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Introduction to Logic
Quiz 3

1. Logic (1 pt) Give two first-order logic statements that are equivalent according to quantificational logic, but not according to truth-functional logic

$$\forall x\ (P(x) \land \neg P(x)$$
$$\big(\neg \forall x\, P(x)\big) \land \big(\exists x\, \neg P(x)\big)$$

2. Symbolization. (4 pts) Symbolize the following claims in predicate logic.

a. Some horses are not gentle (H(x): x is a horse, G(x): x is gentle)

$$\exists x\ \big(G(x) \land\ \neg G(x)\big)$$

b. All fat lawyers are happy (L(x): x is a lawyer, F(x): x is fat, H(x): x is happy)

$$\forall x\ \Big(\big(L(x) \land F(x)\big) \rightarrow H(x)\Big)$$

c. Fruits and vegetables are nourishing (F(x): x is a fruit, V(x): x is a vegetable, N(x): x is nourishing)

$$\forall x\ \Big(\big(F(x) \lor V(x)\big) \rightarrow N(x)\Big)$$

d. Only cubes are small (Cube(x): x is a cube, Small(x): x is small)

$$\forall x\ \big(Small(x) \rightarrow Cube(x)\big)$$

e. Some person donates to all charities (P(x): x is a person, C(x): x is a charity, D(x,y): x donates to y)

$$\exists x\ (P(x) \land (\forall y(C(y) \rightarrow D(x,y))))$$

f. No cube is between two small dodecahedrons (Cube(x): x is a cube, Small(x): x is small, Between(x,y,z): x is between y and z, Dodec(x): x is a dodecahedron)

$$\neg \exists x \exists y \exists z\ \big(Cube(x) \land Small(y) \land Dodec(y) \land Small(z) \land Dodec(z)\big)$$

g. Every cube with nothing to its left has something to its right (Cube(x): x is a cube, RightOf(x,y): x is to the right of y, LeftOf(x,y): x is to the left of y)

$$\forall x \Bigg(\Big(Cube(x) \land \neg \exists y\big(LeftOf(y,x)\big)\Big) \rightarrow \exists y\big(RightOf(y,x)\big)\Bigg)$$

h. Any fish that chases every shiner will be hooked by an angler who uses a shiner for bait (F(x): x is a fish, S(x): x is a shiner, C(x,y): x chases y, H(x,y): x hooks y, A(x): x is an angler, B(x,y): x uses y for bait)

$$\forall x\ \Bigg(\Big(F(x) \land \forall y\big(S(y) \land C(x,y)\big)\Big) \rightarrow\ \exists y\ \Big(A(y) \land H(y,x) \land \exists z\big(S(z) \land B(y,z)\big)\Big)\ \Bigg)$$

3. Rewriting Statements (1 pt) Give a chain of equivalences (do state the principles you use! Also, you are *not* allowed to use the Aristotelean Square of Opposition Equivalences) showing that:

$\neg\exists y\,(E(y) \wedge \forall x\,(F(x) \to G(y, x))) \Leftrightarrow \forall v\,\exists w\,(E(v) \to (F(w) \wedge \neg G(v, w)))$

$$\neg\exists y\,(E(y) \wedge \forall x\,(F(x) \to G(y,x)))$$

$$\forall y\,\neg\left(E(y) \wedge \forall x\,\left(F(x) \to G(y,x)\right)\right) \quad \textit{Negation}$$

