Introduction to Modern Control Systems
Antenna Angular Position Control System Design Project
EE 486 - 01

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1 Introduction

The main objective of this project was to derive a way to stabilize the angular position of a rotating antenna. The problem of stabilizing a rotating antenna is important because antennas need to be accurately positioned to properly receive a signal of interest. An uncontrolled, moving antenna will make it difficult, if not impossible to track the signal of interest.

To effectively solve this problem, a state variable model of the system was derived. Next, controllability and observability of the system was computed and verified. Using this information, two control schemes were designed to solve this problem. The first was a state feedback control system. Next, a Luenberger observer was designed, for future use. The third was a integrated full state feedback and observer, also known as a compensator.

2 Derivation of the State Variable Model

Beginning with the block diagram derived from Fig 2, while neglecting the dynamics of the armature winding, L_a, the following state variables were assigned

$$\dot{\Theta}_a = \Omega_m \tag{1}$$

$$\dot{\Omega}_m = T_m \tag{2}$$

Using the above state variables the following state equations were derived

$$\dot{\Theta}_a = -k_b V_a = \frac{1}{f}\dot{\Omega}_m \tag{3}$$

$$\dot{\Omega}_m = \frac{k_m}{R_a} V_a + N T_d + \frac{1}{f}\Omega_m \tag{4}$$

Using the above state equations, the following state space model of the system was derived

$$\begin{bmatrix} \dot{\Theta}_a \\ \dot{\Omega}_m \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{f} \\ 0 & \frac{1}{f} \end{bmatrix} \begin{bmatrix} \Theta_a \\ \Omega_m \end{bmatrix} + \begin{bmatrix} -k_b \\ \frac{k_m}{R_a} \end{bmatrix} V_a \tag{5}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Theta_a \\ \Omega_m \end{bmatrix} \tag{6}$$

3 Controllability and Observability Analysis

3.1 Controllability Analysis

Starting with the equation of controllability

$$P_c = \begin{bmatrix} B & AB \end{bmatrix} \tag{7}$$

$$P_c = \begin{bmatrix} -k_b & 0 \\ \frac{k_m}{R_a} & \frac{k_m}{R_a} \end{bmatrix} \tag{8}$$

$$\Delta P_c = k_b - \frac{k_m}{R_a} \tag{9}$$

Substituting in the correct values

$$\Delta P_c = 0.55 - \frac{0.55}{1.2} = 0.092 \tag{10}$$

Since $\Delta P_c \neq 0$, the system is controllable.

3.2 Observability Analysis

Starting with the equation of observability

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix} \tag{11}$$

$$P_o = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{f} \end{bmatrix} \tag{12}$$

$$\Delta P_o = \frac{1}{f} \tag{13}$$

Substituting in the correct values

$$\Delta P_o = \frac{1}{10} \tag{14}$$

Since $\Delta P_0 \neq 0$, the system is observable.

4 Derivation of the State Feedback Controller

Starting with the equation for the gain matrix of a full state feedback controller [2]

$$\mathbf{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} P_c^{-1} q(\mathbf{A}) \tag{15}$$

The following substitutions allows the state feedback controller to be designed with an arbitrary pole placed at $\lambda = -7$.

$$q(\lambda) = \lambda + 7 \tag{16}$$

$$P_c^{-1} = \begin{bmatrix} 4.997 & -4.997 \\ 0.000 & -6.000 \end{bmatrix} \tag{17}$$

Solving equation 15 using the above substitutions

$$\mathbf{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 4.997 & -4.997 \\ 0.000 & -6.000 \end{bmatrix} \left(\begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \right) \tag{18}$$

Simplifying the above equation

$$\mathbf{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 4.997 & -4.997 \\ 0.000 & -6.000 \end{bmatrix} \left(\begin{bmatrix} 7 & 17 \\ 0 & 17 \end{bmatrix} \right) \tag{19}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 34.979 & -84.949 \\ 0.000 & 102.000 \end{bmatrix} \tag{20}$$

Results in a gain matrix, $\mathbf{K}$, of

$$\mathbf{K} = \begin{bmatrix} 0.000 & 102.000 \end{bmatrix} \tag{21}$$

Thus the equation of the closed looped system becomes

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}(t) \tag{22}$$

Where

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{1}{10} \\ 0 & \frac{1}{10} \end{bmatrix} \tag{23}$$

$$\mathbf{B} = \begin{bmatrix} -0.55 \\ \frac{0.55}{1.2} \end{bmatrix} \tag{24}$$

$$\mathbf{K} = \begin{bmatrix} 0.000 & 102.000 \end{bmatrix} \tag{25}$$

The results of the MATLAB Simulunk simulation of this controller are presented in the following Simulations section.

