Unemployment and the State-Dependent Effects of Monetary Policy

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Abstract

This paper provides empirical evidence that monetary policy has larger and more persistent effects on output, unemployment, and market tightness during times of high unemployment than during low-unemployment episodes. In contrast, we find little evidence for unemployment-dependence in the response of prices and wages. This result implies that the inflation-unemployment trade-off, also known as the Philips Multiplier, is time-varying and procyclical. These results can be rationalized using a tractable New Keynesian model with frictional labor markets, when solved using global projection methods. The main mechanism operates through strong nonlinearities inherent in firm profit and labor demand. The model, estimated using nonlinear full-information Bayesian methods, endogenously generates state-dependent impulse response functions in line with our empirical evidence, and Philips Multipliers that are up to three times smaller in recessions than during expansions.

Keywords: Numerical methods, discretization filter, labor search

JEL classification codes: C63, C68, D15, E21

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1 Introduction

The objective of this paper is to show empirically that the effects of monetary policy depend on the unemployment rate, and provide a theoretical model to understand the mechanisms behind this phenomenon. Our empirical approach builds on recent advances that allow model-free estimates of the effects of identified monetary policy shocks. We find the real effects of monetary shocks are driven by the response during times of high unemployment, while the effects on prices are largely independent of labor market conditions. To understand these findings, we propose a New Keynesian model with frictional labor markets and wage inertia from bargaining. We preserve and exploit the inherent nonlinearities and state-dependencies in the model by using global solution and nonlinear estimation methods. Our solution and estimation approach allows us to construct state-dependent impulse response functions that measure the time-varying effects of monetary policy over time. We find the model can replicate the countercyclical nature of the impulse response functions of output, employment, and labor demand, and implies a procyclical Philips Multiplier.

The first part of this paper is empirical, providing model-free evidence on how the effects of identified monetary policy shocks depend on the current unemployment rate. We use state-dependent local projections, as in Cloyne, Jordà, and Taylor (2023), combined with regularization techniques. We find that state-dependence is economically and statistically significant for the industrial production index, as well as for labor market variables such as the unemployment rate, the job finding rate, market tightness and the number of vacancies. Specifically, we find that during times of high unemployment, the response of these variables to monetary policy shocks is stronger and more persistent, compared to times with low unemployment rates, where the effects are small and short-lived[1]. In contrast, we find little evidence for state-dependence in the responses of wages and prices.

Next, we propose a structural model that can rationalize these empirical facts. Our model combines a standard small-scale New Keynesian environment (Gali, 2015) with Diamond-Mortensen-Pissarides[2] style labor market frictions, henceforth NKDMP. While this model is small relative to other NKDMP models, e.g. Christiano, Eichenbaum, and Trabandt (2016), the focus of our paper is to show these models have strong nonlinearities, revealed by its global model solution, that can speak to the state-dependent effects of monetary policy we see in the data. The key modeling choices that allow the model to reconcile the empirical results are (i)

[1]We also find a small role for asymmetries: negative monetary policy shocks have smaller and less persistent effects than positive shocks on these variables.

[2]Diamond (1982), Mortensen (1982), and Pissarides (1985)

the assumption that the firm have a working capital constraint that necessitates borrowing to pay the wage bill as in Christiano, Eichenbaum, and Evans (2005), and (ii) wage inertia from bargaining. The first assumption makes that marginal cost move with the interest rate, as is known to be important for New Keynesian models to generate sizable real effects. What is new is that when the environment includes frictional labor markets and bargained wages, the movements to marginal cost have very nonlinear effects on labor demand operating through the flow profit of the firm, or equivalently, the fundamental surplus of the match (Ljungqvist and Sargent, 2017). Wage inertia leads to pro-cyclical firm profits, so a shock to the marginal cost of the firm is large relative to profits in recessions. This state-dependent ratio of marginal costs to profits leads to state-dependent responses of real variables to monetary policy shocks.

It is known that labor search models and NK models are poorly approximated by perturbation solutions away from the steady state.[3] We show that the issues of perturbation methods are aggravated when combining New Keynesian model features with labor market frictions. A global solution method, on the other hand, retains the model's inherent nonlinearities, and, importantly, preserves the state-dependent nature of the policy rules. We find large differences between the linearized and global model solution of the NKDMP model. For example, when evaluated in the same set of parameters, the ergodic sets of the linearized model and global model solution differ substantially. In the linearized economy, there is less volatility in unemployment and there are more vacancies, resulting in higher market tightness. The solution method also matters for the model dynamics; in response to a monetary policy shock, the linearized model implies a response of inflation about four times as large, and responses of employment, output and market tightness are two times smaller compared to the responses that follow from the global model solution. Consequently, for the same parameters, the global model solution generates a smaller Philips Multiplier (i.e., inflation-unemployment trade-off as in Barnichon and Mesters, 2021) than the linearized model, consistent with a flatter Philips curve.

Motivated by the large differences between the linearized and global model solution, rather than calibrating the model parameters, we estimate the global model using Bayesian full-information methods.[4] While this is computationally intensive, we use a methodological

[3]As shown by Petrosky-Nadeau and Zhang (2017), models with labor market frictions feature nonlinearities that are poorly approximated by local solution methods like log-linearization. Similarly, Judd, Maliar, and Maliar (2017) show that perturbation-based solutions to New Keynesian models can be prone to large errors.

[4]An important point made by Petrosky-Nadeau and Zhang (2017) that also applies to our model, is that the ergodic (long-run) mean of the model is considerably different from the steady state. Consequently, standard calibration approaches that target the steady state of the model would lead to unreliable inference, motivating our choice for a formal estimation procedure. Similarly, estimating the linearized model and using those parameter estimates for the global model solution can be unreliable in our application.

advancement to make this exercise feasible on a standard desktop computer. We exploit the discrete nature of our global model solution and combine this with the discretization filter of Farmer (2021), showing that once the model is solved globally, the discretization filter gives an evaluation of the likelihood with little additional cost. The global and linear estimation procedures result in parameter estimates that imply considerably different model dynamics. We find stronger evidence for state-dependence when using the global rather than the linear parameter estimates.

Our main findings from the estimated NKDMP model solved with global projection methods confirm our empirical analysis, showing that the effects of monetary policy depend on the current unemployment rate in the economy. During times of high unemployment, variables such as output, unemployment, and market tightness respond strongly to monetary policy shocks. On the other hand, during times of low unemployment, the effects of monetary policy are smaller and less persistent. The peak responses in low (4%) versus high (11%) unemployment are between two and three times as small. As in the data, there is little evidence for state-dependent effects of monetary policy on inflation and wages. The nonlinearities and frictions in the model generate hump-shaped impulse responses for our main variables of interest, as in our empirical analysis.[5]

An important implication that follows from our results is that the effects of monetary policy vary with the current state of the economy, and in particular, the model implies that the *costs* of monetary policy vary. While we find that the monetary authority is similarly effective controlling inflation and prices during high and low unemployment episodes, however, its effects on labor markets and aggregate output depend strongly on the current state of the labor market. This results in a procyclical Philips Multiplier, where the size can be almost three times as large in expansions than during recessions. By generating time-varying impulse response functions and Philips Multipliers, our model can be used by policy makers to assess the time-varying effects of policy changes on variables of interest.

Related literature Our empirical results add to an existing literature studying asymmetries and state-dependence of impulse response functions as implied by Structural Vector Autoregressions (SVARs) or local projections, as in Barnichon, Debortoli, and Matthes (2022) and Ben Zeev, Ramey, and Zubairy (2023) for government spending shocks, and Ravn and Sola (2004), Angrist, Jordà, and Kuersteiner (2018), Tenreyro and Thwaites (2016), and Cloyne

[5]Our estimated model also generates asymmetries consistent with the empirical evidence, showing positive monetary policy shocks have larger and more persistent effects on output, employment, vacancies and market tightness than negative shocks, but, as in the empirical evidence, finds no strong asymmetries in the response of prices and wages.

et al. (2023) for monetary policy. We focus on the role of unemployment as a source of state-dependence. In particular, our results suggest that state-dependence emerging from unemployment is quantitatively more important than state-dependence from economic growth, and its role also seems more important than the role of asymmetries, which were the focus of previous research. Our proposed regularization technique in the empirical section is similar in spirit as Barnichon and Brownlees (2019) and El-Shagi (2019), that is, a Ridge regression that imposes smoothness on the parameters across different horizons.

Our NKDMP model builds on a long tradition of New Keynesian models and models featuring search and matching à la Diamond (1982), Mortensen (1982), and Pissarides (1985), and more recently, a literature merging both, as in Christiano et al. (2016) with other important contributions from Galí, Smets, and Wouters (2012), Blanchard and Galí (2010), Krause and Lubik (2007) and Gertler, Sala, and Trigari (2008). Our main contribution compared to this literature is to show that the NKDMP model has strong state dependence in the real response to a monetary policy surprise, and the estimation of the global model solution.

Our computational framework adapts Judd, Maliar, Maliar, and Valero (2014), where the most important difference is that we discretize the exogenous state (total factor productivity) using finite-state Markov chain approximation methods, rather than using integration, because this helps us in our estimation procedure. Our computational results contribute to a literature stressing that highly nonlinear models cannot always be solved accurately using perturbation methods, with important contributions from, among others, Petrosky-Nadeau and Zhang (2017), and Judd et al. (2017). The zero-lower-bound episode has renewed interest in global solution methods for New Keynesian models, for example in Fernàndez-Villaverde, Gordon, Guerrùn-Quintana, and Rubio-Ramìrez (2015) and Gust, Herbst, López-Salido, and Smith (2017), but we show that nonlinear dynamics matter also when the zero-lower-bound is not binding.

In terms of structural estimation, to our knowledge, we are the first to demonstrate and leverage the seamless integration of discrete projection methods and the discretization filter for globally solving and nonlinearly estimating a structural macroeconomic model. Farmer (2021) uses the discretization filter to estimate a second-order perturbation of a model, in which case the discretization filter still requires discretizing the state space, which is computationally intensive. In our approach, we already incur this cost while solving the model, after which the likelihood evaluation is of low additional computational cost. On the other hand, Gust et al. (2017) estimate a globally solved model using the particle filter, in which case both the solution and filtering step are computationally expensive, as large models require many particles, and there are no synergies between solution and estimation. Borağan Aruoba, Cuba-Borda, and Schorfheide (2018), as well as Harding, Lindé, and Trabandt (2022) estimate a linearized version of a DSGE model, and then evaluates the model implications using a globally solved version of the model. While computationally attractive, when the differences between the local perturbation and global solution are too large, as is the case for our model, such an approach may be unreliable.

Finally, our paper relates to a long literature studying the Philips (1958) curve, including its nonlinearities and time variation in its slope, in, e.g., Coibion and Gorodnichenko (2015), Smith, Timmermann, and Wright (2023), and others. Smith et al. (2023) (empirically) as well as Benigno and Eggertsson (2023) (model and empirics) argue the Philips curve is steeper at lower unemployment rates (tighter labor markets), consistent with our empirical and model findings. Benigno and Eggertsson (2023) model this in a New Keynesian framework by assuming a kink in wage setting around a market tightness of one, which mechanically results in a kink in the Philips curve. We don't find empirical evidence for unemployment-dependence in the response of wages to monetary policy, and rather find that the response of unemployment is state-dependent, which follows endogenously from our model and results in variation in the Philips Multiplier. Harding et al. (2022) and Harding, Lindé, and Trabandt (2023) argue non-linearities in the Philips curve are driven by state-dependencies in prices, rather than labor markets, results we view as complementary. Also related is Lahcen, Baughman, Rabinovich, and van Buggenum (2022), who focus on the state-dependent effects of inflation on unemployment, while we focus on the state-dependent effects of unemployment. Our empirical and model-based estimates of the Philips Multiplier are in line with the existing literature, see, for example, Cerrato and Gitti (2022)'s pre-Covid estimates, and Barnichon and Mesters (2021) and Blanchard (2016) post-1990's.

Outline The rest of this paper is structured as follows. Section 2 presents empirical evidence on how the effect of monetary policy depends on the current unemployment rate. Section 3 presents our New Keynesian model with labor market frictions. Section 4 discusses solution and estimation methods. Section 5 discusses the comparison between the linear and global model, while Section 6 evaluates the state-dependent nature of the model and its implied impulse response functions. Section 7 discusses procyclical Philip Multipliers. Section 8 concludes.

2 Empirical Evidence: State-dependent Regularized Local Projections

This section presents empirical evidence that the effects of monetary policy depends on current labor market conditions, in particular, the current unemployment level of the economy. We show this using state-dependent Local Projections as in Cloyne et al. (2023), in combination with regularization techniques. In addition, we show that, once controlling for state-dependency in the form of unemployment, the role of other state-dependencies, in particular in the form of economic growth or asymmetries, is limited. These empirical results form the motivation for the structural model presented in the next section.

2.1 Methodological Framework

Consider the framework of Local Projections, proposed in Jordà (2005):

$$y_{t+h} = \alpha_h + \beta_h s_t + w_t \gamma_h + \varepsilon_t, \quad \text{for } h = 0, \ldots, H \text{ and } t = 1, \ldots, T, \tag{1}$$

where s_t is some exogenous shock, y_{t+h} is the outcome of interest, and w_t is a set of controls, which typically includes lagged values of the shock s_t and the outcome variable y_t. Under the appropriate assumptions, β_h, $h = 0, \ldots, H$ traces out the *Impulse Response Function* (IRF) of variable y in response to a shock s. Ramey (2016) gives an excellent overview of commonly used shock series s_t for studying the effects of monetary or fiscal policy.

