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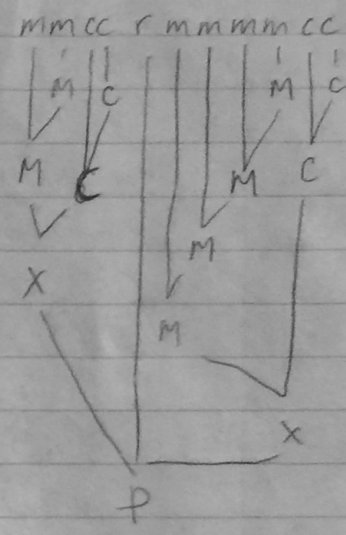
① a) $P \to XrX$
$X \to MC$
$M \to m \mid mM \mid \varepsilon$
$C \to c \mid cC \mid \varepsilon$

b) mmcc r mmmmcc
mMcc r mmmmcc
Mcc r mmmmcc
McC r mmmmcc
MC r mmmmcc
Xr mmmmcc
Xr mmmMcc
Xr mmMcc
Xr mMcc
Xr Mcc
Xr McC
Xr MC
XrX
P

it takes 14 steps



c) Cannot use the same type of solution because we need to keep track of the # of cannibals per missionary. This makes it more difficult

d) $P \to AKJ \mid BRH \mid CRF \mid DRI \mid ERG \mid FRC \mid GRE \mid HRA \mid IRD \mid JRA$

$A \to DI$ $B \to DF$ $C \to DH$ $E \to FN$ $G \to MH$ $J \to \varepsilon$

$M \to m$ $H \to c$
$N \to mM$ $F \to cH$
$D \to mN$ $I \to cF$

$R \to r$

② a) It can start with ab, abb and after 'a' we get 'b' and after 'b' we get 'a'

b) It's ambiguous because there is more than one way to resolve "ab":
$S \to aB \to ab$ or $S \to AB \to aB \to ab$

c) $S \to aO \mid abQ \mid abbR \mid SbR$
$O \to ba \mid baT$
$Q \to baQ \mid \varepsilon$
$R \to abR \mid \varepsilon$
$S \to abR$
$T \to O \mid \varepsilon$

adding the possible start cases prevents the ambiguity (only one parse tree per expression)

3) $\{x \geq 0 \text{ and } y \leq 0\}$ ← post

$\{x \geq 0 \text{ and } y \leq 0\}$ ← negation of 'if' statement

$x = yx \rightarrow \{yx \geq 0 \text{ and } y \leq 0 \Rightarrow x \leq 0\}$

$\{x \leq -10 \text{ and } y < 3\}$ → compatible with previous line condition

For positive if condition

$\{x \geq 0 \text{ and } y \leq 0\}$ ← post

$x = x.x$

$\{y \leq 0$, cannot determine condition on $x \rightarrow$ can be pos. or neg.$\}$

$y = x.y$

$\{x.y \leq 0 \text{ and } x \in \mathbb{R}\}$

$\{x < 0\}$ ← given $\Rightarrow \{y \geq 0\}$

$x = y.x$

$\{y \geq 0 \text{ and } yx < 0 \Rightarrow x < 0\}$

$\{x \leq -10 \text{ and } y \geq 0\}$ ← pre is compatible with previous line condition

4) test: $a = x$, $b = 7$, $b = a$

Loop #	0	1	2	3	4	5	0	1	2	3	4	5
a =	x	x	x^2	x^2	x^4	x^4	x	x	x^2	x^4	x^8	x^9
b =	7	6	3	2	1	0	9	8	4	2	1	0
k =	1	x	x	x^3	x^3	x^7	1	x	x	x	x	x^9

let $M = x$

b: even → $a = x^2$, $b = b/2$, $k = 1$

- b even → $a = x^4$, $b = b/4$, $k = 1$
  - b even → $a = x^8$, $b = b/8$, $k = 1$
  - b odd → $a = x^4$, $b = \frac{b-4}{4}$, $k = x^4$
- b odd → $a = x^2$, $b = \frac{b-2}{2}$, $k = x^2$
  - b even → $a = x^4$, $b = \frac{b-2}{4}$, $k = x^2$
  - b odd → $a = x^2$, $b = \frac{b-3}{2}$, $k = x^3$

b: odd → $a = x$, $b = b - 1$, $k = x$

- b even → $a = x^2$, $b = \frac{b-1}{2}$
  - even k = x → $a = x^4$, $b = \frac{b-1}{4}$, $k = x$
  - odd k = x → $a = x^2$, $b = \frac{b-3}{2}$, $k = x^3$

Post condition: $\{k = M^N\}$

Loop invariant: $\left\{b = \frac{N - \log_M k}{\log_M a}\right\} \Leftrightarrow \left\{k = \frac{M^N}{a^b}\right\}$