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Introduction to Logic
Quiz 4

3. (3 pts) Symbolize the following claims about natural numbers into logic, or translate back into fluent English expressions of mathematical claims about numbers, using the following symbols and interpretations:

Domain of Discourse: Natural Numbers (0,1,2,…)

E(x): x is even	s: successor function	0: zero
O(x): x is odd	+: addition function	x < y: x is smaller than y
P(x): x is prime	×: multiplication function	

a. Three is an odd number

$$\boldsymbol{O}\left(\boldsymbol{s}\left(\boldsymbol{s}\left(\boldsymbol{s}(\boldsymbol{0})\right)\right)\right)$$

b. $\forall x\, \forall y\, ((O(x) \wedge O(y)) \rightarrow O(x \times y))$

The product of any 2 odd numbers is odd

c. The successor of any odd number is even

$$\forall \boldsymbol{x}\ \left(\boldsymbol{O}(\boldsymbol{x}) \rightarrow \boldsymbol{E}\left(\boldsymbol{s}(\boldsymbol{x})\right)\right)$$

d. For every number there is a greater number (i.e. there is no greatest number)

$$\forall \boldsymbol{x}\ \left(\boldsymbol{x} \rightarrow \boldsymbol{s}(\boldsymbol{s})\right)$$

e. $\exists x\, (O(x) \wedge x < s(s(0)) \wedge \forall y\, ((O(y) \wedge y < s(s(0))) \rightarrow y = x))$

1 is the only odd number less than 2

f. Every even number greater than 2 is the sum of two prime numbers

$$\forall \boldsymbol{x}\ \left(\boldsymbol{E}(\boldsymbol{x}) \wedge \boldsymbol{x} > \boldsymbol{s}\left(\boldsymbol{s}(\boldsymbol{0})\right)\ \rightarrow\ \exists \boldsymbol{y} \exists \boldsymbol{z}\ \left(\boldsymbol{P}(\boldsymbol{y}) \wedge \boldsymbol{P}(\boldsymbol{z}) \wedge (\boldsymbol{x} = \boldsymbol{y} + \boldsymbol{z})\right)\right)$$