Practice Exercises for Parameter Optimization

Sean Matthew Nolan

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1 Inequality Problem

Problem

$$\min_{\boldsymbol{u}} J = x^2 - u^2 \tag{1.1}$$

Subject to:

$$g = x^2 + u^2 - 4 \leq 0 \tag{1.2}$$

2 Maximum Steady Rate of Climb for Aircraft[1]

Problem

This problem aims to find the maximum steady rate of climb of am aircraft. The rate of climb is $v \sin \gamma$, so the cost function is

$$\min J = -v \sin \gamma \tag{2.1}$$

For a steady climb, acceleration, thus force, in both directions must be zero:

$$\boldsymbol{f}(v, \gamma, \alpha) = \begin{bmatrix} T(v) \cos(\alpha + \epsilon) - D(v, \alpha) - mg \sin \gamma \\ T(v) \sin(\alpha + \epsilon) + L(v, \alpha) - mg \cos \gamma \end{bmatrix} = 0 \tag{2.2}$$

Where:

$$\begin{aligned} v &= \text{velocity} \\ \gamma &= \text{flight path angle} \\ \alpha &= \text{angle of attack} \\ m &= \text{mass} \\ g &= \text{gravity} \\ \epsilon &= \text{angle between thrust axis and zero-lift axis} \\ T(\alpha) &= \text{thrust} \\ L(v, \alpha) &= \text{lift} \\ D(v, \alpha) &= \text{drag} \end{aligned}$$

Derive first order necessary conditions to minimize Eq. (2.1) at a given altitude.

References

[1] A. E. Bryson and Y.-C. Ho, *Applied optimal control: optimization, estimation, and control*. Washington : New York: Hemisphere Pub. Corp. ; distributed by Halsted Press, rev. printing ed., 1975.