

# Systems Design Guided by Progress Concerns

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## └ Safety vs. Liveness

Safety vs. Liveness	
<b>Safety Properties</b>	<b>Liveness Properties</b>
• Something (bad) <i>never happens.</i>	• Something (good) <i>will happen</i>
• e.g. invariance properties	• e.g. termination, progress

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• Liveness properties are **essential.**

A small cartoon illustration of a character with orange hair and a brown beret, wearing a yellow dress with a white collar, holding a red sign that says "OUT OF ORDER".

## Safety vs. Liveness

## Safety Properties

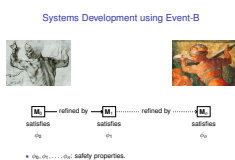
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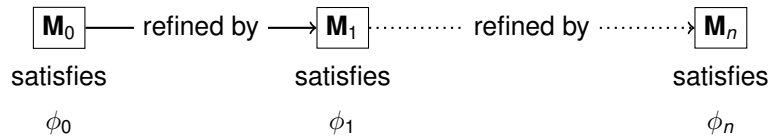
## └ Systems Development using Event-B



## Systems Development using Event-B



- To develop a system **M** satisfying property  $\phi$ , i.e.,  $\mathbf{M} \models \phi$ .
  - **M**: some transition system
  - $\phi$ : some logical formula
- The main challenge: the **complexity of the system**.
- **Refinement** allows the step-by-step design of the system.



- $\phi_0, \phi_1, \dots, \phi_n$ : safety properties.

## Systems Design Guided by Progress Concerns

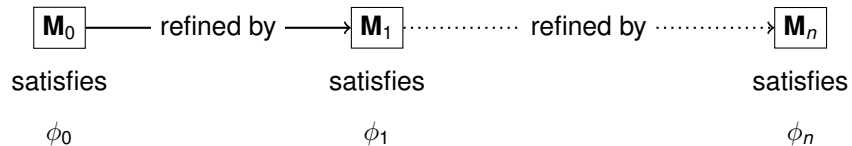
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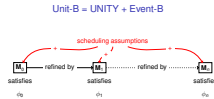


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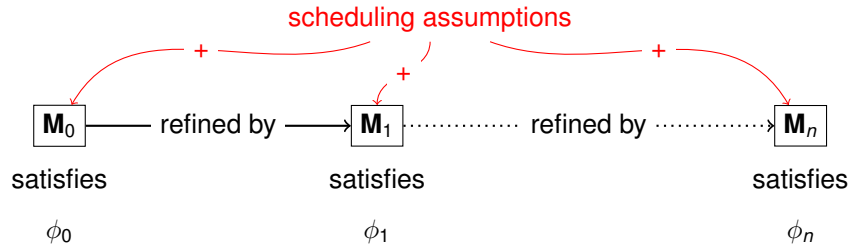
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- Support the reasoning of **liveness properties** (UNITY).
- **Refinement** of transition systems (Event-B style).
- Developments using Unit-B are **guided by both safety and liveness requirements**.



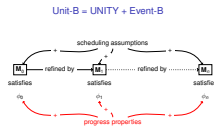
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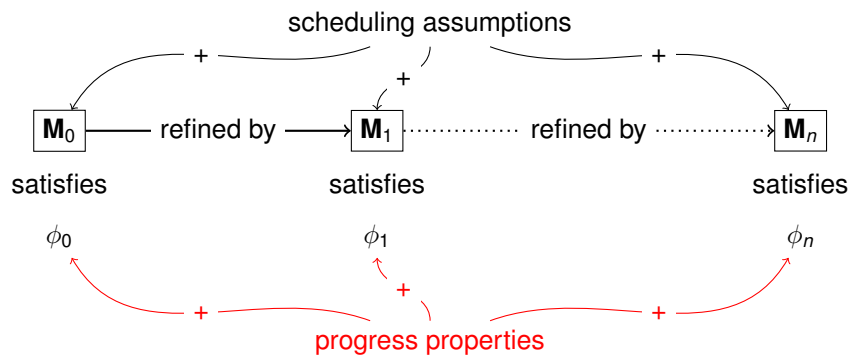


