A Job Ladder Model of Firm, Worker, and Earnings Dynamics

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Abstract

This paper proposes a multiworker firm model with on-the-job search and decreasing returns-to-scale production. A coalition bargaining solution between a firm and its incumbent workers yields tractability, results in privately efficient recruiting decisions, and delivers an explicit expression for the wage function. The paper shows how a stylized calibrated version of the model can replicate untargeted empirical facts on the cross-sectional dispersion in firm growth and on measured elasticities of separation rates, quitting rates and vacancy duration with respect to wages. It can also replicate observed net poaching rates by firm size and firm wage, therefore rationalizing the absence of firm size ladders and the presence of wage ladders. In terms of business cycles, the model can replicate the cyclical properties of job flows and worker flows.

Keywords: Multilateral bargaining, search, matching

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1 Introduction

The labor market is characterized by the joint dynamics of workers and jobs. Firms grow and shrink by hiring and firing workers. Workers are moving between unemployment and employment, as well as switching to better jobs. Over half of new hires are workers switching jobs. To understand these labor market dynamics, we need models featuring multiworker firms and workers who search on-the-job. Key in these models is the value of a job to workers, that is, the present value of wages workers expect to earn. High job values attract workers from other firms and lead to fewer workers quitting. However, the value of a job offer is a complicated object, as it depends on the offered wage, as well as the predicted future path of wages of this and all other firms. Moreover, the distribution of job values matters for firms' labor demand decisions because they determine hiring and quitting rates.

These intricacies make solving for models with multiworker firms and on-the-job search seem intractable. This paper proposes the use of Nash bargaining between firms and workers to obtain a tractable model solution. The contributions of this paper are twofold. First, the paper outlines a set of theoretical results that show how Nash bargaining allows for a reduction of the state space in a multiworker firm model with on-the-job search and delivers a characterization of the wages as a function of the other equilibrium objects in the model. As such, it can deliver predictions for wages in a heterogeneous firm model with on-the-job search in a more general setting than has previously been considered by the literature. Second, this paper shows how a calibrated stylized multiworker firm model with on-the-job search can account for many labor market facts documented in the empirical literature, speaking to firm size distributions, wages, and labor market stocks and flows.

Regarding the first contribution, this paper proposes a bargaining solution that preserves tractability of the model solution. Firms and workers lack commitment, which is why a firm cannot match outside offers, and why workers cannot commit to stay after receiving a better offer. I therefore assume that each period, the firm renegotiates the wage and hiring decision with the coalition of incumbent workers according to a Nash sharing rule, where workers within the firm are all paid a common wage[1]. Under this protocol, job values are determined by the joint surplus of the firm and its incumbent workers. As a result, the state vector needed to define the acceptance set of workers contains only the current level of employment at the firm and the firm's current productivity level. This is the same state vector needed in standard heterogeneous firm models without on-the-job search. I show how this bargaining solution results in an elegant representation of optimal labor demand decisions for the firm in terms of the joint surplus of the firm and its incumbent workers. This results in a single Bellman equation that determines the optimal labor demand decisions of the firm that does not depend on the wage, and an explicit expression for wages can be solved for as a function of equilibrium objects. As such, this paper provides the first characterization of wages in a multiworker firm model with on-the-job search and privately efficient recruiting.

The second contribution of the paper is to provide a quantitative assessment of whether this stylized theoretical model can account for labor market facts documented by the empirical literature. The model is calibrated to match the firm size distribution, average wage gains of job-switchers, and labor markets stocks and flows, and can match these targets despite its parsimoniousness. Aside from these targeted moments, the model can replicate three main (untargeted) empirical facts on labor markets, speaking to (i) firm

[1]In reality, there is a distribution of wages within a firm. In the model, workers are homogeneous and the wage is defined per efficiency unit of labor. I follow the literature (Acemoglu and Hawkins, 2014),Elsby and Michaels (2013),Elsby and Gottfries (2022) on multi-worker firms by assuming the wage is common to all workers.

growth, (ii) worker turnover and wages and (iii) net poaching. This is the first model that is capable of capturing the facts jointly.

First, the model can reproduce several empirical facts on firm growth. It matches the cross-sectional dispersion in firm growth rates, replicating that over half of employment is accounted for by firms growing or shrinking by 5% or more. That is, over half of workers are employed at a firm which grows or shrinks by more that 5% in a quarter. The model can also reconcile the results of Davis, Faberman, and Haltiwanger (2012) and Davis, Faberman, and Haltiwanger (2013), who find, in the data, firms that grow more quickly do so both by posting more vacancies, as well as by receiving a higher vacancy yield per vacancy. While there are existing theories relating the vacancy yield to employment growth, like including an additional margin of recruiting effort Gavazza, Mongey, and Violante (2018), or paying higher wages to attract more workers as in a directed search environment Kaas and Kircher (2015). The presence of on-the-job search provides another natural explanation for this fact. Firm that are growing quickly have high job values and thus can poach from a larger set of currently employed workers. They do, in fact, pay higher wages, but this is a result of their greater levels of productivity and ex-post bargaining. This leads to higher vacancy yields for fast growing firms at the top of the job ladder.

Second, the model also has implications for worker turnover and wages. The elasticity of the separation rate with respect to wages and the elasticity of the voluntary quitting rate with respect to wages is in line with the literature on imperfect labor competition as documented by Manning (2011). The elasticity of vacancy duration with respect to wages is -0.08, consistent with the findings of Mueller, Osterwalder, Zweimülller, and Kettermann (2018). Importantly, the authors take their estimates as evidence against models of wage posting such as Kaas and Kircher (2015). Their argument states that, when hiring from only unemployment, for the vacancy yield to match the data, there must be some firms promising very high wages, and these jobs are filled quickly. As a result, the model implied elasticity of vacancy duration with respect to the wage is very large, and the small elasticity poses a puzzle for this model. The authors suggest that this is elasticity is more consistent with models of recruiting effort such as Gavazza et al. (2018). However, my model is able to reproduce this elasticity with neither wage posting nor a recruiting effort margin, because with on-job-search higher wage firms also have a larger pool of workers to hire from, which leads to higher vacancy-filling rates. This allows for both a low elasticity of vacancy duration with respect to the wage, and high vacancy filling rates for quickly growing firms.

Third, the model can explain empirical facts on net poaching – the difference in hires from other firm and quits to other firms – by firm size and firm wage. In the model, high wage firms gain workers through net poaching, and low wage firms lose workers to poaching, which is consistent with the evidence presented in Haltiwanger, Hyatt, Kahn, and McEntarfer (2018). In contrast, both in the model as in the data, there is little evidence for a net poaching ladder in firm size. The model in this paper has the essential ingredients needed to explain these empirical facts on net poaching: decreasing returns to scale, a firm size distribution that the matches the data, on-the-job search, and a prediction for the wage.

In the last section, I explore the model's implication for the propagation of business cycle shocks. From the results in Shimer (2005), we know that standard search-and-matching models can fail to reproduce the transmission of productivity shocks to labor market variables that are observed in the data. This is also the case for the model in this paper. The decline in vacancies with respect to labor productivity is roughly half of what is seen in the data. However, the decline in worker reallocation, through falling job-to-job transition rates, and convex recruiting costs, leads to slower recoveries than implied by labor productivity alone. Output recovers more slowly than productivity, reflecting the costly reallocation of workers up the job ladder, showing that incorporating on-the-job search in a multiworker firm setting does improve its ability to speak to business cycles compared to more standard search-and-matching models. Moreover, the presence of on-the-job search leads to the vacancy-to-unemployment ratio being a misleading indicator of labor market tightness. I show that if one were to use a matching function in only unemployment and vacancies, my model can generate an implied drop in matching efficiency of 4% during a recession.

Outline. The remainder of the paper is structured as follows. Section 2 outlines the dynamic model of on-the-job search with multiworker firms. Section 3 discusses the calibration strategy. Section 4 presents results of the calibrated model with untargeted moments in the cross-section. Section 5 conducts a business cycle experiment. Finally, section 6 gives concluding remarks.

1.1 Related Literature

This paper presents a model of multiworker firms with on-the-job search and firm dynamics through entry and exit, and relates to a number of existing strands of literature.

The first is a large literature beginning with Hopenhayn (1992) and Hopenhayn and Rogerson (1993) who consider job-reallocation with decreasing returns to scale firms. In these models there is a clear notions of firm boundaries and firm size, but the labor market does not have search frictions, so wages are set competitively.

Next, there are models of on-the-job search with constant returns to scale production technologies. Beginning with Burdett and Mortensen (1998), and the extensions out of steady state by Moscarini and Postel-Vinay (2013) and Coles and Mortensen (2016), these models have productivity dispersion and on-the-job search in a random search environment. However, with constant returns to scale, larger firms are more productive and gain more workers through poaching, which is at odds with the evidence in Haltiwanger et al. (2018) that there is little evidence for a job ladder in firm sizes. My model includes decreasing returns to scale which breaks the link between firm size and average labor productivity and can address these facts.

Next, my paper builds on the literature extending firm dynamics models to include search frictions in the labor market. Elsby and Michaels (2013) and Acemoglu and Hawkins (2014) study random search models without on-the-job search, so their models do not face the issue of determining the acceptance set of job offers that on-the-job search presents. Kaas and Kircher (2015) invoke a convex vacancy adjustment cost, so firms face a tradeoff between posting hiring wages and posting more vacancies to attract workers. In my model, firms that pay high wages also have larger vacancy yields, but this is the result of on-the-job search and bargaining. Schaal (2017) allows for on-the-job search in a directed search environment. The paper by Schaal (2017) allows for on-the-job search with a linear vacancy posting cost and the ability of the firm to commit to posted contracts, and shows the equilibrium is block-recursive, allowing for a computationally attractive approach to the model. Both Kaas and Kircher (2015) and Schaal (2017) can capture firm employment dynamics in the data well. There are two limitations of directed search that my model is able to overcome. First, in a directed search environment with on-the-job search, firms must be able to commit to job values when they meet new workers, this leaves the wage the firm pays as indeterminate. Second, directed search does not make a prediction as to whether hires are from unemployment or other firms, so they cannot speak to the data on poaching by firm size or wage in Haltiwanger et al. (2018), while my model does.

Most closely related to my paper are Elsby and Gottfries (2022) and Bilal, Engbom, Mongey, and Violante (2022) which combine random search on and off the job with multiworker firms. Elsby and Gottfries (2022) provide a set of sufficient conditions on model primitives and wage bargaining such that the only state variable in the model is the marginal product of labor. They can solve the model analytically, allowing for a number of insights related to job flows at the firm level, and how firm dynamics are influenced by firm pay. I show in Appendix B that their environment is a restricted case of the model presented in this paper. Furthermore, I show the main analytical results of Elsby and Gottfries (2022) hold under coalition bargaining, when I restrict the hiring cost, technology, and idiosyncratic productivity process to their environment. The contribution of my paper relative to theirs is that I propose a computationally tractable solution for the firm problem, and wage determination, that holds in more general environments without restrictions on technology, recruiting costs functions, or the idiosyncratic shock process.

Bilal et al. (2022) solve a multiworker firm with on-the-job search in a random search environment by using the bargaining protocol of Postel-Vinay and Robin (2002), extended to a multiworker setting. They show that this protocol implies the firm problem is equivalent to a joint surplus maximization problem for all firm decisions. Due to the use of the Postel-Vinay and Robin (2002) bargaining protocol, wages are not determined in their model. The authors suggest wages can be recovered with a stronger set of assumptions on how wages a renegotiated internally, and, furthermore, they argue the bargaining solution they build upon in Postel-Vinay and Robin (2002) provides empirically plausible wage dynamics. However, recent empirical work by Di Addario, Kline, Saggio, and Solvsten (2022) finds little evidence for the wage implications of Postel-Vinay and Robin (2002). Moreover, the bargaining solution of Bilal et al. (2022) leads to wage dispersion *only if* there is on-the-job search. In the absence of on-the-job search, firms have all the bargaining power, and the wage distribution is degenerate. The contribution of my paper relative to Bilal et al. (2022) is to show that under a different set of conditions on bargaining, which nests both the textbook one-worker one-firm model and the multiworker firm model without on-the-job search, both a joint surplus representation and predictions for wages can be obtained.

