Assignment 3: Numerical solution of an ordinary differential equation

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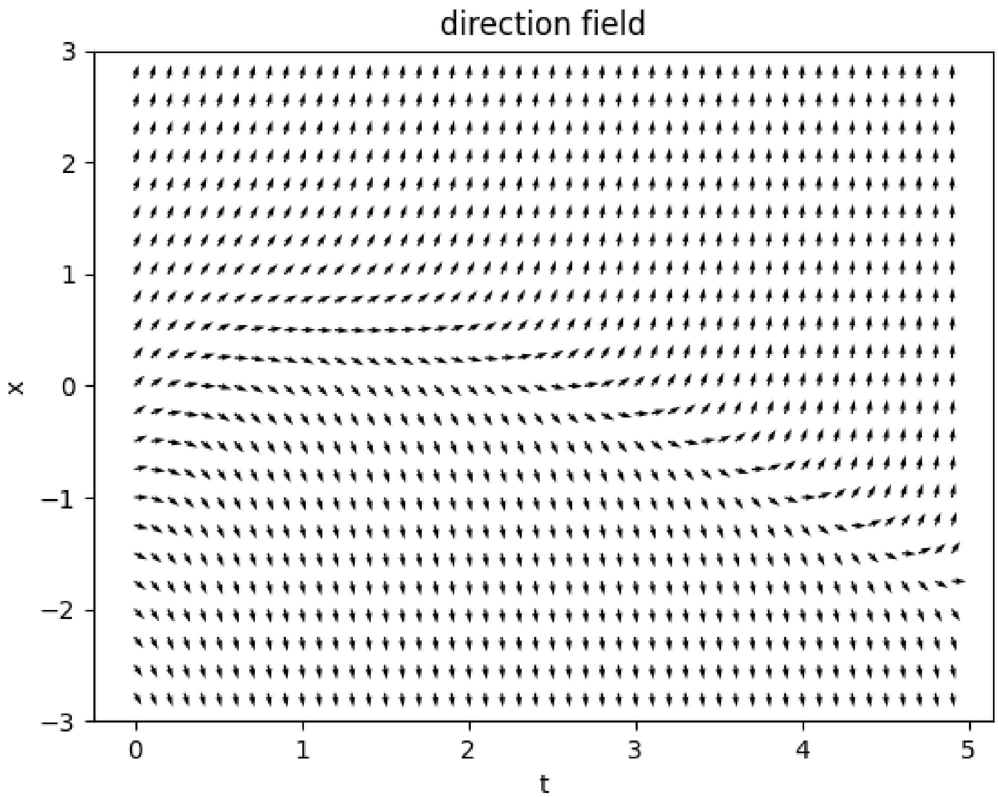
Exercise 1

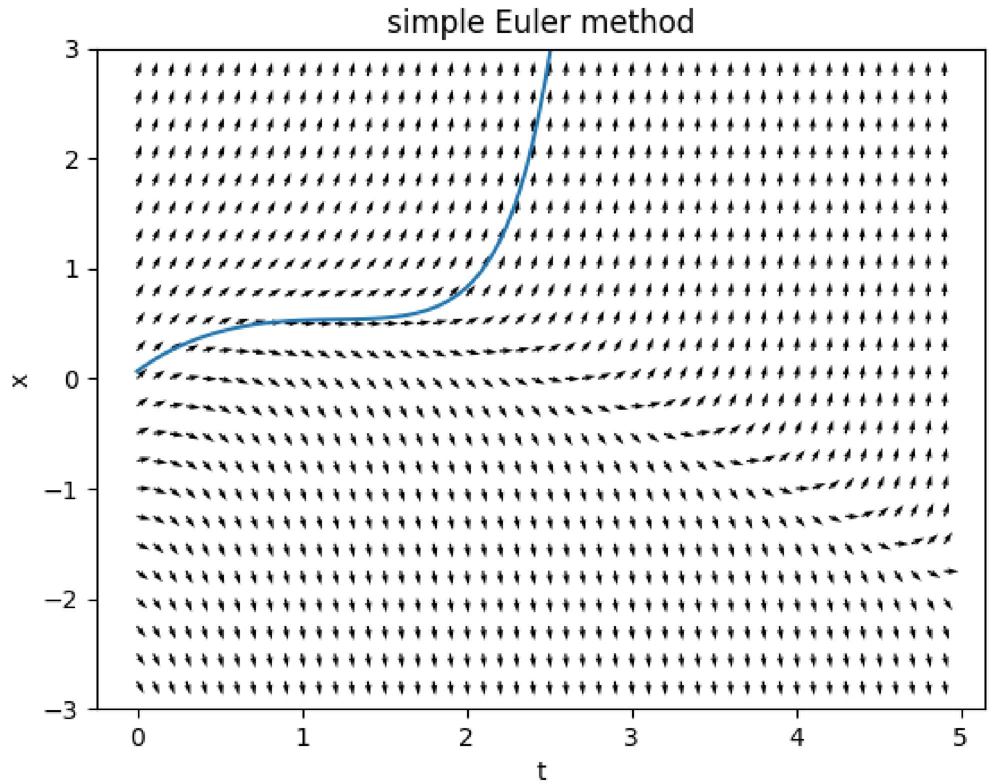
The first exercise was to plot the direction field for the differential equation given. A direction field plots the slope dx/dt to points (t, x) on a x-t grid. The direction field was made using the np.mesh function and the ax.quiver function. The mesh was set to have a range from -3 to 3 for t and 0 to 6 for x. The arrows were generated with the ax.quiver function and were normalized to have unit length by dividing the x direction and y direction arguments by the vectors magnitude. The graph is shown below.