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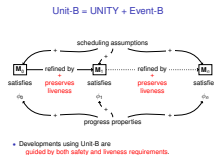
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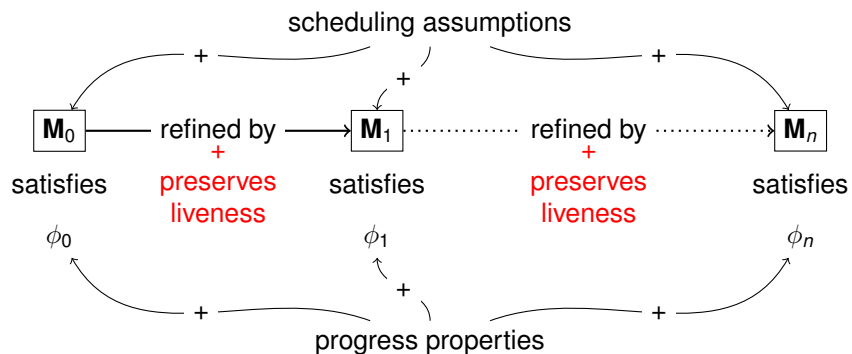


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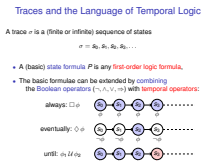
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# Traces and the Language of Temporal Logic



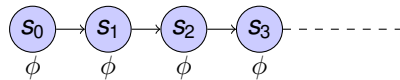
## Traces and the Language of Temporal Logic

A trace  $\sigma$  is a (finite or infinite) sequence of states

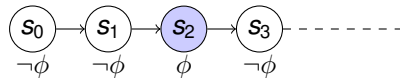
$$\sigma = s_0, s_1, s_2, s_3, \dots$$

- A (basic) **state formula**  $P$  is any **first-order logic formula**,
- The basic formulae can be extended by **combining** the **Boolean operators** ( $\neg, \wedge, \vee, \Rightarrow$ ) with **temporal operators**:

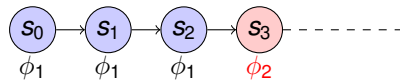
always:  $\Box \phi$



eventually:  $\Diamond \phi$



until:  $\phi_1 \mathcal{U} \phi_2$



## Unit-B Models. Guarded and Scheduled Events

- $e.t$  is **enabled** when  $G.t.v$  holds.
- Execution of  $e.t$ :  $v$  is **updated** according to the action  $S.t.v.v'$ .
- $e.t$  corresponds to a formula  $act.(e.t)$ .

