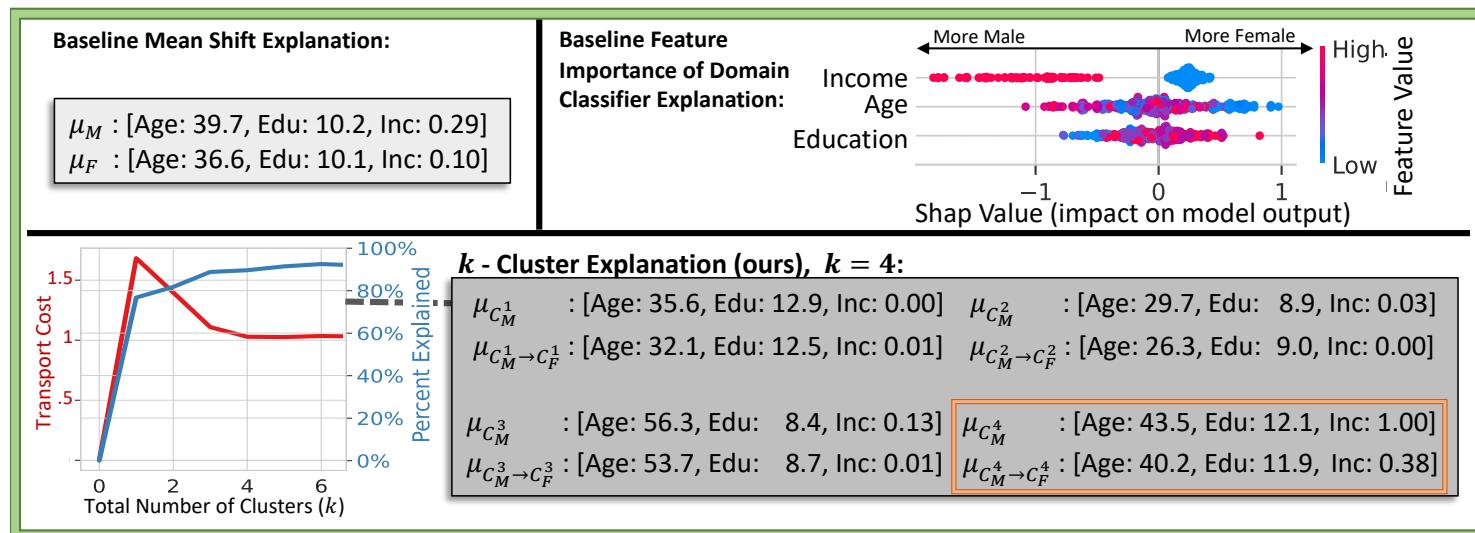
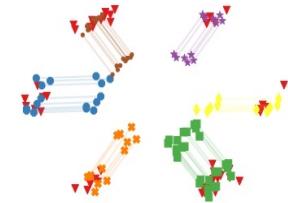
# $k$ -Cluster Mappings can show how heterogenous subgroups have shifted

- $\Omega_{\text{cluster}}^k$ : Find  $k$ -cluster-specific transport maps which maximizes alignment between  $P_{T(P_{tgt})}$  and  $P_{tgt}$ 
  - We can restrict per cluster transport maps to a specific class of transport functions



# $T_{IIT}$ can be used to gain actionable insights from explanations of complex shifts

- Using our  $k$ -cluster mappings  $\Omega_{\text{cluster}}^k$ , we can see how heterogeneous groups (clusters) moved differently under a distribution shift



Example of 6-cluster mapping

**Insight 1 :**  
Income is largest predictor between M and F

**Insight 2 :**  
The income difference is largest in  $M_{C^4}$ , middle-aged adults with a bachelor's degree

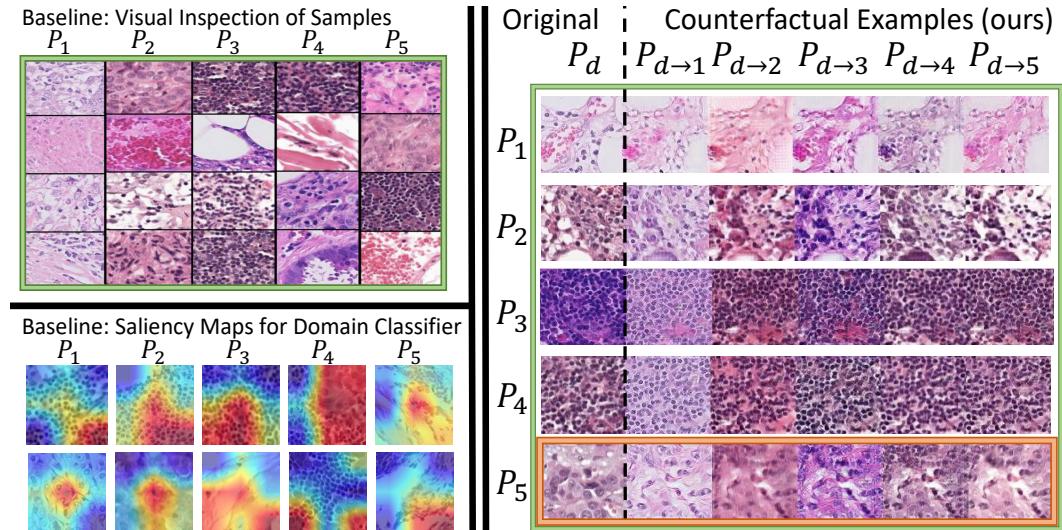
Using  $\Omega_{\text{cluster}}^k$  to compare male and female response to the US 1994 Census

