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ABSTRACT
The VXX option market has grown in popularity alongside the VXX ETN market in activity and size of oustanding positions, yet there is no complete VXX option pricing model. This paper is the first to document and analyze the implied volatility (IV) curves of the VXX options market, by applying the methodology of Zhang and Xiang, providing a necessary benchmark for developing a VXX option pricing model. The IV curves of the VXX options market do not exhibit the typical smirk shape, as for S&P 500 options, but rather an upward-sloping almost linear curve.

KEYWORDS
Implied volatility (IV); IV smirk; VIX futures ETN; VXX; VXX options

JEL CLASSIFICATION
G13

I. Introduction

This is the first paper to quantify and analyze the implied volatility (IV) curve of the VXX options market. Quantifying the VXX's IV curve provides a benchmark for creating a VXX option pricing model founded on the empirical observations of the VXX return risk-neutral moments. Understanding the dynamics of the VXX options market will help to determine the correct starting assumptions for such a model as the IV has all the information of market option prices and reflects the risk-neutral distribution of the underlying asset returns over different horizons.

The market for trading volatility derivatives has developed swiftly over the past 15 years. The VIX index was revised in 2003 to track the IV of S&P 500 OTM options. Later, on 26 March 2004, the CBOE launched the first futures contracts on the VIX index providing the first access to VIX exposure, which was desirable for its possible hedging/diversification benefits. In 2006, VIX options started to trade. In 2009, Standard & Poor's started calculating VIX futures indices, such as the S&P 500 VIX Short-Term Futures Total Return index (SPVXSTR). The VIX futures indices track a daily rebalanced position of VIX futures contracts to achieve an almost constant one-month maturity. Shortly after, on 29 January 2009, Barclays Capital iPath launched the first VIX futures exchange-traded product (ETP), the VXX exchange-traded note (ETN).[1] Finally, on 28 May 2010, VXX and VXZ options markets were launched by the CBOE. Now options markets exist for many of the popular VIX futures ETPs.[2]

The VIX futures ETPs and their option markets have become very popular. The VXX is the most popular VIX futures ETP with an average market cap of about \$1 billion and average daily dollar trading volume of \$1.4 billion.[3] The VXX options market has grown to an average daily trading volume of 328,884 and average daily open interest of 3,097,297 contracts.[4] In Figure 1 we can see the growth of the VXX options market over our sample. The open interest in VXX options has grown consistently from 2010 to 2016, reaching about three million contracts recently, while the trading volume has hovered around 300,000 contracts per day since 2013. The other VIX futures ETP options have also grown in popularity, but their trading volume and open interest are still quite low, relative to the VXX options. This is why our study focuses on the VXX option market's IV curve.

The VIX futures ETPs have recently been making headlines as a spike in volatility led to

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[1]An ETN is a non-securitized debt obligation, similar to a zero-coupon bond, but with a redemption value that depends on the level of something else, i.e. the SPVXSTR index for the VXX ETN.
[2]The VXZ is the much less liquid mid term maturity version of the VXX, tracking the S&P 500 VIX Mid-Term Futures Total Return index (SPVXMTR).
[3]Averages described here are taken over the last year of our sample.
[4]For a detailed comparison between S&P 500 index (SPX), S&P 500 index ETF (SPY), VIX and VIX futures ETP option markets, please refer to Table 1.

