Tree Diagram

Tree diagram to show the probabilities when a coin is tossed twice.

2nd Toss

1st Toss

$\frac{1}{2}$ Head   P(head, head) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2}$ Head

$\frac{1}{2}$ Tail   P(head, tail) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2}$ Head   P(tail, head) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2}$ Tail

$\frac{1}{2}$ Tail   P(tail, tail) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P(A)

A

P(B|A)   P(A ∩ B)

P(B'|A)   P(A ∩ B')

P(A')

A'
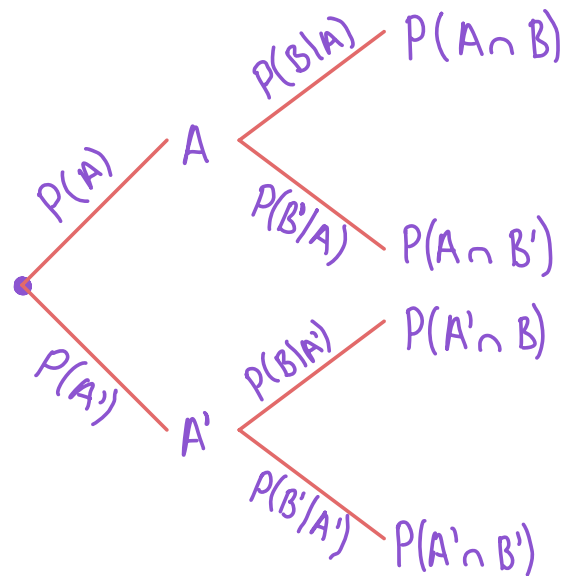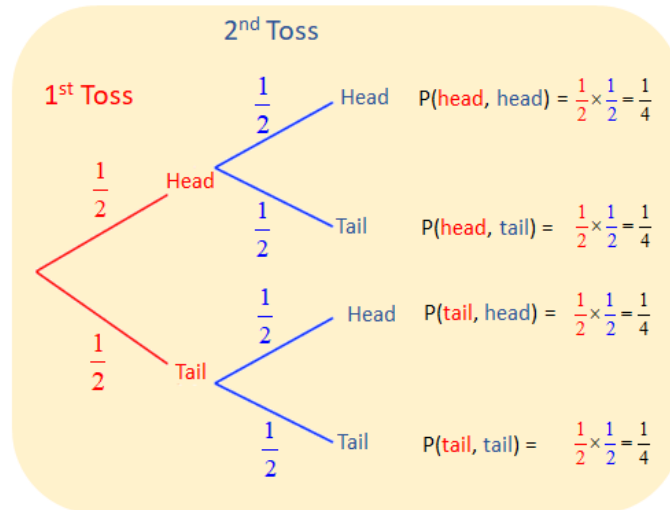
P(B|A')   P(A' ∩ B)

P(B'|A')   P(A' ∩ B')

Conditional Probability Formula

$$\frac{P(A \cap B)}{P(A)} = \frac{P(A) \cdot P(B|A)}{P(A)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

☆ $P(B) > 0$

A math teacher gave her class two tests. 25% of the class passed the both tests and 42% of the class passed the first test. What percentage of those who passed the first test also passed the second test?

F - first
S - second

$P(F) = 0.42$ $P(F \cap S) = 0.25$

Find $P(F|S) = ?$

$$P(S|F) = \frac{P(F \cap S)}{P(F)} = \frac{0.25}{0.42} = 60\%$$

ex) Find the probability of drawing 2 Kings from a deck

Event A: drawing a King first

Event B: drawing a King second

$P(A) = \frac{4}{52}$

$P(B|A) = \frac{3}{51}$

$\Rightarrow$ $P(A \cap B) = P(A) \cdot P(B|A)$

$= \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = 0.4\%$

ex) 70% of your friends like Chocolate Ice Cream, and 35% like Chocolate and Strawberry. What percent of those who like Chocolate also like Strawberry?

$P(C) = 0.7$ $P(C \cap S) = 0.35$

$P(S|C) = \dfrac{0.35}{0.7} = 0.5$

The probability that it is Friday and that a student is absent is 3%. What is the probability that a student is absent given that today is Friday?

F - Friday
A - Absent

To Find $P(A|F)$ we need $P(F \cap A)$, $P(F)$

We know $P(F \cap A) = 0.03$

$P(F) = \frac{1}{5} = 0.2$ Since there are 5 school days

$$P(A|F) = \frac{P(F \cap A)}{P(F)} = \frac{0.03}{0.2} = 0.15 = 15\%$$