# Transport Maps can also explain distribution shifts in high-dimensional regimes (images)

- When raw features are not semantically meaningful, but samples are (e.g., images), we can use *domain counterfactuals* to understand a complicated  $T$

- Distributional-Counterfactuals :=

$$\{x, T(x) : x \sim P_{src}, T(x) \sim P_{tgt}\}$$

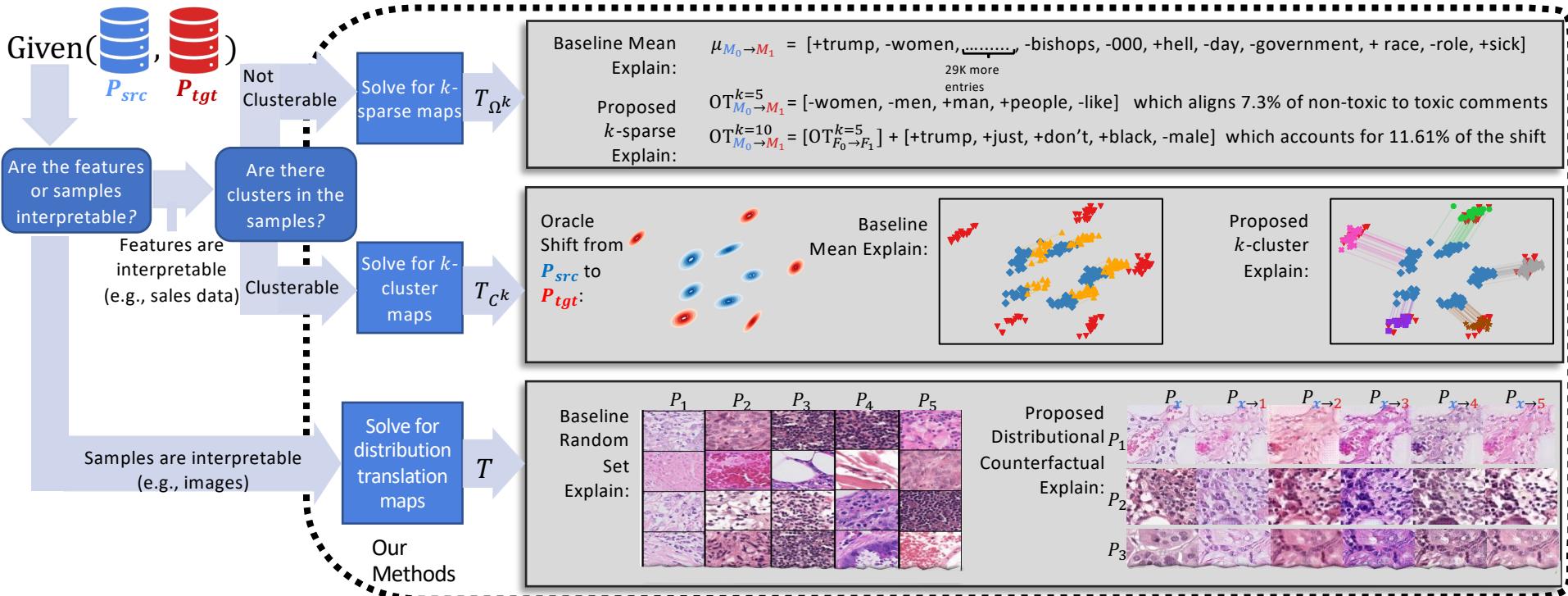


Insight 1: There seems to be a difference in staining across hospitals

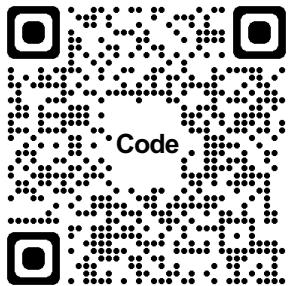
Insight 2: There is a clear difference in staining, and it seems to be unique to each hospital

Using StarGAN to show the difference between tissue samples across 5 hospitals

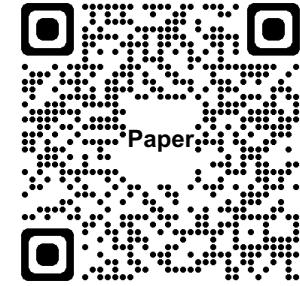
# Methodology for solving for a shift explanation



# Thank you for listening!



# Towards Explaining Distribution Shifts



Sean Kulinski



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