2 Model

This section presents the multiworker firm model with on-the-job search studied by this paper. In this model, firms maximize the present value of profit earned by operating a decreasing returns-to-scale production technology. Firms face idiosyncratic stochastic productivity shocks that result in hiring and firing of workers. Hiring occurs in a frictional labor market. Firms hire by posting costly vacancies that randomly contact unemployed and employed workers. Unemployed workers become employed workers, but when vacancies contact a worker employed at another firm, job-switching will only occur if the value of the job offer at the new firm exceeds the value at the existing firm.

The next sections introduce the model environment in more detail, derive the values to firms and workers, explain the bargaining solution between firms and workers that results in privately efficient allocations, and provide the resulting equilibrium definition.

2.1 Environment

Time is continuous and goes forever. There is no aggregate uncertainty. There are two types of agents in the model, firms and workers. There is a single final good produced by firms using labor as the only input. Labor markets are frictional, and firms meet workers through random matching by posting vacancies.

Firms. There is a large mass of potential firms and a finite mass of operating firms. Firms pay a fixed cost c to enter. Upon entry, firms start with n_0 workers and draw a productivity level z from the discrete set $\mathcal{Z} = \{\underline{z}, \ldots, \bar{z}\}$ according to the distribution $\pi(z)$. Operating firms are subject to idiosyncratic shocks that arrive at random times according to a Poisson process. This Poisson process is defined such that a shock that moves the firm down one level on the productivity ladder arrives at rate λ_-, and one that moves firms up the productivity ladder arrives at rate λ_+. At the highest or lowest levels of productivity, firms can only move down and up, respectively. Firms are destroyed and exit at exogenous rate σ. Operating firms use a decreasing returns-to-scale production function $y(n, z)$ which takes labor as an input. Firms can fire workers at no cost, but to hire workers, firms must post vacancies v at cost $c(v, n)$ which contact workers at random according to the meeting technology described below.

Workers. There is a fixed labor force with mass L of workers who are risk-neutral and discount the future at rate ρ. Workers can be either employed or unemployed, with aggregate mass $E = L - U$ and U, respectively. Unemployed workers receive flow benefit $\tilde{b}$, and employed workers receives a wage w which is firm specific and determined according to the bargaining protocol discussed below. In the following, $u = U/L$ and $e = E/L$ denote the unemployment and employment rates, respectively.

Matching. Employed workers search with relative efficiency ζ, consequently the mass of effective searchers is given by $U + \zeta(L - U)$. With probability $\phi = U/(U + \zeta(L - U))$ a vacancy contacts an unemployed worker, and with probability $1 - \phi$ a vacancy contacts an employed worker. Hiring firms post vacancies which contact workers through a constant returns-to-scale meeting function $M(V, U + \zeta(L - U))$. Under the assumption of constant returns-to-scale, the market tightness, defined as the ratio of vacancies to searchers $\theta = V/(U + \zeta(L - U))$, determines the contact rates. Vacancies contact workers at rate $q(\theta) = M/V$. Unemployed workers receive offers at rate $\lambda^u(\theta) = M/(U + \zeta(L - U))$, and employed workers receive offers at rate $\lambda^e(\theta) = \zeta\lambda^u(\theta)$.

Distributions. There is an equilibrium distribution of firms over states $M(x)$. In principle, the state variable x could include the entire distribution of wages within the firm, and across other firms. This would make solving the firm problem highly intractable. A result in the subsequent sections will show the state vector of the firm needs only contain employment and productivity (n, z) as in a standard heterogeneous firm model (Hopenhayn, 1992). As an additional consequence of bargaining, the job value to a worker when meeting a firm $V(n,z)$ is also only a function of these two state variables.

Using this result, define the offer distribution, that is, the distribution of job values over vacancies as

$$F(V) = \frac{1}{\mathbf{v}} \int_{\{(n,z):V(n,z)<V\}} v(n,z) dM(n,z) \tag{1}$$

where $\mathbf{v}$ denotes aggregate vacancies. Similarly, define the distribution of job values over currently employed workers as

$$G(V) = \frac{1}{E} \int_{\{(n,z):V(n,z)<V\}} n dM(n,z) \tag{2}$$

where E is aggregate employment. These distributions are key in defining the hiring and separation rates for firms in the subsequent analysis.

2.2 Hiring and Separations

Hiring and separation rates depend on a firm's state (n,z) through the job value $V(n,z)$ a worker expects to receive when contacted by the firm. The vacancy filling rate is given by

$$h(V) = q(\theta)[\phi + (1-\phi)G(V)] \tag{3}$$

where $q(\theta)$ is the rate at which a vacancy meets a worker, with probability ϕ being the probability the contacted worker is currently unemployed, and $(1-\phi)$ the probability the vacancy contacts a currently employed worker who accepts with probability $G(V)$, that is, they accept the job offer if they are currently employed at a lower job value than they expect to receive at the hiring firm. The separation rate is given by

$$s(V) = \delta + \lambda^{e}(\theta)[1 - F(V)] \tag{4}$$

where δ is an exogenous separation rate, $\lambda^e(\theta)$ is rate at which incumbent workers are contacted by firms, and $F(V)$ is the distribution of job offers which determines the probability that the worker is contacted by a firm with a higher expected job value.

Note, the model without on-the-job search is a special case with $\phi = 1$ and $\lambda^e(\theta) = 0$ where

$$h(V) = q(\theta), \qquad s(V) = \delta \tag{5}$$

that is, the hiring and separation rates are independent of firm characteristics, so that the vacancy filling is constant across firms.

2.3 Values

This section describes the values to employed and unemployed workers, as well as the firm. These values, along with the surplus sharing rule implied by bargaining, are used to define the joint surplus representation in the next section.

Unemployed workers receive flow output $\tilde{b}$ while unemployed. They contact hiring firms at rate $\lambda^u(\theta)$, resulting in the following value function:

$$\rho U = \tilde{b} + \lambda^u(\theta) \int V dF(V) \equiv b$$

The term $V = W - U$ is the surplus to the worker of becoming employed. Using this expression, define b to be the value of unemployment inclusive of the option value of searching, i.e. the opportunity cost of the workers time while employed.

A firm decides how many vacancies v to post or separations d to initiate to maximize the present value of profit subject to the firm level law of motions for hiring. The firm value is given by:

$$\rho\Pi(n,z) = y(n,z) - w(n,z)n - c(v,n)$$
$$+ \Pi_n[h(V)v - s(V)n - d]$$
$$+ \lambda_+(\Pi_+ - \Pi) + \lambda_-(\Pi_- - \Pi) - \sigma\Pi$$

Here $y(n,z)$ is the production function of the firm, $w(n,z)$ denotes the wage paid to a single worker, and $c(v,n)$ denotes the cost of posting v vacancies. $\Pi_n[h(V)v - s(V)n - d]$ is the capital gain to the firm of acquiring an additional worker. The terms $\lambda_+(\Pi_+ - \Pi) + \lambda_-(\Pi_- - \Pi)$ represents the Poisson process of idiosyncratic productivity shocks, and the gain in value from changing productivity states. Note, the wage is a function of employment and productivity.

Finally, the worker's value, denoted $W(n,z)$, is given by:

$$\rho W(n,z) = w(n,z) + \lambda^e(\theta)\int_V^\infty (V' - V)dF(V')$$
$$+ W_n[h(V)v - s(V)n - d]$$
$$- \left[\delta + \frac{d}{n}\right]V$$
$$+ \lambda_+(V_+ - V) + \lambda_-(V_- - V) - \sigma V$$

As we can see, the worker's value depends on its current wage $w(n,z)$, which is a function of the firm's state. Additionally, $\lambda^e(\theta)\int_V^\infty(V'-V)dF(V')$ captures the gain of being poached by a firm with a higher job value. The term $W_n[h(V)v - s(V)n - d]$ captures the effect of the number of workers at the firm on the worker's value. When the firm hires an additional worker, the value to an incumbent worker changes by W_n. Finally, $-\left[\delta + \frac{d}{n}\right]V$ captures the capital loss from moving to nonemployment. Job loss occurs at exogenous job destruction with rate δ or because the firm is firing workers. When a firm with n workers fires d

workers, an individual workers faces separation rate d/n, which follows from the random layoff assumption. Finally, $\lambda_+(V_+ - V) + \lambda_-(V_- - V)$ captures the effect of the idiosyncratic productivity shocks to the firm on the worker's job value, and $-\sigma V$ is the effect of firm exit on the worker's value.

2.4 Bargaining

This section outlines the bargaining process between the firm and its incumbent workers.

Now let's go back to our original model outlined in Section 2.1. Assume that wages are continuously renegotiated by the firm and coalition of workers according to the surplus sharing rule:

$$\eta\Pi(n,z) = (1-\eta)nV(n,z) \tag{6}$$

where $V(n,z) = W(n,z) - U$ is the surplus to a worker from employment at the firm, and η denotes the bargaining power of the coalition of workers. As before, this surplus sharing rule is the result of a Nash Bargaining between the firm and the coalition of incumbent workers

$$\max_w \big(nV\big)^\eta \big(\Pi\big)^{1-\eta} \tag{7}$$

Equivalently, surplus sharing satisfies

$$\begin{aligned} nV(n,z) &= \eta S(n,z) \\ \Pi(n,z) &= (1-\eta)S(n,z) \end{aligned}$$

where $S(n,z) = \Pi(n,z) + nV(n,z)$ is the joint surplus of the firm and incumbent workers. There are two key results from this bargaining solution that allow for a tractable solution which are summarized in the following propositions.

Result 1. *Given the surplus sharing rule* (6)*, the joint surplus* $S(n, z)$ *determines the job value* $V(n, z)$ *as*

$$V(n, z) = \eta \frac{S(n, z)}{n}$$

Proposition 1. *Firm decisions – the vacancy and separation policies* $v(n, z)$*,* $d(n, z)$ *– maximize the joint surplus of the firm and incumbent workers.*

This result allows for a computationally tractable approach. The first result states that knowledge of the joint surplus $S(n, z)$ at the current firm and poaching firm is sufficient to determine the job value, and hence the poaching set the worker chooses. Consequently, the joint surplus function $S(n, z)$ is sufficient to determine the hiring $h(V)$ and separation rate $s(V)$ the firm faces. The proposition states that in order to solve for the optimal allocation of employment across firms, the wage can be ignored, and one can solve the joint surplus directly. This is not to say wages are indeterminate as in directed search. There is an expression for the wage given in the following result.

Result 2. *Given the surplus sharing rule* (6)*, the wage a worker is paid is defined in terms of equilibrium objects as:*

$$w(n, z) = \eta \left(\frac{y(n, z) + c_v(v, n)v - c(v, n)}{n} \right) + (1 - \eta) \left(b - \lambda^e \mu(V) \right)$$

where b *is the value of unemployment,* $c(v, n)$ *is the vacancy cost function evaluated at optimal choices, and* $\mu(V)$ *captures the value of on-the-job search given by*

$$\mu(V) = [1 - F(V)] E[V' | V' > V].$$

The first term is intuitive, workers capture a fraction of average output net of vacancy costs according to the coalition bargaining weight η. The second wage term reflects the tradeoffs that occur if bargaining breaks down. If bargaining breaks down, workers gain their outside option b, but lose the opportunity to search on the job.

2.5 Joint Value Representation

Let $\Omega(n,z) = nW(n,z)+\Pi(n,z)$ denote the joint value of the firm and incumbent workers. It follows that $\Omega_n = W + nW_n + \Pi_n$. Substitution of the previous expressions gives

$$\begin{aligned}\rho\Omega(n,z) =& y(n,z) - c(v;n,z)\\ &+ n\lambda^e(\theta)\int \max\left\{W(n',z') - W(n,z), 0\right\} dF(W')\\ &+ [\Omega_n - W]\left[h(W)v - s(W)n - d\right]\\ &+ [\delta n + d](U - W)\\ &+ \lambda_+(\Omega_+ - S) + \lambda_-(\Omega_- - S) - \sigma\Omega\end{aligned}$$

2.6 Joint Surplus Representation

Let $S(n,z) = \Omega(n,z) - nU$ be the joint surplus of the firm and incumbent workers, and $V(v,z) = W(n,z) - U$ the surplus of an individual worker. It follows that $S_n(n,z) = \Omega_n - U$. Substitution into the joint value representation gives the joint surplus representation

$$\begin{aligned}\rho S(n,z) =& y(n,z) - c(v,n) - nb\\ &+ n\lambda^e(\theta)\int \max\left\{V' - V, 0\right\} dF(V')\\ &- [\delta n + d]V\\ &+ [S_n - V]\left[h(V)v - s(V)n - d\right]\\ &+ \lambda_+(S_+ - S) + \lambda_-(S_- - S) - \sigma S\end{aligned} \tag{8}$$

where the first line is the flow surplus to the coalition, the second is the value to incumbent workers of being contacted by new firms, the third is the loss in surplus by incumbent workers due to separations, the fourth is the values of new hires $S_n - V$ which is the marginal value to the coalition net of the value obtained by the new hire, and the last line is the evolution of the shocks.

