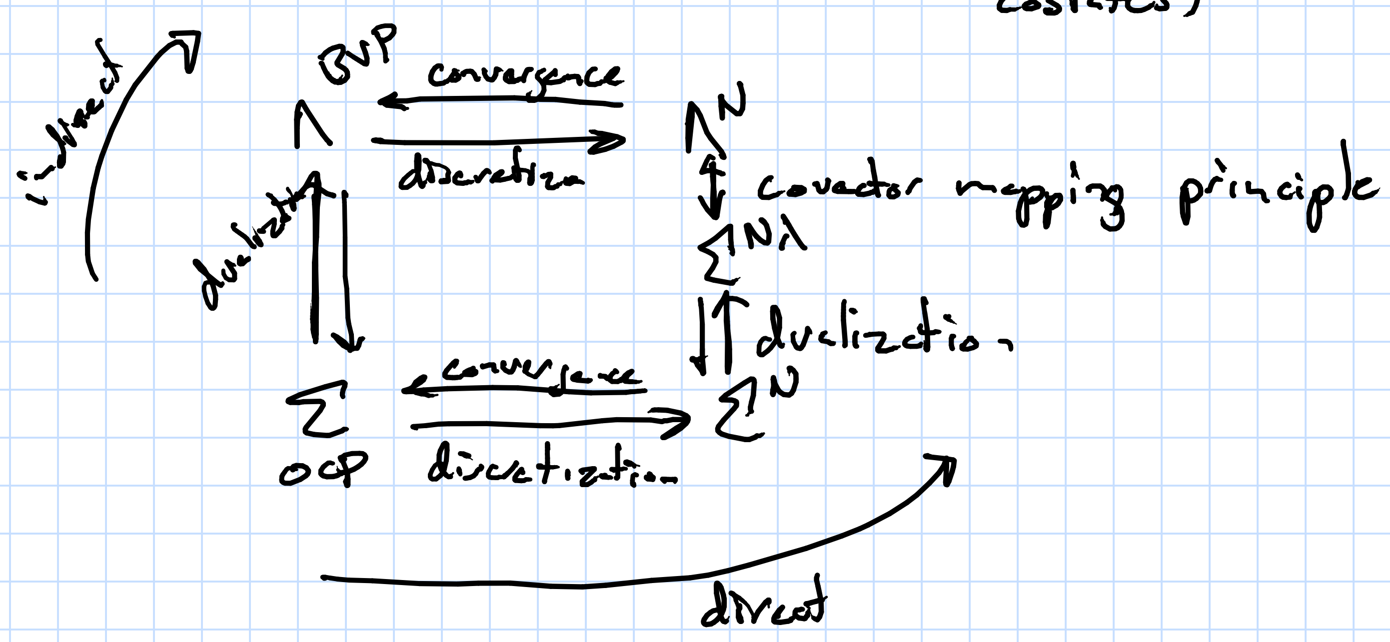
JWJ

Why Beluga?

- By using the additional information provided by the "dual" problem, indirect methods can rapidly generate high quality optimal trajectory solutions
- Beluga is one of a very few available indirect, general purpose trajectory optimizers

- Indirect vs. direct
  - Direct methods discretize the problem first, then optimize the objective directly manipulating the control variables within a NLP solver (in which dualization occurs)
  - Indirect methods apply calculus of variations to convert the OCP into a BVP (dualization) in which the control is specified indirectly via the control law. The BVP is solved numerically through a BVP solver