As in Cloyne et al. (2023), we extend this framework to allow for state-dependent impulse responses, based on the so-called Kitagawa-Blinder-Oaxaca (KBO) decomposition. Consider a *state* x_t. The state-dependent local projection then becomes:

$$y_{t+h} = \alpha_h + \beta_h s_t + \beta_h^x s_t (x_t - \bar{x}) + w_t \gamma_h + \varepsilon_t, \quad \text{for } h = 0, \ldots, H \text{ and } t = 1, \ldots, T, \tag{2}$$

where the set of controls w_t is now augmented with $(x_t - \bar{x})$, as well as an interaction of all the controls with $(x_t - \bar{x})$. Compared to the local projection in Equation (1), the difference is the KBO term $\beta_h^x s_t (x_t - \bar{x})$, capturing state dependence. The time-varying impulse response function is then traced out by the sum of the direct effect of the shock, $\hat{\beta}_h$, and the state-dependent effect $\hat{\beta}_h^x (x_t - \bar{x})$. As pointed out in both Cloyne et al. (2023) and Gonçalves, Herrera, Kilian, and Pesavento (2023), it is important that the shock s_t is exogenous to both x_t and y_{t+h}, which we assume to be the case in our setting.

Allowing for state-dependence introduces many additional parameters; Equation (2) has more than double the number of parameters than Equation (1). Cloyne et al. (2023) uses crosscountry variation to aid inference. Our data are available on monthly frequency, and we find that this, in combination with regularization techniques, gives us sufficient statistical power to study state dependence.

For our framework, we propose using a simple regularization technique, similar in spirit to El-Shagi (2019) and Barnichon and Brownlees (2019), that is, a Ridge regression that imposes smoothness on the parameters across different horizons. In this Ridge regression framework, we estimate the local projections as one large system, instead of equation-by-equation. Rather than using smoothing splines, as in Barnichon and Brownlees (2019), or penalizing second differences, as in El-Shagi (2019), our method simply includes a small penalty term for large deviations in β_h, as well as β_h^x, from one horizon to the next. That is, we have penalty terms $\lambda_1 \sum_{h=2}^{H-1}(\beta_h - \beta_{h-1})^2$, $\lambda_2 \sum_{h=2}^{H-1}(\beta_h^x - \beta_{h-1}^x)^2$ and $\lambda_3 \sum_{j=1}^{k} \sum_{h=2}^{H-1}(\gamma_h^j - \gamma_{h-1}^j)^2$, that we add to the total sum of squares. Here k is the number of control variables in w. So instead of a Ridge regression that shrinks all parameters to zero, we (mildly) shrink the difference between parameters from one horizon to the next, to avoid big jumps in the impulse response function. We leave the first coefficients, i.e., β_1, β_1^x and $\{\gamma_1^j\}_{j=1}^k$, unrestricted.

For simplicity, only consider regularization of β_h. Let Y_t be the (column) vector containing y_{t+h} across all horizons $h = 1, ..., H$. X_t is a matrix containing the shock, the element one (for the constant) and the controls w_t, repeated for H rows. Next, write X as the vertically stacked regressors X_t, for $t = 1, ..., T - H$, Y the vertically stacked dependent variables Y_t, and P a matrix with $H - 1$ rows, and $H \times K$ columns, where K is the number of columns of X_t. P has 1's on the diagonal, and -1 on the first upper off-diagonal. Note that in X, the shocks are ordered first. Our Ridge regressor can then be written as the following estimator:

$$\hat{\theta} = (X'X + \lambda P)^{-1} X'Y \tag{3}$$

where the first elements of $\hat{\theta}$ contain the estimates of β_h, $h = 0, 1, ..., H$. We can trivially extend this to allow for regularization on the other parameters.

We follow Barnichon and Brownlees (2019) and use a heuristic asymptotic Newey-West variance estimator:

$$\hat{V}(\hat{\theta}) = T \left[\sum_{t=1}^{T-1} X_t' X_t + \lambda P \right]^{-1} \left[\hat{\Gamma}_0 + \sum_{l=1}^{L} b_l \left(\hat{\Gamma}_l + \hat{\Gamma}_l' \right) \right] \left[\sum_{t=1}^{T-1} X_t' X_t + \lambda P \right]^{-1}$$

where $b_l = 1 - l/(1 + L)$ and $\hat{\Gamma}_l = \sum_{l+1}^{T-1} X_t' \hat{U}_t \hat{U}_{t-l}' X_{t-l}$ with $\hat{U}_t$ the regression residuals. As in Barnichon and Brownlees (2019), we set $L = H$.

Both El-Shagi (2019) and Barnichon and Brownlees (2019) use k-fold cross validation to choose the penalty terms λ. That can be done in our approach too, but in our applications, we opt for choosing λ conservatively low: enough to smooth out the kinks, but not affecting the overall shape of the impulse response function.

2.2 Empirical evidence on state-dependence

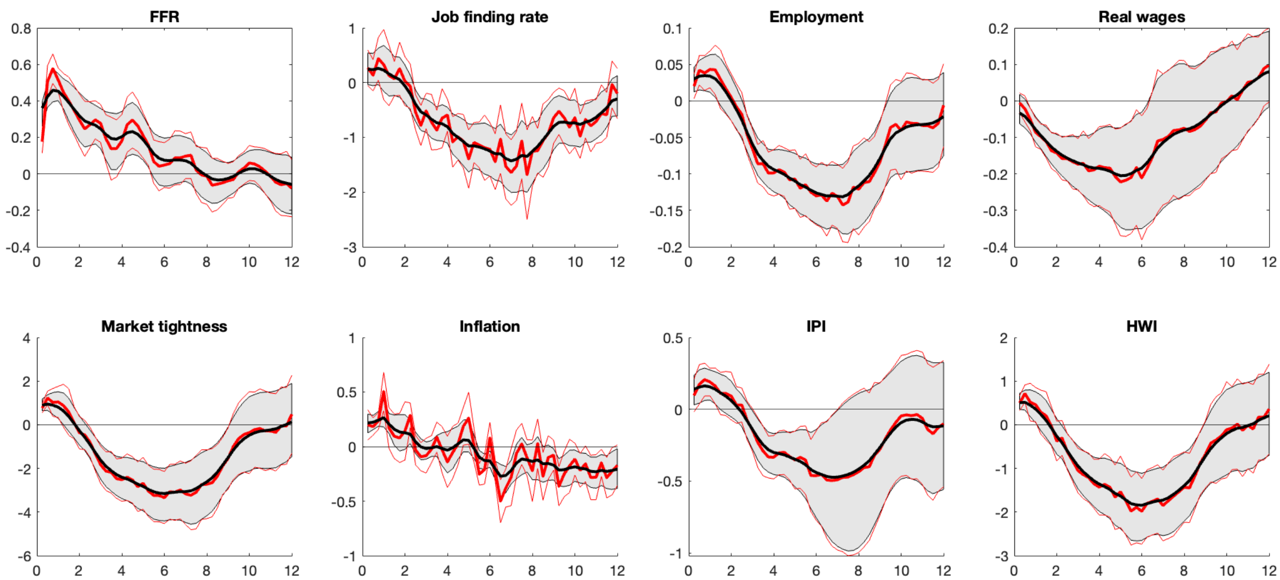
In this subsection, we use the framework presented above to analyze the state-dependent effects of monetary policy on various variables. First, we present our baseline framework, based on the standard Local Projection regressions in Equation (1), and compare them with the regularized LP regressions of Equation (3). For these, we use the monetary policy shock series by Romer and Romer (2004), extended by Wieland and Yang (2020) to 2007. These series capture monetary policy surprises identified using quantitative and narrative records around the FOMC meetings of the Federal Reserve.

The dependent variables of interest are the unemployment rate, the Federal Funds Rate (FFR), inflation based on the Consumer Price Index (CPI), wages, the Industrial Production Index (IPI), the job finding rate, market tightness, and vacancies (HWI: help-wanted-index). More details on the data are available in Appendix A. We follow Ramey (2016), and our control variables include the current values of IPI, CPI, PPI (production price index), and the unemployment rate, as well as four lags of the own variable, IPI, CPI, PPI, the unemployment rate, the federal funds rate, and the shock.

Baseline specification The baseline results are visualized in Figure 1. We scale all IRFs to be a one standard deviation shock of the respective shock series used. As can be seen from Figure 1, we replicate the shape of the IRFs for unemployment, the Federal Funds Rate (FFR), and the Industrial Production Index (IPI) documented in Ramey (2016). As can be seen, our regularization does not affect the shape, but merely smooths out the kinks, as desired. The other variables, not studied in Ramey (2016), move as expected: the job finding rate, market tightness and vacancies (HWI) all go down in response to an unexpected increase in the FFR.

Unemployment-dependent monetary policy Next, we study state-dependence of monetary policy shocks using the regression framework of Equation (2). As our state, that is, x_t in Equation (2), we use (log) unemployment. The assumption for the framework of Equation (2) to be valid is that our monetary policy shocks are exogenous to both the dependent variables as well as to the state, which we believe to be appropriate given the rationale of the monetary

Figure 1: Baseline Impulse Response Functions using Romer-Romer shocks



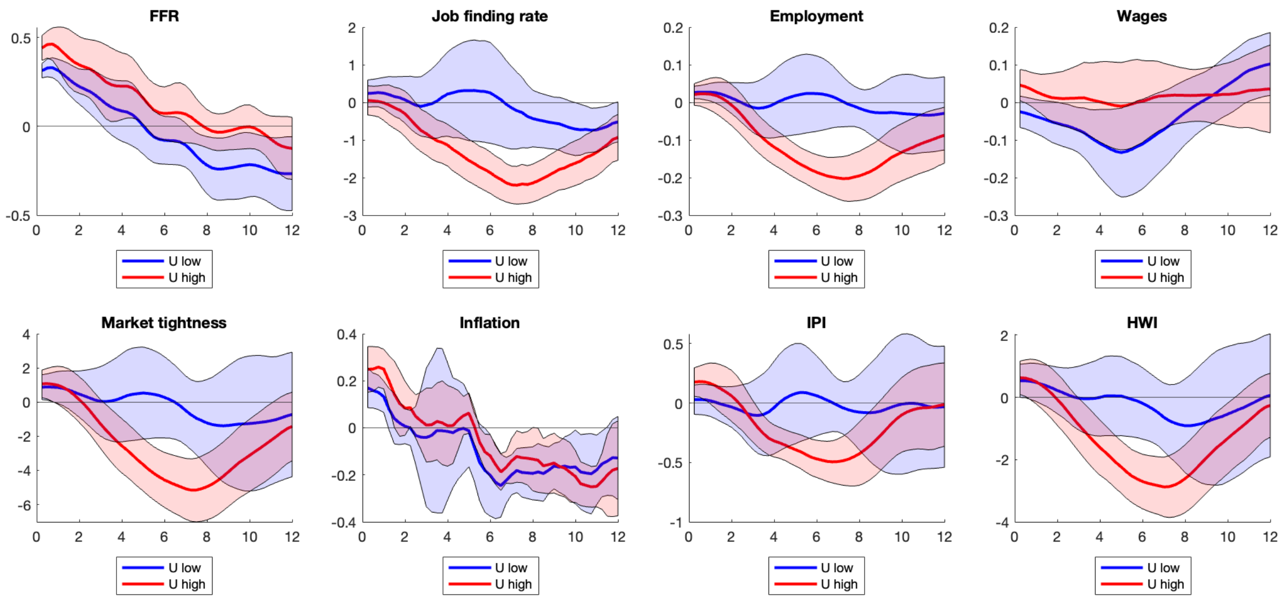
Notes: Thick red line shows the IRF without regularization, thin red lines show the asymptotic confidence interval. Thick black line is the regularized IRF with $\lambda_1 = 100$, there is no state-dependence, so no λ_2, and $\lambda_3 = 0$. The grey bands show the heuristic 90% confidence intervals based on the regularized estimator. Sample 1969-2007, monthly frequency.

policy shock. The resulting state-dependent Local Projections are visualized in Figure 2.[6] For the purpose of this figure, we compute the implied impulse response functions for two different realizations of the state: the 30$^{\text{th}}$ percentile of unemployment and the 70$^{\text{th}}$ percentile.

As follows from Figure 2, we find strong evidence for state dependence. Focusing on the labor market variables, we find that in times of low unemployment, a positive unexpected monetary policy shock has a non-significant effect on the job finding rate, the unemployment rate, market tightness as well as the number of vacancies. In times of high unemployment, on the other hand, we find large economic effects: the job finding rate, market tightness, employment and vacancies go down. As for industrial production, we find a similar result: in times of low unemployment, there is no significant effect of unexpected monetary policy shocks on the Industrial Production Index, while in high unemployment, a positive one-standard deviation shock lowers the IPI by about 0.4 % around 2 years after impact. We find little evidence for unemployment-dependence in inflation and wages.

[6]For consistency with the specifications presented in the next subsection, we also include lagged values of Total Factor Productivity (TFP) as control variables, but this does not affect the current results.

Figure 2: State-Dependent Impulse Response Functions using Romer-Romer shocks: low and high unemployment



Notes: 90% confidence intervals (asymptotic). Sample 1969-2007, monthly frequency. $\lambda_1 = 200, \lambda_2 = 20$, $\lambda_3 = 0.1$. For the purpose of this figure, the local projections are evaluated in the 30$^{\text{th}}$ percentile (low unemployment, blue line) and the 70$^{\text{th}}$ percentile of unemployment (high unemployment, red line) over the sample period.

Robustness Our findings are robust to the inclusion of additional lags, shown for six lags in Figure A1 in the Appendix. In addition, we find qualitatively similar results for the monetary policy shock series constructed in Gertler and Karadi (2015) over the period 1990-2012, for which we present results in the Appendix in Figure A2. The shock series of Gertler and Karadi (2015) are noisier, and available over a shorter period, which is why we present the inference based on the Romer and Romer (2004) shocks as our main results. Compared to the Romer and Romer (2004) series, the Gertler and Karadi (2015) series show a somewhat later peak response (around 10 quarters rather than 6-8). State-dependence also seems less strong than for the Romer and Romer (2004) shocks. Except for this difference, we find our main conclusions on state-dependence are confirmed by both shock series.