The surplus can be simplified further by noting the optimality condition for separations.

$$S_n = 0$$

that is, the firm will fire workers whenever the marginal surplus to the coalition is zero. This implies for each level of productivity, there is a threshold level of employment $\bar{n}(z)$ implicitly define as

$$S_n(\bar{n}(z), z) = 0 \tag{9}$$

such that the firm chooses to fire workers if $n > \bar{n}(z)$. That is,

$$d(n, z) = n - \bar{n}(z) \quad \text{if } n > \bar{n}(z) \tag{10}$$

Using this result the joint surplus maximization problem can be reformulated as

$$\begin{aligned}
\rho S(n, z) = {} & y(n, z) - c(v, n) - nb \\
& + n\lambda^e(\theta) \int \max\{V' - V, 0\}\, dF(V') - \delta n V \\
& + [S_n - V]\,[h(V)v - s(V)n] \\
& + \lambda_+(S_+ - S) + \lambda_-(S_- - S) - \sigma S \\
\text{s.t. } & S_n \geq 0
\end{aligned}$$

From the first order condition for vacancy posting, the optimal choice of vacancies satisfies

$$\frac{c_v(v,n)}{h(V)} = S_n - V. \tag{11}$$

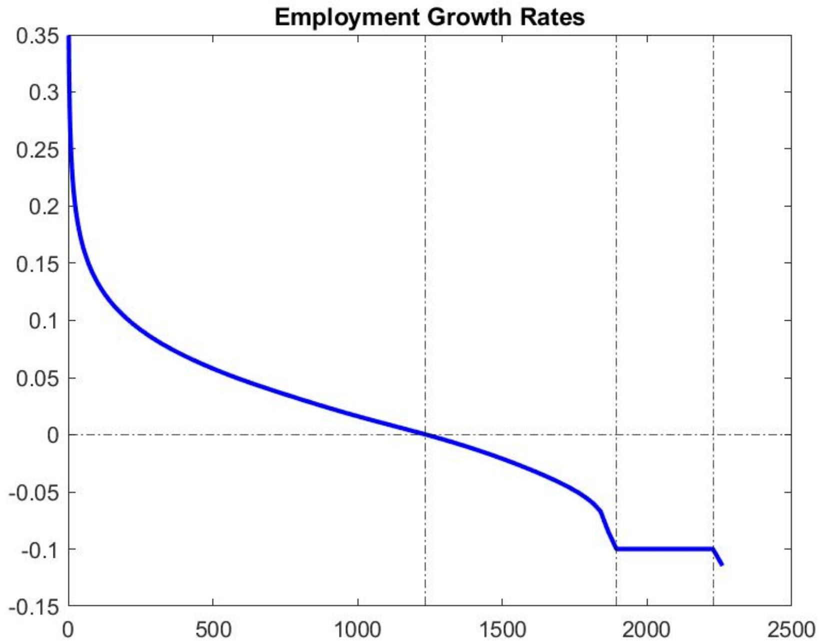
If the marginal surplus S_n to the coalition exceeds the value given to the marginal worker V, the firm finds it optimal to hire by posting vacancies.

There is an inaction region when $0 \leq S_n \leq V$ where the firm finds it is neither optimal to fire workers, nor post vacancies. The existence of these different hiring regions is visualized in Figure 1. As can be seen, the optimal firm size is obtained when employment growth is zero. If the firm is smaller than the optimal size, it will hire by posting vacancies at a rate such that hiring exceeds separations. If the firm is larger than the optimal size, but the marginal surplus exceeds the worker value $S_n > V$, the firm will still post vacancies, but not enough to prevent the firm from shrinking due to exogenous quits and poaching. If the marginal surplus is positive, but less than a worker value $0 \leq S_n \leq V$, it will not post vacancies, and the employment size at the firm shrinks with near constant rate reflecting this firm's location at the bottom of the job ladder. Then, if the number of employees at the firm is large enough such that the marginal surplus is negative $S_n < 0$, it will actively destroy jobs and fire workers, so that employment shrinks faster than the separation rate.

Taking the optimal labor demand choices at the firm level together, gives the firm level law of motion for employment

$$\frac{\partial n}{\partial t} = \begin{cases} h(V)v(n,z) - s(V)n & \text{if } \; n \leq \bar{n}(z) \\ \bar{n}(z) - n & \text{if } \; n > \bar{n}(z) \end{cases} \tag{12}$$

Figure 1: *Example of firm employment growth by level of employment*



Or in term of growth rates,

$$\frac{\dot{n}}{n} = \begin{cases} h(V)\frac{v(n,z)}{n} - s(V) & \text{if } n \leq \bar{n}(z) \\ \frac{\bar{n}(z)}{n} - 1 & \text{if } n > \bar{n}(z) \end{cases} \tag{13}$$

2.7 Equilibrium

First, define the aggregate law of motion for unemployment. There are three ways a worker may lose their job (i) exogenous separation δ (ii) endogenous firing $d(n,z)$, and (iii) firm exits σ. Firms fire workers when they are hit with a negative productivity shock, so the aggregate separation rate is given by

$$\chi = \delta + \sigma + \lambda_{-}\frac{\int(\bar{n}(z) - n)dM(n,z)}{E} \tag{14}$$

The last term captures the flows into unemployment from endogenous separations. This leads to the standard law of motion for unemployment:

$$\dot{u} = \chi(1-u) - \lambda^{u}(\theta)u \tag{15}$$

A stationary equilibrium consists of: (1) a joint surplus function $S(n,z)$, (2) vacancy posting and separation policies $v(n,z)$, $d(n,z)$, (3) a stationary distribution of firms $M(n,z)$, (4) distributions of job values over vacancies $F(v)$ and employment $G(V)$, (5) a law of motion for firm level employment $\dot{n}$, (6) meeting rate $q(\theta)$ for firms' vacancies and probability of meeting an unemployed worker ϕ, (7) and a positive mass of new entrants m, such that the following hold:

i The surplus function $S(n,z)$ solves the Hamilton-Jacobi-Bellman equation (8) described above

ii Worker values are determined through the surplus sharing rule $nV(n,z) = \eta S(n,z)$

ii The vacancy policy satisfies the optimality condition

$$c_v\big(v(n,z),z\big) = h\big(V(n,z)\big)\left[S_n(n,z) - V(n,z)\right]$$

iii Given the vacancy and separation policies the law of motion for firm employment satisfies (12)

iv The distributions of job values over vacancies $F(v)$ and employment $G(v)$ are consistent with (1) and (2)

v The mass of entrant m is such that the zero profit condition holds given the entry cost c

$$c = \sum_{z} S(n_0,z)\pi(z)$$

vi The stationary distribution of firms $M(n, z)$ admits density $m(n, z)$ that satisfies

$$0 = -\frac{\partial}{\partial n}\left(\frac{\partial n}{\partial t} g(n, z)\right) + \lambda_+\big(g(n, z_-) - g(n, z)\big) + \lambda_-\big(g(n, z_+) - g(n, z)\big) + m\pi(z)\mathbf{\Delta}(n_0) \tag{16}$$

vii The meeting rate $q(\theta)$ and probability ϕ are consistent with the aggregate matching function, given a steady state level of unemployment u_{ss}

3 Calibration

Having described the model in the preceding section, this section describes how the model is calibrated to match key moments in the labor market. The model is calibrated to the quarterly frequency. First, I outline the functional form assumptions used in the quantitative model. Next, I discuss which parameters are set externally, and which are set internally to match a choice of targeted moments. The computational details of solving the model can be found in Appendix C.

Functional forms. The production function is Cobb-Douglas $y(n, z) = e^z n^\alpha$. The vacancy cost function is $c(v, n) = \frac{\kappa}{1+\gamma} v \left(\frac{v}{n}\right)^\gamma$ following Kaas and Kircher (2015). The matching function is Cobb-Douglas with vacancy elasticity β. This implies the meeting rate is $q(\theta) = A\theta^{\beta-1}$ and the job-finding rate for unemployed workers is $\lambda^u(\theta) = A\theta^\beta$, where A is matching efficiency. The initial distribution of productivity π is based on a Pareto distribution with minimum value one and shape parameter ξ. The distribution is discretized using 15 equally spaced quantiles, which implies new firms draw uniformly from the set $\mathcal{Z} = \{\underline{z}, \dots, \bar{z}\}$.

Externally Set. The discount rate ρ is set based on an annual interest rate of 5%. The estimate for the vacancy cost elasticity $\gamma = 3.45$ is taken from Bilal et al. (2022) who estimate the parameter using the relationship between the vacancy rate and the vacancy-filling rate using JOLTS microdata. The exogenous exit rate σ is set to 0.021 to match the annual exit rate of firms in 2019 from 8% from the Business Dynamics Statistics (BDS). The production function parameter α is set using an estimated of returns to scale of 0.9 from Burnside (1996). This is the estimated degree of returns for a production function using capital and labor. Interpreting $y(n,z)$ as the value-added of labor from a Cobb-Douglas production function, that is,

$$y(n,z) = \max_k ak^{\alpha_1}n^{\alpha_2} - rk, \quad \alpha_1 + \alpha_2 = 0.9$$

allows for the calibration of α. Straightforward algebra yields

$$y(n,z) = zn^{\frac{\alpha_1}{1-\alpha_2}}.$$

Setting the labor share $\alpha_1 = 0.6$, gives $\alpha = 0.857$. Finally, the vacancy cost parameter κ is set such that $\kappa/(1+\gamma) = 20$. In previous work Kaas and Kircher (2015) pick κ to match the job-filling rate, but as noted in Bilal et al. (2022), the cost function parameter κ and the matching function efficiency A can not be identified independently from the first order condition for vacancy creation, and they treat κ as a normalization as well. As a check, I verify that the choice of κ leads to an average vacancy cost per hire that is equal to 84% of monthly wages in the calibrated model which is close to the value of 92.8% found by Gavazza et al. (2018) using recruiting cost data. Finally, the option value of time spent unemployed is $b = 0.65$ which is within the range of values used in the literature. With a decreasing returns-to-scale production technology, this is effectively a normalization as larger or smaller values of b will lead to different optimal firm sizes, after which the productivity distribution can be scaled up or down accordingly to recover the original firm size distribution. The calibration of the externally-set parameters is summarized in Table 1.

Table 1: Externally set

	Parameter	Value	Moment
ρ	Discount rate	0.0146	5% Annual real interest rate
α	Production	0.857	Burnside (1996)
σ	Exit Rate	0.021	8% Annual exit rate
γ	Vacancy cost elasticity	3.45	Bilal et al. (2022)
$\kappa/(1+\gamma)$	Vacancy cost scale	20	Normalization
b	Value of unemployment	0.65	Literature

Model parameters set externally. Discussion of sources for the moments contained in the text.

Internally Estimated. The remaining parameters $\{\lambda_+, \lambda_-, \xi, \eta, \zeta, \delta, A, \beta, \tilde{b}\}$ are calibrated internally. Although, each parameter is affected by each targeted moment, I will provide an intuitive discussion on which specific moments are informative for each specific parameter. The parameters governing the initial draw of productivity and its subsequent evolution $\{\lambda_+, \lambda_-, \xi\}$ are set to target the distribution of employment by firm size in 2019 from the Business Dynamics Statistics (BDS) data set of the U.S. Census Bureau. The Pareto tail ξ informs how much productivity dispersion there is in the model, which determines the optimal firm size. The values of $\lambda_- = 0.104$ and $\lambda_+ = 0.026$ also affect the distribution of employment by firm size. A lower probability of moving up the ladder than down allows the model to match the employment share of small and medium size firms.