Unit-B Models. Guarded and Scheduled Events

```

e
  any / where
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  then
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  end
  
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Unit-B Models. Guarded and Scheduled Events

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any t where
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• Execution of  $e.t$  corresponds to a formula  $act.(e.t)$ .

•  $C.t.v$ : coarse-schedule.

•  $F.t.v$ : fine-schedule.

**Liveness (Scheduling) Assumption**

If  $C.t.v$  holds infinitely long and  $F.t.v$  holds infinitely often then eventually  $e.t$  is executed when  $F.t.v$  holds.

$$sched.(e.t) = \Box(\Box C \wedge \Box \Diamond F \Rightarrow \Diamond(F \wedge act.(e.t)))$$

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## Schedules vs. Fairness

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$e \triangleq$  any  $t$  where  $G.t.v$  during  $C.t.v$  upon  $F.t.v$  then ... end

• Schedules are a **generalisation** of weak- and strong-fairness.

• Weak-fairness:  
If  $e$  is **enabled infinitely long** then  $e$  eventually occurs.  
• Let  $C$  be  $G$  and  $F$  be  $\top$ .

• Strong-fairness:  
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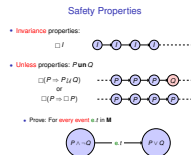
- **Unscheduled** events (without **during** and **upon**):  $C$  is  $\perp$ .
- When only **during** is present (no **upon**),  $F$  is  $\top$ .
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## Safety Properties



Invariance properties are proved using induction technique.

- $I$  holds for every reachable state.
- Proved using the standard **induction technique**.

If  $P$  holds at some point then

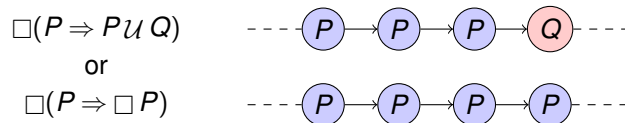
- $Q$  never holds and  $P$  continues to hold forever, or
- $Q$  holds eventually and  $P$  continues to hold at least until  $Q$  holds.

## Safety Properties

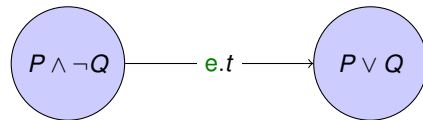
- Invariance** properties:



- Unless** properties:  $P \text{ un } Q$



- Prove: For every event  $e.t$  in  $\mathbf{M}$





## └ Liveness Properties

## Liveness Properties

## • Progress properties

$$P \rightsquigarrow Q \triangleq \Box(P \Rightarrow \Diamond Q)$$

## • Some important rules

$$\begin{array}{ll} (P \Rightarrow Q) \Rightarrow (P \rightsquigarrow Q) & \text{(Implication)} \\ (P \rightsquigarrow Q) \wedge (Q \rightsquigarrow R) \Rightarrow (P \rightsquigarrow R) & \text{(Transitivity)} \end{array}$$

## Liveness Properties

- Progress properties

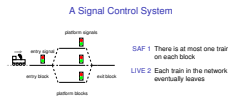
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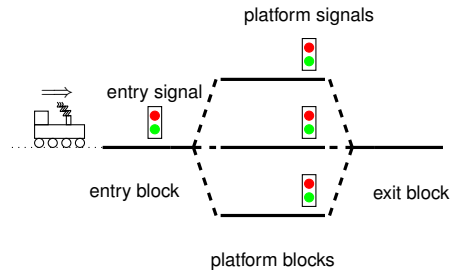
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# A Signal Control System



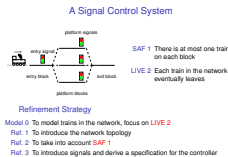
## A Signal Control System



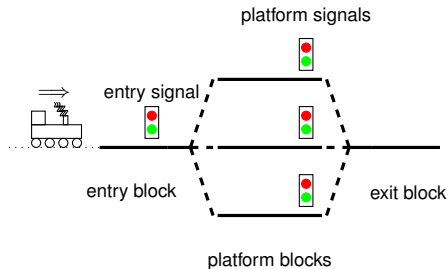
**SAF 1** There is at most one train on each block

**LIVE 2** Each train in the network eventually leaves

## A Signal Control System



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### Refinement Strategy

Model 0 To model trains in the network, focus on LIVE 2

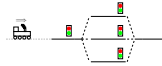
Ref. 1 To introduce the network topology

Ref. 2 To take into account SAF 1

Ref. 3 To introduce signals and derive a specification for the controller

# └ A Signal Control System. The Initial Model

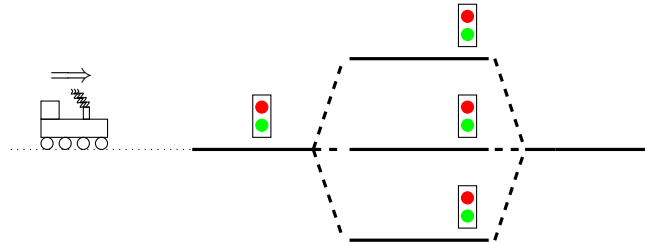
A Signal Control System. The Initial Model  
Sketch  
 LIVE 2 Each **train in the network** eventually leaves



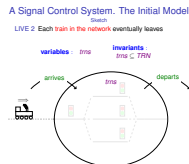
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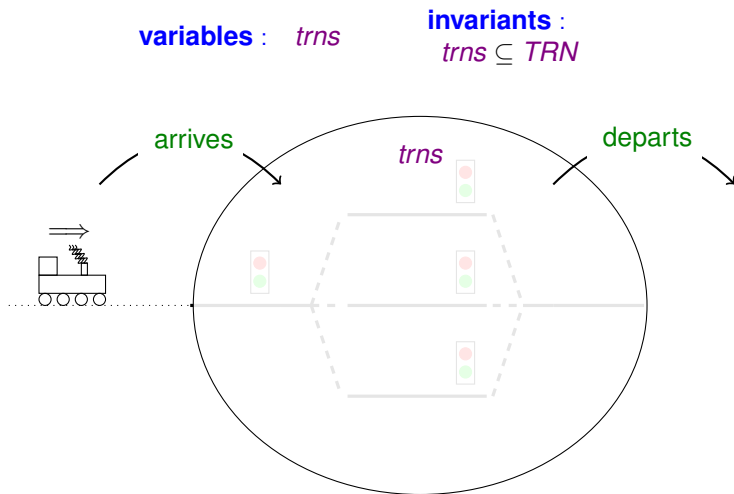
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A Signal Control System. The Initial Model  
Sketch

LIVE 2 Each train in the network eventually leaves

