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Intro to AI – Homework 1

Problem 1

    a.   There are k variables, each representing one of the knights in the game.

    b.   The values of the variables are the x and y coordinates of the knight on the board.

    c.   The variables are constraint so that the square of the distance between any two variables cannot be equal to 5. So for each pair of variables $X_1 = (x_1, y_1)$ and $X_2 = (x_2, y_2)$, $(x_1 - x_2)^2 + (y_1 - y_2)^2 \mathrel{!=} 5$

Problem 2

$$P(A_1 \cap A_2) = P(A_1)P(A_2 \mid A_1)$$

$$Let\ B = A_1 \cap \ldots \cap A_k$$

$$P(B) = P(A_1)P(A_2 \mid A_1) \ldots P(A_k \mid A_1 \cap \ldots \cap A_{k-1})$$

$$P(B \cap A_{k+1}) = P(B)P(A_{k+1} \mid B)$$

$$= P(A_1)P(A_2 \mid A_1) \ldots P(A_k \mid A_1 \cap \ldots \cap A_{k-1})P(A_{k+1} \mid A_1 \cap \ldots \cap A_k)$$

q.e.d.

Problem 3

$$P(x, y|z) = P(x|z)p(y|z)$$

$$P(x, y|z) = P(x|y, z)p(y|z)$$

$$P(x|z)P(y|z) = P(x|y, z)P(y|z)$$

$$P(x|z) = P(x|y, z)$$

Problem 4

1. Assume a coinflip. P(A) is the probability of heads on first flip. P(B) is probability of heads on second flip. A and B are independent variables. Assume P(C) is the probability of the two coinflips being different. Although A and B are independent, they are not conditionally independent with C since they both affect the outcome of C.

2.

$$P(x, y|z) = P(x|z)P(y|z)$$

$$= \frac{P(x \cap z)P(y \cap z)}{P(z)P(z)}$$

$$= \frac{P(x)P(y)P(z)P(z)}{P(z)P(z)}$$

$$= P(x)P(y)$$

Therefore x and y are unconditionally independent.

Problem 5

1. Yes, observing Intelligent blocks influence to good test taker and understands material.
2. No, seeing the high test score influences explanation of good test taker and understands material.
3. Yes, observing the cause blocks influence to its effects.
4. Yes, understanding material blocks the influence.
5. Yes, understanding material blocks the influence.
6. No, high exam score influences understands material which influences both intelligent and hardworking
7. Yes, good test taker doesn't influence hardworking
8. Yes, observing intelligent blocks influence to hardworking.
9. Yes, high exam score influences both good test taker but is blocked by not observing understands material.
10. No, high exam score influences both good test taker and understands material which influences hardworking.

Problem 6

$$P(+U|+E) = \sum_h P(h) + \sum_i P(i)P(+U|i,h) + \sum_t P(+E|t,+U)P(t|i)$$

$$f_2(h,+I) = P(+E|+T,+U)P(+T|+I) + P(+E|-T,+U)P(-T|+I) = (0.9)(0.8) + (0.7)(0.2) = 0.86$$

$$f_2(h,-I) = P(+E|+T,+U)P(+T|-I) + P(+E|-T,+U)P(-T|-I) = (0.9)(0.5) + (0.7)(0.5) = 0.8$$

$$f_1(+H) = P(+I)P(+U|+I,+H)f_2(+H,+I) + P(-I)P(+U|-I,+H)f_2(+H,-I)$$
$$= (0.7)(0.9)(0.86) + (0.3)(0.5)(0.8) = 0.6618$$

$$f_1(-H) = P(+I)P(+U|+I,-H)f_2(-H,+I) + P(-I)P(+U|-I,-H)f_2(-H,-I)$$
$$= (0.7)(0.3)(0.86) + (0.3)(0.1)(0.8) = 0.2046$$

$$P(+U|+E) = P(+H)f_1(+H) + P(-H)f_1(-H) = (0.6)(0.6618) + (0.4)(0.2046) = \mathbf{0.47892}$$