2.3 Other sources of state-dependence

Economic growth The literature has considered other sources of state-dependence, in particular, Tenreyro and Thwaites (2016) analyze whether monetary policy has larger or smaller effects during episodes of high or low economic growth. We use our empirical framework to ask whether, once we control for state-dependence in unemployment, we find evidence for state-dependence in economic growth. For economic growth, we use the TFP series of

Fernald (2014). This comes down to having two states x_t, one for (log) unemployment, one for TFP growth. We evaluate the Impulse Response Functions in two percentiles again, the 30$^{\text{th}}$ percentile and 70$^{\text{th}}$ percentile of TFP and (log) unemployment, respectively, giving rise to four different Impulse Response Functions for all different combinations. These IRFs are visualized in Figure A3 in the Appendix.

As follows from Figure A3, controlling for state-dependence in unemployment, we do not find statistically significant evidence for state-dependence in TFP growth, and state-dependence in unemployment is many times larger than state-dependence in TFP growth. In particular, the IRFs of high and low TFP growth fall well within the confidence bands of IRFs with median TFP growth. We see this as a refinement of the results in Fernald (2014), decomposing its findings, by saying business cycles consist of both growth (TFP) and employment, and we find it is unemployment that drives most of this state-dependence in monetary policy, with a smaller role for TFP.

Asymmetry Several papers have asked the question whether the effects of fiscal or monetary policy are asymmetric, that is, do unexpected fiscal or monetary policy shocks have different effects when they are negative versus positive? Consider, for example, Barnichon and Matthes (2018), Ben Zeev et al. (2023), Barnichon et al. (2022), Angrist et al. (2018), and others. We ask whether, in addition to state-dependence of unemployment, we can find evidence for asymmetry of monetary policy shocks, by including two (three) x variables: the (log) unemployment rate, an indicator for whether the shock is nonnegative (and an interaction term between the two). The resulting impulse response functions are visualized in Figure A4 and A5 in the Appendix.

Figure A4 shows the IRFs, evaluated when unemployment is at its median. As can be seen, there is no statistical evidence for asymmetry when we control for state-dependence in unemployment. Figure A5 looks at asymmetry and state-dependence jointly. We compute IRFs for positive and nonpositive monetary policy shocks, and high and low unemployment, resulting in four different IRFs. When we do so, we still find strong evidence for unemployment-dependence in the job finding rate, employment, market tightness, IPI and vacancies. We also find evidence for asymmetry, but only when unemployment is high, in which case we find that negative shocks are less persistent than nonnegative monetary policy shocks. As before, there is no strong evidence for state-dependence, nor asymmetry, in inflation and wages.

3 A New Keynesian Model with Search Frictions

This section outlines a small-scale New Keynesian DSGE model with search frictions in the labor market a là Diamond-Mortensen-Pissarides. The model economy consists of a perfectly competitive final-goods-producing sector, a continuum of monopolistically competitive intermediate goods producers that set prices subject to an adjustment cost à la Rotemberg (1982) and hire workers by posting vacancies, a continuum of identical households that consume, save in bonds, and supply labor when employed, and a monetary authority that determines interest rates according to an inertial Taylor rule subject to the zero lower bound constraint.

The model uses the following timing convention, which is similar to Christiano et al. (2016). Households can lose a job and find a new job within the same period, and engage in production and earn wages as soon as they are hired. This assumption is used because the model will be estimated using quarterly data, and requiring workers to spend one model period in unemployment would result in unemployment spells that are too long relative to the data[7]. Details on the solution method and estimation are provided in the next section. Model derivations are provided in Appendix C.

3.1 Households

The economy is populated by a continuum of identical households $i \in [0,1]$ who derive utility from consumption, and supply one unit of labor inelastically to the labor market. The preferences of the representative household are an equally weighted average of its members. The household maximizes

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{C_t^{1-\tau}-1}{1-\tau}\right], \tag{4}$$

subject to a budget constraint

$$P_tC_t + B_{t+1} = W_tN_t + R_tB_t + D_t, \tag{5}$$

and the law of motion for employment

$$N_{t+1} = N_t - \delta(1-f_t)N_t + f_t(1-N_t). \tag{6}$$

Here β is the discount factor, $1/\tau$ is the intertemporal elasticity of substitution, N_t is the number of household members who are employed at the beginning of the period, P_t is the price of the

[7]According to the estimate of Ahn and Hamilton (2022), two-thirds of workers exit unemployment within the first quarter of an unemployment spell.

final good, B_t denotes one-period risk-free bonds with gross return R_t. D_t are dividends paid to the household from flow profits of the firm. The law of motion for employment represents the current stock of employed workers N_t net of those who separate and do not find a job within the same period $\delta(1-f_t)N_t$, plus those who were unemployed and found a job $f_t(1-N_t)$, which depend on the exogenous separation rate δ and the endogenous job-finding rate f_t. We define the stochastic discount factor as

$$m_{t+1} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\tau}.$$

3.2 Final Good Producers

The finals good producer aggregates intermediate goods, indexed by $j \in [0, 1]$, using the technology

$$Y_t = \left(\int_0^1 Y_{jt}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}, \tag{7}$$

where $\nu \in (0, 1)$. The representative firm chooses intermediate inputs $Y_{j,t}$ to maximize profits,

$$P_t Y_t - \int_0^1 P_{jt} Y_{jt} dj, \tag{8}$$

taking input prices P_{jt} and output prices P_t as given. Profit maximization implies that the demand for input j is given by:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\frac{1}{\nu}} Y_t. \tag{9}$$

We define inflation as $\Pi_t = P_t / P_{t-1}$.

3.3 Intermediate Goods Producers and the Labor Market

Intermediate good j is produced by a monopolistic firm with production function

$$Y_{jt} = Z_t N_{jt}^{\alpha}$$

choosing prices and vacancies. Price adjustments are subject to a Rotemberg (1982) adjustment cost given by

$$\frac{\phi}{2} \left(\frac{P_{jt}}{P_{jt-1}} - \Pi^* \right)^2 Y_t,$$

where Π^* is long-run, i.e. target, inflation. Our motivation for working with the Rotemberg approach to sticky prices instead of the more common approach of Calvo (1983) is to keep the state space manageable. In a global solution, Calvo pricing requires keeping track of price dispersion as an additional state variable, which increases the computational burden.

Goods are produced using the following production technology:

$$Y_{jt} = Z_t N_{jt}^{\alpha}$$

with where Z_t is a productivity term than is common to all firms. Productivity Z_t evolves according to

$$\ln Z_{t+1} = \rho \ln Z_t + \varepsilon_{t+1}.$$

with $\varepsilon_t \overset{i.i.d.}{\sim} (0, \sigma_\varepsilon^2)$.

Firms contact workers by posting vacancies. We assume vacancies are a stock, and firms choose how much flow X_{jt} to add to the vacancy stock. The firm-level vacancy stock V_{jt} has the following law of motion:

$$V_{jt+1} = (1-\xi)(1-q_t)\left(V_{jt} + X_{jt}\right).$$

Here ξ is the depreciation rate of the vacancy stock, and q_t is the vacancy filling rate. The firm incurs a vacancy cost $k\left(X_{jt}\right)$ to create new job openings, and cost κ to keep an existing vacancy open, which they finance by borrowing at rate R_t. We assume vacancy posting cost are quadratic:

$$k(X_{jt}) = \kappa X_{jt} + \frac{\psi}{2}\left(X_{jt}\right)^2,$$

such that $k'\left(X_{jt}\right) - \kappa = \psi X_{jt}$, which is convenient for tractability. Vacancies contact workers with probability q_t, with $q_t = \chi(1+\theta_t^{\iota})^{-1/\iota}$, where the market tightness is given by

$$\theta_t = \frac{V_t + X_t}{\delta + (1-\delta)(1-N_t)}. \tag{10}$$

In recursive form, the firms' optimization problem is given by:

$$\Omega_t = \max_{X_{jt}, P_{jt}} \left\{ \frac{P_{jt}}{P_t} Z_t N_{jt}^{\alpha} - R_t \left(\frac{W_t}{P_t} N_{jt} + k\left(X_{jt}\right) + \kappa V_{jt} \right) - \frac{\phi}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \Pi^* \right)^2 Y_t + \mathbb{E}_t[m_{t+1}\Omega_{t+1}] \right\}$$

subject to the constraints

$$
\begin{aligned}
Z_t N_{jt}^{\alpha} &= \left(\frac{P_{jt}}{P_t}\right)^{\frac{-1}{\gamma}} Y_t \\
N_{jt+1} &= (1-\delta)N_{jt} + q_t\left(V_{jt} + X_{jt}\right) \\
V_{jt+1} &= (1-\xi)(1-q_t)\left(V_{jt} + X_{jt}\right)
\end{aligned}
$$

The firm faces a working capital constraint and finances wage payments and hiring costs with debt it pays back after production has occurred. Consequently, the gross nominal interest rate R_t scales these variables. The firm also faces three constraints: (i) the price it charges for output $Z_t N_{jt}^{\alpha}$ must be on the demand curve, (ii) the law of motion for employment, and (iii) the law of motion for vacancies.

3.4 Wage Bargaining

Wages are determined using Nash bargaining, that is, defining $S_t^f = \frac{\partial Q_t}{\partial N_{jt}}$ as the surplus to the firm from a match, i.e. the present value of the marginal worker to the firm , and S_t^h as the surplus to the worker, we have:

$$\eta S_t^f = (1-\eta) S_t^h \tag{11}$$

where η is the bargaining weight, and

$$S_t^h = w_t - \nu + \mathbb{E}\left[m_{t+1}(1-\delta)(1-f_t)S_{t+1}^h\right], \tag{12}$$

where ν the outside option value of the worker and $w_t = W_t/P_t$ is the real wage.

3.5 Monetary Authority

Monetary policy is determined by a Taylor Rule for the nominal interest rate subject to a ZLB constraint:

$$R_{t+1} = \max\left\{1, \tilde{R}_{t+1} e^{\epsilon_{R,t}}\right\}, \tag{13}$$

where $\tilde{R}_t$ is the systematic component of monetary policy which reacts to the state of the economy, and $\epsilon_{R,t}$ is the shock. We consider the following Taylor rule:

$$\tilde{R}_{t+1} = \left[R_* \left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_1} \left(\frac{Y_t}{Y^*}\right)^{\psi_2}\right]^{1-\rho_R} R_t^{\rho_R} \tag{14}$$

stating the central bank reacts to deviations in inflation and output from their targeted values Π_* and Y^*. In the linear model, we ignore Equation (13), and look at $R_{t+1} = \tilde{R}_{t+1}e^{\varepsilon_{R,t}}$ instead.

3.6 Equilibrium Conditions

In equilibrium, all firms make the same choices, so we omit the index j. The household optimization problem results in a consumption Euler equation

$$C_t^{-\tau} = \beta \mathbb{E}\left(C_{t+1}^{-\tau} \frac{R_{t+1}}{\Pi_{t+1}} \right). \tag{15}$$

The first order conditions of the intermediate goods producers result in the following pricing condition

$$1 - \phi(\Pi_t - \Pi^*)\Pi_t - \frac{\lambda_t}{\gamma} + \phi \mathbb{E}\left(m_{t+1} \left((\Pi_{t+1} - \Pi^*)\Pi_{t+1} \left(\frac{Z_{t+1}N_{t+1}^{\alpha}}{Z_t N_t^{\alpha}} \right) \right) \right) = 0, \tag{16}$$

where λ_t is the multiplier on the demand equation constraint, and a job-creation (i.e. labor demand) equation

$$x_t = \frac{1}{\psi R_t} \mathbb{E}_t \left[m_{t+1}\{ q_t S_{t+1}^f + (1 - \xi(1 - q_t))\psi R_{t+1} x_{t+1} \} \right] - \frac{\kappa}{\psi} \tag{17}$$

The envelope condition gives an expression for the surplus to the firm we can rewrite as

$$S_t^f = (1 - \lambda_t)\alpha Z_t N_t^{\alpha-1} - R_t w_t + R_t \kappa + \mathbb{E}_t \left[m_{t+1}\{ (1 - \delta - q_t) S_{t+1}^f + ((1 - q_t)\xi - \delta) R_{t+1} \phi x_{t+1} \} \right] \tag{18}$$

We have the following Laws of Motion of the aggregate employment and vacancy stock:

$$N_{t+1} = (1 - \delta)N_t + q_t(V_t + x_t)$$
$$V_{t+1} = (1 - \xi)(1 - q_t)(V_t + x_t)$$

The aggregate resource constraint becomes:

$$C_t = Z_t N_t^{\alpha} \left(1 - (\Pi_t - \Pi^*)^2 \frac{\phi}{2} \right) - \kappa V_t - k(x_t)$$

In equilibrium, these hold together with Equations (11)-(14). For details on the derivations, consider Appendix C.

4 Computation and Estimation

First, we outline our computational algorithm that solves the for the global solutions to the model, given a set of parameter values. Second, we discuss the Bayesian sampler and nonlinear filter we use along with the global solution algorithm to estimate the model parameters.

4.1 Computational Algorithm

In this subsection, we elaborate upon our computational algorithm. The algorithm is based on Judd et al. (2014), who use their algorithm to globally solve the multi-country real business cycle model. We describe the algorithm in the main text, because while of lower dimensionality, our policy updating rules are somewhat more complex than in their model. In addition, we deviate from their algorithm in how we deal with the exogenous state Z_t. Rather than integration, we use finite-state Markov chain approximations to deal with the exogenous state, as this is convenient for our estimation procedure detailed in the next subsection.

Our model has four states: (R_t, N_t, V_t, Z_t), that is, interest rates, the employment stock, the vacancy stock and total factor productivity. We need to solve for five policies functions: $g^{\Pi}(Z, N, R, V)$ for inflation, $g^{\lambda}(Z, N, R, V)$ for the multiplier, $g^{x}(Z, N, R, V)$ for the vacancy flow, $g^{S_f}(Z, N, R, V)$ for the firm's surplus, and $g^{w}(Z, N, R, V)$ for the wage.