The bargaining weight η is set to target an average wage increase of 8% among job-switchers found in Barlevy (2008). The search intensity of employed workers is set to match the quarterly job-to-job transition rate of 0.048 from Bilal et al. (2022). The exogenous separation rate δ is set to 0.009 to match the average quarterly separation rate into nonemployment of 0.069 from the Census Job-to-Job flows data from 2000-2019. The contact rate $q(\theta)$ depends on matching function parameters, and is set to 0.935 targeting a vacancy filling rate of 0.794. This vacancy-filling rate is derived from the daily job-filling rate of 5.2% in Davis et al. (2013). To convert the daily vacancy-filling rate to a quarterly

Table 2: *Internally Calibrated*

	Parameter	Value	Moment	Model	Data
Productivity					
λ_+	Positive shock	0.026	Emp. share 1-20	0.146	0.168
λ_-	Negative shock	0.104	Emp. share 100+	0.693	0.669
ξ	Pareto tail	4.42	Emp. share 500+	0.533	0.529
			Emp. share 1000+	0.409	0.475
Bargaining and Search					
η	Bargaining weight	0.036	Avg. EE wage gain	0.082	0.080
ζ	Employed search intensity	0.121	EE transition	0.049	0.048
δ	Exogenous separations	0.009	EN transition	0.068	0.069
$q(\theta)$	Contact rate	0.935	Vacancy-filling rate	0.789	0.794
Residual					
A	Matching Efficiency	0.728			
β	Matching Function Elasticity	0.612			
$\tilde{b}$	Flow value of unemployment	0.479			

Distribution of employment is from the Census Business Dynamics Statistics (BDS) in 2019. Wage gain among job-switchers is from Barlevy (2008). Quarterly job-switching rate is from Bilal et al. (2022). Separation rate is from the Census Business Dynamics Statistics (BDS) 2000-2019. Vacancy-filling rate is derived from results in Davis et al. (2013). Further discussion the sources of the targeted moments and their construction is contained in the text.

value, I assume vacancies are posted uniformly across days within a quarter, so that earlier vacancies are filled with a higher probability, and calculate the implied share of vacancies that are filled by the end of a given quarter.

The steady state unemployment rate is set to $u_{ss} = 0.1$ to solve for the matching function parameters. Given the targeted separation rate and the unemployment rate, the law of motion for unemployment implies $\lambda^u = 0.621$. This value, along with the contact rate, is used to solve for the values of matching efficiency, resulting in $A = 0.728$, and the elasticity of the matching function with respect to vacancies $\beta = 0.612$. Finally, the flow value of unemployment $\tilde{b} = 0.479$ is solved as a residual from the value of an unemployed worker given our previous normalization $b = 0.65$. The internally calibrated parameters are summarized in Table 2.

4 Cross Section Results

This section uses the calibration presented in the previous section to illustrate the performance of the multiworker firm model with on-the-job search in terms of a large set of untargeted moments on firm dynamics and other labor market statistics.

4.1 Firm Dynamics

Table 3: *Cross-sectional growth rate distribution*

Growth rate interval	Data JOLTS	Data BED	Model
< -0.2	4.3	7.6	8.7
$(-0.2, -0.05]$	13.2	16.7	12.7
$(-0.05, -0.02]$	9.5	9.7	2.8
$(-0.02, 0]$	11.6	7.8	8.4
0.0	17.1	15.7	9.5
$[0, 0.02)$	13.1	8.0	8.2
[0.02.0.05)	11.7	10.0	12.1
[0.05.0.20)	15.1	16.9	26.1
> 0.2	4.5	7.6	11.4

Notes: Distribution of quarterly employment growth rates by employment shares. Last column is model generated data. Data from JOLTS and BED are reproduced from Table 5.2 in Davis, Faberman, Haltiwanger, and Rucker (2012) who use micro data from the Job Openings and Labor Turnover Survey (JOLTS) and Business Employment Dynamics (BED) from 2001 to 2006.

Table 3 reproduces the cross-sectional distribution of employment shares over employment growth rates. Davis, Faberman, Haltiwanger, and Rucker (2012) use microdata from the Job Openings and Labor Turnover Survey (JOLTS) and Business Employment Dynamics (BED) from 2001 to 2006 to measure quarterly employment growth rates. As Davis, Faberman, Haltiwanger, and Rucker (2012) note in their study, the JOLTS data and BED differ significantly in that the JOLTS data tend to represent more stable establishments. The share with employment growth $(-0.02, 0.02)$ in JOLTS is 41.8% while in BED it is only 31.5%. The last column is the model implied distribution of employment shares by employment growth. To make the model comparable the data, we consider a firm to have no employment growth if the absolute value of employment growth is less than 0.5%. Notice this does not change the share with employment growth $(-0.02, 0.02)$, which is 26.1%. Overall, the model does well in matching the distribution, with one notable exception. The share with growth $[0.05, 20)$ in the model is 26.1% versus 15.1% and 16.9% in the data. The model does capture that a large share of workers are employed at firms that are growing or shrinking significantly within a quarter. If the role of search frictions was trivial in the model, then the only firms that would adjust would be those whose productivity received a shock, roughly 12% of firms in the model, and then move to their new optimal firm size. This would imply a large share of firms with stable employment. The addition of search frictions and convex recruiting technology accounts for the additional dispersion in employment growth rates in the model.

The work of Davis, Faberman, and Haltiwanger (2012) and Davis et al. (2013) elucidate how job flows and worker flows evolve at the firm level in the data.[2] First, employment growth and the gross hiring rate are correlated and follow a "hockey stick" shape. Firms that are shrinking hire fewer workers, while firms that are growing are doing so by increasing gross hiring. Panel A of Figure 2 shows this gross hiring mechanism is present in the model, and is quantitatively similar to the rate of gross hiring in the data. Similarly, layoffs are correlated with shrinking firms and also follow a "hockey stick" shape. Firms that shrink do so partly by laying off workers. Panel B of Figure 2 shows the relation between firms shrinking and layoffs is present in the model, and the lines almost overlap. While the "hockey stick" shape of gross hiring and layoff are present in canonical labor search models without on-the-job search, as in, for example, Elsby and Michaels (2013), the ability to relate these facts to the behavior of vacancies is unique to models with on-the-job-search.

[2]The data Davis, Faberman, and Haltiwanger (2012) and Davis et al. (2013) use are for establishments. The model was calibrated in terms of the firm size distribution, but I will adopt the convention of referring to establishment as firms when comparing the model results to avoid excessive switching between terms.

Figure 2: *Worker flows and job flows as a function of employment growth*

Notes: Dashed lines are data, solid lines are their model generated counterparts. Data on the hiring rate, layoff rate, and quit rate in Panel A and Panel B are from Davis, Faberman, and Haltiwanger (2012) who use Business Employment Dynamics (BED) microdata. Data on vacancy rates and vancancy yields in Panel C and Panel D are from Davis et al. (2013) who use Job Openings and Labor Turnover Survey (JOLTS) microdata.

Panel C of Figure 2 shows how in the data, as well as in the model, the quit rate is declining in firm growth. In the model, as in the data, the quit rate includes both separations to nonemployment, and job-switching. The decline in the quit rate is accounted for by the latter, as firms that are shrinking are lower in the job value distribution and see a larger share of their workers quitting to other firms, while firms that are growing offer workers a higher job values and lose fewer worker to job-switching. Notice, the quitting rate is higher the Davis, Faberman, and Haltiwanger (2012) data than in the model. This is partly due to time aggregation. The model is calibrated to match an average quarterly employment-to-nonemployment (EN) transition rate of 0.069, but the data Davis, Faberman, and Haltiwanger (2012) use have a monthly frequency. Given the average duration of unemployment in the US is shorter than one quarter, many job losers find new employment within a quarter and are not counted as EN by the Census. However, the qualitative relationship of quits declining with employment growth is present in the model.

Panels E and F of Figure 2 show the relationship between vacancy yield and employment growth and vacancy rate and employment growth. In the data, vacancies are measured as the stock of outstanding job available at the end of a month. To make the model comparable with the data, we need to adjust our measure of vacancies. In the model, the vacancy policy $v(n, z)$ includes all vacancies posted in a quarter. We assume vacancies are posted continuously throughout the period, so the outstanding stock of vacancies in the last month of the quarter are the remaining vacancies that went unfilled. We use this as the model implied measure of the stock of vacancies to compare with the data. The relationship between the vacancy rate and vacancy filling rate highlights the importance of on-the-job search. In the absence of poaching the vacancy yield would be flat, and equal to the rate at which workers are hired from nonemployment.

As Panel E in Figure 2 shows, the vacancy yield rises with net employment growth. The model mechanism that delivers this result is on-the-job search. Firms that are higher in the job value distribution are small relative to their optimal size and receive a higher yield on their vacancies by poaching from other firms. This employment growth cannot be accounted for by higher vacancy posting alone. As Panel F in Figure 2 shows the difference in the vacancy rate at firms with little employment growth and those with high employment growth is 4% in the data. In the absence of on-the-job search the vacancy rate maps one-to-one with the hiring rate. The model is able to reproduce both a higher vacancy yield and a higher vacancy rate for fast growing firms, which leads to gross hiring

Table 4: Non-targeted moments

Description	Model	Data
Std. log wage	0.12	-
Mean-Min Wage Ratio	1.07	-
90-10 Wage Ratio	1.10	-
95-05 Wage Ratio	1.18	-
Std. Marginal Product	0.21	-
Pass-trough (Avg. Product to Wage)	0.33	[0.2,0.4]
Elasticity (Wage to Avg. Product)	0.41	[0.4,0.7]
Elasticity (Separation rate to Wage)	-1.02	[-0.25,-3]
Elasticity (Quit rate to Wage)	-2.97	-
Elasticity (Vacancy duration to Wage)	-0.087	[-0.04,-0.08]

rates close to those found in the data. Overall, the success of the model in explaining firms dynamics highlights the importance of poaching to understand firm growth.

4.2 Wages and Turnover

Now we turn to the model's ability to replicate labor market facts that relate wages to productivity and worker flows. Because the model is able to deliver a prediction for wages, these moments provide a test of the implications of the multiworker firm model with ex-post bargaining against the data.

The first panel in Table 4 reports the residual wage dispersion generated by the model. The standard deviation of log wages across workers is 12%, the 90-10 ratio is 10%, and the 95-05 ratio is 18%. This is generated by a long right tail of wages that are paid by highly productive firms. Hornstein, Krusell, and Violante (2011) suggest the use of the mean-min ratio of wages to show how structural labor search models tend to deliver too little wage dispersion. For a model with a job-ladder, Hornstein et al. (2011) find that the mean-min ratio depends on the ratio of the average wage to the value of non-market time, which is $\bar{w}/\tilde{b} = 0.71$ in the model. Based on this value, the low mean-min ratio in the model is consistent with what Hornstein et al. (2011) find in models without decreasing returns-to- scale production.

The second panel in Table 4 reports estimates of the pass-through and elasticity of wages to a unit increase in value added. The data counterparts in the table are those reported in Elsby and Gottfries (2022) based on the estimates from Kline, Petkova, Williams, and Zidar (2019). The pass-through measures the direct effect of an additional unit of value added, while the elasticity scales up this pass-through by the ratio of value added to wages. The model implied estimates of both the pass-through and the elasticity are within the ranges reported in the literature. This is striking because the bargaining power of workers is only 0.036 which in a static model would imply little rent-sharing between the firm and workers. However, in this model the wage equation depends on equilibrium values that capture additional competitive forces through search frictions and recruiting effort that increase these measures.

The elasticity of the separation rate with respect to wages and the elasticity of the voluntary quitting rate with respect to wages is in line with the literature on imperfect labor competition as documented by Manning (2011). High wage firms face lower quit rates through job-switching. The estimated elasticity of quitting with respect to wages is -1.02, well within the estimates reported in Manning (2011). The model allows the decomposition of separations into voluntary quits and exogenous separations. The elasticity of quits to wages is larger, namely -2.97, which drives the overall separation elasticity. Finally, the elasticity of vacancy duration with respect to wages is -0.08, consistent with the findings of Mueller et al. (2018) using Austrian data, who report an elasticity of -0.04 for the starting wage and -0.10 to the establishment wage. The authors take their estimates as evidence against models of wage posting such as Kaas and Kircher (2015), and suggest that this is consistent with model of recruiting effort such as Gavazza et al. (2018). The model is able to reproduce this fact, with neither wage posting nor a recruiting effort margin, because spot wages are only an imperfect proxy for job values, and the relationship between job values and vacancy duration is non-linear.