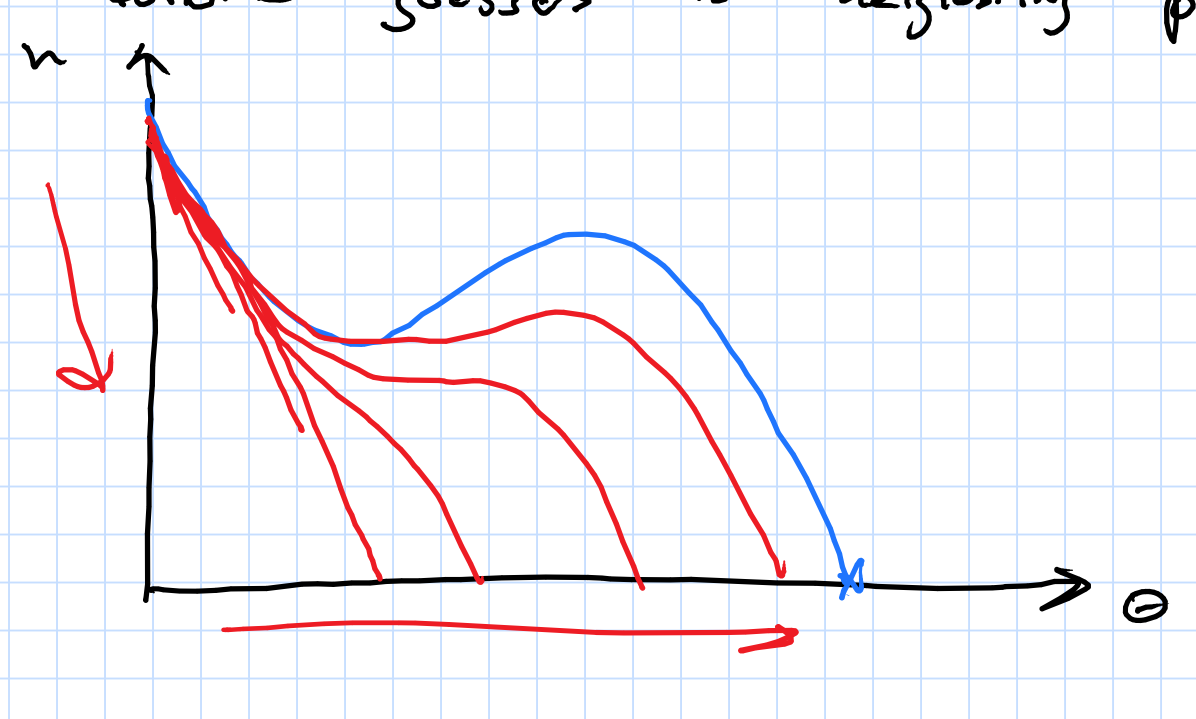
* Note: Dualization is the adjoining of constraints with adjoint variables (Lagrange mult., KKT, costates)



- Direct methods have been dominate since the advent of sufficiently fast computers
- Traditional difficulties w/ indirect methods
  1. Deriving analytical expressions for complicated nonlinear problems is difficult
     - Computer Algebra System
     - Index-Reduction
  2. The region of convergence for BVP solvers can be very small especially considering costates can be difficult to guess
     - Continuation
  3. Solving problems w/ inequality path constraints requires knowing the sequence of constrained and unconstrained subarcs beforehand
     - UTM and other piecewise methods