5 Derivation of the Luenberger Observer

Starting with the equation for the gain matrix of a Luenberger observer.

$$\mathbf{L} = q(\mathbf{A})P_o^{-1} \begin{bmatrix} 0 & 1 \end{bmatrix}^T \tag{26}$$

The following substitutions allows the Luenberger observer to be designed with an arbitrary poles placed at $\lambda = 60 \pm j3$.

$$q(\lambda) = (\lambda + 60 + j3)((\lambda + 60 - j3) = \lambda^2 + 120\lambda + 3609 \tag{27}$$

$$P_o^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{28}$$

Solving equation 26 using the above substitutions

$$\mathbf{L} = (\begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}^2 + \begin{bmatrix} 120 & 0 \\ 0 & 12 \end{bmatrix} + \begin{bmatrix} 3609 & 0 \\ 0 & 3609 \end{bmatrix}) \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}^T \tag{29}$$

Simplifying the above equation

$$\mathbf{L} = \begin{bmatrix} 3370 & 3729 \\ 37293729.1 & \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{30}$$

Results in a gain matrix, $\mathbf{L}$, of

$$\mathbf{L} = \begin{bmatrix} 372.90 \\ 372.91 \end{bmatrix} \tag{31}$$

6 Derivation of the Compensator

Combing the full state feedback controller and the Luenberger observer results in a compensator. Combing both these controllers gives the following equation of the closed looped system [1].

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{BK} - \mathbf{LC})\mathbf{x}(t) + \mathbf{L}y(t) \tag{32}$$

$$u(t) = \textbf{-K}\mathbf{x}(t) \tag{33}$$

Where

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{1}{10} \\ 0 & \frac{1}{10} \end{bmatrix} \tag{34}$$

$$\mathbf{B} = \begin{bmatrix} -0.55 \\ \frac{0.55}{1.2} \end{bmatrix} \tag{35}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{36}$$

$$\mathbf{K} = \begin{bmatrix} 0.000 & 102.000 \end{bmatrix} \tag{37}$$

$$\mathbf{L} = \begin{bmatrix} 372.90 \\ 372.91 \end{bmatrix} \tag{38}$$

The results of the MATLAB Simulunk simulation of this controller are presented in the following Simulations section.

7 Simulations

Using the previously derived state models and MATLAB, the two controllers were simulated. Using this information and tools, four simulations were performed. The first simulation performed was of the full state feedback controller with no disturbance. The second simulation was a full state feedback controller with a disturbance. The third simulation was a compensator with no disturbance. The final simulation was a compensator with a disturbance.

7.1 Simulation of Full State Feedback Controller

The following plots in this section show the simulation of the full state feedback controller. From these plots, it is clear the full state feedback controller is able to successfully stabilize the angular position, θ_A. However, despite being stable, the controller is not able to drive the position to the desired value.

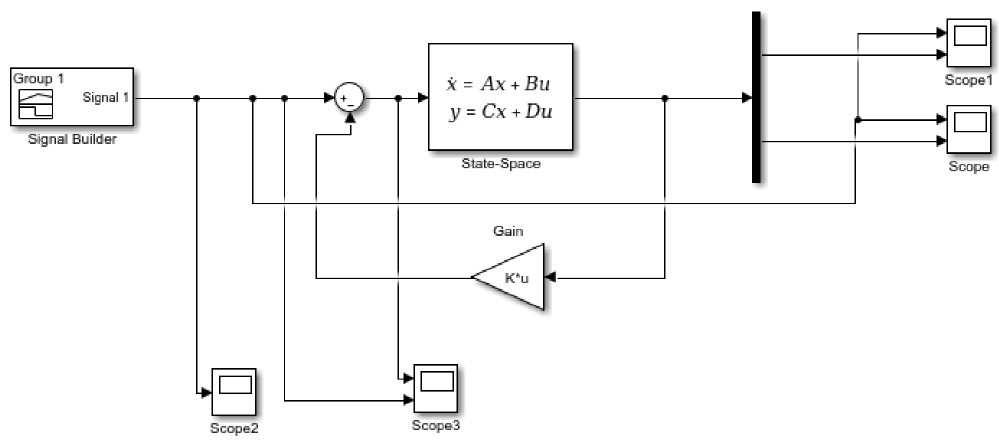

Figure 1: Block diagram of the full state feedback controller

The following three plots are the result of the simulation of the full state feedback controller.