Given that none of these untargeted moments on wages and turnover are explicit functions of model primitives, I see this as evidence in favor of the wage bargaining solution used in the model. That is, not only does it result in a tractable representation for the firm problem, but additionally the implications for wages that arise from the bargaining solution accord well with the data.

4.2.1 Job Ladder

The model implied job-ladder is in terms of job values which involve both the present value of wages and the opportunities to search for better jobs. In this section, I explore how the job ladder in the model relates to empirical measures of the job ladder in Haltiwanger et al. (2018) who use matched employer-employee data from the Longitudinal Employer Household Dynamics (LEHD) database 1998-2011 to measure poaching rates by firm size and firm wage. Their key finding is that there is a stronger job ladder in firm wage than in firm size. This is at odds with existing theories of on-the-job search that use constant returns to scale, as in , for example, Burdett and Mortensen (1998), because in these models firms that are more productive also pay higher wages, and are large because they face low worker turnover through poaching. Instead, Haltiwanger et al. (2018) find almost no evidence for a firm size ladder, but more evidence for a firm wage ladder.

Haltiwanger et al. (2018) use the following accounting identity for labor flows LF at the firm level:

$$LF = H^p + H^n - S^p - S^n$$

Flows are determined by hires through poaching H^p, hires from nonemployment H^n, poaching separations S^p, and separations into nonemployment S^n. They then define the net poaching rate NP as the fraction of current employment gained or lost through poaching:

$$NP = \frac{H^p - S^p}{n}$$

The model equivalent to the net poaching rate is

$$NP = \frac{v}{n} q(\theta)(1-\phi)G(V) - \lambda^e(1 - F(V))$$

where the first term is the arrival rate of new workers from other firms, and the second term is the separation rate of workers quitting to a new firm.

Table 5: *Job ladder firm size and by wage*

Description	Model	Data
Low-wage firms	-6.1	-1.2
High-wage firms	4.0	0.7
Emp. 0-50	-0.52	0.23
Emp. 500+	-0.51	-0.16

Notes: Entries are quarterly net poaching rates in percentage terms. Data on net poaching come from Haltiwanger et al. (2018). Low wage firms are in the bottom quintile of the wage distribution and high wage firms are in the top two quintiles.

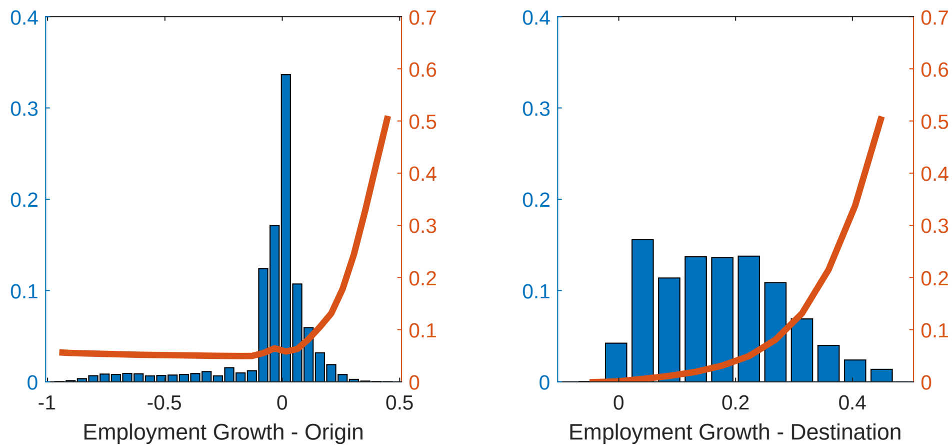
Table 5 reports the model results against those reported in Haltiwanger et al. (2018). First, I compute the average net poaching rate among large (> 500 workers) and small (< 50 workers). In the model, as in the data, the net poaching rates by firm size are small. Large firms lose 0.52% of employment through net poaching in the model and large firms lose 0.16% in the data. Similarly small firms lose 0.52% of employment in the model, almost identical to that of larger firms, while in the data small firms gain 0.23% of employment through poaching. The model has much stronger implications for a wage ladder. Low wage firms, measured as being in the bottom quintile of wages among firms, gain 4% of employment through poaching in the model, and in the data, high wage firms gain 0.7% of employment through poaching. Low wage firms, on the other hand, lose 6.1% of their workers to poaching, while in the data, low wage firms lose 1.2% of their workers to poaching each quarter. While the existence of a wage ladder in the model is to be expected, since the job value is related to the present value of wages, the lack of job ladder in firm size is by no means automatic. In the model there are two types of small firms; firms which have just entered and pay a high wage to attract workers, and mature firms that are moving down the productivity ladder after a sequence of bad shocks. Similarly, some large firms are highly productive and are growing, while others are not, because either they have reached their optimal size, or have received a negative productivity shock and are shrinking. What this table shows is that, without this being an explicit target, the two effects offset to create a lack of a job ladder in firm size.

4.2.2 Earnings Growth and Firm Growth

This section explores the relationship between firm growth and wage growth among job-switchers. Recent evidence from matched data in Tanaka, Warren, and Wiczer (2022) shows that employment growth at the origin and destination firm of job-switchers is predictive of larger wage gains from switching jobs.

Figure (3) is the model output of growth and job switching. The left panel reports the distribution of job-switchers by the employment growth at the origin firm. Most job switchers come from firms that are shrinking or have stable employment (employment growth near zero). Firms that are firing workers, and firms with stable employment have lower average productivity. The line refers to the average wage gain job job-switchers in each employment growth bin. Workers at stable or shrinking firms are at low rungs on the job ladder, and will accept a larger fraction of job offers. At firms with positive employment growth, workers are further up the ladder, and accept fewer offers. The upward slope in the line is a result of workers at higher growth firms accepting job with larger wage gains. This is not immediate from the model, as these workers also earn a higher spot wage and, while they may accept fewer offers, the wage gains could be smaller for these workers if the set of poaching firms is offering wages near the current wage. The left panel shows the same statistics by the growth rate of employment at the destination firm that the worker moves to. The wage is increasing in the employment growth rate of the poaching firm. Taken together this is evidence that employment growth at the origin and destination firm matter for the wage gains of job-switchers.

Figure 3: *Distribution of Earnings Growth of Job Switchers*



Notes: Left-panel plots distribution of job changers by origin firm employment growth rate and average wage gain from switching. Right-panel plots distribution of job-switchers by destination firm employment growth rate and average wage gain from switching. The majority of job switchers come from origin firms with stable or shrinking employment, while poaching firms are growing. Wage gain from switching are increasing in origin firm and destination firm employment growth rates.

To test the strength of the relationship between wage growth and employment growth at the origin and destination firm, I run the following regression from Tanaka et al. (2022) using the model:

$$\Delta w = \beta_0 + \beta_1 \Delta L_o + \beta_2 \Delta L_d.$$

here Δw is the wage gain of a job-switcher, ΔL_o is employment growth at the origin firm , and ΔL_d is employment growth at the destination firm. In the model, I consider the immediate wage gain upon switching jobs, while Tanaka et al. (2022) consider the difference in earnings in the year prior to a switch and after switching. The results of the regression are summarized in Table (6). A range of estimates for different controls in the data is given in the last column.

Table 6: *Job-to-Job Flows and Wage Growth by Firm Growth*

	Model	Data
ΔL_o	0.004	[0.026,0.065]
ΔL_d	0.044	[0.066,0.091]

Notes: Regression of wage growth on firm growth rates. Data is from Tanaka et al. (2022).

In the data, both employment growth at the origin and destination firm are predictive of wage growth, while in the model, employment growth at the origin firm has almost no effect, and employment growth at the destination firm is predictive of larger wage gains. The lack of origin firm effect largely comes from the model's prediction that few workers are poached from growing firms. The qualitative prediction of the model is similar to the data, employment growth at the origin and destination firm matters for expected earnings growth, but the growth of the destination firm is a more important predictor.

The model's prediction for worker reallocation from low productivity to high productivity firms result in switching patterns that are more stark than in the data. Table (7) shows the distribution of job-switchers and wage gains in the data and in the model. Tanaka et al. (2022) find many more than half of job-switchers are moving to growing firms, while the model predicts growing firms account for almost all job-switchers. In the data, job-switchers across all firms see an increase in the wage, but the increase is larger for workers moving to growing firms and coming from growing firms. In the model, the only wage gains come from moving to a growing firm, and shrinking firms do not poach any workers. To be sure, there are several mechanisms that can explain the difference in the model and the data. The first is due to time aggregation. Firms which are growing in the quarter of the job change may not increase net employment over a two year horizon. Second, the model is one of homogeneous workers and decreasing returns, so employment growth is highly correlated with labor productivity and wages, which makes it sub-optimal to switch to a shrinking firm. Finally, in the data, not all job-switches are driven by wage gains, workers switch firms a number of idiosyncratic reasons outside the model. Overall, these results do show that the reallocation mechanism in the model can account for part of the patterns observed in the data, specifically, it can explain why the bulk of job switchers are moving to growing firms. For these workers, the model provides evidence that the wage gains they receive may be driven by the frictional component of wage dispersion.

Table 7: *Job-to-Job Flows and Wage Growth by Firm Growth*

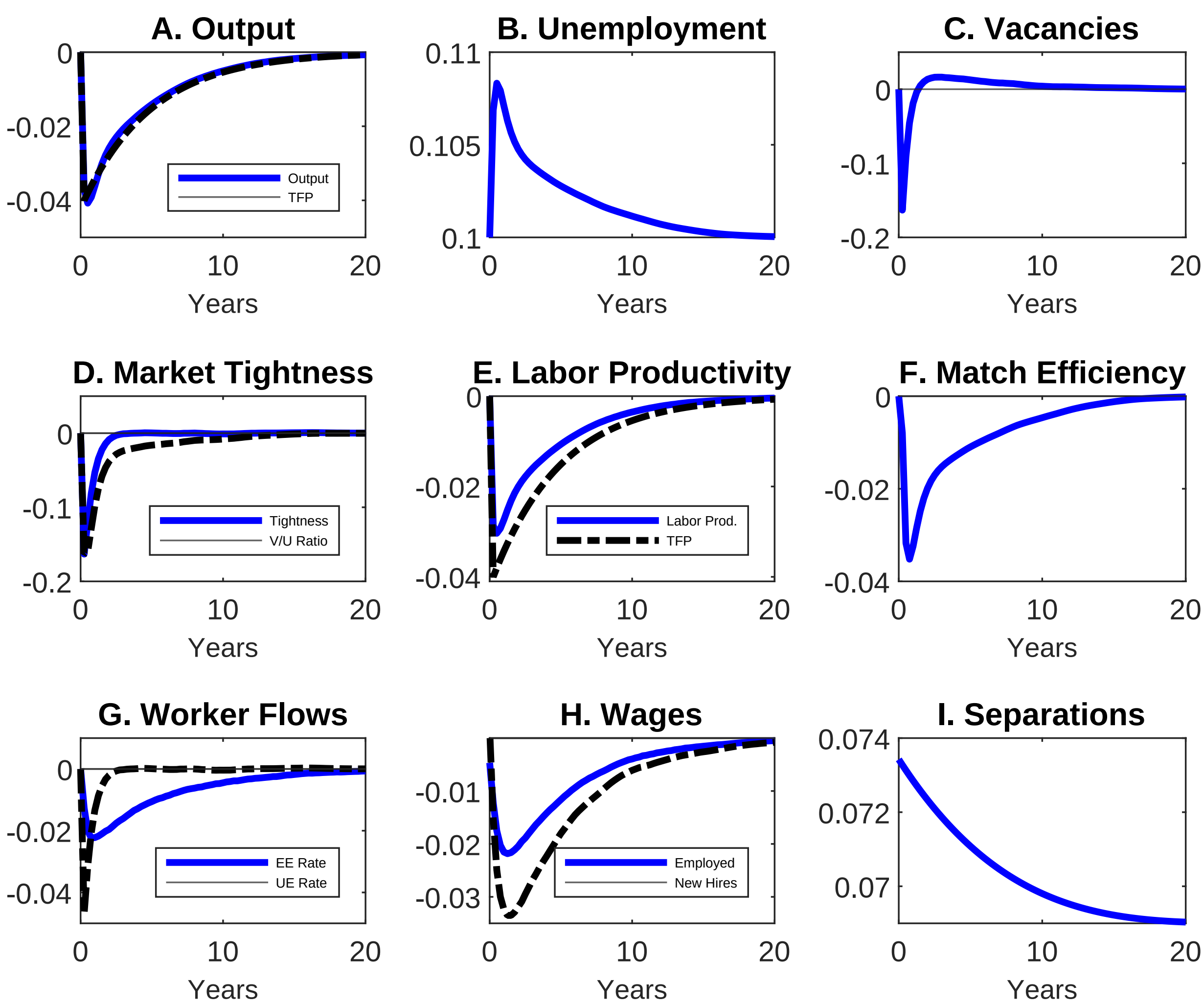
	Data					
	Fraction			Wage Growth		
Origin \Destination	Growing	Stable	Shrinking	Growing	Stable	Shrinking
Growing	0.238	0.084	0.134	0.132	0.119	0.089
Stable	0.090	0.040	0.052	0.122	0.110	0.082
Shrinking	0.175	0.066	0.120	0.107	0.094	0.061
	Model					
	Fraction			Wage Growth		
Origin \Destination	Growing	Stable	Shrinking	Growing	Stable	Shrinking
Growing	0.211	0.018	0	0.117	-0.011	·
Stable	0.409	0.098	0	0.099	0.001	·
Shrinking	0.182	0.081	0	0.101	0.012	·

Notes: Distribution of job-switchers by origin firm employment growth and destination firm employment growth. Wage growth is the average increase in wage. Data is from Tanaka et al. (2022).