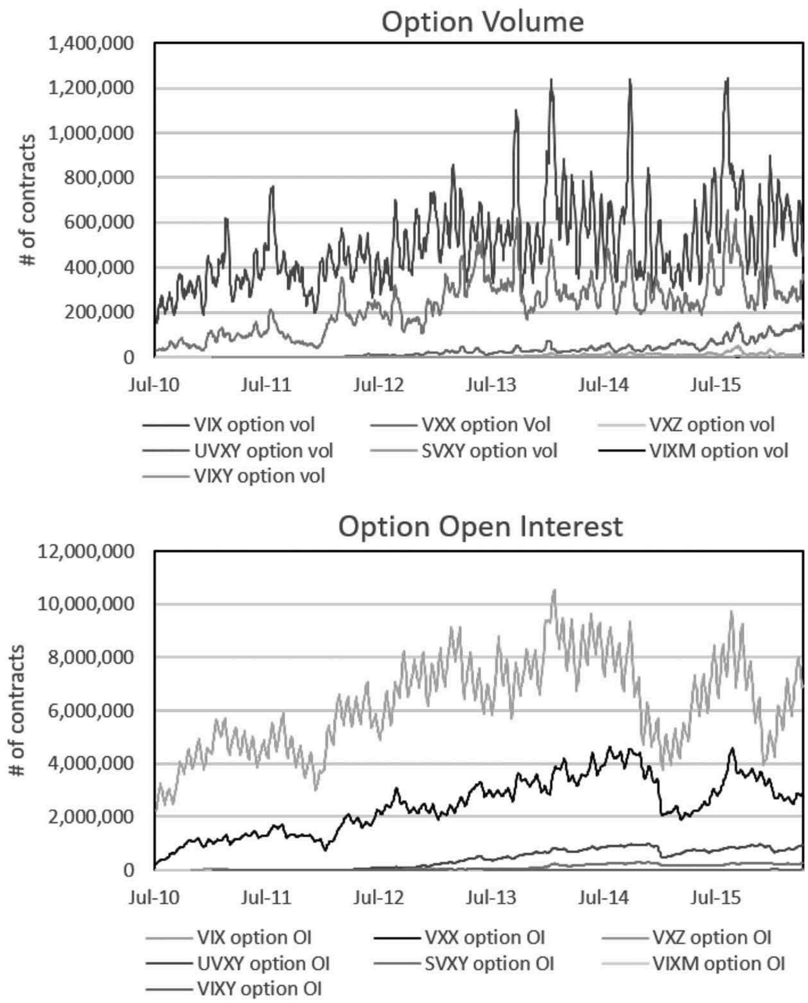

Figure 1. Option volume and open interest. This figure shows the 10-day moving average of the daily trading volume and open interest of the VIX, VXX, VXZ, UVXY, SVXY, VIXM and VIXY option markets. The values for the VXZ, SVXY, VIXM and VIXY markets are very small and therefore can hardly be observed in this figure.

unprecedented losses in the inverse exposure ETPs, some of which were even terminated. One article states "The problem with ETFs is that many of them appeal to retail investors, but are really meant for institutions" (Dillian 2018). However, even after the increased media attention on the complexity and risk in these products, trading activity in their options is picking up again as noted by a recent article by Bloomberg (Kawa 2018). These products are traded like stocks and accessible to retail investors, but possess complexities that even academics and highly trained institutional investors do not yet fully understand. Providing some more insight on these products is of the utmost importance in order to avoid unexpected outcomes, often not even considered as a possibility for retail investors.

We use the method of Zhang and Xiang (2008) to quantify the VXX IV curve for any maturity each day by three factors; the level, slope and curvature. This allows us to summarize the often vast number of IV-moneyness (strike price) data points with just three numbers. We can then examine the dynamics of these factors to draw conclusions on how the VXX options market behaves. Zhang and Xiang (2008) also provide a link between the IV curve factors and the risk-neutral moments and demonstrate how this can be used to calibrate option pricing models. They develop this methodology and demonstrate its application for a very small sample, whereas we apply the methodology to the VXX options market and extend it by studying the term structure and time series of the quantified IV curve. Fajardo

Table 1. Summary of SPX, SPY, VIX and VIX futures ETP option markets.

	SPX options	SPY options	VIX options	VXX options	VXZ options	UVXY options	SVXY options	VIXM options	VIXY options
Style	European	American	European	American	American	American	American	American	American
Expiration	3rd Friday	3rd Friday	30 days before 3rd Friday	3rd Friday	3rd Friday	3rd Friday	3rd Friday	3rd Friday	3rd Friday
Settlement	Cash	Physical	Cash	Physical	Physical	Physical	Physical	Physical	Physical
Underlying	SPX	SPY	VIX futures with same maturity	VXX	VXZ	UVXY	SVXY	VIXM	VIXY
Multiplier	$100 × index	100 × price	$100 × index	100 × price	100 × price	100 × price	100 × price	100 × price	100 × price
Average Daily Option Volume	1,071,517	2,715,706	643,266	328,884	312	91,349	17,254	52	1,278
Average Daily Open Interest	12,388,390	19,429,209	6,785,566	3,097,297	7,304	810,048	223,809	759	15,707
Underlying Average Daily Volume (000,000's)		$24,641	–	$1,444	$6	$799	$257	$1	$40
Underlying Average Market Cap. (000,000's		$176,045	–	$965	$53	$492	$476	$28	$126