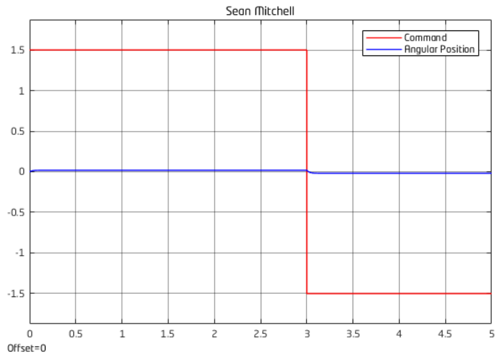

Figure 2: Simulation of $\theta_a^C(t)$ (rad) and $\theta_a(t)$ (rad) vs time (s)

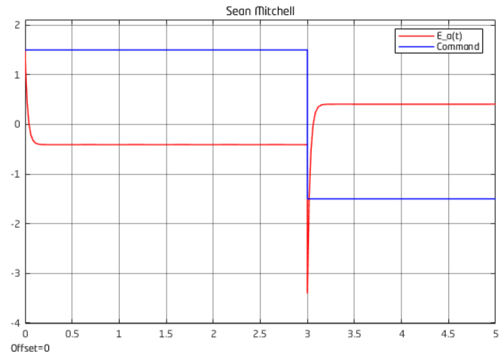

Figure 3: Simulation of $e_a(t)$ (rad) vs time (s)

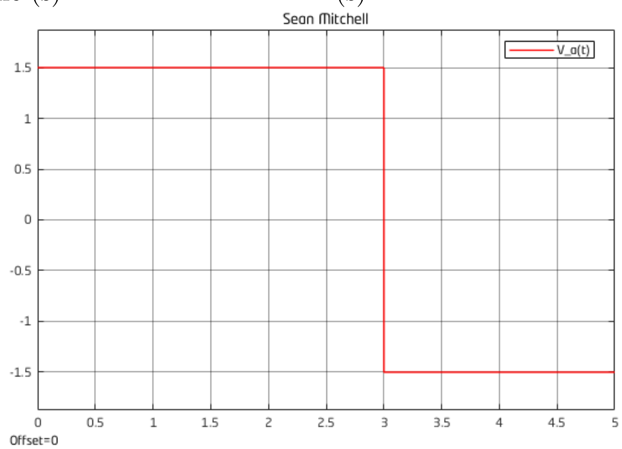

Figure 4: Simulation of $V_a(t)$ vs time (s)

7.2 Simulation of Full State Feedback Controller with Disturbance

The following plots in this section show the simulation of the full state feedback controller with a applied disturbance. From these plots, it is clear the full state feedback controller is able to successfully stabilize the angular position, θ_A, after the disturbance occurs, but the controller is not able to stabilize it to the value of the command signal.

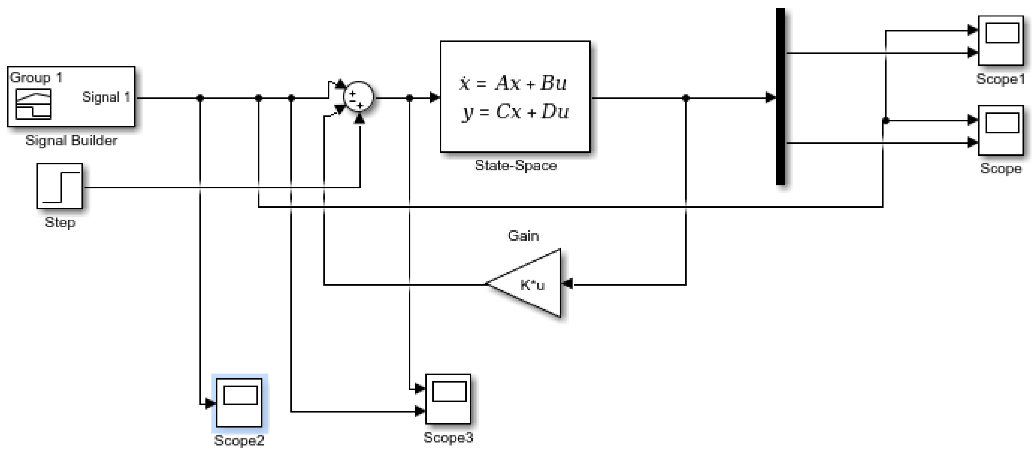

Figure 5: Block diagram of the full state feedback controller with an applied disturbance