It can be seen at the middle of the graph where x is zero moving right, the arrows curve upwards at a higher initial value of x, and downwards at a lower initial value of x. This is the critical point, the point at which solutions either tend towards plus or minus infinity if the initial value is above or below.

Exercise 2

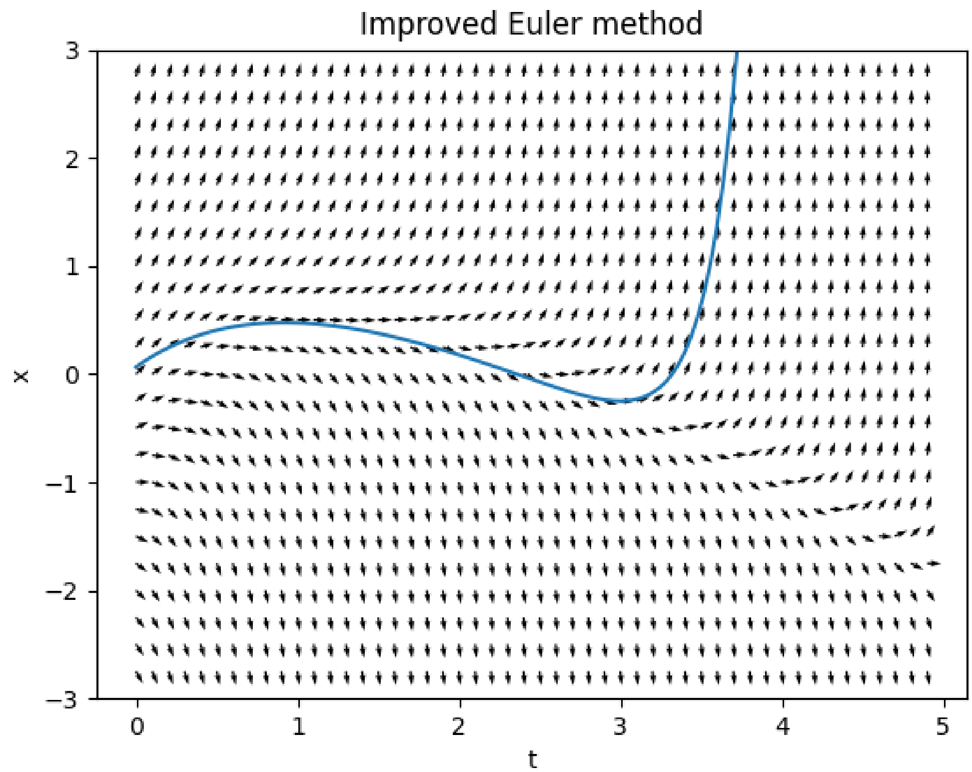
The second exercise was to plot a numerical solution for x onto the direction field using the simple Euler method. An initial value of x(0) = 0.0655 was chosen, as this value is close to the critical value of 0.065923. If above the critical value the solution tends to plus infinity, and if below the critical value it will tend to minus infinity. A step size of 0.04 was chosen. The results are shown below.