Because we discretize total factor productivity, we have $Z \in \mathcal{Z} = \{z_1, ..., z_{n_z}\}$ with P_z its transition probability matrix. Because of this, a policy function becomes: $g_z^{\cdot}(R, N, V)$ for each $z \in \mathcal{Z}$. We approximate each policy function using a three-dimensional Chebychev polynomial:

$$g_z^{\pi}(R, N, V) = \sum_{i=1}^{N_R} \sum_{j=1}^{N_N} \sum_{k=1}^{N_V} c_{ijk}^{z,\Pi} \phi_i(R) \phi_j(N) \phi_k(V)$$

and the same for $g_z^{\lambda}(R, N, V)$, $g_z^{x}(R, N, V)$, $g_z^{S_f}(R, N, V)$ and $g_z^{w}(R, N, V)$.

Given some initial guess for the coefficients $c^{\cdot}_{ijk}$, using our policies, we update our states using the model equations:

$$
\begin{aligned}
N' &= (1-\delta)N + g_z^q (V + g_z^x) \\
V' &= (1-\xi)(1-g_z^q)(V + g_z^x) \\
R' &= \left[R^* \left(\frac{g_z^\pi}{\Pi^*}\right)^{\psi_1} \left(\frac{ZN^\alpha}{Y^*}\right)^{\psi_2} \right]^{1-\rho_R} R^{\rho_R} e^{\epsilon_R} \\
\ln Z' &= \rho_z \ln Z + \sigma_z \varepsilon_z
\end{aligned}
$$

where $g_z^\theta = (V + g_z^x)/(\delta + (1-\delta)(1-N))$ and $g_z^q = \chi(1 + (g_z^\theta)^\iota)^{-1/\iota}$. Define $g_z^f = g_z^q g_z^\theta$. We can now evaluate our policies in N', R', V', Z', resulting in $g_{z'}^{\Pi'}$, etc.

Next, we update our policies. To do so, we first need to update the stochastic discount factor.

$$
\tilde{m} = \beta \left(\frac{Z'(N')^\alpha (1 - (g_z^{\pi'} - \Pi^*)^2 \frac{\phi}{2}) - \kappa V' - k(g_z^{x'})}{ZN^\alpha (1 - (g_z^\pi - \Pi^*)^2 \frac{\phi}{2}) - \kappa V - k(g_z^x)} \right)^{-\tau}
$$

Our policy rule for inflation can be updated using the Euler Equation:

$$
\tilde{g}_z^\pi = \mathbb{E}_Z \left(\tilde{m} \frac{R'}{g_{z'}^{\pi'}} g_z^\pi \right)
$$

The updating equation for the multiplier follows from the pricing equation:

$$
\tilde{g}_z^\lambda = \gamma \left(1 - \phi(g_z^\Pi - \Pi^*) g_z^\Pi + \phi \mathbb{E}_Z \left(\tilde{m} \left(g_{z'}^{\Pi'} - \Pi^* \right) g_{z'}^{\Pi'} \frac{Z'N'^\alpha}{ZN^\alpha} \right) \right)
$$

The job creation condition gives an update for g_z^x:

$$
\tilde{g}_z^x = \frac{1}{\psi R} \mathbb{E}_Z \left(\tilde{m} \left(g_z^q g_{z'}^{S_f'} + (1 - \xi(1 - g_z^q)) \psi R' g_{z'}^{x'} \right) \right) - \frac{\kappa}{\psi}
$$

The surplus of the firm is updated using:

$$
\tilde{g}_z^{S_f} = (1 - g_z^\lambda) \alpha Z N^{\alpha-1} - R g_z^w + R\kappa + \mathbb{E}_Z \left(\tilde{m} \left\{ (1 - \delta - g_z^q) g_{z'}^{S_f'} + ((1 - g_z^q)\xi - \delta) \psi R' g_{z'}^{x'} \right\} \right)
$$

The wage is updated using:

$$\tilde{g}_z^w = \left(g_z^{S_f} - \mathbb{E}_Z \left(\tilde{m}(1-\delta)(1-g_z^f) g_{z'}^{S'_f} \right) \right) \frac{\eta}{1-\eta} + \nu$$

Note that the expectations are computed using the transition probability matrix for Z, P_Z. Next, we again compute the coefficients $c_{ijk}^{z,\Pi}$ etc., using a regression of the updated policy rules and the Chebychev nodes. Repeat this, by again updating the states and policies. Iterate until the convergence criterion is met. One can use damping to make the algorithm more stable.

Computational details We use three Chebychev nodes for N_R and N_V, and four for N_N. We use the linearized solution as our initial guess,[8] evaluated in the Chebychev nodes, after which a regression of the Chebychev polynomials on the first guess of the policy gives the coefficients $c_{z,ijk}^{\Pi}$, $c_{ijk}^{z,w}$, ... We use fixed grids and fixed grid widths for our state variables N, R and V, which is a deviation from Judd et al. (2014) who iteratively update the grid to deal with the curse of dimensionality, but we choose not to, because our state space is relatively small. To discretize the stochastic process of TFP, Z_t, we use the Tauchen (1986) method, with 18 grid points for Z_t, and a grid width of three standard deviations. We modify the transition probability matrix that follows from this discretization by correcting the transition probabilities such that the discretized process matches the conditional means of the underlying process, evaluated in each grid point. We choose a relatively large value for the number of grid points n_z because TFP is typically quite persistent, which requires more grid points, as shown in Janssens and McCrary (2022).

4.2 Estimation Approach

We compare the results of three estimation procedures. All three approaches use standard Bayesian estimation methods (Random Walk Metropolis Hastings Sampler) to construct the posterior distribution, and the differences between the approaches come from the solution method and filtering method. For the Random Walk Metropolis Hastings Sampler, we split the parameters in three fixed blocks: (i) structural parameters $(\eta, \nu, \xi, \kappa, \psi, \chi, \psi_1, \psi_2)$, (ii) stochastic parameters $(\rho_r, \sigma_r, \rho_z, \sigma_z)$ and (iii) the standard deviations of the measurement error terms.

[8]We find that it stabilizes the algorithm to scale down the perturbation derivatives for the initial guess, and we use a scaling factor of 50.

The three different procedures are as follows. In the first approach, we log-linearize the NKDMP model and use the Kalman filter to evaluate its likelihood. In the second approach, we solve the model using the global method outlined in the previous subsection, evaluated in the parameter estimates of the linearized model, and use a nonlinear filtering technique to evaluate the likelihood, which we use to re-estimate the standard errors of the measurement errors of the model. In the third approach, we re-estimate all parameters of interest using the global model solution and nonlinear filter. All three procedures use the same sets of priors, which are in line with the existing literature, and selected to cover a reasonable parameter range and not be too influential. We elaborate on our choice for the nonlinear filter in the next subsection.

4.2.1 Nonlinear Filtering

Our estimation approach uses the discretization filter of Farmer (2021). The idea behind the discretization filter is that by approximating the state space using a discrete-valued Markov chain, the likelihood can be evaluated recursively and fast using simple matrix multiplications. This results in an approximate likelihood of nonlinear models that can be used in classical (frequentist) or Bayesian estimation procedures.

Denote the states of the model at time t by $X_{t,M}$ with M different realizations on a discrete grid, with transition probability matrix $P_{\theta,M}$ which has elements $P_{\theta,M}(m,m') = \mathbb{P}_\theta(X_{t,M} = x_{m',M}|X_{t-1,M} = x_{m,M})$. With some abuse of notation relative to the previous section, θ is a vector summarizing the parameters of the structural model. We assume $X_{t,M}$ is governed by this transition probability matrix $P_{\theta,M}$, and we have measurement equations

$$Y_t|X_{t,M} \sim g_\theta(Y_t|X_{t,m})$$

that map the unobserved states $X_{t,m}$ into observables. As is common, we assume each variable is observed with Gaussian i.i.d. measurement error relative to its model counterpart, with standard errors we estimate jointly with our other parameters.

The likelihood of the model is now easy to evaluate. Let $\hat{\zeta}_{t,M|t} = \mathbb{E}_\theta[\zeta_{t,M}|Y_1^t]$ be the vector of filtered probabilities of being in each state, using information up to time t. The forecast equation is then:

$$\hat{\zeta}_{t,M|t-1} = \mathbb{E}_\theta[\zeta_{t,M}|Y_1^{t-1}] = P'_{\theta,M}\hat{\zeta}_{t-1,M|t-1}$$

Defining $\eta_{t,M}$ as an $M \times 1$ vector containing the likelihoods of observing Y_t conditional on being at state m at time t, for $m = 1, ..., M$, the marginal likelihood of Y_t given Y_1^{t-1} is given by

$p_{\theta,M}(Y_t|Y_1^{t-1}) = \mathbf{1}'(\eta_{t,M} \odot \hat{\zeta}_{t,M|t-1})$ with $\odot$ element-wise multiplication and $\mathbf{1}$ a vector of ones. We then update $\hat{\zeta}_{t,M|t} = \frac{\eta_{t,M} \odot \hat{\zeta}_{t,M|t-1}}{p_{\theta,M}(Y_t|Y_1^{t-1})}$. The log likelihood is then the sum over $t = 1, ..., T$ of the logs of the marginal likelihoods.

This method may suffer from the curse of dimensionality, in the sense that if there are many state variables X_t, M has to be large, and, given that most discretization methods rely on tensor products, $P_{\theta,M}$ quickly becomes larger than a typical computer can store. With macro models featuring both exogenous and endogenous states, this issue can be partially aided. In the case of our model, recall that we have three endogenous state variables: N_t, V_t and R_t, and one exogenous state variable: Z_t. Let N_{endo} be the number of discrete values the endogenous states can take, and N_{exo} the total number of discrete values the exogenous states can take. We have $M = N_{\text{endo}} \times N_{\text{exo}}$. For the purpose of likelihood evaluation, we only need to store P_{endo}, which is $M \times N_{\text{endo}}$, that is, the transition probability matrix of all possible states to the N_{endo} different endogenous states, and P_{exo} which is the transition probability matrix governing the exogenous variables. Define $D = I_{N_{\text{exo}}} \otimes \mathbf{1}_{1\times N_{\text{endo}}}$ and $B = P'_{\text{exo}} D$. Here $I_{N_{\text{exo}}}$ is an identity matrix of size N_{exo}. We can update our forecast ζ in two steps:

$$F_{t-1} = P_{\text{endo}} \cdot \hat{\zeta}_{t-1,M|t-1}$$

and

$$\hat{\zeta}_{t,M|t-1} = F'_{t-1} B'.$$

Using this two-step update allows us to use more grid points and therefore get a better approximation of the likelihood. In particular, we will use 18 grid points for our exogenous variable Z_t, consistent with our computation approach, 15 grid points to discretize N, 7 grid points to discretize V, and 11 grid points to discretize R.[9] To construct P_{endo}, we use the policy rules to compute next period's states, and let P_{endo} be a weighted probability of the distance of next period's state to the grid points. To construct our grids for N, V and R, we use the same grid widths as used in the global model solution, which we pick to be slightly wider than the ergodic sets.

As shown in Farmer (2021), most of the computation time of the discretization filter goes to the construction of the discrete Markov chains. However, as explained in the previous subsection, our computational approach already relies on a discretization of the exogenous variable Z_t,

[9]Note that this is more precise than Farmer (2021), who used 5 grid points for the exogenous variables, and 7 for the endogenous variables.

and the transition dynamics for the other states and variables are easy to compute and follow directly from the computed policy rules. As such, the discretization filter can be used to compute (an approximation of) the likelihood of our model with little additional cost once the model has been solved. This is an important advantage compared to particle filters used by for example, Gust et al. (2017), to estimate a globally solved model. In their application, the computational costs for solving the model and evaluating the likelihood are of the same order of magnitude.[10]

4.3 Data

Data used for estimation are summarized in Table A2 in the Appendix. This description also includes the data transformations used. In particular, we use the Hamilton filter (2018) to remove long-run trends. Our six variables of interest are: the unemployment rate, a proxy for wages, output per worker, market tightness, the federal funds rate and inflation. We consider the period 1967-2019, with a quarterly frequency. Note that there is a difference between the market tightness in the model and the data. In the model, it follows Equation (10), in the data, it is simply vacancies over unemployment, which is also how we construct our measurement equation.

5 Comparing the Linear and Nonlinear NKDMP Model

In this section, we compare the different implications of the linearized and nonlinear model. First, we present and compare the estimation results from the linear and global model solution. Next, we compare the differences in model dynamics and other model solution properties between the linear and global model.

5.1 Comparing Estimation Results

Table 1 summarizes the model parameters, the prior selection, and the parameter estimates for the linearized NKDMP model and the NKDMP model solved using global projection methods. We use the same priors for the linearized and nonlinear model to ensure differences in posteriors are driven by the model solution method and not by the prior choice. Figure 3 visualizes the prior and posterior distributions, for the linearized model and the global model solution.