(2017) adds a torsion factor into the polynomial regression of Zhang and Xiang (2008), which quantifies the IV curve, but this is a model-based factor and we want to keep our quantification simple. Our method captures close to 100% of the variation of the daily VXX IV curves without this extra factor.

This study contributes to the literature by being the first empirical study of the dynamics of the VXX options market. We document and provide a comprehensive study of the VXX option IV dynamics as a starting point for developing VXX option pricing models in the future.[5] We show that the IV curve of the VXX is usually an upward-sloping line with some convexity. As the options maturity increases the at-the-money (ATM) IV increases, the IV curve's slope decreases and it becomes more convex. Our quantification of the VXX's IV curve performs well with an average R-squared value of 94.55%. The fit is best for shorter maturity options, with an average R-squared of 98.49% for less than and 83.65% for more than 180 days to maturity. Our results suggest that a VXX option pricing model must be flexible enough to exhibit a volatility of arounr 69%, positive skewness and very little kurotosis in the risk-neutral distribution of VXX returns. Although during market turmoil it has to be able to produce brief periods where the term-structures of the risk-nuetral moments invert, negative skewness and larger magnitude kurtosis. The risk-neutral volatility and skewness of such a model need to be mean-reverting over time.

Although there is a growing literature on pricing volatility derivatives (Zhang and Zhu 2006; Zhang, Shu, and Brenner 2010; Lu and Zhu 2010; Chung et al. 2011; Wang and Daigler 2011; Zhu and Lian 2012; Mencía and Sentana (2013); (Huskaj and Nossman 2013; Lian and Zhu 2013; Bardgett, Gourier, and Leippold 2019; Papanicolaou and Sircar 2014; Eraker and Wu 2017; Gehricke and Zhang 2018a); and many more) and their empirical dynamics (Shu and Zhang 2012; Whaley 2013; Bordonado, Molnáar, and Samdal 2017; Bollen, O'Neill, and Whaley 2017; Gehricke and Zhang 2018b; and many more), there is only one published paper looking at a VIX futures ETP options

[5]The model of (Gehricke and Zhang 2018a) could be useful to price VXX options.

market. Bao, Li, and Gong (2012) provide the only study on VXX options proposing and horse-racing several models for pricing the contracts. The authors, however, ignore the underlying relationships among the VXX VIX futures, VIX index and S&P 500 index, which are essential to understanding the VXX options market fully.

Many studies have documented the S&P 500 option IV shape and/or its dynamics (Rubinstein 1985, 1994; A¨ıt-Sahalia and Lo 1998; Skiadopoulos, Hodges, and Clewlow 2000; Cont and Da Fonseca 2002; Carr and Wu 2003; Foresi and Wu 2005; Garleanu, Pedersen, and Poteshman 2009). Some authors have tried to explain the shape/dynamics of the IV curve through other market and economic factors (Pena, Rubio, and Serna 1999; Pan 2002; Dennis and Mayhew 2002; Bollen and Whaley 2004). The predictability power of option market IV for the underlying assets return has also been explored (Xing, Zhang, and Zhao 2010; Cremers and Weinbaum 2010; Conrad, Dittmar, and Ghysels 2013; Lin and Lu 2015).

In the next section we describe the methodology used to quantify the IV curve. In section 3 we describe our sample data and cleaning procedure. Then in section 4 we present the results and describe the dynamics of the VXX's IV. Lastly, in section 5 we conclude.