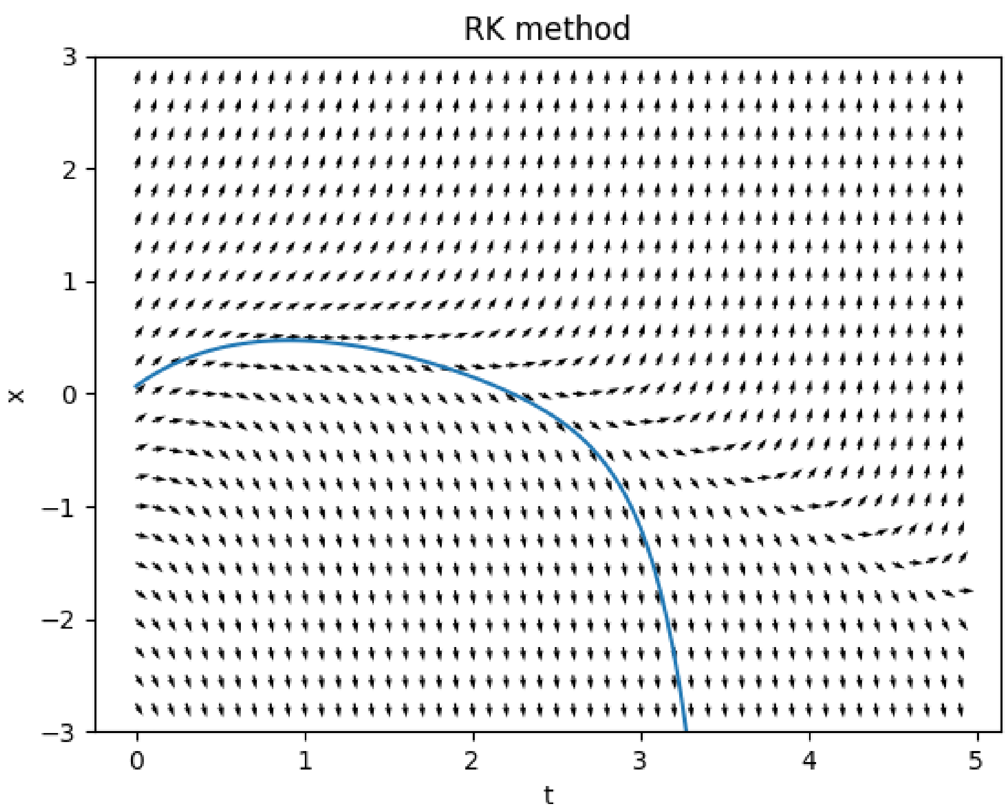


The above graph shows the simple euler method following the direction field although with some error.