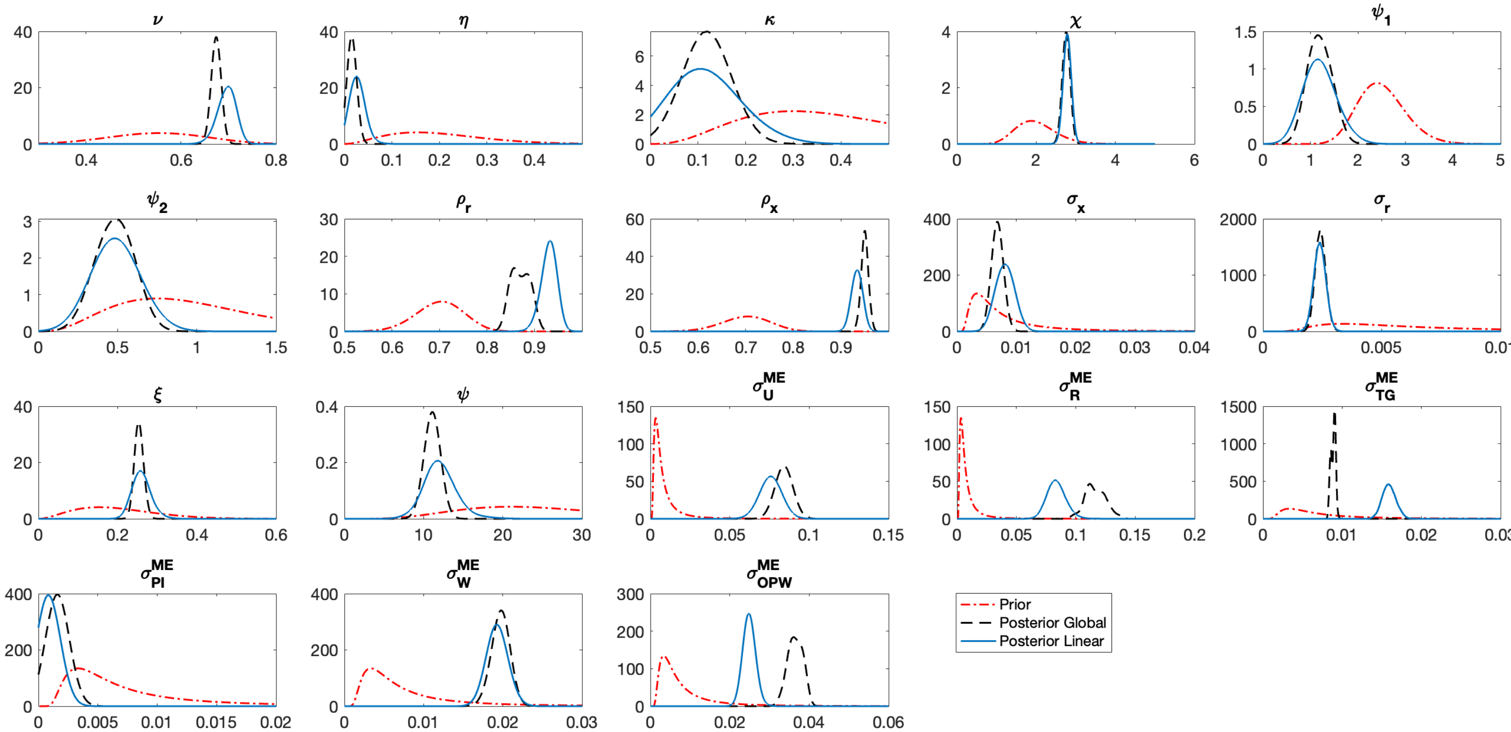
[10] Admittedly, the model in Gust et al. (2017) has a larger state space and it would therefore likely be infeasible to use the discretization filter to estimate it, even after using the two-step decomposition of exogenous and endogenous states.

Table 1: Parameters, priors (mean and std), posterior (mean and std) of linearized model, and nonlinear model

Param.	Description	Prior	Linear		Global (only ME)		Global (all param.)	
			Calibration					
α	Production elasticity	-	0.90		0.90		0.90	
β	Discount factor	-	0.9876		0.9876		0.9876	
γ	Demand elasticity	-	0.10		0.10		0.10	
δ	Separation rate	-	0.08		0.08		0.08	
ι	Matching function elasticity	-	0.45		0.45		0.45	
τ	Risk aversion	-	2.0		2.0		2.0	
ϕ	Price adjustment cost	-	65		65		65	
			Estimation structural parameters					
η	Bargaining weight	B(0.2,0.1)	0.027	(0.013)	0.027		0.015	(0.003)
ν	Outside option of the worker	B(0.55,0.1)	0.696	(0.017)	0.682		0.674	(0.003)
ξ	Vacancy depreciation rate	B(0.2,0.1)	0.261	(0.022)	0.261		0.254	(0.006)
κ	Vacancy maintenance cost	G(0.4,0.2)	0.114	(0.060)	0.114		0.118	(0.014)
ψ	Vacancy creation cost	G(25,10)	12.1	(1.88)	12.1		11.1	(0.324)
χ	Matching efficiency	G(2,0.5)	2.78	(0.021)	2.78		2.75	(0.006)
ψ_1	Taylor rule weight inflation	G(2.5,0.5)	1.20	(0.296)	1.20		1.19	(0.171)
ψ_2	Taylor rule weight output gap	G(1,0.5)	0.490	(0.121)	0.490		0.48	(0.076)
ρ_r	Inertia of Taylor rule	B(0.7,0.05)	0.930	(0.014)	0.930		0.87	(0.018)
σ_r	Standard deviation MP shocks	IG(0.01,1)	0.002	(0.0002)	0.002		0.002	(0.0001)
ρ_z	Persistence of TFP shocks	B(0.7,0.05)	0.934	(0.011)	0.934		0.950	(0.006)
σ_z	Standard deviation TFP shocks	IG(0.01,1)	0.008	(0.001)	0.007		0.007	(0.0002)
			Estimation measurement errors standard deviations					
$\sigma^{\theta}_{\text{ME}}$	Measurement Error θ	IG(0.01,1)	0.016	(0.001)	0.150	(0.009)	0.009	(0.0003)
σ^{W}_{ME}	Measurement Error W	IG(0.01,1)	0.019	(0.001)	0.019	(0.001)	0.020	(0.001)
σ^{Π}_{ME}	Measurement Error Π	IG(0.01,1)	0.0008	(0.0001)	0.009	(0.001)	0.0016	(0.0001)
σ^{OPW}_{ME}	Measurement Error Y/N	IG(0.01,1)	0.025	(0.001)	0.042	(0.002)	0.037	(0.002)
σ^{U}_{ME}	Measurement Error U	IG(0.01,1)	0.076	(0.005)	0.087	(0.005)	0.084	(0.003)
σ^{R}_{ME}	Measurement Error R	IG(0.01,1)	0.084	(0.008)	0.002	(0.0001)	0.117	(0.009)

Notes: **These estimates are very preliminary**. If there are no standard errors in between brackets, it means that the parameters are calibrated/fixed instead of estimated. G stands for the Gamma distribution, B for Beta, IG for inverse-Gamma. Linearized model estimates are based on 200,000 draws from a block Random-Walk-Metropolis-Hasting sampler using the Kalman filter, with a burn-in of 40,000 draws. For first column of the global model, we re-estimate the measurement error standard deviations, but use the modes from the linear model for all other parameter. Note that in this exercise, we use slightly lower values for the global model for ν and σ_x, coinciding with the 20% quantile of the posterior, to ensure stability of the computational algorithm. The global parameter estimates for the measurement error only are based on 10,000 draws with 1000 burn-in. The global parameters in the last column are based on 22,000 draws, after 18,000 burnin, obtained from 7 independent chains.

Figure 3: Priors (red dash dot) and posteriors of linearized model (blue solid) and the global model (black dashed)



Notes: Details on the estimation are given in Section 4 and Table 1.

From Table 1 and Figure 3, we see there are some important differences between the local and global parameter estimates. The following parameters stand out and are particularly important for the model dynamics. First, the outside option ν is lower in the global model estimation than in the linearized model, and the posterior mode decreases from 0.696 to around 0.67. While this seems like a small decrease, the outside option ν is important for the labor market dynamics and volatility, and, as we will see in the results below, changes the model dynamics considerably. Second, the posterior mode of ρ_r, the persistence of the Taylor rule, is lower in the global model estimation than in the linearized model. This difference comes from the fact that in the global (nonlinear) model, the interest rate is bounded below by zero but governed by an unbounded shadow rate. Because of the long zero-lower-bound period in the data, the linear model interprets this as a highly inertial interest rate rule, while the nonlinear model understands that the shadow rate may have moved while the actual policy rate was stuck at the zero lower bound. The volatility of TFP σ_x is lower in the global model estimation than the linearized model, because the global model generates more volatility in unemployment with a lower variance of the TFP shocks. Finally, note that for several parameters, the posterior of the global model estimates is tighter than for the linearized model. This is because the model is more sensitive to parameter changes, leading to better identification of the parameters and consequently tighter posteriors.

5.2 Comparing Ergodic Sets

In this section, we compare ergodic sets as simulated from the linearized NKDMP model with those from the the global model solution. In Figure 4, we visualize the ergodic distributions from both model solutions, evaluated at the parameters in Table 1 in the column "global (only ME)", which corresponds to the mode of the linearized model posteriors, except for ν and σ_x where we use the 20^{th} quantile of the posterior for stability of the global model solution[11]. As can be seen, the two model solutions, when evaluated in the same parameters, result in considerably different ergodic sets. This is consistent with the findings of Petrosky-Nadeau and Zhang (2017) for the DMP model, but the large differences suggest that New Keynesian model elements seem to introduce important additional nonlinearities.

From Figure 4, we see that the largest differences come from the labor market variables: the ergodic distribution of unemployment and TFP from the global model solution is fan-shaped, and lies strictly above the distribution following from the linearized model solution, which is an ellipse. The global model solution has more volatility in unemployment than

[11]The global model evaluated at the posterior mode of the linear estimates of ν and σ_x displays an even larger and more volatile unemployment rate.

Figure 4: Ergodic distributions for the linearized (red) and global (blue) NKDMP model, evaluated in the linearized model parameter estimates of Table 1.

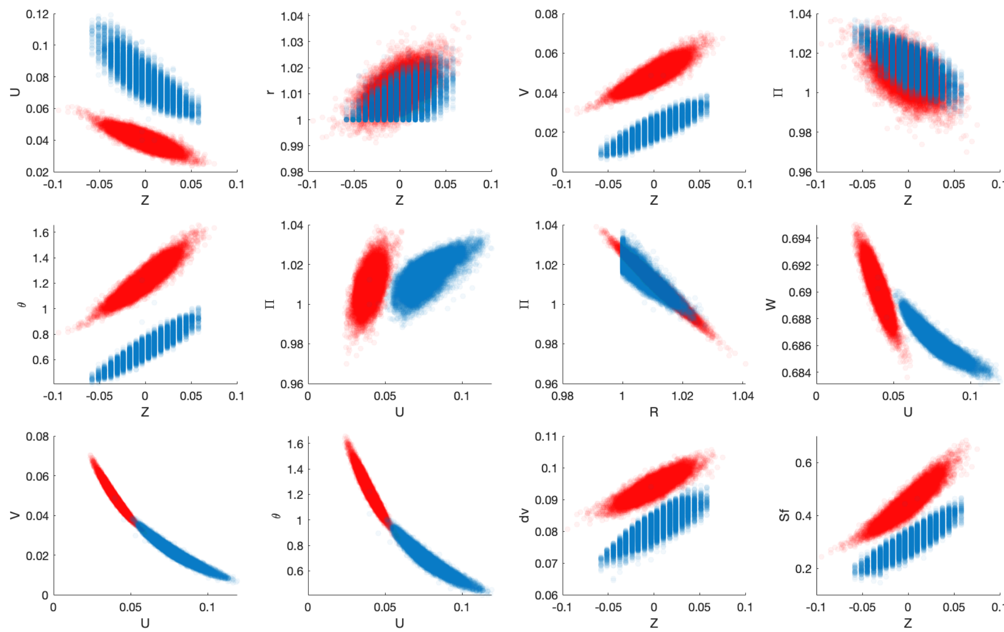
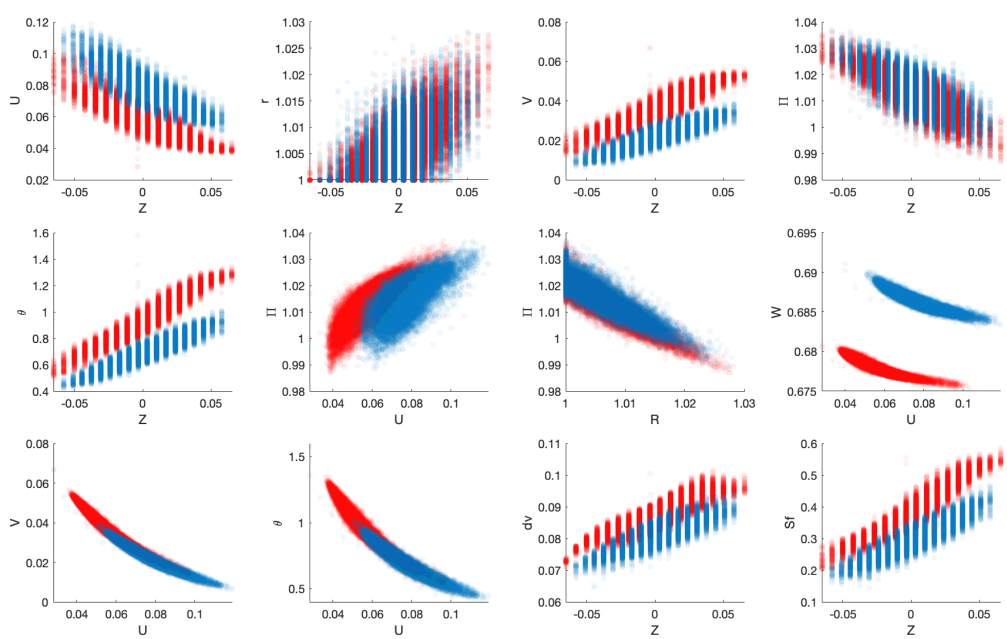


Figure 5: Ergodic distributions for the global model in the linear-model parameter estimates (blue) and the global model in the nonlinear parameter estimates (red) of Table 1..



the linearized solution. The vacancy stock and vacancy flow distributions of the global model solution, on the other hand, lie strictly below that of the linearized model solution. This, consequently, results in large differences in the market tightness of the two different model solutions, and, for example, the relationship between wages and unemployment. In the global model solution, the ergodic distribution of wages and unemployment is half-moon-shaped, and is less steep: wages rise less when unemployment goes down, relative to the linearized model solution, which generates an ellipse-shaped ergodic distribution for wages and unemployment. Comparing interest rates and inflation, note that the global model solution generates less deflation than the linearized model.

Table 2 summarizes the difference between the steady state and the ergodic mean, again evaluated in the posterior mean of the linearized model solution. As can be seen, the ergodic mean of the global model solution deviates considerably from the steady state (which coincides with the ergodic mean of the linear model solution). This further motivates our choice for a full-information estimation approach based on the global model solution, rather than considering a calibration approach, or relying on using the linearized model parameter estimates. We also see the model solutions generate different correlations between parameters: inflation and interest rates are less correlated in the global model solution, while the correlation between inflation and unemployment is stronger than in the linearized model.