II. Methodology

Implied forward price and ATM IV

In this paper we employ the methodology developed by Zhang and Xiang (2008) in order to summarize the VXX option IV curve (IV as a function of option moneyness), every day and for each maturity. For this we first calculate the implied forward price based on the ATM call and put prices as follows:[6]

$$F_t^{T_i} = K^{ATM} + e^{r_{i,t}\tau_{i,t}}(c_{i,t}^{ATM} - p_{i,t}^{ATM}), \tag{1}$$

where $F_t^{T_i}$ is the implied forward price, K is the ATM strike price, $r_{i,t}$ is the risk-free rate, $\tau_{i,t}$ is the annualized time to maturity, $c_{i,t}^{ATM}$ is the ATM call option price and $p_{i,t}^{ATM}$ is ATM put option price, for maturity T_i on day t.

Moneyness of options

We use the implied forward price to measure the moneyness of an option as follows:

$$\xi = \frac{\ln \frac{K}{F_t^{T_i}}}{\sigma\sqrt{\tau_i}}, \tag{2}$$

where K is the strike price we are calculating the moneyness for and $F_t^{T_i}$ is the implied forward price for maturity T_i on day t. σ is the average volatility of the underlying, which we proxy by the 30-day ATM IV.[7] Lastly, $\tau_i = (T_i - t)/365$ is the annualized time to maturity of the given expiry option contracts.

Quantified IV curve

Having calculated the moneyness of the options, we can quantify the IV curve by fitting the regression:

$$IV(\xi) = \alpha_0 + \alpha_1\,\xi + \alpha_2\,\xi^2, \tag{3}$$

where IV is the IV and ξ is the moneyness of the option.[8] The regression is fitted separately each day and for each maturity. Here, the estimated coefficients $\hat{\alpha}_0$, $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are termed the intercept, unscaled slope and unscaled curvature, respectively. We estimate this quadratic function to the IV by minimizing the volume-weighted mean-squared error given by:

$$VWMSE = \frac{\sum_\xi Vol(\xi) \times [IV_{MKT}(\xi) - IV_{MDL}(\xi)]^2}{\sum_\xi Vol(\xi)}, \tag{4}$$

where $Vol(\xi)$ is the volume, $IV_{MKT}(\xi)$ is the IV from market prices and $IV_{MDL}(\xi)$ is the model IV,

[6]ATM is defined as the strike price where the difference between call and option prices is the smallest. This is not exactly at the money and we will be providing an estimate of exactly ATM IV using Equation 3, which we call the ``exactly ATM IV''.

[7]The 30 day ATM VXX IV is calculated by linearly interpolating the two nearest to 30 day maturity ATM implied volatilitys as $IV^{\tau} = IV^{\tau_1} w_1 + IV^{\tau_2}(1 - w_1)$, where $w = \frac{\tau - \tau_2}{\tau_1 - \tau_2}$.

[8]The IV is supplied by OptionMetrics Ivy DB and is calculated using the Cox, Ross, and Rubinstein (1979) binomial tree model and a proprietary algorithm to speed up convergence.

for the option with moneyness ξ, on a particular day for a given maturity. When estimating the IV function we only use OTM options, as is industry practice.[9] This means that when the strike price is above (below) the implied forward price, that is, $K_{i,t} > F_{i,t}$ ($K_{i,t} < F_{i,t}$), we only use call (put) options in estimating the IV curve.

Efficiently estimating the parameters in Equation (3) should allow us to describe the entire volatility smirk for a given maturity on a - certain day with just three parameters. We then document these parameters across time and maturities in order to describe and explore the dynamics of the VXX options market. We will present the results of the regressions with and without a constraint forcing the line to go through the ATM IV point in section 4[10].

We can also present the parameters in a dimensionless form as follows:

$$IV(\xi) = \gamma_0(1 + \gamma_1\xi + \gamma_2\xi^2), \tag{5}$$

where

$$\gamma_0 = \alpha_0, \quad \gamma_1 = \frac{\alpha_1}{\alpha_0}, \quad \gamma_2 = \frac{\alpha_2}{\alpha_0},$$

where γ_0 is the level, γ_1 is the slope and γ_2 is the curvature factor. We can interpret the level coefficient ($\alpha_0 = \gamma_0$) as the exact ATM IV where moneyness is actually equal to zero, which will be slightly different to the ATM IV available in the market data, whose moneyness is the closest to zero available.