The following three plots are the result of the simulation of the full state feedback controller with a applied disturbance.

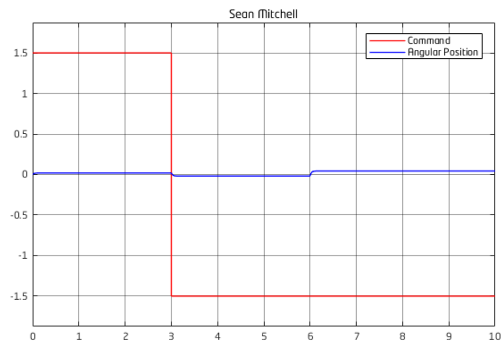

Figure 6: Simulation of $\theta_a^C(t)$ (rad) and $\theta_a(t)$ (rad) vs time (s)

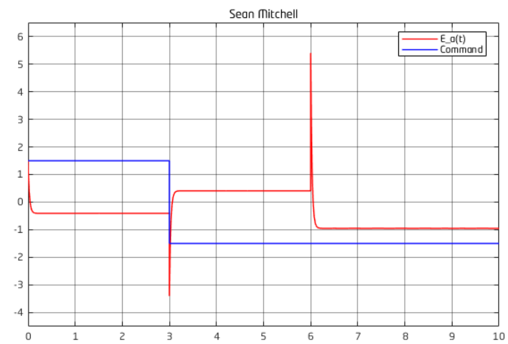

Figure 7: Simulation of $e_a(t)$ (rad) vs time (s)

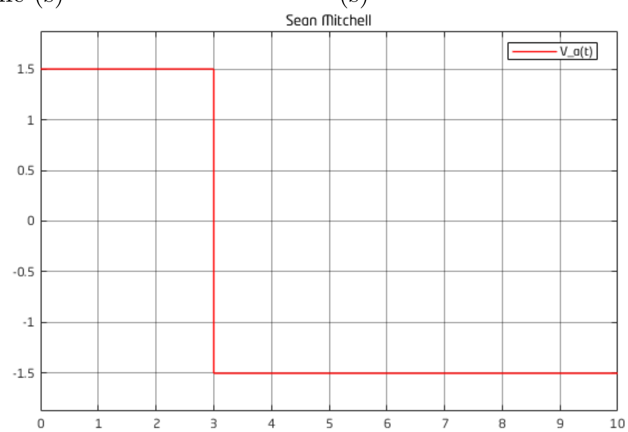

Figure 8: Simulation of $V_a(t)$ vs time (s)

7.3 Simulation of Compensator

The following image shows the block diagram of the compensator. This controller was unable to be simulated due to Simulink errors. Simulink had errors computing some of the gain values due to matrix dimension mismatches. Probable causes for these errors are further discussed in the conclusion.