Table 2: Moments in the linear and global model: evaluated in the estimates of the linearized model in Table 1

	Linear	Global	Linear	Global
	Ergodic mean		Standard deviations	
Unemployment rate	0.0403	0.0786	0.0048	0.0101
Output	0.9636	0.9294	0.0221	0.0266
Inflation (quarterly)	0.0084	0.0155	0.0079	0.0068
Wage level	0.6904	0.6865	0.0014	0.0009
Market tightness	1.2161	0.6811	0.1111	0.0890
Vacancy Stock	0.0485	0.0216	0.0054	0.0049
Vacancy Flow	0.0939	0.0813	0.0032	0.0038
Firm Surplus	0.4411	0.2966	0.0574	0.0443
Interest rate	1.0126	1.0061	0.0056	0.0049
	Selected Correlations			
(U,R)	-0.4652	-0.4396		
(Π,R)	-0.9799	-0.8724		
(Π,U)	0.5224	0.6873		
(W,U)	-0.8731	-0.9142		

Comparing ergodic sets also allows us to visualize how different the global and linear parameter estimates are, through the lens of the global model solution. Particularly, Figure 5 visualizes the ergodic distributions of the global model, evaluated in the linear and global parameters respectively. The large differences between the two global ergodic distributions show that, while the differences between posterior distributions in Figure 3 may seem small, the global model solution is sensitive and relatively small parameter changes result in big changes in the ergodic distributions, as well as in the model dynamics, which is the focus of the next subsection.

5.3 Comparing Model Dynamics

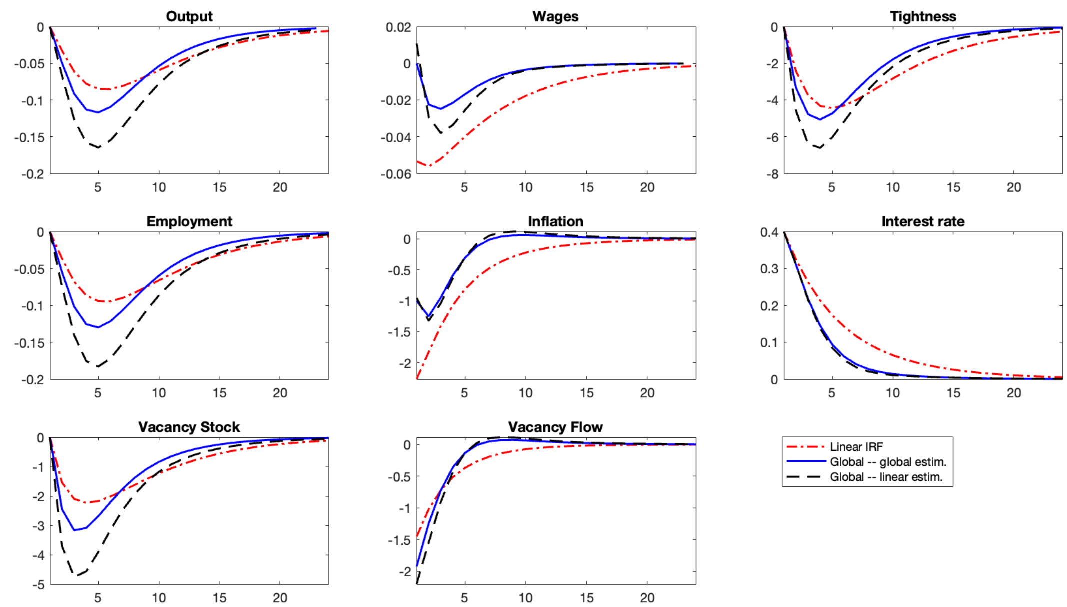
Next, we compare the differences in model dynamics between the linearized and global model. The red dash-dot line and the black dashed line in Figure 6 show the impulse response functions in response to a 40bp monetary policy shock, for the linearized and global model solution respectively, both evaluated in the parameter estimates for the linearized model. For the impulse response functions of the global model in this figure, we assume the variables start in their ergodic mean rather than in their steady state. Comparing the red dash-dot and black dashed lines shows the linearized and global model result in different dynamics: output is three times as responsive to a monetary policy shock in the global model than in the linearized model. Employment and the vacancy stock are more than twice as responsive. Inflation and wages, on the other hand, are about twice as responsive to a monetary policy shock in the linear model than in the global model solution.

Comparing the solid blue line and black dashed line in Figure 6 shows the differences between the parameters estimated for the linear model and the global model, through the lens of the global model solution. This again confirms that while the estimates for the linear and global model may look similar, they result in strong differences in model dynamics. Estimating the full global model, rather than using the linearized model parameters, brings the responses of output, employment and vacancies closer to the linearized model solution, but the dynamics of wages and inflation are little affected, and still differ strongly from the linear model.

6 Endogenous State-Dependent Impulse Response Functions

In this section, we assess the ability of our estimated model to generate state-dependent impulse response functions, consistent with the evidence presented in our empirical section. For this purpose, consider Figure 7, which visualizes the impulse response functions to a 40bp monetary policy shock, for the linearized model in the linear parameter estimates, which we

Figure 6: Impulse Response Functions to a 40bp MP shock for the linearized (red line) and global NKDMP model evaluated in the linear parameter (black dashed) and in the global parameters (blue solid).

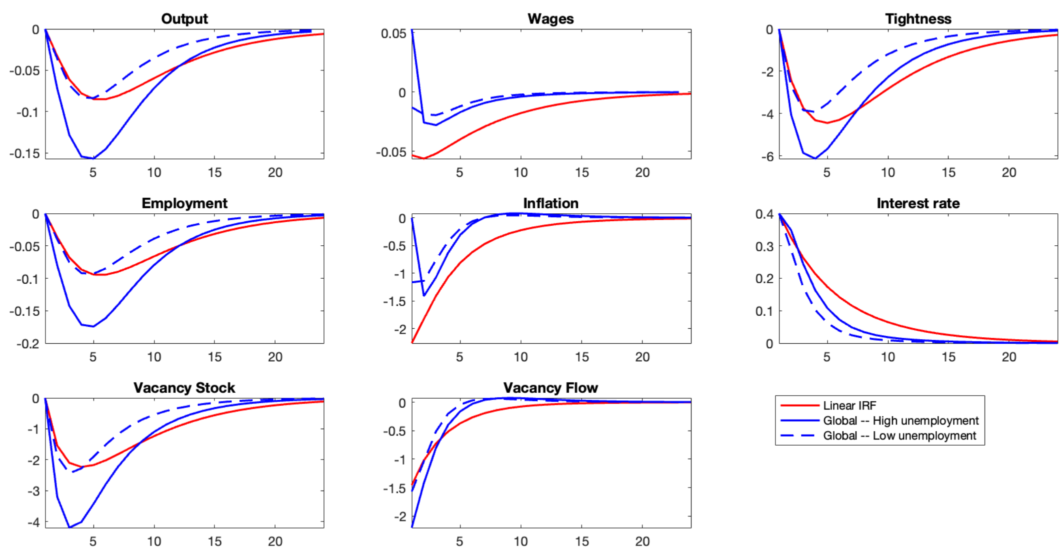


Notes: Based on parameter estimates of the linearized model, summarized in Table 1.

include for reference, and the global NKDMP model in the global parameter estimates. For the global model, we have two settings: we either start the economy in a high unemployment state (10% unemployment), or a low unemployment (4%) state, and compare responses. As can be seen, the model features strong state-dependence: the responses to output, employment, vacancies and market tightness are at its peak almost twice as large when the monetary policy shock hits the economy during a high unemployment state, compared to when it hits during a low unemployment state. Similar to our empirical model-free findings, inflation and wages show little state-dependence.

Also interesting to point out are the similarities between the sizes of the impulse response functions of the model and the empirical evidence from our local projections in Figures 1-2. Note that these are not explicitely targeted, in contrast to an estimation approach based on impulse response function matching, used in, for example, Christiano et al. (2016). Particularly, we find that the impulse response functions for employment, market tightness and the vacancies stock are of similar size and shape in the data and model. The empirical impulse response functions peak around 6-8 quarters, while our model-based impulse response functions peak around 5 quarters. Output and employment in the model are linked through the aggregate production function which limits how different the response of the two can be in the model. As a consequence, the model response of output with the empirical response of industrial production share similarities, but the effect is smaller relative to what we see in the data. Inflation in our model is more responsive than in the local projections; compared to Figure 1 the response in the model is about twice as large and immediate while in the data, inflation only falls after 4-6 quarters. Note, however, that this is an improvement compared to the linearized model, for which the response is about twice too large, and does not feature a hump. A similar disparity in the relative size of the effects on inflation is also true for wages which fall by less in the global model.

Figure 7: State Dependence: Impulse Response Functions to a 40bp MP shock for the linear and global NKDMP model, for a high (10%) and low unemployment (4%) state.

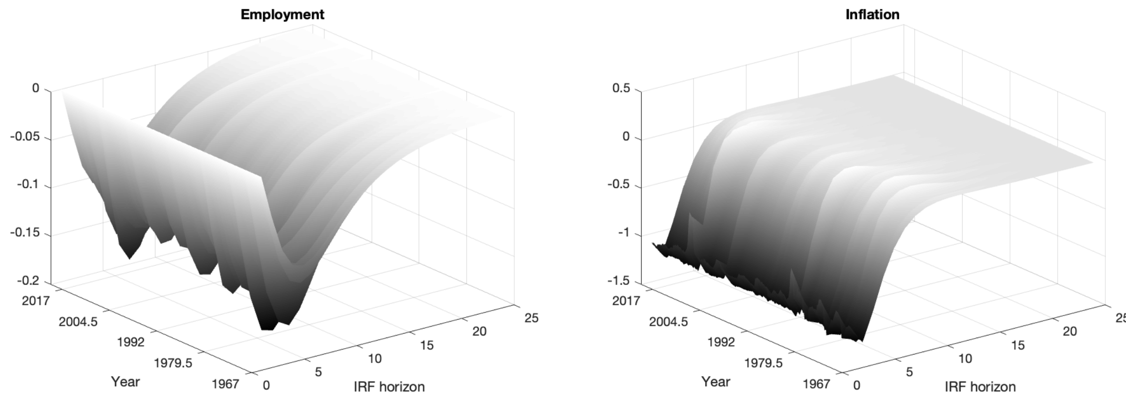


Notes: Based on parameter estimates of the global model as summarized in Table 1 (all parameters re-estimated) for the global IRFS, and the linearized estimates for the linear IRF.

Not only does our global model solution endogenously generate impulse response functions that depend on the current unemployment rate, we can use our estimation procedure and the discretization filter to filter the data, yielding estimates of the underlying states of the economy, and use this to compute time-varying impulse response functions. To illustrate this, we focus on the time-varying impulse response functions of employment and inflation in response to a 40 bp monetary policy shock. These are visualized in Figure 8, based on the parameter estimates of the global model. The figure shows that the impulse response function for inflation does not vary strongly over time. On the other hand, the impulse response function of employment varies strongly over time, with countercyclical size and persistence:

the effects of a monetary policy shock on employment are larger and longer-lasting during recessions than expansions.

Figure 8: Time-Varying Impulse Response Functions to a 40bp MP shock for the global NKDMP model.



Notes: Based on parameter estimates of the global model as summarized in the last column of Table 1 (all parameters re-estimated).

Our model also generates asymmetric responses to monetary policy shocks, similar in spirit to the empirical findings of Figure A5 in the Appendix. The model impulse response functions are shown in Figure 9. As in our empirical results, negative shocks are less persistent than positive monetary policy shocks.

7 A State-Dependent Philips Multiplier

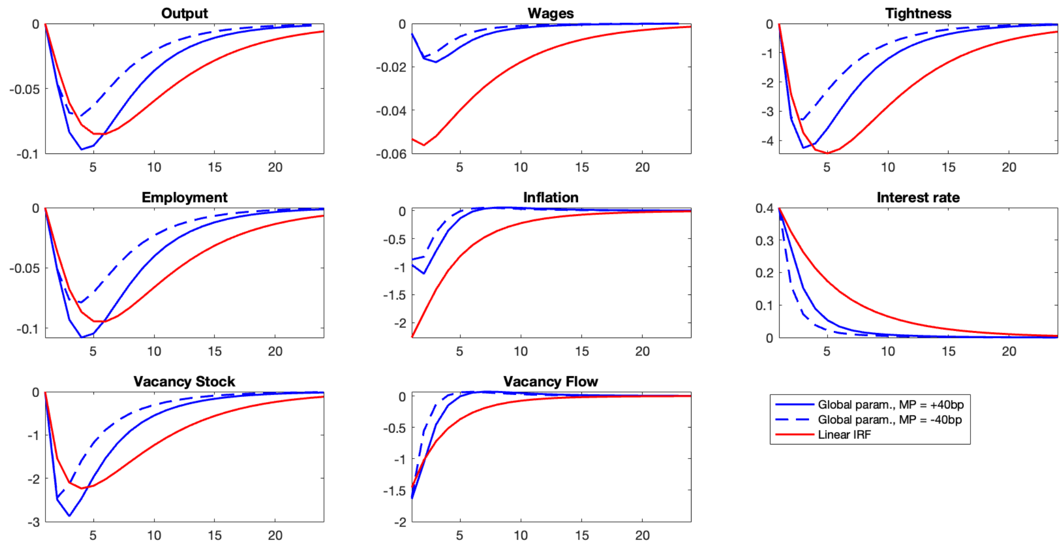
What are the policy implications of state-dependent impulse response functions? One statistic that policy makers, in particular, central banks, care about is the trade-off between inflation and unemployment. As in Mankiw (2001) and Barnichon and Mesters (2021), this trade-off is characterized by the average change in inflation caused by an interest rate change lowering the unemployment rate by one percentage point over h periods:

$$T_h = \frac{\partial \bar{\Pi}_{t:t+h}}{\partial R_t}\Big|_{\varepsilon_t} \Big/ \frac{\partial \bar{U}_{t:t+h}}{\partial R_t}\Big|_{\varepsilon_t}$$