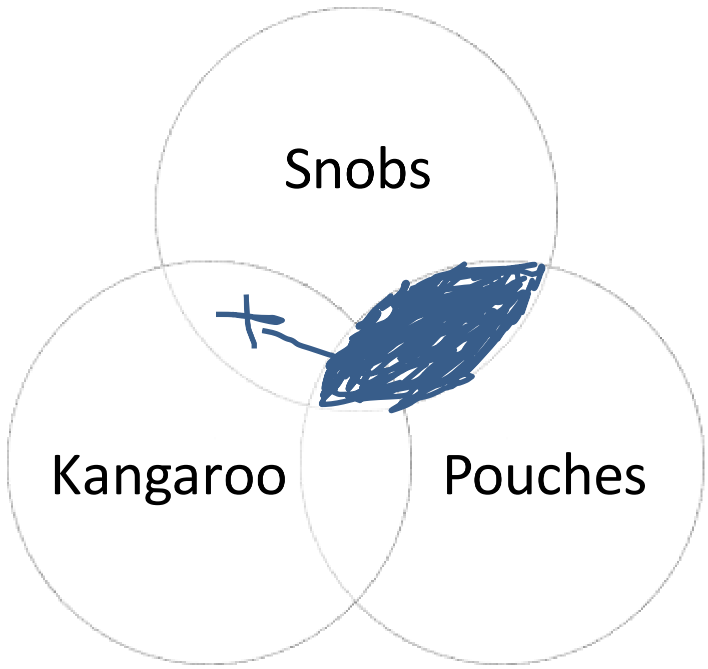
$$\forall y\,\left(\neg E(y) \vee \neg\forall x\,\left(F(x) \to G(y,x)\right)\right) \quad \textit{DeMorgan's}$$

4. Equivalence. (2 pts) Given that R is some arbitrary binary predicate, determine for each of the following pairs of statements whether they are equivalent according to quantificational logic or not. (You don't have to give any kind of proof. Just say whether the two statements mean the same thing.)

a. $\forall x\,\exists y\, R(y,x)$, $\forall y\,\exists x\, R(x,y)$ **Equivalent**
b. $\neg\exists x\,\exists y\, R(x,y)$, $\forall x\,\forall y\,\neg R(y,x)$ **Equivalent**
c. $\exists x\,\neg\exists y\, R(x,y)$, $\neg\forall y\,\exists x\, R(y,x)$ **Not Equivalent**
d. $\exists x\,\neg\exists y\, R(x,y)$, $\exists y\,\neg\exists x\, R(x,y)$ **Not Equivalent**

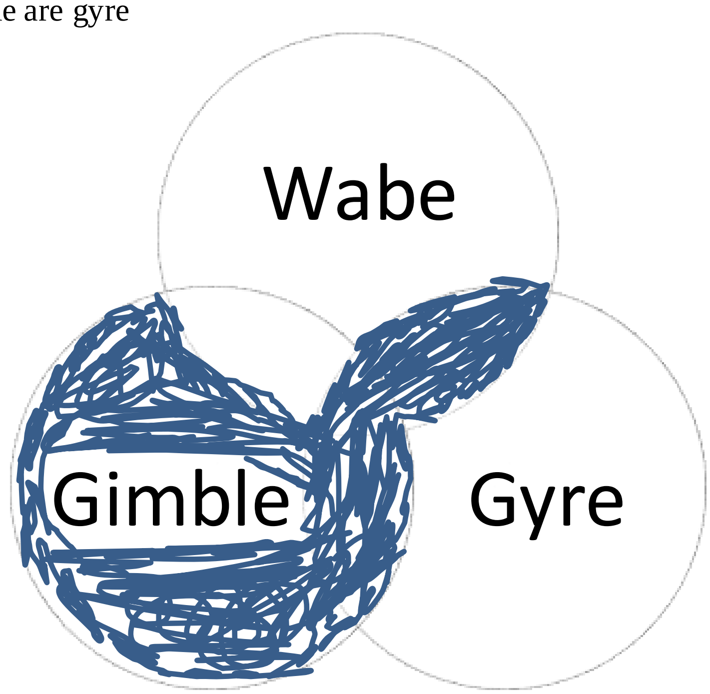
5. Venn Diagrams (2 pts) Use a Venn Diagram to determine for each of the following two syllogisms whether it is valid or invalid. Make sure to explain your answer, by referring to the Venn diagram and indicating what you are seeing in the diagram.

   a. Some snobs are kangaroos
      No snobs have pouches
      --
      Some kangaroos have pouches



This is not a valid argument. There is no ‘x’ in the intersection of kangaroo and pouches, so it is not necessarily true that some kangaroos have pouches.

b. No gyre are wabe
   All gimble are wabe
   --
   No gimble are gyre



This is a valid argument. The intersection of gimble and gyre is shaded.