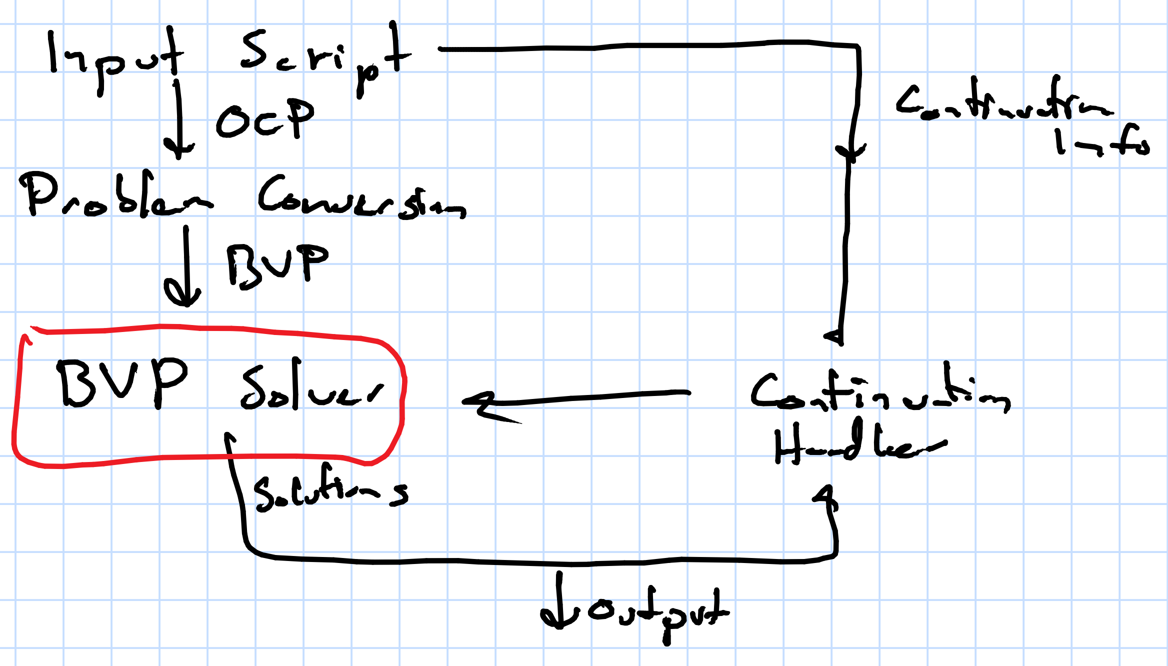
jws

BVP Solvers Overview



Two-point BVP

Find $y(t)$ such that

$$\dot{y}(t) = f(t, y)$$
$$\Psi_0(t_0, y(t_0)) = 0$$
$$\Psi_f(t_f, y(t_f)) = 0$$

In this case, the OCP is collapsed into the BVP such that

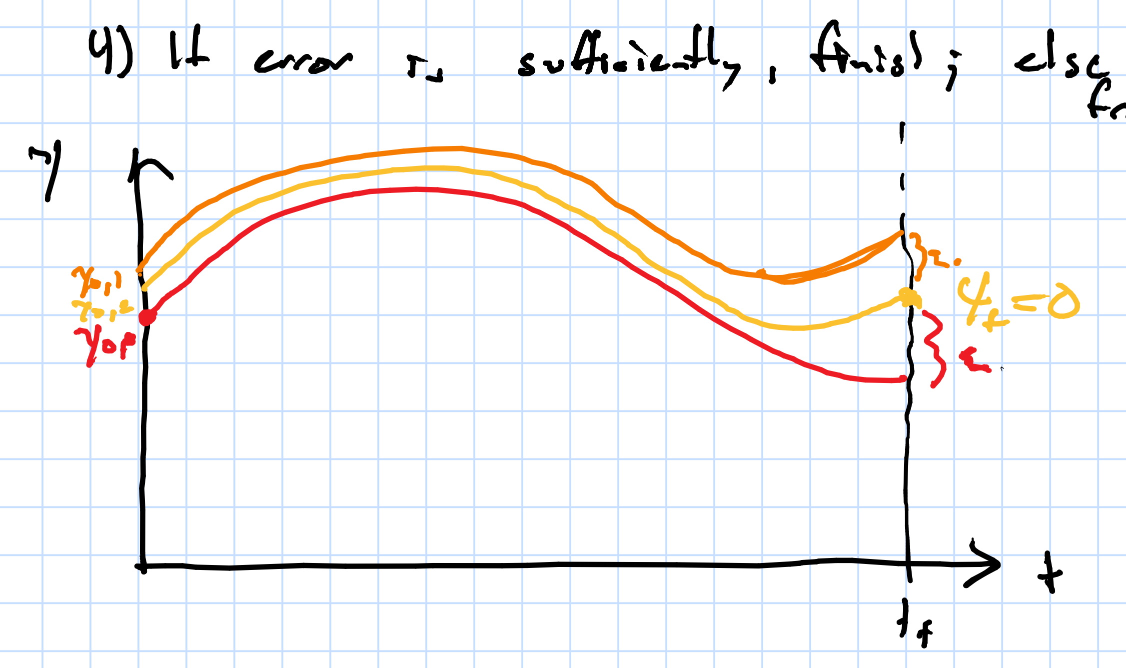
$$y = [x^T, \lambda^T]^T$$

Single Shooting Method ⟷ Collocation

Multiple Shooting

Shooting Method

- Shooting methods reduce BVPs into corresponding IVPs
- IVPs can be solved relatively easily by sequential propagation (RK45)
- Shooting method steps
  1) Choose (guess) values of free initial states (y_0)
  2) Propagate the trajectory forward to terminal point
  3) Compute the error in the terminal condition
  4) If error is sufficiently, finish; else repeat from 1)



- How do we adjust y_0?
  - For a linear system:

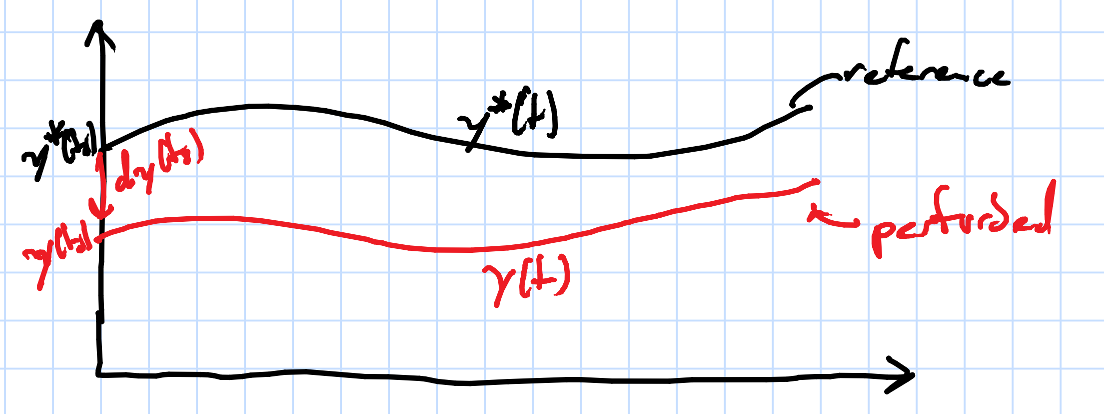
$$\Delta y_0 \Rightarrow \Psi_f = 0$$
$$\underset{\text{step}}{d y_0} \rightarrow \Psi_f = 0$$

$$\frac{d\Psi_f}{dy_0} = \frac{\partial \Psi_f}{\partial y_f} \frac{dy_f}{dy_0}$$

$$d\Psi_f = \frac{\partial \Psi_f}{\partial y_f} \frac{dy_f}{dy_0} dy_0$$

ϕ state transition matrix

Define $\quad \phi(t_f, t_0) = \dfrac{dy(t_f)}{dy(t_0)}$



$$y(t) = y^*(t) + dy(t) \qquad \text{let } \dot{y} = f(y)$$

$$\frac{d}{dt}(y^* + dy) = f(y^* + dy)$$

$$\cancel{\dot{y}^*} + d\dot{y} = \cancel{f(y^*)} + \left.\frac{\partial f}{\partial y}\right|_{y^*} dy + \cancel{HOT}(dy) \quad \text{ignore}$$

$$d\dot{y} = \left.\frac{\partial f}{\partial y}\right|_{y^*} dy$$

$$d\dot{y} = F^* dy$$

Above describes the time evolution of a "small" perturbation in the reference trajectory