5 Business Cycles

In this section I explore the model's implications for business cycles. To do so, I subject the model to a set an MIT shocks and solve for the perfect foresight transition (Achdou, Han, Lasry, Lions, and Moll, 2022). This amounts to solving the model backwards in time from the initial steady state via value function iteration followed by solving the model forward to recover the equilibrium distributions of job values, and repeating these steps until the sequence of aggregate distribution $G(V_t, t)$ and $F(V_t, t)$ converge. Additional details on computation are given in Appendix C. I consider the response of the economy to an aggregate productivity shock that effects each firm $y(n, z, a) = e^a e^z n^\alpha$, where a is the aggregate component of productivity. The shock lowers TFP by 4% upon impact and reverts to 0 with an auto-correlation of 0.95.

Figure 4: *Response of Aggregates to Business Cycle Shock*



Notes: Response of the economy to a negative 4% aggregate productivity shock with autocorrelation 0.95.

Figure 4 summarizes the response of aggregates to the shocks. Panel A of Figure 4 shows that output falls by 4.2% following the shocks and recovers roughly at the same rate as productivity. Panel B of Figure 4 shows the unemployment, or nonemployment rate, rises by 0.075, or a 7.5% increase from the steady state value. Panel C of Figure 4 shows that aggregate vacancies fell by 15% relative to the steady state, but recover quickly. Taken together Panel D shows the decline vacancies and rise in unemployment combine to a decline in effective market tightness of 15%. The effect on the vacancy unemployment ratio V/U, which is the relevant object in canonical models without on-the-job search, falls slightly more with a trough of close to 16%, and recovers more slowly than market tightness.

Panel E shows that labor productivity, measured as output per worker, does not decline as much as TFP, and recovers much more quickly. This is due to the decreasing returns to scale technology, as firms shrink, individual workers become more productive. Panel I shows the increase in the aggregate separation rate, which rises by 0.04 and recovers only slowly. The ability of the model to generate a persistently high separation rate drives the increase and slow recovery in unemployment, as we can see from panel C, labor demand recovers quickly. Panel G shows the effect of the job-finding rate of unemployed workers (UE) and the job-switching rate (EE). The job-finding rate falls by slightly more than 4% reflecting the drop in vacancies, but recovers more slowly than vacancies, reflecting the persistently high unemployment. Panel G also shows the EE rate falls by less than the UE rate, and recovers more slowly, reflecting the process of reallocation across firms following a recession. Taken together, the model's qualitative predictions for the cyclical behavior of labor market variables is consistent with the data, but the model generates less movement in aggregates than we see in the data.

The model also has a prediction for the cyclical behavior of wages. Panel H shows the deviation in the average wage of currently employed workers, and the deviation in the average wage of new hires. Both wage measures are pro-cyclical, but the new hires exhibit larger drop in average wage. The size of the decline in wages in smaller than the drop in labor productivity. The wages of new hires are driven both by hires from unemployment and job-switchers. During a recession, firms higher up the job ladder decrease hiring, and EE rates fall, leading to lower average wages for new hires than in normal times.

The model can also shed light on why measured matching efficiency falls during recessions, or equivalently, why the Beveridge curve may shift out. Consider the standard Cobb-Douglas matching function with only vacancies and unemployment as inputs $\mu_t V_t^{\beta} U_t^{1-\beta}$. I take the value of β to be 0.66 based on the estimates of Barnichon and Figura (2015). Panel F plots the implied decline in matching efficiency if one were to use the standard matching function. It appears that matching efficiency falls by 3.5% . Because the vacancy unemployment ratio falls by more than the effective market tightness, it appears that matching efficiency has declined. This highlights the importance of considering on-the-job search when thinking about match efficiency over the business cycles.

6 Conclusion

I consider a framework where multiworker firms enter and exit the market and hire and fire workers that transition between employment and nonemployment, as well as between jobs through random on-the-job search. The firms produce using a diminishing returns-to-scale technology. This is a challenging framework due to its dependence on job values, which depend on current and future wages of all firms, while at the same time the distribution of job values matters for firms' labor demand decisions because they determine hiring and quitting rates.

I show that tractability can be obtained by assuming that each period the firm and its incumbent workers negotiate over the firm wage through Nash bargaining between the firm and the coalition of incumbent workers. This assumption results in an equivalence result between the firm deciding on its optimal labor decision and maximizing the joint surplus of the firm and its incumbent workers. This offers computational tractability of the equilibrium problem. In addition, I am able to solve for the wage in terms of the other equilibrium objects in the model. This innovates on the existing literature, where Bilal et al. (2022) solve a multiworker firm with on-the-job search using the bargaining protocol of Postel-Vinay and Robin (2002). The Postel-Vinay and Robin (2002) protocol in this framework, however, has the disadvantage that wages become indeterminate, and the Bilal et al. (2022) modeling framework therefore remains silent about how wages behave in the multiworker firm model with a job ladder.

I use a stylized calibrated model to show how the multiworker model with on-the-job search can replicate many untargeted empirical facts, including statistics on firm growth, measured elasticities of separation rates, quitting rates and vacancy duration with respect to wages and measured net poaching rates by firm size and firm wage. The model can also speak to the role of firm growth on wage growth of job-switchers. I also explore the model's implications for business cycles. The model is able to replicate the qualitative cyclical patterns of job flows and wages in response to a productivity shock. The magnitudes remain smaller than those in the data, but larger than the textbook matching model.

The framework is flexible and allows for many potential extensions as long as these extensions retain the tractability of the joint surplus representation and keep the state space manageable for computation. Heterogeneity in worker productivity, or capital with an adjustment cost, can be accommodated in the model. Extensions like this can make the model more realistic and would allow one to study the effect of labor market policies on firm and labor market dynamics.

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Appendices

A Derivations

Wage Derivation

Consider the joint value of the firm and its incumbent workers after separations, but before hiring has taken place, and let s_z denote the Ito terms (for brevity)

$$(\rho + \sigma)S = y - c - nb + n\lambda^e I(V) - n\delta V + [S_n - V][hv - sn] + s_z \tag{A.1}$$

where $\mu(V) = \int_V (V' - V)dF(V')$ to economize on notation. Similarly the job-value to a worker is given by

$$(\rho + \sigma)V = w - b + \lambda^e I(V) - \delta V + V_n[hv - sv] + v_z \tag{A.2}$$

where v_z captures the evolution of productivity, and $I(V) = \int_V (V' - V)dF(v')$. Using the surplus sharing rule $\eta S = nV \Rightarrow \eta S_n = nV_n + V$, yields the following

$$n(w - b) - \eta(y - c - nb) + (1 - \eta)n\lambda^e I(V) - (1 - \eta)\delta nV - (1 - \eta)V[hv - sn] = 0 \tag{A.3}$$

Moreover, the option value of on-the-job search can be written as

$$I(V) = \int_V (V' - V)dF(V') = [1 - F(V)][E(V'|V' > V) - V] \tag{A.4}$$

where $E[V'|V' > V]$ is the expected value of a new job, conditional on exceeding the value of the current job. Noting $s = \delta + \lambda^e[1 - F(V)]$, and let $\mu(V) = [1 - F]E(V'|V' > V)$ we can further simplify the expression to

$$n(w-b)-\eta(y-c-nb)+(1-\eta)n\lambda^{e}\mu(V)-(1-\eta)Vhv=0. \tag{A.5}$$

which can be rearranged for the wage as

$$w(n,z)=\eta\left(\frac{y-c}{n}\right)+(1-\eta)\left(b+\frac{1}{n}Vh(V)v-\lambda^{e}\mu(V)\right) \tag{A.6}$$

Clearly, if the firm is not posting vacancies, the wage reduces to

$$w=\eta\frac{y}{n}+(1-\eta)(b-\lambda^{e}\mu(V)) \tag{A.7}$$

The wage is a fraction η of the marginal product, and a fraction $1-\eta$ of the value of time spent unemployed net of the option value of searching on the job which is lost if bargaining breaks down. This expression, has not made use of any optimizing behavior of the hiring firm, doing so allows for further simplification. Noting the optimal hiring decision satisfies

$$\begin{aligned} S_n - V &= c_v/h \\ \Pi_n &= c_v/h \end{aligned} \tag{A.8}$$

and using the surplus sharing rule $(1-\eta)S=\Pi \Rightarrow (1-\eta)S_n=\Pi_n$, along with the definition of marginal surplus $S_n = nV_n + V + \Pi_n$, we can see that the job value at a hiring firm is

$$V=\frac{\eta}{1-\eta}\frac{c_v}{h} \tag{A.9}$$

Substituting this into the previous expression yields

$$w(n,z)=\eta\left(\frac{y+vc_v-c}{n}\right)+(1-\eta)\Big(b-\lambda^{e}\mu(V)\Big) \tag{A.10}$$

With the functional form $c(v, c) = \kappa/(1+\gamma)v(v/n)^{\gamma}$ this expression becomes

$$w(n,z) = \eta\left(\frac{y+\gamma c}{n}\right) + (1-\eta)\Big(b - \lambda^{e}\mu(V)\Big) \tag{A.11}$$

B Relation to the Literature

Here I discuss the coalition bargaining solution in the context of two closely related papers in the literature Elsby and Michaels (2013) and Elsby and Gottfries (2022). I will show that these environments are particular cases of the model presented in the main text, and the coalition bargaining solution gives tractability in each case.

B.1 Relation to Elsby and Michaels (2013)

Here, I present a continuous time version of the environment presented in Elsby and Michaels (2013) with the coalition bargaining solution. This is a special case of the model presented in the main text with three simplifications: (i) workers do not search on-the-job, (ii) the hiring cost is linear, and (iii) there is no exogenous job-destruction.

The problem of a firm is given by:

$$\rho\Pi(n,z)dt = \max_{dX\geq 0}\left\{\left[y(n,z) - wn + \mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz}\right]dt - (c - \Pi_n)dH - \Pi_n dX\right\} \tag{B.1}$$

where the idiosyncratic productivity process follows an arbitrary diffusion (the results extend to jump-diffusion as well), $dH = q(\theta)vdt$ is the measure of hires, and $dX = xdt$ is the measure of separations. The optimal firing decision satisfies,

$$\Pi_n dX^{*} = 0 \tag{B.2}$$

either the firm fires workers until $\Pi_n = 0$, or the solution is inaction $dX^* = 0$. This condition implies the value of the firm, after separations, is given by:

$$\begin{aligned}\rho\Pi(n,z)dt &= \left[y(n,z) - wn + \mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz}\right]dt - (c - \Pi_n)dH - \Pi_n dX^* \\ &= \left[y(n,z) - wn + \mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz}\right]dt - (c - \Pi_n)dH\end{aligned} \tag{B.3}$$

Now consider the value to unemployed and employed workers. Unemployed workers receive flow benefit b and search for a job

$$\begin{aligned}\rho U &= b + \lambda \int (W - U)dF(W) \\ &= b + \lambda \int \tilde{V} dF(\tilde{V})\end{aligned} \tag{B.4}$$

where $\tilde{V} = \tilde{W} - U$ the surplus from finding a job. Employed workers receive wage w, receive capital gains from the firm changing size, and face a risk of separation

$$\rho W dt = \left[w + \mu(z)W_z + \frac{\sigma(z)^2}{2}W_{zz}\right]dt + W_n(dH^* - dX^*) - (W - U)\frac{dX^*}{n} \tag{B.5}$$

Note, the worker would in theory quit if the value of employment falls below the value of being unemployed, but this won't happen with the coalition bargaining solution, so I omitted this possibility when writing the employment value.