Here $\bar{\pi}_{t:t+h}$ is the average over variable Π over h time periods, and ε_t is the exogenous change in the interest rates. The statistical counterpart of T_h is the so-called Philips multiplier (Barnichon and Mesters, 2021):

$$\mathcal{P}_h = \mathcal{R}_h^{\bar{\Pi}} / \mathcal{R}_h^{\bar{U}}, \quad h = 0, 1, 2, \ldots,$$

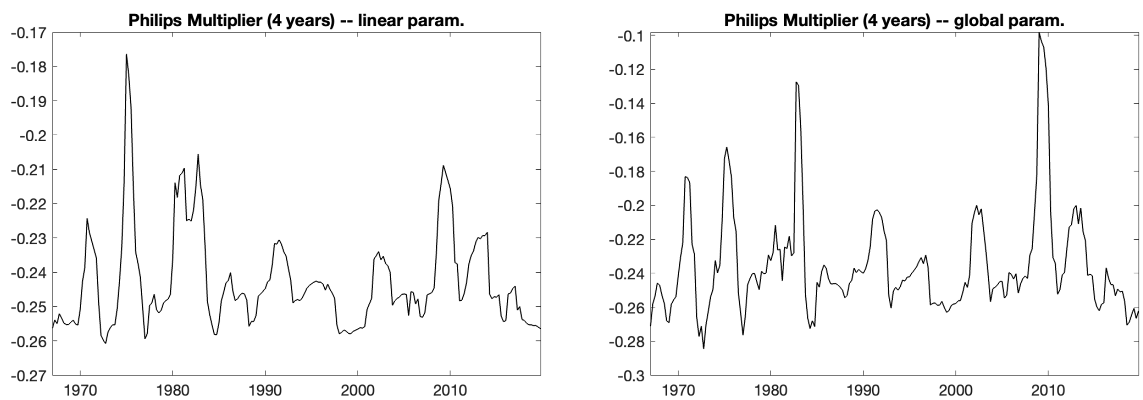
Figure 9: Asymmetry: Impulse Response Functions to a 40bp MP shock for the linear and global NKDMP model, for a negative and positive MP shock (sign of the negative MP shock IRF's is flipped to ease comparison).



Notes: Based on parameter estimates of the global model as summarized in Table 1 (all parameters re-estimated).

that is, the ratio of the sum of the impulse response function (up to horizon h) of inflation over unemployment. The Philips Multiplier has been interpreted loosely as the slope of the Philips curve. In the case of state-dependent impulse response functions, the Philips Multiplier is naturally also state-dependent.

Figure 10: Philips Multiplier based on Time-Varying Impulse Response Functions to a 40bp MP shock for the global NKDMP model.



Notes: Parameter estimates summarized in Table 1. Left panel uses the linear parameter estimates, with re-estimated measurement errors for the nonlinear filter, right panel uses the global parameter estimates.

Figure 10 visualizes the Philips Multiplier based on the global NKDMP model, evaluated in the linear versus global parameter estimates, four years after shock impact. The global model evaluated in the global parameter estimates features stronger state-dependence of the Philips Multiplier than the model based on the linear parameter estimates, again stressing the large differences between the two models. Focusing on the right-hand panel based on the global parameter estimates, we see that the Philips Multiplier is strongly procyclical, which, informally, means that the Philips curve is considerably flatter during recessions than expansions. During the great financial crisis, at its peak, the Philips Multiplier was around -0.1, while in 2019, it was as large as -0.27.

We can compare these values with the Philips Multiplier of the linearized model, which, after four years of shock impact, equals -0.11. The linear model thus overestimates the costs of lowering inflation during low unemployment episodes, and its estimate is comparable to the Philips Multiplier in recession periods in the global model.

In the local projection estimates from Figure 2, we find a Philips Multiplier after four years of -0.097. With the effects of state-dependence, we find that when we're in the 40th quantile (low unemployment), the Philips Multiplier is -0.26, while in the 60th quantile (high unemployment), it is -0.06. It follows that state-dependence in the Philips Multiplier is somewhat stronger in the model than in the data, but the estimates are quite comparable.

8 Conclusion

The paper provided model-free estimates of the dependence the real response of identified monetary shock on the unemployment rate. We proposed a New Keynesian model with labor market frictions, and showed it can generate state-dependent impulse response functions and Philips multipliers consistent with empirical evidence identified monetary policy shocks. A global model solution and nonlinear estimation procedure expose the strong non-linearities inherent in the model, and preserves its state-dependent nature, while a linear model solution and estimation method generate different model dynamics, and fail to match empirical evidence. Overall we find evidence and theory point to the real effects of monetary policy being larger when unemployment is high.

On the methodological side, we show that using the discretization filter of Farmer (2021), in combination with a global model solution, results in a fast evaluation of the likelihood of our globally solved model, and allows for filtering our data, such that we can construct time-varying impulse response functions that can be used to measure the state-dependent effects of monetary policy at any point in time. Particularly, we show our model endogenously generates Philips Multipliers (i.e., inflation-unemployment trade-off) that are three times as large in expansions than in recessions. A linearized solution does not feature state-dependence, and overestimates the costs of fighting inflation during low unemployment episodes.

In line with existing literature, e.g. Gust et al. (2017), our paper stresses the importance of solving models using global rather than perturbation methods, and of estimating the nonlinear model rather than relying on calibration or the perturbed model estimates. We hope by showing that the combination of the discretization filter with a global model solution results in faster filtering than traditional nonlinear filters such as the particle filter, making estimation of small-scale NK models feasible on standard desktop computers, more researchers will employ these methods, and will better explore the inherent nonlinearities in these models.

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Supplementary Appendix to “Unemployment and the State-Dependent Effects of Monetary Policy”

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February 15, 2024

A Data

Table A1: Data details: local projections

Variable	Details	Transformation	Source
Unemployment Rate	Percent, Monthly, Seasonally Adjusted	None	LNS14000000
Wages	Average Hourly Earnings of Production and Nonsupervisory Employees, Total Private, Dollars per Hour, Monthly, Seasonally Adjusted	$100 \times \log$	CES0500000008
Federal Funds Rate	Percent, Monthly, Not Seasonally Adjusted	None	DFF
Market tightness	Vacancy over unemployments	$100 \times \log$	Barnicon (2010) & FRED
Job finding rate		$100 \times \log$	Shimer (2005)
CPI	Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Index 1982-1984=100, Monthly, Seasonally Adjusted	$100 \times \log$	CPIAUCSL
PPI	Producer Price Index by Commodity: All Commodities, Index 1982=100, Monthly, Not Seasonally Adjusted	$100 \times \log$	PPIACO
IPI	Industrial Production: Total Index, Index 2017=100, Monthly, Seasonally Adjusted	$100 \times \log$	IP.B50001.S
HWI	Help-Wanted-Index from Barnicon (2010), JOLTS post 2001	$100 \times \log$	Barnicon (2010)
TFP	1 quarter moving average of TFP growth	None	Fernald (2014)

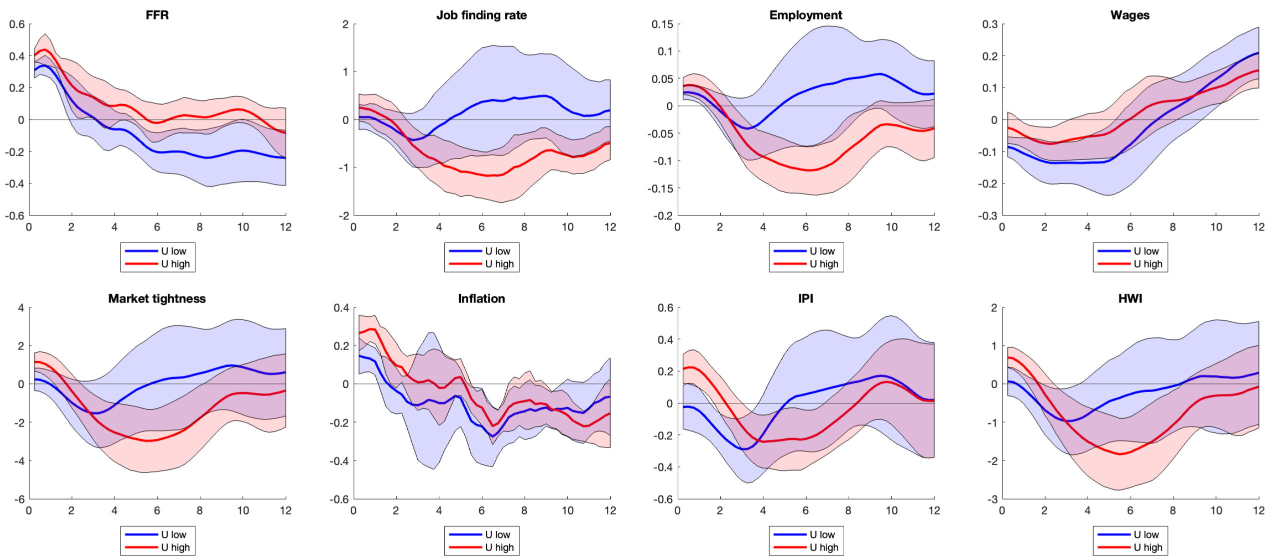
Table A2: Data details: estimation and filtering

Variable	Details	Transformation	Source
Unemployment Rate	Percent, Quarterly, Seasonally Adjusted	Detrended*	UNRATE (FRED)
Average Weekly Earnings of Production and Nonsupervisory Employees Total Private	Dollars per Week, Quarterly, Seasonally Adjusted	Detrended* & demeaned	CES0500000030 (FRED)
Nonfarm Business Sector: Output per Worker	Index 2017=100 Quarterly Seasonally Adjusted	Detrended* & demeaned	PRS85006163
Market tightness	Vacancy over unemployments	Detrended*	Barnicon (2010) & FRED
Federal Funds Effective Rate	Percent	Quarterly**	DFF
CPI	Consumer Price Index for All Urban Consumers: All Items in U.S. City Average Index 1982-1984=100, Monthly, Seasonally Adjusted	Quarter-to-quarter % ch., detrended*	CPIAUCSL

Notes: (*): we use the Hamilton filter to remove long-run trends (Hamilton, 2018). (**) DFF converted to a quarterly interest rate $(100 * ((1 + DFF/100)^{(1/4)} - 1))$.

B Additional results for the Local Projections

Figure A1: State-Dependent Impulse Response Functions using Romer-Romer shocks

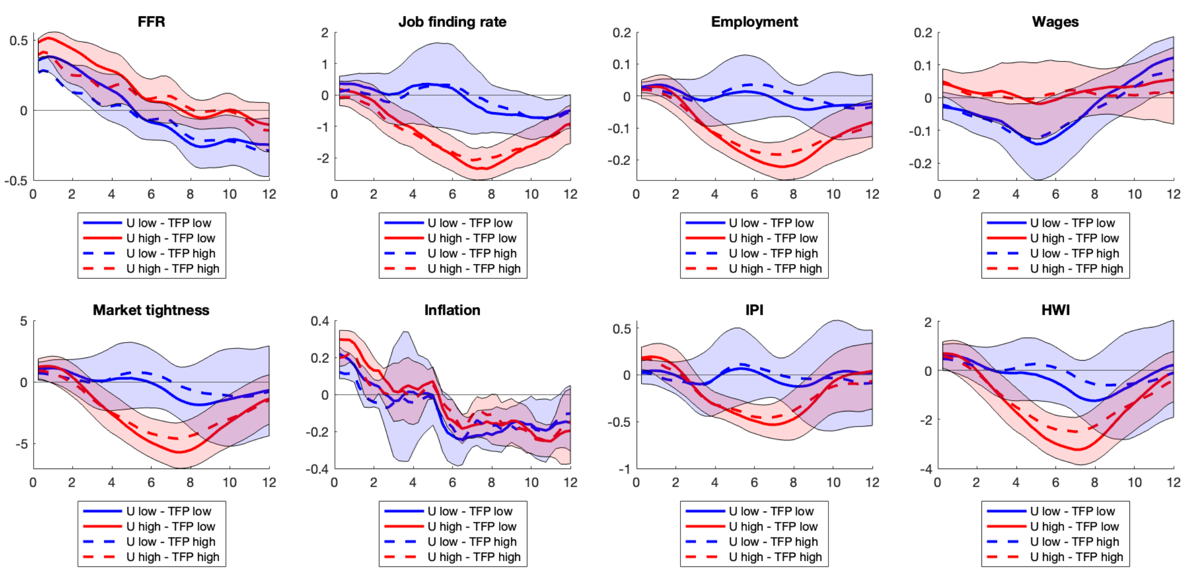


Notes: Six lags, 90% confidence intervals (asymptotic). Sample 1969-2007, monthly frequency. $\lambda_1 = 200, \lambda_2 = 20$, $\lambda_3 = 0.1$. For the purpose of this figure, the local projections are evaluated in the 30$^{\text{th}}$ percentile (low unemployment) and the 70$^{\text{th}}$ percentile of unemployment (high unemployment) over the sample period.