```

variables : trns
invariants : trns ⊆ TRN

arrives
any t where
t ∈ TRN
then
trns := trns ∪ {t}
end

departs
any t where
t ∈ TRN
then
trns := trns \ {t}
end

properties :
prg0_1 : t ∈ trns ⇔ t ∉ trns

```

Note: Free variables are universally quantified.

## A Signal Control System. The Initial Model

Sketch

LIVE 2 Each train in the network eventually leaves

**variables :**  $trns$

**invariants :**  
 $trns \subseteq TRN$

**arrives**

**any**  $t$  **where**  
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**then**

$trns := trns \cup \{t\}$

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**then**

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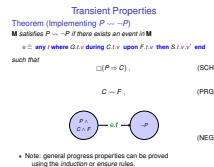
**end**

**properties :**

$prg0\_1 : t \in trns \rightsquigarrow t \notin trns$

Note: Free variables are universally quantified.

## └ Transient Properties



- (SCH) is trivial when  $C$  is  $P$
- (PRG) is trivial when  $F$  is  $\top$

## Transient Properties

Theorem (Implementing  $P \rightsquigarrow \neg P$ )

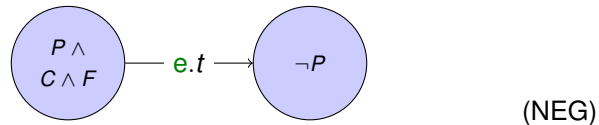
$\mathbf{M}$  satisfies  $P \rightsquigarrow \neg P$  if there exists an event in  $\mathbf{M}$

$e \triangleq$  any  $t$  where  $G.t.v$  during  $C.t.v$  upon  $F.t.v$  then  $S.t.v.v'$  end

such that

$$\Box(P \Rightarrow C), \quad (\text{SCH})$$

$$C \rightsquigarrow F, \quad (\text{PRG})$$



- Note: general progress properties can be proved using the *induction* or *ensure* rules.

# └ A Signal Control System. The Initial Model

$\text{prg0}_1$  holds no matter what the other events of the system are. In particular, we did not need to say that eventually, depart is the only event enabled. It is going to be executed because it is scheduled to. This is one strength of (weak) fairness.

A Signal Control System. The Initial Model  
Properties

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any t where
  t ∈ TRN

prg0_1 : t ∈ trns ⇒ t ∉ trns

then
  trns := trns \ {t}
end

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- (SCH) is trivial.
- No fine-schedule ( $F$  is  $\top$ ) hence (PRG) is trivial.
- The event falsifies  $t \in \text{trns}$  (NEG)

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# Refinement

## Refinement

• **Event-based reasoning.**

