Recall

$S(t)$ - position

$\frac{ds}{dt} = V(t)$ - velocity

$\frac{dv}{dt} = a(t)$ - acceleration

ex) $v(t) = t^2 - \frac{8}{(t+1)^2}$ m/s

How far did the car travel from its initial position from $t = 0$s to $t = 5$s?

★ units

$\frac{m}{s} \cdot \cancel{dt} = m$ ($\Delta t \to s$)

$$\Rightarrow \int_0^5 t^2\,dt - \int_0^5 \frac{8}{(t+1)^2}\,dt$$

$$\Rightarrow \int_0^5 t^2\,dt - 8 \cdot \int_0^5 (t+1)^{-2}\,dt \Rightarrow \left[\frac{1}{3}t^3\right]_0^5 - 8\left[-(t+1)^{-1}\right]_0^5$$

$$\Rightarrow \left[\frac{125}{3} - 0\right] - 8\left[\frac{-1}{6} + 1\right] = \boxed{35m}$$

Displacement

What is the car's initial position was at $-9m$? $S(0) = -9$ → Initial Position

★ Think of a car traveling on the real number line

We know from $t=0$ to $t=5$, the car traveled $+35m$ (i.e. $35m$ to the right)

The initial position was $-9m$ (i.e. $9m$ to the left of origin point)

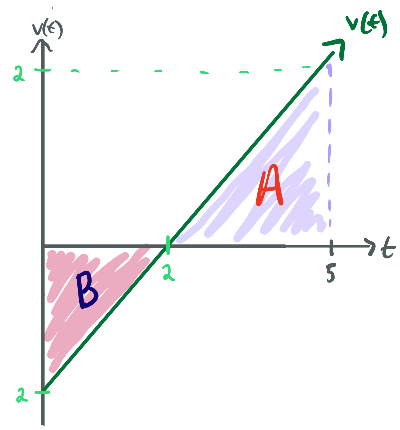
Thus, $35 + (-9) = \boxed{26m}$ After 5s, the car is located 26m to the right of origin point.

Displacement = New Position − Initial Position $= \int_a^b v(t)\,dt = A + B$

Total Distance traveled $= \int_a^b |v(t)|\,dt = A + |B|$

If $\xrightarrow{+}$ then $\xleftarrow{-}$

B - car is moving left
A - car is moving Right
Acceleration is constant



$A = \frac{1}{2}(3)(2) = 3$

$B = \frac{1}{2}(2)(-2) = -2$

ex | A car moving with an initial velocity of 5 mph accelerates at a rate of $a(t) = 2.4t$ mph per second for 8 seconds.

ⓐ How fast is the car moving after 8 seconds? We know $v(0) = 5$ mph

Asking for velocity $= \int a(t)\,dt = \int_0^8 2.4t\,dt = \left[1.2t^2\right]_0^8 = 1.2(64-0) = 76.8$ mph

$\frac{\text{mph}}{\cancel{s}} \cdot \cancel{s} = \text{mph}$

The acceleration over $t=0$ to $t=8$ adds 76.8 mph to the initial $v(0) = 5$mph. Thus, the car is moving $5 + 76.8 = 81.8$ mph

$\boxed{81.8 \text{ mph}}$

ⓑ How far did the car travel during those 8 seconds?

Asking for Displacement $= \int v(t)\,dt = \int_0^8 (1.2t^2 + c)\,dt = \int_0^8 (\overbrace{1.2t^2 + 5}^{\text{mph}})\,d\overbrace{t}^{s}$

We know $v(0) = 5$
$v(t) = 1.2t^2 + C$
$5 = C$
$\Rightarrow v(t) = 1.2t^2 + 5$

$= \left[0.4t^3 + 5t\right]_0^8 = 244.8$ mph·s

$\frac{mi}{h} \cdot s = \frac{mi}{h} \cdot \frac{1}{3600} h = \frac{mi}{3600}$

$244.8 \frac{mi}{3600} = \boxed{0.068 \text{ mi}}$

ex | From 1970 to 1980, the rate of potato consumption in U.S. was $C(t) = 2.2 + 2^t$ million bushels per year (t). How many bushels were consumed from the beginning of 1972 to the end of 1973?

$$\int_2^4 \underbrace{(2.2 + 2^t)\,dt}_{\frac{mb}{\cancel{y}} \cdot \cancel{y} = mb \text{ "million bushels"}} = \Big[2.2t + ?\Big]_2^4 \cdots\cdots = 21.71 \text{ million bushels}$$

$$\frac{d}{dx}\left[a^x\right] = a^x \ln(a) + c$$

$$\int a^x\,dx = \frac{a^x}{\ln(a)} + c \quad \Rightarrow \quad \int 2^x\,dx = \boxed{\frac{2^x}{\ln(2)}} + c$$

Physics Ex | It takes a force of 10N to stretch a spring 2m beyond its natural length. How much work is needed to stretch it 4m?

Recall:
Work = Force · Displacement
Hook's Law: $F = kx$
N · M = 1 Joule
Work is area under Force graph

We know $F(2) = 10$ $\quad \underbrace{10 = k \cdot 2}_{\text{Hook's Law } F = kx} \Rightarrow k = 5 \Rightarrow \underline{F = 5x}$

$$\int_0^4 F(x)\,dx = \int_0^4 \underbrace{5x\,dx}_{N \cdot m = J} = \Big[2.5x^2\Big]_0^4 = \boxed{40\,J}$$