Risk-neutral moments

Transforming the coefficients of the regressions (α's) to the dimensionless factors(γ's), as above, allows us to calculate the moments of the risk-neutral distribution of the VXX, as in Zhang and Xiang (2008). They show that the risk-neutral standard deviation, skewness and excess kurtosis (σ, λ_1, λ_2) are related to the level, slope and curvature (γ_0, γ_1 and γ_2) by:

$$\gamma_0 \approx \left(1 - \frac{\lambda_2}{24}\right)\sigma, \quad \gamma_1 \approx \frac{1}{6}\lambda_1, \quad \gamma_2 \approx \frac{1}{24}\lambda_2.$$

These relationships can be used to approximate the risk-neutral moments of the VXX from the IV curve factors. Once one has the risk-neutral moments these can be used to calibrate VXX option pricing models, as Zhang and Xiang (2008) demonstrate for S&P 500 options.

III. Data

Our sample is from 1 June 2010 to 29 April 2016. The options data are sourced from OptionMetrics, a widely used and very reliable source. The VXX options are American style; therefore, the IV is computed using an algorithm based on the binomial tree model of Cox, Ross, and Rubinstein (1979), by OptionMetrics. We obtain the Treasury yield data from the U.S. Department of the Treasury website.[11]

We apply the following standard option data filters to the option data, following previous work by Bakshi, Cao, and Chen (1997), Zhang and Xiang (2008) and the VIX index option data cleaning methodology.

- We remove option quotes where the open interest, bid price or IV is zero or missing.
- We remove option quotes with a maturity of less than six days.

In Table 2 we summarize the trading activity of the VXX options market overall and by maturity category after cleaning the data, as above. In the table we can see that as the maturity of the options contracts increases the number of observations, mean number of strikes, mean daily trading volume and mean open interest all decrease substantially. Most of the trading in VXX options happens in the shorter maturity contracts.

[9]This is because OTM options are more liquid and are more sensitive to pricing models.
[10]We constrain the regression to go through the ATM IV because then the fitted volatility curve gives the same price as the market for ATM options. This means that there is no arbitrage between the model price and market price for ATM options.
[11]If there are no yield data on a day where there are option data we use the previous days value.

Table 2. Summary of the VXX option market activity. This table shows the mean and median daily number of strikes, trading volume and open interest for the VXX option market. The statistics are shown overall and for each maturity category. The statistics for the daily open interest or volume are calculated as the mean/median of the daily trading volume for each maturity, either overall or by the maturity category grouping.

		By Maturity (days)						
	Overall	< 30	$30-90$	$90-180$	$180-360$	>360	<180	>180
Number of observations	12,701	3,226	3,680	2,141	1,846	1,808	9,047	3,654
Mean number of strikes	38	47	42	38	26	21	43	24
Median number of strikes	36	45	41	37	24	20	42	22
Mean volume	17,035	37,149	19,229	6,744	3,444	2,745	22,664	3,098
Median volume	5,319	24,315	9,351	3,490	1,350	1,000	10,114	1,174
Mean open interest	168,518	220,756	186,588	149,706	121,605	108,708	190,044	115,224
Median open interest	82,613	90,839	95,233	100,745	31,819	71,735	94,148	54,131

Table 3. Summary of implied volatility function estimation. This table shows summary statistics of the estimated implied volatility function.

$$IV(\xi) = a_o + a_1\xi + a_2\xi^2$$

where IV is the implied volatility and ξ is the moneyness of the option. The regression is fitted separately each day and for each maturity. To estimate we minimize the volume-weighted squared errors. Here, $\hat{a}_0$, $\hat{a}_1$ and $\hat{a}_2$ are the unscaled level, slope and curvature coefficients, respectively. The mean, median and standard deviation values are calculated overall and by maturity category[a]. The percentage of significant parameter estimates is the percentage of parameter estimates that are significant at the 5% level of significance over the entire sample, in each maturity category. The mean volume is calculated as the mean of the daily sum of the trading volume for each maturity, either overall or by the maturity category grouping. The volume in this table is different than in Table 2 because it includes only the OTM options used to create the implied volatility curves. We also present the mean and standard deviation of the forward price overall and by maturity category.