Combining the previous two equations gives the job values (i.e. worker surplus) $V = W - U$

$$\rho V dt = \left[w - \rho U + \mu(z)V_z + \frac{\sigma(z)^2}{2}V_{zz}\right]dt + V_n(dH^* - dX^*) - V\frac{dX^*}{n} \tag{B.6}$$

Now consider the wage determination rule given by coalition bargaining over the joint surplus $S = nV + \Pi$

$$(1-\eta)nV = \eta\Pi \Rightarrow \eta S = nV, \quad (1-\eta)S = \Pi \tag{B.7}$$

together with the marginal surplus

$$S_n = nV_n + V + \Pi_n \tag{B.8}$$

to get the joint surplus maximization problem

$$\rho S dt = \max_{dH\geq 0, dX\geq 0}\left\{\left[y(n,z)-bn-n\lambda\int \tilde{V}dF(\tilde{V})+\mu(z)S_z+\frac{\sigma(z)^2}{2}S_{zz}\right]dt-(c-S_n+V)dH-S_n dX\right\} \tag{B.9}$$

with optimal hiring and firing decisions given by

$$(c - S_n + V)dH^* = 0 \qquad S_n dX^* = 0 \tag{B.10}$$

which gives the maximized value of the joint surplus as

$$\rho S = y(n,z) - bn - n\lambda\int \tilde{V}dF(\tilde{V}) + \mu(z)S_z + \frac{\sigma(z)^2}{2}S_{zz} \tag{B.11}$$

This can be simplified further by noting that optimal hiring implies

$$V = S_n - c \tag{B.12}$$

Using the surplus splitting rule and the firm's optimal hiring condition gives

$$\Pi = (1-\eta)S \Rightarrow \Pi_n = (1-\eta)S_n \Rightarrow c = (1-\eta)S_n \tag{B.13}$$

Together with the previous expression, this implies the job value is equalized across hiring firms

$$\tilde{V} = \frac{\eta}{1-\eta}c \tag{B.14}$$

Substitution into the joint surplus expression gives

$$\rho S = y(n,z) - bn - nc\lambda\frac{\eta}{1-\eta} + \mu(z)S_z + \frac{\sigma(z)^2}{2}S_{zz} \tag{B.15}$$

Using the surplus splitting rule $\Pi = (1-\eta)S$ we can solve for the wage:

$$w(n,z) = \eta\left(\frac{y(n,z)}{n} + c\lambda\right) + \big(1-\eta\big)b \tag{B.16}$$

Here workers get a fraction of their average product y/n and the hiring cost c, and a fraction of their outside option b according to the coalition bargaining weight η. In Elsby and Michaels (2013) , the authors pick a Cobb-Douglas form for production $y(n,z) = zn^{\alpha}$ which leads to

$$w(n,z) = \eta\left(zn^{\alpha-1} + c\lambda\right) + \big(1-\eta\big)b \tag{B.17}$$

Note, with only one productive worker $n = 1$ (a severe form of decreasing returns to scale) or constant returns $\alpha = 1$, this becomes the textbook bargaining solution. Substitution of the wage back into the firm's value gives

$$\rho\Pi = (1-\eta)\big(y(n,z) - nb\big) - \eta nc\lambda - +\mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz} \tag{B.18}$$

Differentiating this expression and letting $J = \Pi_n$ represent the value of a marginal job yields

$$\rho J = (1-\eta)\big(y_n(n,z) - b\big) - \eta c\lambda + \mu(z)J_z + \frac{\sigma(z)^2}{2}J_{zz} \tag{B.19}$$

Suppose the technology is Cobb-Douglas and define the marginal product $m = \alpha zn^{\alpha-1}$. Furthermore following Elsby and Gottfries (2022), assume that productivity follows a geometric Brownian motion $dz = \mu z dt + \sigma z dz$. It follows that when the firm is neither hiring or firing, the marginal product follows a geometric Brownian motion with drift μ and volatility σ^2. Using a change of variables, the value of a marginal job can be written in terms of the marginal product as

$$\rho J(m) = (1-\eta)\big(m-b\big) - \eta c\lambda + \mu m J'(m) + \frac{1}{2}\sigma^2 m^2 J''(m) \tag{B.20}$$

There are two boundaries for hiring and firing in terms of the marginal product (m_l, m_h). The smooth-pasting and super-contact conditions are

$$J(m_h) = c, \quad J(m_l) = J'(m_l) = J'(m_h) = 0$$

which can be used, as in the original paper of Elsby and Michaels (2013), to fully characterize the firm's optimal employment policy. Note this is a special case of the "m"-equilibrium concept of Elsby and Gottfries (2022) applied to the model without on-the-job search. Hence, the "m"-equilibrium concept holds with coalition bargaining in a model without on-the-job search.

B.2 Relation to Elsby and Gottfries (2022)

Here I discuss the coalition bargaining solution with respect to the environment in Elsby and Gottfries (2022). The environment is similar to that of Elsby and Michaels (2013), with the additional possibility of on-the-job search. That is, this is a special case of the model presented in the main text with two simplifications: (i) the hiring cost is linear, and (ii) there is no exogenous job-destruction. Search on-the-job makes hiring and quitting rates a function of equilibrium distributions (as in the main text) which are given by

$$\delta(V) = \lambda^e[1 - F(V)], \quad h(V) = q(\theta)[\phi + (1-\phi)G(V)] \tag{B.21}$$

where $F(\cdot)$ is the distribution of job values over vacancies, and $G(\cdot)$ is the distribution of job-values over current employment. The problem of the firm is similar to the case without on-the-job search, except now $dH = hvdt$. Formally, the problem of a firm is given by:

$$\rho\Pi(n,z)dt = \max_{v\geq 0, x\geq 0}\left\{\left[y(n,z) - wn - \delta n\Pi_n + \mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz}\right]dt - (c - \Pi_n)dH - \Pi_n dX\right\} \tag{B.22}$$

where the idiosyncratic productivity process follows an arbitrary diffusion (the results extend to jump-diffusion as well), $dH = hvdt$ is the measure of hires, and dX is the measure of separations. The optimal hiring and firing decisions satisfy,

$$(c - \Pi_n)dH^* = 0 \qquad \Pi_n dX^* = 0 \tag{B.23}$$

otherwise the solution is inaction $dH = 0$, i.e. $v = 0$, and $dX = 0$, Note, it is never optimal to simultaneously post vacancies and fire workers. Taken together, these conditions imply the maximized value of the firm satisfies

$$\rho\Pi = y(n,z) - wn + -\delta n\Pi_n + \mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz} \tag{B.24}$$

which is identical to the problem without on-the-job search, except now the quitting rate $\delta = \delta(\cdot)$ depends on the job-value. The value of unemployment is identical to the case without on-the-job search as well:

$$\rho U = b + \lambda^u \int (W - U)dF(W) \tag{B.25}$$

Now employed workers receive wage w, receive capital gains from the firm changing size, face a risk of separation, and the opportunity to search on-the-job:

$$\rho W dt = \left[w + \lambda^e \int_W (\tilde{W} - W) dF(\tilde{W}) - \delta n W_n + \mu(z) W_z + \frac{\sigma(z)^2}{2} W_{zz} \right] dt + W_n (dH^* - dX^*) - W \frac{dX^*}{n} \tag{B.26}$$

Note, the worker would in theory quit if the value of employment falls below the value of being unemployed, but this won't happen with the coalition bargaining solution, so I omitted this possibility when writing the employment value. Combining the previous two equations gives the job values (i.e. worker surplus) $V = W - U$

$$\begin{aligned} \rho V dt = & \left[w - b - \delta n V_n + \mu(z) V_z + \frac{\sigma(z)^2}{2} V_{zz} \right. \\ & \left. + \lambda^e \int_V (\tilde{V} - V) dF(\tilde{V}) - \lambda^u \int \tilde{V} dF(\tilde{V}) \right] dt \\ & + V_n (dH^* - dX^*) - V \frac{dX^*}{n} \end{aligned} \tag{B.27}$$

Now consider the wage determination rule given by coalition bargaining over the joint surplus $S = nV + \Pi$

$$(1 - \eta) n V = \eta \Pi \Rightarrow \eta S = nV, \quad (1 - \eta) S = \Pi \tag{B.28}$$

together with the marginal surplus

$$S_n = n V_n + V + \Pi_n \tag{B.29}$$

to get the joint surplus maximization problem

$$
\begin{aligned}
\rho S dt = \max_{v\geq 0, x\geq 0} \Bigg\{ \Bigg[& y(n,z) - bn - cv - \delta n(S_n - V) + \mu(z) S_z + \frac{\sigma(z)^2}{2} S_{zz} \\
& + n\lambda^e \int_V (\tilde{V} - V) dF(\tilde{V}) - n\lambda^u \int \tilde{V} dF(\tilde{V}) \Bigg] dt \\
& - (c - S_n + V) dH - S_n dX \Bigg\}
\end{aligned}
\tag{B.30}
$$

with optimal hiring and firing decisions given by

$$
(c - S_n + V) dH^* = 0 \qquad S_n dX^* = 0 \tag{B.31}
$$

which gives the maximized value of the joint surplus as

$$
\rho S = y(n,z) - bn - \delta n(S_n - V) + n\lambda^e \int_V (\tilde{V} - V) dF(\tilde{V}) - n\lambda^u \int \tilde{V} dF(\tilde{V}) + \mu(z) S_z + \frac{\sigma(z)^2}{2} S_{zz} \tag{B.32}
$$

which is identical to the case without on-the-job search, except for the additional term capturing the option value of searching on-the-job to incumbent workers. As in the case without on-the-job search, the value of openings are equalized at hiring firms as

$$
\tilde{V} = \frac{\eta}{1-\eta} c \tag{B.33}
$$

which can be used to simplify the joint surplus considerably. To do so, note the following terms can be rewritten as

$$
\begin{aligned}
& - \delta n(S_n - V) + n\lambda^e \int_V (\tilde{V} - V) dF(\tilde{V}) - n\lambda^u \int \tilde{V} dF(\tilde{V}) \\
= & - \delta n S_n + \delta n V + n\lambda^e [1 - F] \frac{\eta}{1-\eta} c - n\lambda^e [1-F] V - n\lambda^u \frac{\eta}{1-\eta} c \\
= & - \delta n S_n - n(\delta - \lambda^u) \frac{\eta}{1-\eta} c
\end{aligned}
$$

where we use the fact $\delta = \lambda^e[1-F]$. So the maximized value of the joint surplus becomes

$$\rho S = y(n,z) - bn - \delta n S_n - n(\delta - \lambda^u)\frac{\eta}{1-\eta}c + \mu(z)S_z + \frac{\sigma(z)^2}{2}S_{zz} \tag{B.34}$$

Using the surplus sharing rule $(1-\eta)S = \Pi$ gives an expression for the wage

$$w(n,z) = \eta\left(\frac{y(n,z)}{n} + c(\lambda^u - \delta)\right) + \big(1-\eta\big)b \tag{B.35}$$

where workers get a fraction of their average product y/n and the hiring cost c, and a fraction of their outside option b according to the coalition bargaining weight η. Substitution of the wage back into the firm's value gives

$$\rho\Pi = (1-\eta)\big(y(n,z) - nb\big) - \eta n c \lambda^u + \delta n(\eta c - \Pi_n) + \mu(z)\Pi_z + \frac{\sigma(z)^2}{2}\Pi_{zz} \tag{B.36}$$

Differentiating this expression and letting $J = \Pi_n$ represent the value of a marginal job yields

$$\rho J = (1-\eta)\big(y_n(n,z) - b\big) - \eta c \lambda^u + \frac{\partial(\delta n)}{\partial n}(\eta c - J) - \delta n J_n + \mu(z)J_z + \frac{\sigma(z)^2}{2}J_{zz} \tag{B.37}$$