Figure A2: State-Dependent Impulse Response Functions using Gertler-Karadi shocks

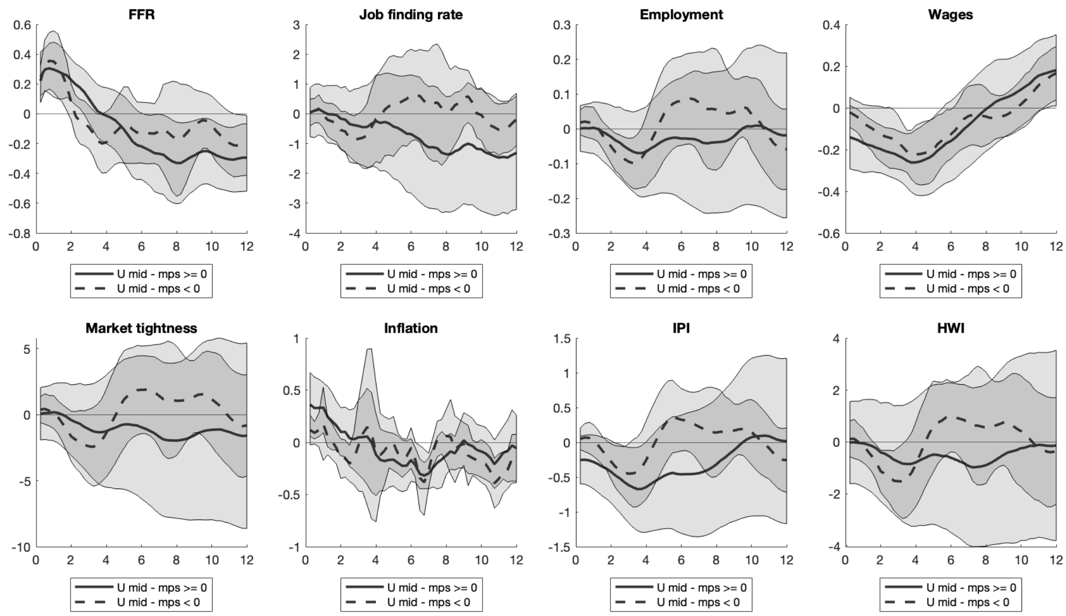
Notes: 90% confidence bands (asymptotic). Two lags. Shocks used are the residuals of the original shock series, regressed on 12 months of changes in FFR. Sample 1990-2012, monthly frequency. Shrinkage parameters: $\lambda_1 = 4, \lambda_2 = 0.05$, $\lambda_3 = 0$. The local projections are evaluated in the 30$^{\text{th}}$ percentile (low unemployment) and the 70$^{\text{th}}$ percentile of unemployment (high unemployment) over the sample period.

Figure A3: State-Dependent Impulse Response Functions using Romer-Romer shocks: unemployment and TFP



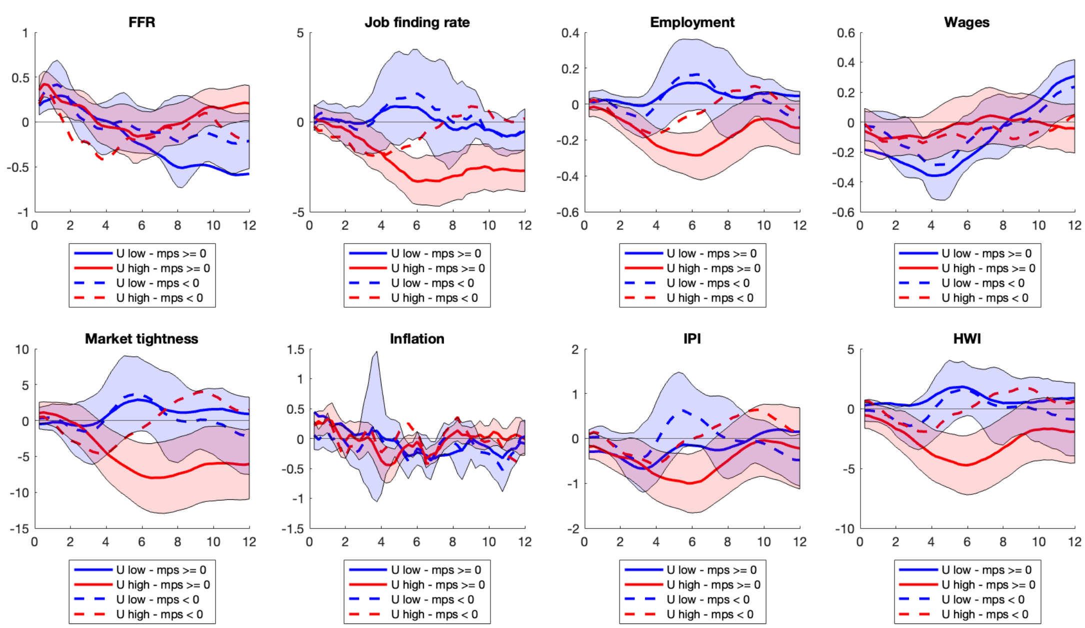
Notes: 4 lags, sample 1969-2007, monthly frequency. $\lambda_1 = 200, \lambda_2 = 20, \lambda_3 = 0.1$. Local projections are evaluated in the 30$^{\text{th}}$ percentile (low unemployment and low TFP growth respectively) and the 70$^{\text{th}}$ percentile of unemployment (high unemployment and high TFP growth respectively). Dashed lines are high TFP growth, solid lines are low TFP growth. Red is High Unemployment, Blue is Low Unemployment. 90% confidence intervals (asymptotic) correspond to the IRF of having high unemployment and low unemployment, respectively, with median TFP growth (not visualized).

Figure A4: Asymmetric Impulse Response Functions using Romer-Romer shocks, controlling for state-dependence in unemployment



Notes: 4 lags, sample 1969-2007, monthly frequency. $\lambda_1 = 50, \lambda_2 = 1, \lambda_3 = 50, \lambda_4 = 10$. The local projections are evaluated in the 50$^{\text{th}}$ percentile of unemployment. 90% confidence intervals (asymptotic). Solid line is positive monetary policy shock, dashed line non-positive.

Figure A5: Asymmetric and State-dependent Impulse Response Functions using Romer-Romer shocks



Notes: 4 lags, sample 1969-2007, monthly frequency. $\lambda_1 = 50, \lambda_2 = 1, \lambda_3 = 50, \lambda_4 = 10$. The local projections are evaluated in the 30$^{\text{th}}$ and 70$^{\text{th}}$ percentile of unemployment. 90% confidence intervals (asymptotic) visualized for the red solid line and the blue dashed line.

C Derivations

Household. The recursive form of the household problem is,

$$V(B_t, \cdot) = \max_{B_{t+1}, C_t} \{u(C_t) + \beta\mathbb{E}[V(B_{t+1}, \cdot)]\} \tag{C.1}$$

subject to a budget constraint

$$C_t = \frac{W_t}{P_t} N_t + R_t \frac{B_t}{P_t} - \frac{B_{t+1}}{P_t} + \frac{D_t}{P_t}. \tag{C.2}$$

The first order condition for B_{t+1} is

$$\frac{1}{P_t} u'(C_t) = \beta\mathbb{E}[V'(B_{t+1}, \cdot)]. \tag{C.3}$$

The envelope condition implies

$$V'(B_t) = \frac{R_t}{P_t} u'(C_t). \tag{C.4}$$

Iterating this equation forward and substituting it in the first order condition gives

$$u'(C_t) = \beta\mathbb{E}\left[\frac{R_{t+1}}{\Pi_{t+1}} u'(C_{t+1})\right], \tag{C.5}$$

where $\Pi_{t+1} = P_{t+1}/P_t$. In steady state, this becomes

$$R_{ss} = \frac{1}{\beta}\Pi_{ss}. \tag{C.6}$$

Intermediate goods producers. In recursive form, the firms' optimization problem is given by:

$$\Omega(S_{jt}) = \max_{X_{jt}, P_{jt}} \left\{ \frac{P_{jt}}{P_t} Z_t N_{jt}^{\alpha} - R_t \left(\frac{W_t}{P_t} N_{jt} + k\left(X_{jt}\right) + \kappa V_{jt} \right) - \frac{\phi}{2} \left(\frac{P_{jt}}{P_{jt-1}} - \Pi^* \right)^2 Y_t + \mathbb{E}_t[m_{t+1}\Omega(S_{jt+1})] \right\}$$

subject to the constraints

$$
\begin{aligned}
Z_t N_{jt}^{\alpha} &= \left(\frac{P_{jt}}{P_t}\right)^{\frac{-1}{\gamma}} Y_t \\
N_{jt+1} &= (1-\delta)N_{jt} + q_t\left(V_{jt} + X_{jt}\right) \\
V_{jt+1} &= (1-\xi)(1-q_t)\left(V_{jt} + X_{jt}\right)
\end{aligned}
$$

where the states S_t include prices, employment, and vacancies $(P_{jt-1}, N_{jt}, V_{jt})$. Letting λ_{jt} be the multiplier on the first constraint, the first order condition for price-setting is

$$
\frac{Z_t N_{jt}^{\alpha}}{P_t} - \phi\left(\frac{P_{jt}}{P_{jt-1}} - \Pi^*\right)\frac{Y_t}{P_{jt-1}} + \mathbb{E}_t\left[m_{t+1}\Omega_P(S_{jt+1})\right] + \lambda_{jt}\left(-\frac{1}{\gamma}\left(\frac{P_{jt}}{P_t}\right)^{\frac{-1}{\gamma}-1}\frac{Y_t}{P_t}\right) = 0. \tag{C.7}
$$

The envelope condition for prices is

$$
\Omega_P(S_{t+1}) = \phi\left(\frac{P_{jt+1}}{P_{jt}} - \Pi^*\right)\frac{P_{jt+1}}{P_{jt}^2}Y_{t+1}. \tag{C.8}
$$

Focusing on a symmetric equilibrium, substituting the envelope condition, and multiplying through by P_t/Y_t yields

$$
1 - \phi\left(\Pi_t - \Pi^*\right)\Pi_t - \frac{\lambda_t}{\gamma} + \phi\mathbb{E}_t\left[m_{t+1}\left(\Pi_{t+1} - \Pi^*\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = 0 \tag{C.9}
$$

where $\Pi_{t+1} = P_{t+1}/P_t$. In steady state, this becomes

$$
\lambda_{ss} = \gamma. \tag{C.10}
$$

The first order condition for flow vacancies X_{jt} is

$$
R_t k'(X_{jt}) = \mathbb{E}_t\left[m_{t+1}\left(q_t\Omega_N(S_{jt+1}) + (1-\xi)(1-q_t)\Omega_V(S_{jt+1})\right)\right]. \tag{C.11}
$$

which under the functional form $k(X_{jt}) = \kappa X_{jt} + \frac{\psi}{2}(X_{jt})^2$ becomes

$$
R_t\kappa + R_t\psi X_{jt} = \mathbb{E}_t\left[m_{t+1}\left(q_t\Omega_N(S_{jt+1}) + (1-\xi)(1-q_t)\Omega_V(S_{jt+1})\right)\right]. \tag{C.12}
$$

The envelope conditions for N_{jt} and V_{jt} are

$$\Omega_N(S_{jt}) = \left(\frac{P_{jt}}{P_t} - \lambda_{jt}\right)\alpha Z_t N_{jt}^{\alpha-1} - R_t\frac{W_t}{P_t} + \mathbb{E}_t[m_{t+1}(1-\delta)\Omega_N(S_{jt+1})] \tag{C.13}$$

$$\Omega_V(S_{jt}) = -R_t\kappa + \mathbb{E}_t\left[m_{t+1}\left(q_t\Omega_N(S_{jt+1}) + (1-\xi)(1-q_t)\Omega_V(S_{jt+1})\right)\right]. \tag{C.14}$$

Note, the first order and envelope conditions for vacancies imply

$$\Omega_V(S_{jt}) = R_t\psi X_{jt}, \tag{C.15}$$

that is, the value of the marginal stock vacancy relative to the marginal flow vacancy is the lower cost associated with keeping a job open once it is in the stock of vacancies.

Surplus and Wages. We define the surplus of a new worker to a firm as $S_t^f = \Omega_N(S_{jt}) - \Omega_V(S_{jt})$. That is, the firm gains the marginal value of an additional employee, but loses the value of the vacant job[1]. Focusing on a symmetric equilibrium, and letting $w_t = W_t/P_t$ be the real wage yields:

$$\begin{aligned} S_t^f = \Omega_N(S_t) - \Omega_V(S_t) = (1-\lambda_t)\,\alpha Z_t N_t^{\alpha-1} - R_t w_t + R_t\kappa \\ + \mathbb{E}_t\left[m_{t+1}\left((1-\delta-q_t)S_{t+1}^f + [1-\delta-q_t-(1-\xi)(1-q_t)]\Omega_V(S_{jt+1})\right)\right] \end{aligned}$$

which, using the expression for Ω_V implied by optimizing behavior, and simplifying, becomes

$$S_t^f = (1-\lambda_t)\alpha Z_t N_t^{\alpha-1} - R_t w_t + R_t\kappa + \mathbb{E}_t\left[m_{t+1}\left((1-\delta-q_t)S_{t+1}^f + [(1-q_t)\xi - \delta]\psi R_{t+1}X_{jt+1}\right)\right] \tag{C.16}$$

Similarly, we can write the job-creation condition (first-order condition for vacancies) as

$$R_t\kappa + R_t\psi X_{jt} = \mathbb{E}_t\left[m_{t+1}(q_t S_{t+1}^f + [(1-\xi)(1-q_t) + q_t]\Omega_V(S_{jt+1}))\right]. \tag{C.17}$$

which becomes

$$R_t\kappa + R_t\psi X_{jt} = \mathbb{E}_t\left[m_{t+1}(q_t S_{t+1}^f + [1-\xi(1-q_t)]\psi R_{t+1}X_{jt+1})\right]. \tag{C.18}$$

[1]This nests the textbook labor search model without stock vacancies as $\Omega_V(S_{jt}) = 0$ in that case.

The surplus to the household is the difference between the flow value of employment V_t^e and unemployment U_t which are given by:

$$\begin{aligned} U_t &= \nu + \mathbb{E}_t\left[m_{t+1}(f_t V_{t+1}^e + (1-f_t)U_t)\right] \\ V_t^e &= w_t + \mathbb{E}_t\left[m_{t+1}((1-\delta(1-f_t))V_{t+1}^e + \delta(1-f_t)U_t)\right], \end{aligned}$$

where $f_t = \theta_t q_t$ is the job-finding probability. Defining $S_t^h = V_t^e - U_t$, we have

$$S_t^h = w_t - \nu + \mathbb{E}_t\left[m_{t+1}(1-\delta)(1-f_t)S_{t+1}^h\right] \tag{C.19}$$

The wage is determined via the Nash sharing condition

$$\eta S_t^f = (1-\eta)S_t^h. \tag{C.20}$$