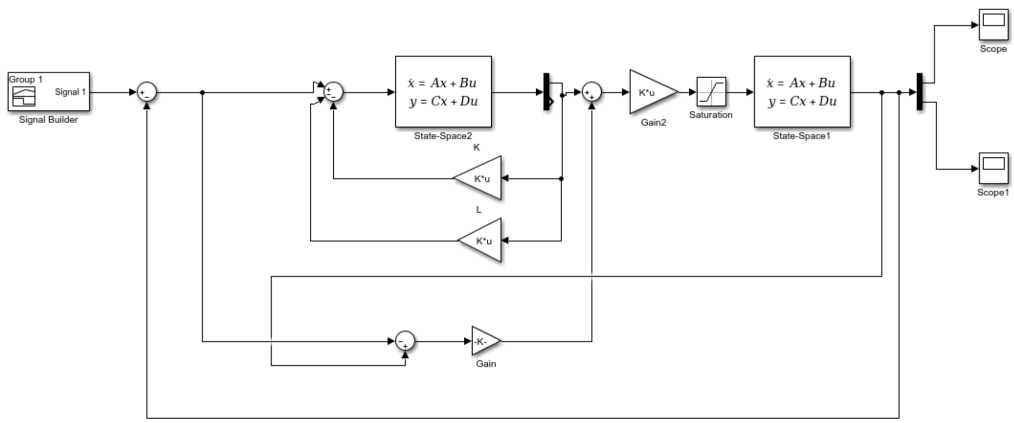

Figure 9: Block diagram of the compensator

7.4 Simulation of the Compensator with Disturbance

The following image shows the block diagram of the compensator with a disturbance applied. This controller was unable to be simulated due to Simulink errors. Simulink had errors computing some of the gain values due to matrix dimension mismatches. Probable causes for these errors are further discussed in the conclusion.

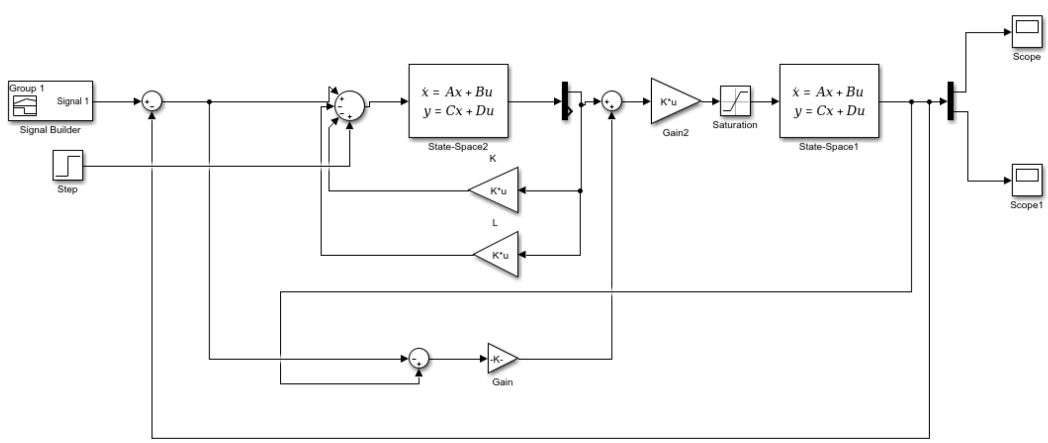

Figure 10: Block diagram of the compensator with an applied disturbance

8 Conclusion

In conclusion, the goal of this project was to derive a way to stabilize the angular position of an antenna. This goal was attempted using two different controllers. The first controller designed was a full state feedback controller. This controller stabilized the antenna's angular position but did not accurately track the command signal. When a wind disturbance was introduced, the full state feedback controller was able to stabilize the antenna's angular position, but not to the desired value. Next, a Luenberger observer was designed for the state model. Finally, using the feedback controller and observer, a compensator was designed. The compensator was unable to be properly simulated due to errors using Simulink.

For properly designed controllers, the full state feedback controller would have minimal oscillation, at the cost of settling time. Introducing the observer into the controller reduces settling time, but increases the percent overshoot due to initial state estimation errors. Both of these controllers should be able to drive the output of the system to a given input. Both of these controllers should also be able to handle an applied disturbance and return to nominal value.

Considering the results of the full state feedback controller, the controller could track a given input signal, but at a reduced magnitude. The output of the controller was incorrect by a factor of five. Since the controller itself was stabilizing the signal, but not to the required levels, the error is likely in the state model itself. Further evidence of this is supported by the Simulink errors when simulating the compensator. Simulink had errors computing some of the compensator gain values due to matrix dimension mismatch. The results of simulating both of the controllers point towards an incorrect state model. Further checking of the state model is required to properly design controllers that meet the project specifications.

9 Appendix

9.1 References

[1] R. Dorf and R. Bishop, Modern Control Systems. Boston, MA: Pearson, 2016, pp. 801.

[2] D. Wiberg, State Space and Linear Systems. Los Angeles, CA: McGraw-Hill, 1971, pp. 129.

[3] Y. Shtessel, Class Lecture, Topic: "The Design of State Variable Feedback Systems." EE486, College of Engineering, Universty of Alabama in Huntsville, Huntsville, AL, Aug. 26, 2018.