Now consider the case where production is Cobb-Douglas $y(n,z) = zn^{\alpha}$, and z follows a geometric Brownian motion $dz = \mu z dt + \sigma z dz$. Suppose, as in Elsby and Gottfries (2022), the separation rate δ can be written as a function of the firm's marginal product[3], then by

[3]This can be proved by showing the job value W is monotone in m as in *Lemma* 1 of Elsby and Gottfries (2022). Applying the same logic here is not so straightforward if both the destination firm and origin firm are hiring, given that job values are equalized across hiring firms. One needs to make the additional assumption that workers use the marginal product (i.e. the spot wage) as a tie-breaking rule when deciding whether to leave a firm. Thus, even though the distribution of worker values across hiring firms is degenerate, a non-degenerate distribution of marginal products is sufficient to create turnover.

the chain rule $\delta_n = \delta_m \frac{\partial m}{\partial n}$, and $J_n = J_m \frac{\partial m}{\partial n}$, which in the Cobb-Douglas case implies

$$\begin{aligned} \frac{\partial(\delta n)}{\partial n} &= \delta - (1-\alpha)\delta_m m \\ \delta n J_n &= -\delta(1-\alpha) m J_m \end{aligned} \tag{B.38}$$

Furthermore, m also follows a geometric Brownian motion, as in the case without on-the-job-search. Substitution into the previous expression gives

$$\begin{aligned} \rho J(m) =&(1-\eta)\big(m-b\big) - \eta c \lambda^u + [\delta(m) - (1-\alpha)\delta'(m) m]\big(\eta c - J(m)\big) \\ &+ [\mu + \delta(m)(1-\alpha)] m J'(m) + \frac{1}{2}\sigma^2 m^2 J''(m) \end{aligned} \tag{B.39}$$

As in Elsby and Gottfries (2022), the job value lies in the set $[0, c]$, and there is a region with boundaries for hiring and separations given by (m_l, m_h), where the firm is inactive. In this region, the firm is at the bottom of the job ladder, so $\delta(m) = \lambda^e$, and $\delta'(m) = 0$. In the inaction region, the marginal value of a job becomes

$$(\rho + \lambda^e) J(m) = (1-\eta)\big(m-b\big) - \eta c(\lambda^u - \lambda^e) + [\mu + \lambda^e(1-\alpha)] m J'(m) + \frac{1}{2}\sigma^2 m^2 J''(m) \tag{B.40}$$

Let $\omega_1 = \eta$ and $\omega_0 = (1-\eta)b + \eta c(\lambda^u - \lambda^e)$, then the value of a marginal job becomes

$$(\rho + \lambda^e) J(m) = (1-\omega_1) m - \omega_0 + [\mu + \lambda^e(1-\alpha)] m J'(m) + \frac{1}{2}\sigma^2 m^2 J''(m) \tag{B.41}$$

which is identical to Equation (17) in Elsby and Gottfries (2022). The smooth-pasting and super-contact conditions are

$$J(m_h) = c, \quad J(m_l) = J'(m_l) = J'(m_h) = 0$$

which can be used to solve the previous equation analytically. Second, there is a region (m_h, m_u) where the firm value is constant $J = c \Rightarrow J' = J'' = 0$ and the firm hires optimally. In this region, the value of a marginal job becomes

$$\rho c = (1-\eta)\big(m-b\big) - \eta c \lambda^u - [\delta(m) - (1-\alpha)\delta'(m)m](1-\eta)c \tag{B.42}$$

Let $\tilde{c} = (1-\eta)c$, $\omega_1 = \eta$, and $\omega_0 = (1-\eta)b + \rho\eta c + \eta c\lambda_u$, then the equation becomes

$$\rho\tilde{c} = (1-\omega_1)m - \omega_0 - [\delta(m) - (1-\alpha)\delta'(m)m]\tilde{c} \tag{B.43}$$

which is identical to Equation (22) in Elsby and Gottfries (2022). The boundary conditions $\delta(m_h) = \lambda^e$ and $\delta(m_u) = 0$ give the solution to the quit rate as in Proposition 2 of Elsby and Gottfries (2022). While it is beyond the scope of this paper to reproduce the additional results in Elsby and Gottfries (2022), the point was to show that the concept of an "m"-equilibrium holds with coalition bargaining with an additional assumption on the tie-breaking rule for the workers' quitting decision.

C Computation

The following outlines the computational details for the steady state model, and the business cycles transitions.

C.1 Steady State

Recall, the Hamilton-Jacobi-Bellman equation for the joint surplus of an operating coalition is given by,

$$
\begin{aligned}
\rho S = & y(n,z) - c(v,n) - nb \\
& + n\lambda^e(\theta) \int \max\left\{\tilde{V} - V, 0\right\} dF\left(\tilde{V}\right) \\
& - \delta n V \\
& + [S_n - V]\,[h(V)v - s(V)n] \\
& + \lambda_+(S_+ - S) + \lambda_-(S_- - S)
\end{aligned}
$$

where the worker surplus (i.e. job value) is given by

$$
V = \frac{\eta}{n} S \tag{C.1}
$$

and vacancies solve

$$
\frac{c_v(v,n)}{h(V)} = S_n - V. \tag{C.2}
$$

The hiring and separation rates are given by

$$
\begin{aligned}
h(V) &= q[\phi + (1-\phi)G(V)] \\
s(V) &= \delta + \lambda^e[1 - F(V)]
\end{aligned}
$$

where G is the distribution of job values over employed workers, and F is the distribution of job values over vacancies. It will be useful to work with the log transformation $y = \log n$, where we define

$$m(y, z) = S(n, z) \tag{C.3}$$

which implies

$$S_n = m_y e^{-y}$$
$$V = \eta m e^{-y}$$

and

$$\frac{dn}{dt} = hv - sn = e^y(h\tilde{v} - s) \tag{C.4}$$

where $\tilde{v} = v/n$ is the vacancy-employment ratio. Substitution into the HJB yields

$$\begin{aligned}
\rho m(y) = y(e^y, z) - c(e^y\tilde{v}, e^y) - e^y b \\
&+ e^y \lambda^e(\theta) \int \max\left\{\tilde{V} - V, 0\right\} dF(\tilde{V}) \\
&- \delta\eta m(y) \\
&+ [m'(y) - \eta m(y)]\,[h\tilde{v} - s]
\end{aligned}$$

Since a worker only leaves if $\tilde{V} > V$, we can simplify to

$$\begin{aligned}
\rho m(y) = y(e^y, z) - c(e^y\tilde{v}, e^y) - e^y b \\
&+ e^y \lambda^e(\theta) \int_V^\infty \tilde{V} dF(\tilde{V}) \\
&- s(V)\eta m(y) \\
&+ [m'(y) - \eta m(y)]\,[h\tilde{v} - s]
\end{aligned}$$

From the first order condition for vacancies with the cost function $\frac{\kappa}{1+\gamma} v(v/n)^{\gamma}$ we have

$$\tilde{v} = \left[\frac{h}{\kappa} e^{-y} \left[m'(y) - \eta m(y) \right]^{+} \right]^{\frac{1}{\gamma}} \tag{C.5}$$

where $[\cdot]^{+} = \max\{\cdot, 0\}$. I solve this model in two steps: (i) there is an inner loop which solves the joint surplus taking as given F and G, and (ii) an outer loop which uses the policies to update the distribution $g(e^{y}, z)$ which determines F and G. Let k denote the iteration of the inner loop and τ denote the iteration of the outer loop.

I discretize $m(y, z)$ onto a grid $y_i \in \{y_1, y_2, \ldots, y_I\}$ of I nodes for employment and $z_1, z_2, \ldots, z_J$ of J nodes for productivity. I approximate the value function with Chebychev polynomials of degree $N - 1$.

$$m_{ij} = \sum_{n=1}^{N} c_{nj} \phi_n(y_i) = \phi(y_i) c_j \tag{C.6}$$

which implies the derivative is given by

$$D_y m_{ij} = \sum_{n=1}^{N} c_{nj} \phi'_n(y_i) = \phi'(y_i) c_j. \tag{C.7}$$

I begin by describing the inner loop that solves the coalition problem taking the aggregate distributions as given. I use an implicit updating scheme. Let $\tilde{v}_i^k$ be the optimal vacancy employment ratio from the previous iteration, and V_{ij}^k be the job value from the previous iteration. With on-the-job search we define the hiring and separation rates based on the previous iteration as

$$\begin{aligned} h_{ij}^k &= h\left(V_{ij}^k\right) = q\left[\phi + (1-\phi) G^{\tau}\left(V_{ij}^k\right)\right] \\ s_{ij}^k &= s\left(V_{ij}^k\right) = \delta + \lambda^{e}(\theta)\left[1 - F^{\tau}\left(V_{ij}^k\right)\right] \end{aligned}$$

and define the flow payoff and drift as,

$$
\begin{aligned}
\pi_{ij}^k &= y\left(e^{y_i, z_j}\right) - e^{y_i} b - c\left(e^{y_i}\tilde{v}_i^k, e^{y_i}\right) + e^{y_i}\lambda^e(\theta)\int_{V_{ij}^k}^{\infty} \tilde{V} dF^{\tau}\left(\tilde{V}\right) \\
d_{ij}^k &= h_{ij}^k \tilde{v}_{ij}^k - s_{ij}^k.
\end{aligned}
$$

I update $c_j^k \longrightarrow c_j^{k+1}$ according to

$$
\frac{m_{ij}^{k+1} - m_{ij}^k}{\Delta} + \left(\rho + \eta h_{ij}^k \tilde{v}_{ij}^k\right) m_{ij}^{k+1} = \pi_{ij}^k + (m_{ij}^{k+1})' d_{ij}^k \tag{C.8}
$$

After substitution of the polynomial expression and rearrangement we have,

$$
\left[\left(\frac{1}{\Delta} + \rho + \eta h_{ij}^k \tilde{v}_{ij}^k\right)\phi(y_i) - \left(h_{ij}^k \tilde{v}_{ij}^k - s_{ij}^k\right)\phi'(y_i)\right] c_j^{k+1} = \pi_{ij}^k + \frac{1}{\Delta}\phi(y_{ij}) c^k. \tag{C.9}
$$

Stacking the expressions, we have a system of equations

$$
\left[B^k \Phi_0 - A^k \Phi_1\right] c^{k+1} = \boldsymbol{\pi}^k + \frac{1}{\Delta}\Phi_0 c^k \tag{C.10}
$$

which are solved via least squares to update $c^k \longrightarrow c^{k+1}$.

The law of motion for the equilibrium density $g(\cdot)$ is given by

$$
\partial_t g = (A^k)^T g - \Sigma g + m \tag{C.11}
$$

where Σ is a diagonal matrices with entries σ, and m is the distribution of new entrants over employment and productivity. In steady state $\delta_t g = 0$ which implies

$$
g = \left[\Sigma - (A^k)^T\right]^{-1} m \tag{C.12}
$$

The distribution F and G are updated using g according to their definitions in the main text.

C.2 Transition

The HJB for the joint surplus along the transition is given by

$$\begin{aligned}
\rho S = & y(n,z) - c(v,n) - nb \\
& + n\lambda^e(\theta) \int \max\left\{\tilde{V} - V, 0\right\} dF\left(\tilde{V}\right) \\
& - \delta n V \\
& + [S_n - V]\,[h(V)v - s(V)n] \\
& + \lambda_+(S_+ - S) + \lambda_-(S_- - S) \\
& + S_t
\end{aligned}$$

which is identical to before, except for the addition of a time derivative, and can be solved in a similar fashion.

The transitions for the equilibrium density $g(\cdot)$ is given by

$$\partial_t g = A^T g - \Sigma g + m \tag{C.13}$$

Using an implicit discretization this becomes

$$\begin{aligned}
\frac{g^{t+1} - g^t}{\Delta t} &= A^T g^{t+1} - \Sigma g^{t+1} + m \\
\Rightarrow g^{t+1} &= (\Delta t)\left[A^T - \Sigma\right] g^{t+1} + (\Delta t) m + g^t \\
\Rightarrow g^{t+1} &= \left[I - \Delta t (A^T - \Sigma)\right]^{-1} \left[g^t + (\Delta t) m\right]
\end{aligned}$$