$(abs\_e) \triangleq \text{any } t \text{ where } G \text{ during } C \text{ upon } F \text{ then } S \text{ end}$

$(cnc\_f) \triangleq \text{any } t \text{ where } H \text{ during } D \text{ upon } E \text{ then } R \text{ end}$

• **Safety:**

- Guard strengthening:  $H \supset G$
- Action strengthening:  $R \supset S$

• **Liveness:**

- Scheduling assumptions strengthening.
- Schedules weakening:

$$(\Box C \wedge \Box \Diamond F) \Rightarrow \Diamond(\Box D \wedge \Box \Diamond E) \quad (REF\_LIVE)$$

## Refinement

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## └ Schedules Weakening

This rule

$$D \wedge E \Rightarrow F \quad (1)$$

Schedules Weakening  
Practical Rules

$$(\Box C \wedge \Box \Diamond F) \Rightarrow \Diamond(\Box D \wedge \Box \Diamond E) \quad (\text{REF\_LIVE})$$

## Schedules Weakening

Practical Rules

$$(\Box C \wedge \Box \Diamond F) \Rightarrow \Diamond(\Box D \wedge \Box \Diamond E) \quad (\text{REF\_LIVE})$$

## └ Schedules Weakening

This rule

$$D \wedge E \Rightarrow F \quad (2)$$

Schedules Weakening		
Practical Rules		
	$(\Box C \wedge \Box \Diamond F) \Rightarrow \Diamond(\Box D \wedge \Box \Diamond E)$	(REF_LIVE)
Practical rules		
• Coarse-schedule following	$C \wedge F \rightsquigarrow D$	(C_FLW)
• Coarse-schedule stabilising	$D \text{ un } \neg C$	(C_STB)
• Fine-schedule following	$C \wedge F \rightsquigarrow E$	(F_FLW)

## Schedules Weakening

## Practical Rules

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## Practical rules

- Coarse-schedule following

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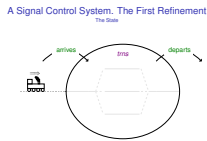
- Coarse-schedule stabilising

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- Fine-schedule following

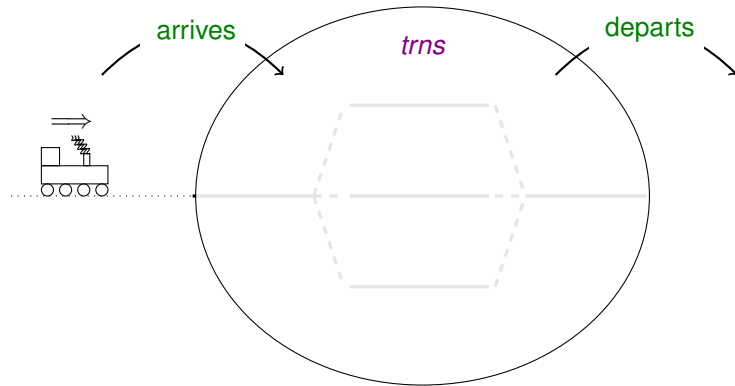
$$C \wedge F \rightsquigarrow E \quad (\text{F\_FLW})$$

## └ A Signal Control System. The First Refinement

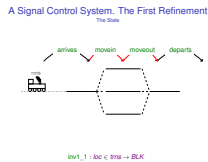


## A Signal Control System. The First Refinement

The State

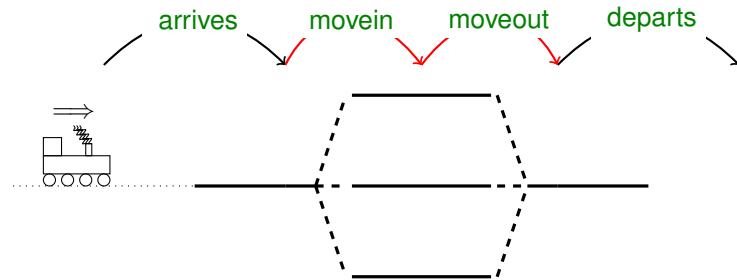


# A Signal Control System. The First Refinement



## A Signal Control System. The First Refinement

The State



# A Signal Control System. The First Refinement

A Signal Control System. The First Refinement  
Refinement of **departs**

```

(abs_)departs
any t where
  t ∈ TRN
during
  t ∈ trns
then
  trns := trns \ {t}
end