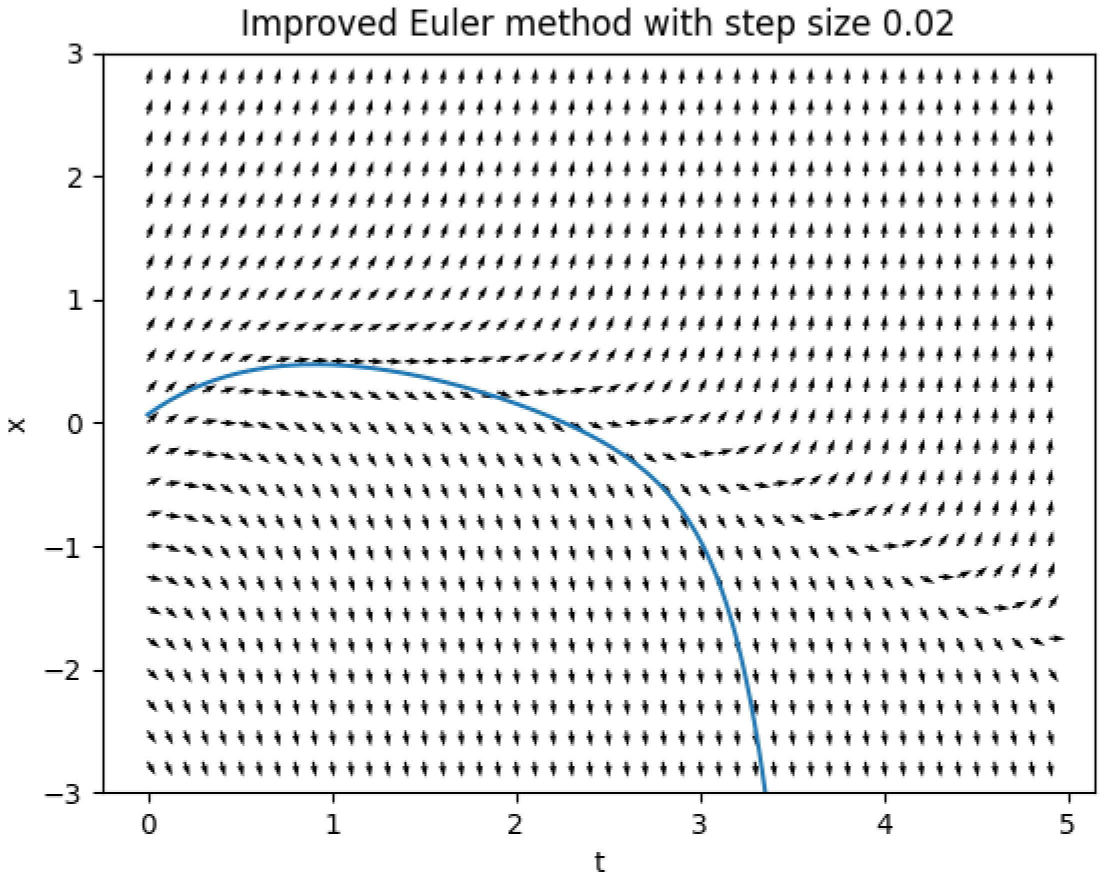
Exercise 3

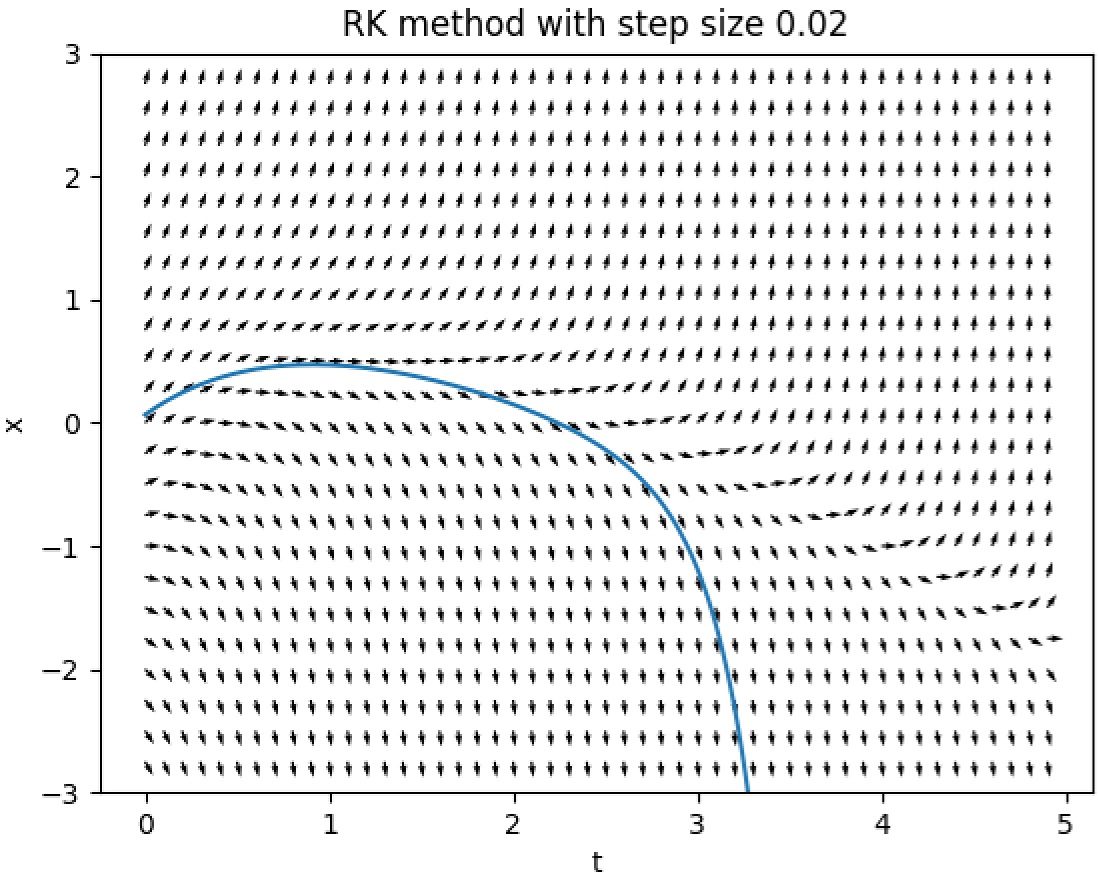
The third exercise was to use the improved Euler method and Runge-Kutta method to numerically solve the ODE and compare with the direction field and simple Euler method to analyse the accuracy of each method. Once again a step size of 0.04 was used. It can be seen from the graphs that the improved Euler method and Runge-Kutta method both show better alignment with the direction field.





The step size was then reduced to 0.02


Improved Euler method with step size 0.02
3
2
1
0
−1
−2
−3
x
0
1
2
3
4
5
t



It is clear that accuracy is improved with the more sophisticated integration schemes and lower step size. The local truncation error for the simple Euler method is proportional to the square of the step size. This is the error introduced after each iteration of the scheme. This is a much greater error than the improved Euler method or the Runge-Kutta method. The Runge-Kutta scheme is a fourth order method, which makes the local error proportional to the fifth power of the step size.

With a smaller step size local error is reduced, however global error can remain high over many iteration of an integration scheme as error accumulates. The advantage of these high accuracy integration schemes is that they can minimize error for large ranges of values and provide a more accurate solution.