		By Maturity (days)						
	Overall	< 30	$30-90$	$90-180$	$180-360$	>360	<180	>180
Mean								
$F_t^{T_i}$	26.2128	26.4525	26.2454	25.6720	26.3190	26.2624	26.1846	26.2911
$\hat{a}_0$	0.6873	0.6445	0.6847	0.7134	0.7077	0.7243	0.6770	0.7159
$\hat{a}_1$	0.0998	0.1322	0.1298	0.1010	0.0683	−0.0032	0.1239	0.0330
$\hat{a}_2$	0.0034	0.0034	−0.0010	0.0005	0.0038	0.0169	0.0009	0.0102
γ_0	0.6873	0.6445	0.6847	0.7134	0.7077	0.7243	0.6770	0.7159
γ_1	0.1527	0.2151	0.1949	0.1441	0.0989	−0.0029	0.1902	0.0487
γ_2	0.0051	0.0054	−0.0013	0.0008	0.0056	0.0240	0.0016	0.0147
Standard deviation								
$F_t^{T_i}$	9.9458	9.6291	9.3908	9.8575	10.6165	11.1207	9.5925	10.8666
$\hat{a}_0$	0.1338	0.1807	0.1346	0.0988	0.0808	0.0595	0.1486	0.0716
$\hat{a}_1$	0.0627	0.0358	0.0409	0.0395	0.0430	0.0627	0.0408	0.0644
$\hat{a}_2$	0.0247	0.0108	0.0140	0.0159	0.0226	0.0538	0.0136	0.0416
γ_0	0.1338	0.1807	0.1346	0.0988	0.0808	0.0595	0.1486	0.0716
γ_1	0.0975	0.0634	0.0640	0.0575	0.0643	0.0864	0.0679	0.0915
γ_2	0.0341	0.0167	0.0210	0.0212	0.0300	0.0732	0.0198	0.0564
% significant coefficients								
$\hat{a}_0$	99.99%	99.97%	100.00%	100.00%	100.00%	100.00%	99.99%	100.00%
$\hat{a}_1$	94.23%	99.53%	99.75%	98.76%	90.98%	68.34%	99.44%	79.81%
$\hat{a}_2$	64.73%	73.58%	69.07%	61.67%	50.98%	55.07%	68.96%	53.00%
Daily R-Squared								
mean R^2	94.55%	98.98%	98.88%	97.07%	91.20%	75.91%	98.49%	83.65%
std. dev. R^2	13.79%	2.65%	3.23%	8.17%	15.83%	24.91%	4.80%	22.17%
Daily trading volume								
mean Volume	14,296	29,529	16,114	5,391	3,065	2,569	18,435	2,820

[a]There can be more than one expiry in the same maturity category on any given day.

IV. Empirical results

Quantified IV curve

In this section we present and analyze the dynamics of the quantified IV curve of the VXX options market, as well as those of the option implied VXX forward price.

Table 3 shows a summary of the implied VXX forward price, the quantified IV curve coefficients (α_0, α_1 and α_2) and the proportion of curves for

Table 4. Summary of implied volatility function ATM constrained estimation. This table shows summary statistics of the estimated implied volatility function.

$$IV(\xi) = \alpha_o + \alpha_1\xi + \alpha_2\xi^2,$$

when it is forced to pass through the point at-the-money. Here, IV is the implied volatility and ξ is the moneyness of the option. The regression is fitted separately each day and for each maturity. To estimate we minimize the volume-weighted squared errors. Here, $\hat{\alpha}_0$, $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the unscaled level, slope and curvature coefficients, respectively. The mean, median and standard deviation values are calculated overall and by maturity category[a]. The percentage of significant parameter estimates is the percentage of parameter estimates that are significant at the 5% level of significance over the entire sample, in each maturity category. The mean daily trading volume is calculated as the mean of the daily sum of the trading volume for each maturity category. The volume in this table is different than in Table 2 because it includes only the OTM options used to create the implied volatility curves. We also present the mean and standard deviation of the forward price for each maturity category.