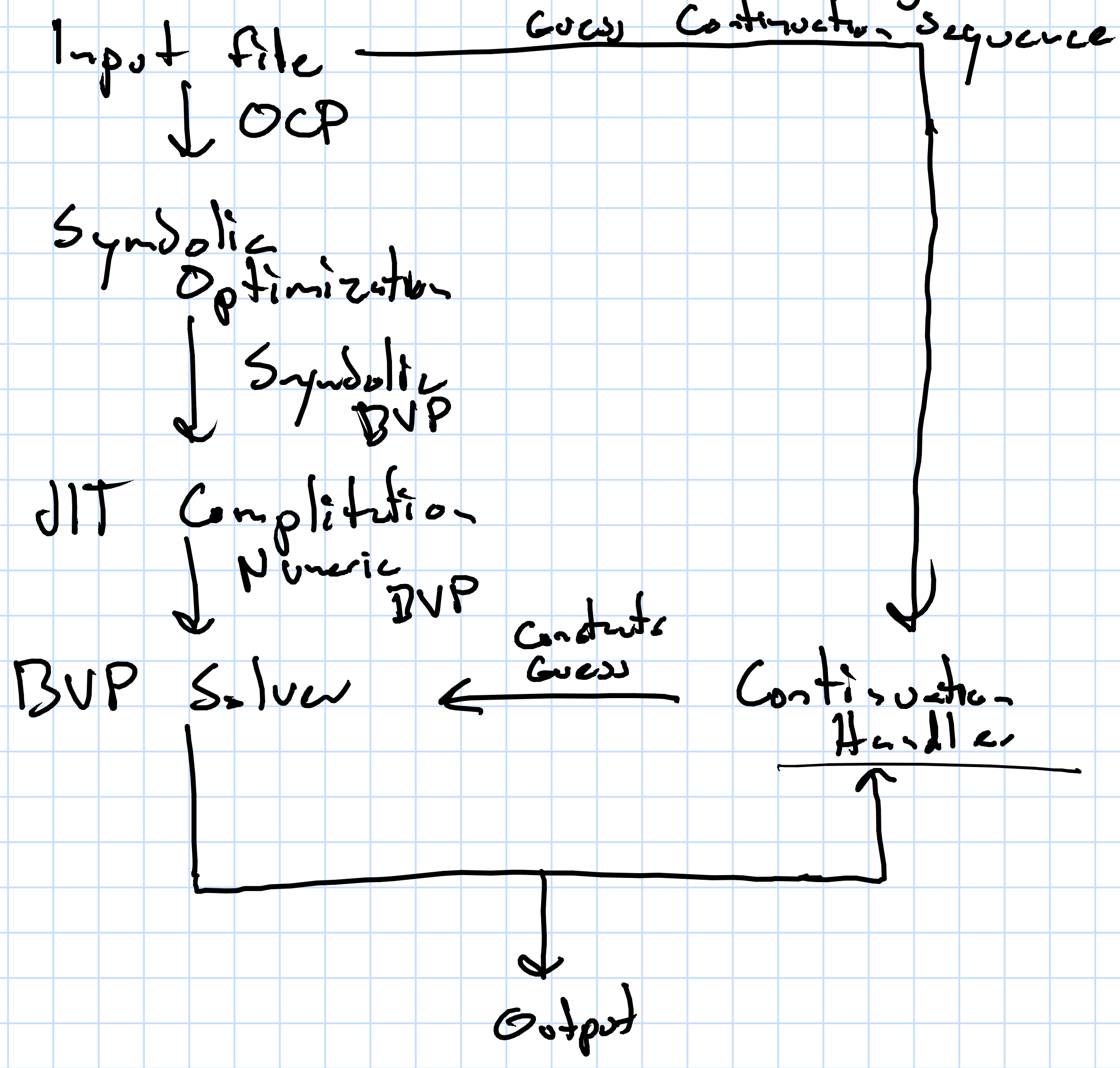
Homotopy Continuation Methods

- It may be practically impossible to form an initial guess for a complex problem that converges in the BVP solver
- But, finding an initial guess to a shorter/simpler/unconstrained/etc. may be relatively easy
- Continuation methods allow the transformation of a "simple" solution to a complex one by gradually evolving the problem and using previously obtained solutions as future guesses for neighboring problems



JMS

Flowchart of beluga



Brachistochrone Problem

Find shape of slope that minimize time for object to reach the bottom

$$\min \; J = t_f = \int_0^{t_f} dt$$

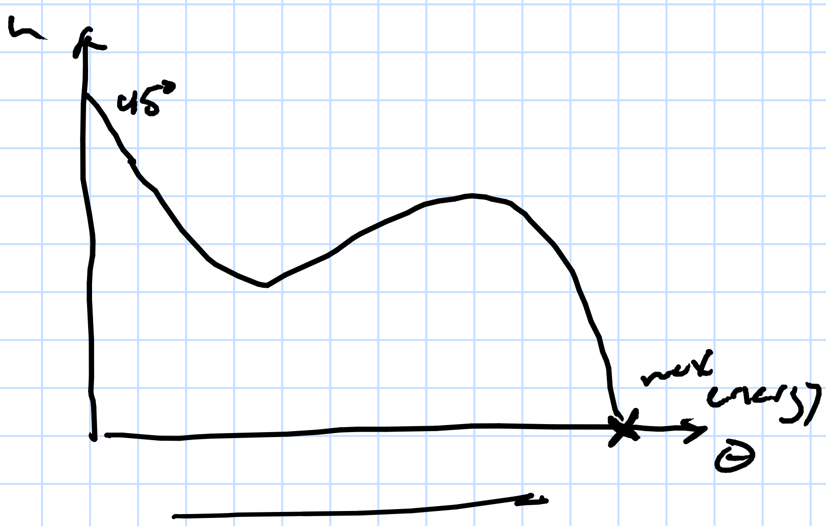
s.t.

$$\dot{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ g\sin\theta \end{bmatrix} \qquad g = 32.17\,\frac{ft}{s^2}$$

$$\psi_0 = \begin{bmatrix} t \\ x \\ y \\ v \end{bmatrix}_{t=t_0} = 0$$

$$\psi_f = \begin{bmatrix} x - x_f \\ y - y_f \end{bmatrix}_{t=t_f} = 0 \qquad x_f, y_f = 10\,ft; 10\,ft$$

Hypersonic Example



$$\min \; J = -v_f^2$$

$$x = \begin{bmatrix} h \\ \theta \\ v \\ \gamma \end{bmatrix}$$

$$\psi_0 = \begin{bmatrix} t \\ h - h_0 \\ \theta_0 \\ v - v_0 \\ \gamma - \gamma_0 \end{bmatrix} \qquad \psi_f = \begin{bmatrix} h - h_f \\ \theta - \theta_f \end{bmatrix}$$