Define state transition matrix

$$\frac{dy(t)}{dy(t_0)} = \phi(t, t_0) \Rightarrow dy(t) = \phi(t, t_0)\, dy(t_0)$$

ϕ is square matrix

Differentiate wrt t

$$d\dot{y}(t) = \dot{\phi}(t, t_0)\, dy(t_0)$$
$$\Downarrow$$
$$F^* dy(t) = \dot{\phi}(t, t_0)\, dy(t_0)$$
$$\Downarrow$$
$$F^* \phi(t, t_0) \cancel{dy(t_0)} = \dot{\phi}(t, t_0) \cancel{dy(t_0)}$$

$$\dot{\phi}(t, t_0) = F^* \phi(t, t_0)$$

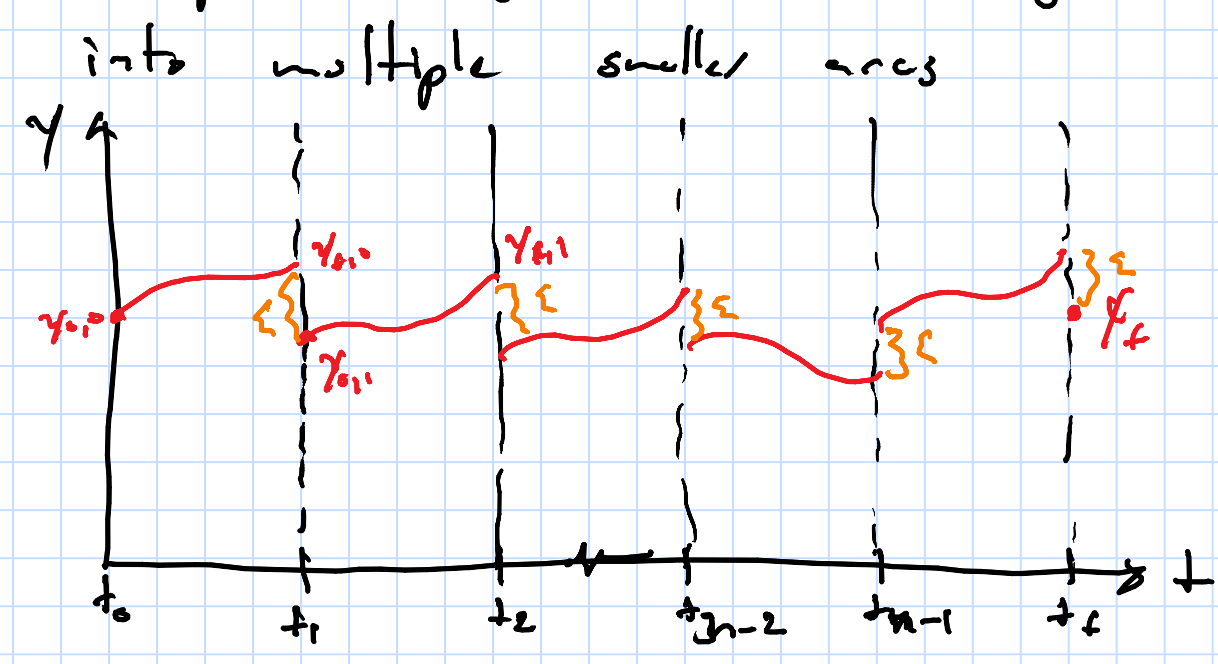
The STM can be propagated with y

$$\phi(t_0, t_0) = \frac{dy(t_0)}{dy(t_0)} = I$$

$$\phi(t_2, t_0) = \phi(t_2, t_1)\, \phi(t_1, t_0)$$
$$= \frac{dy(t_2)}{dy(t_0)} = \frac{dy(t_2)}{dy(t_1)} \frac{dy(t_1)}{dy(t_0)}$$

Multiple Shooting Method

- Single shooting is often numerically unstable
- Stability can be improved by decreasing the time errors have to develop
- Multiple shooting divides the single arc into multiple smaller arcs



Now we need to find not just y_0 but $y_{0,n}$ for $n = 1 \ldots n$

In extreme case, each arc would only contain a single propagation step, this is a collocation

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