		By Maturity (days)						
	Overall	<30	$30-90$	$90-180$	$180-360$	>360	<180	>180
Mean								
$F_t^{T_i}$	26.2128	26.4525	26.2454	25.6720	26.3190	26.2624	26.1846	26.2911
$\hat{\alpha}_0$	0.6854	0.6413	0.6848	0.7123	0.7064	0.7187	0.6756	0.7125
$\hat{\alpha}_1$	0.0982	0.1322	0.1293	0.0993	0.0654	−0.0091	0.1233	0.0286
$\hat{\alpha}_2$	0.0049	0.0035	−0.0012	0.0015	0.0061	0.0248	0.0011	0.0153
γ_0	0.6854	0.6413	0.6848	0.7123	0.7064	0.7187	0.6756	0.7125
γ_1	0.1508	0.2165	0.1942	0.1421	0.0949	−0.0114	0.1899	0.0425
γ_2	0.0077	0.0060	−0.0014	0.0025	0.0093	0.0366	0.0022	0.0228
Standard deviation								
$F_t^{T_i}$	9.9458	9.6291	9.3908	9.8575	10.6165	11.1207	9.5925	10.8666
$\hat{\alpha}_0$	0.1356	0.1827	0.1358	0.1005	0.0824	0.0663	0.1504	0.0751
$\hat{\alpha}_1$	0.0673	0.0368	0.0416	0.0414	0.0473	0.0789	0.0421	0.0748
$\hat{\alpha}_2$	0.0388	0.0128	0.0167	0.0237	0.0382	0.0882	0.0176	0.0683
γ_0	0.1356	0.1827	0.1358	0.1005	0.0824	0.0663	0.1504	0.0751
γ_1	0.1040	0.0662	0.0652	0.0603	0.0699	0.1083	0.0704	0.1053
γ_2	0.0532	0.0195	0.0247	0.0309	0.0498	0.1208	0.0249	0.0929
% significant parameters								
$\hat{\alpha}_1$	92.67%	99.47%	99.69%	97.78%	84.89%	64.16%	99.16%	74.66%
$\hat{\alpha}_2$	67.95%	73.94%	67.83%	64.41%	59.88%	69.05%	69.23%	64.41%
Daily R-Squared								
mean R^2	92.78%	98.51%	97.84%	94.51%	86.14%	74.20%	97.30%	80.25%
std. dev. R^2	15.40%	3.44%	5.32%	11.04%	20.81%	24.49%	6.85%	23.47%
Daily trading volume								
mean Volume	14,296	29,529	16,114	5,391	3,065	2,569	18,435	2,820

[a]There can be more than one expiry in the same maturity category on any given day.

which they are significant, the quantified IV curve factors (γ_0, γ_1 and γ_2), the goodness of fit of the regressions (R-squared) and the trading volume. The summary statistics are provided overall and by maturity category.[12] Table 4 shows the same statistics but for the regressions that are constrained to pass through the ATM IV.

In Tables 3 and 4 we can see that the mean implied VXX forward price across the entire sample and for all maturities is 26.21. Examining the mean forward price by maturity category shows that the implied forward price decreases as the maturity increases, from 26.45 to 26.26, for less than 30- and more than 360-day maturities, respectively. Therefore, the term structure of the implied forward price is in backwardation (downward sloping), on average. Also, the variation (standard deviation) of the implied forward price is 9.95 overall and tends to increase as the maturity becomes longer.