(cnc_)departs
any t where
  t ∈ trns ∧ loc.t = Exit
during
  t ∈ trns ∧ loc.t = Exit
then
  trns := trns \ {t}
  loc := {t} ≺ loc
end

```

- Guard and action strengthening are trivial.
- Coarse-schedule following (amongst others):

$$t \in trns \rightsquigarrow t \in trns \wedge loc.t = Exit \quad (\text{prg1\_1})$$

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## └ A Signal Control System. The First Refinement

A Signal Control System. The First Refinement

New Event **moveout**

```

moveout
  any  $t$  where
     $t \in trns \wedge loc.t \in PLF$ 
  during
     $t \in trns \wedge loc.t \in PLF$ 
  then
     $loc.t := Exit$ 
  end

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# └ A Signal Control System. The Second Refinement

A Signal Control System. The Second Refinement

The State

SAF 1 There is at most one train on each block

invariants :

$$\forall t_1, t_2 \cdot t_1 \in trns \wedge t_2 \in trns \wedge loc.t_1 = loc.t_2 \Rightarrow t_1 = t_2$$

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A Signal Control System. The Second Refinement  
Refinement of *moveout*

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- Neither weak- nor strong-fairness is satisfactory.
  - Weak-fairness requires *Exit* to be free infinitely long.
  - Strong-fairness is too strong assumption.

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## Summary

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## The Unit-B Modelling Method

- Guarded and **scheduled** events.
- Reasoning about **liveness (progress)** properties.
- **Refinement** preserving safety and liveness properties.
- Developments are **guided by safety and liveness requirements**.

## Future Work

- Decomposition / Composition
- Tool support

## Summary

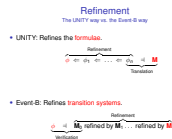
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# Refinement



## Refinement

The UNITY way vs. the Event-B way

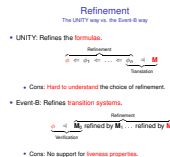
- UNITY: Refines the **formulae**.

$$\underbrace{\phi \Leftarrow \phi_1 \Leftarrow \dots \Leftarrow \phi_n}_{\text{Refinement}} \models \underbrace{\mathbf{M}}_{\text{Translation}}$$

- Event-B: Refines **transition systems**.

$$\underbrace{\phi}_{\text{Verification}} \models \underbrace{\mathbf{M}_0 \text{ refined by } \mathbf{M}_1 \dots \text{ refined by } \mathbf{M}}_{\text{Refinement}}$$

# Refinement



## Refinement

The UNITY way vs. the Event-B way

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Refinement  
Translation

- Cons: **Hard to understand** the choice of refinement.
- Event-B: Refines **transition systems**.

$$\phi \Leftarrow \mathbf{M}_0 \text{ refined by } \mathbf{M}_1 \dots \text{ refined by } \mathbf{M}$$

Refinement  
Verification

- Cons: No support for **liveness properties**.

## └ Execution of Unit-B Models

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$$ex.M \equiv saf.M \wedge live.M \quad (3)$$

$$saf.M \equiv \text{init}.M \wedge \Box step.M \quad (4)$$

$$step.M \equiv (\exists e.t \in M.act.(e.t)) \vee SKIP \quad (5)$$

$$live.M \equiv \forall e.t \in M.sched.(e.t) \quad (6)$$

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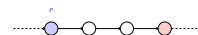
# The Ensure Rule

## The Ensure Rule

Theorem (The ensure-rule)

For all state predicates  $p$  and  $q$ ,

$$(P \text{ un } Q) \wedge ((P \wedge \neg Q) \rightsquigarrow (\neg P \vee Q)) \Rightarrow (P \rightsquigarrow Q) \quad (\text{ENS})$$

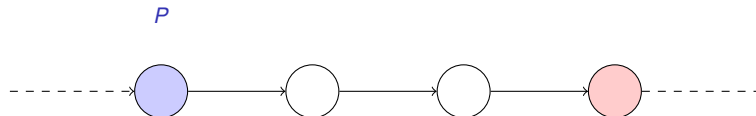


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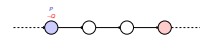
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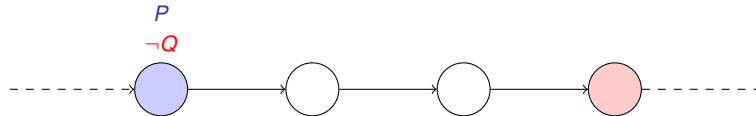


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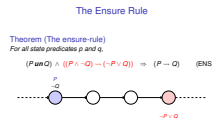
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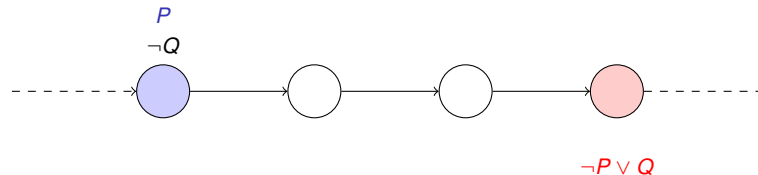


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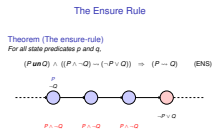
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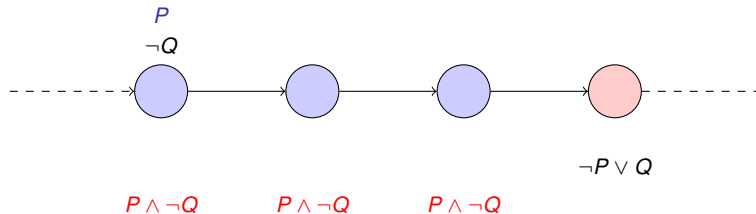


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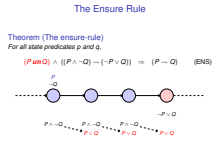
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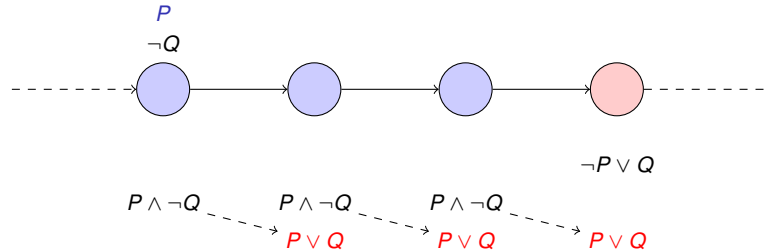


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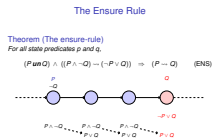
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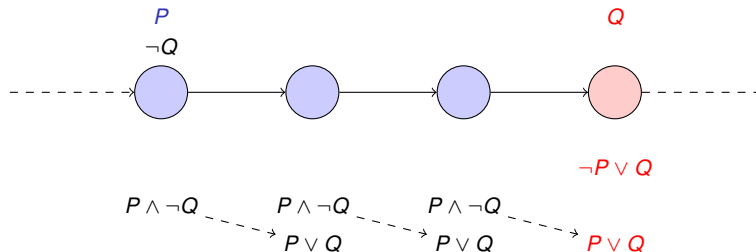


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# └ The specification of the controller

The specification of the controller

```
ctrl_platform
any p where
  p ∈ PLF ∧ p ∈ ran.loc ∧ Exit ∉ ran.loc ∧
  ∀q·q ∈ PLF ⇒ sgn.q = RD
during
  p ∈ PLF ∧ p ∈ ran.loc ∧ sgn.p = RD
upon
  Exit ∉ ran(loc) ∧ ∀q·q ∈ PLF ∧ q ≠ p ⇒ sgn.q = RD
then
  sgn.p := GR
end
```

## The specification of the controller

ctrl\_platform

any p where

$$p \in PLF \wedge p \in \text{ran}.loc \wedge \text{Exit} \notin \text{ran}.loc \wedge$$

$$\forall q \cdot q \in PLF \Rightarrow \text{sgn}.q = RD$$

during

$$p \in PLF \wedge p \in \text{ran}.loc \wedge \text{sgn}.p = RD$$

upon

$$\text{Exit} \notin \text{ran}(loc) \wedge \forall q \cdot q \in PLF \wedge q \neq p \Rightarrow \text{sgn}.q = RD$$

then

$$\text{sgn}.p := GR$$

end