The level coefficient ($\hat{\alpha}_0 = \gamma_0$), which is an estimate of the exact ATM IV, is 0.6873 (0.6854) on average, for the un-constrained (constrained) regressions.[13] The mean level monotonically increases from 0.6445 (0.6413) to 0.7243 (0.7187), for less than 30 and more than 360 days to maturity, respectively. Therefore, the term structure of the exact ATM IV is usually in contango. Its standard deviation is 0.1338 (0.1356) overall and decreases as maturity increases. This shows us that, on average,

[12]The maturity categories are based on the days to maturity of the contracts; therefore, for some days there will be multiple maturities in one category.
[13]$\alpha_0 = \gamma_0$ is an estimate of the exact ATM IV, whereas the market ATM IV is the IV where the call and put prices are closest, that is, the closest available strike price to ATM.

the long-term projections of VXX volatility by option traders are higher than the short term and that their long term volatility projections are more consistent throughout the sample. This would be consistent with VXX option traders believing that the ATM VXX volatility mean-reverts to some long-run level. The level coefficient is significant at the 5% level for essentially 100% of the fitted IV curves. This can be expected, as the exact ATM implied volatility should never be zero or negative.

Looking at the slope factor we can see that, on average and over all maturities, the IV curves are upward sloping, as the overall mean γ_1 is positive for the unconstrained and constrained regressions. This is different to the usual left skewed 'smirk' shape usually found (Foresi and Wu 2005) for equity and equity index option markets. On average, as the VXX option maturity increases the slope becomes less steep and can even turns downward sloping for maturities over 360 days. The unconstrained (constrained) slope, γ_1 goes from 0.2151 (0.2165) to −0.0029 (−0.0114), for less than 30 and more than 360 days to maturity curves, respectively. The term structure of the slope factor is, on average, in backwardation. The unscaled slope coefficient is highly significant with 99.44% of fitted IV curves, with less than 180 and 79.81% of fitted IV curves with over 180 days to maturity showing significant slope coefficients, at the 5% level .

The last quantified IV curve factor is the curvature, γ_2. We can see that, on average and for all maturities, it is positive, meaning the VXX IV curves are usually convex. However, it is also very small in magnitude, so the convexity is not very prominent. The overall average curvature factor is 0.0051 (0.0077) for the un-constrained (constrained) regression. The unscaled curvature coefficient is significant for 64.73% (67.95%) of the fitted IV curves overall. The proportion of quantified IV curves with significant curvature coefficients decreases slightly for longer-maturity categories. The magnitude of the mean curvature factor estimates increases with maturity, meaning that as maturity increases the IV curves tend to become more convex. The average curvature factor is 0.0054 (0.0060) for less than 30-day and 0.0240 (0.0366) for more than 360-day maturity curves.

Constraining the regressions to fit the ATM IV exactly results in a lower level, flatter and more convex quantified IV curves on average, overall and for most maturity categories, but these differences are of small magnitude.

The reason both the slope and curvature become less significant and the R-squared values become much lower for maturities over 180 days, as seen in Tables 3 and 4, may be that traders' opinions on volatility are less reliable resulting in less consistently shaped IV curves. The lower trading volume may also be indicative of less efficient/informative prices at longer maturities.

Figures 2, 3 and 4 show the IV curves, the trading volume of each contract and the fitted line for the unconstrained regression on 27 July 2011, 2 August 2013 and 27 May 2015, respectively. Figures 5, 6 and 7 show the same information but for the constrained quantified IV curves. We can see good examples of the usually upward-sloping and almost linear curves at shorter maturities. As the maturity increases the fitted lines become more convex, consistent with the mean results discussed above.

In Figure 8 we show the average fitted IV curves, that is, the predicted curves resulting from the mean factors presented in Tables 3 and 4. We can clearly see the pattern described above; as the maturity increases the IV curve's slope decreases and they become more convex. Most maturity average IV curves are upward-sloping lines with some convexity, but the longer than 360 days to maturity lines look like the IV smirk found in the S&P 500 options market. We can also see that when the regressions are constrained to cross the ATM IV point, they become more smirked (skewed to the left), this is most apparent in the line for maturities longer than 360 days.

These findings, combined with the theoretical link presented by Zhang and Xiang (2008), tell us that, on average, a VXX option pricing model should be able to produce a risk-neutral distribution with a volatility of around 69%, positive skewness and some small positive kurtosis. The average term-structures of the risk-neutral volatility, skewness and kurtosis should be in contango, backwardation and contango, respectively.

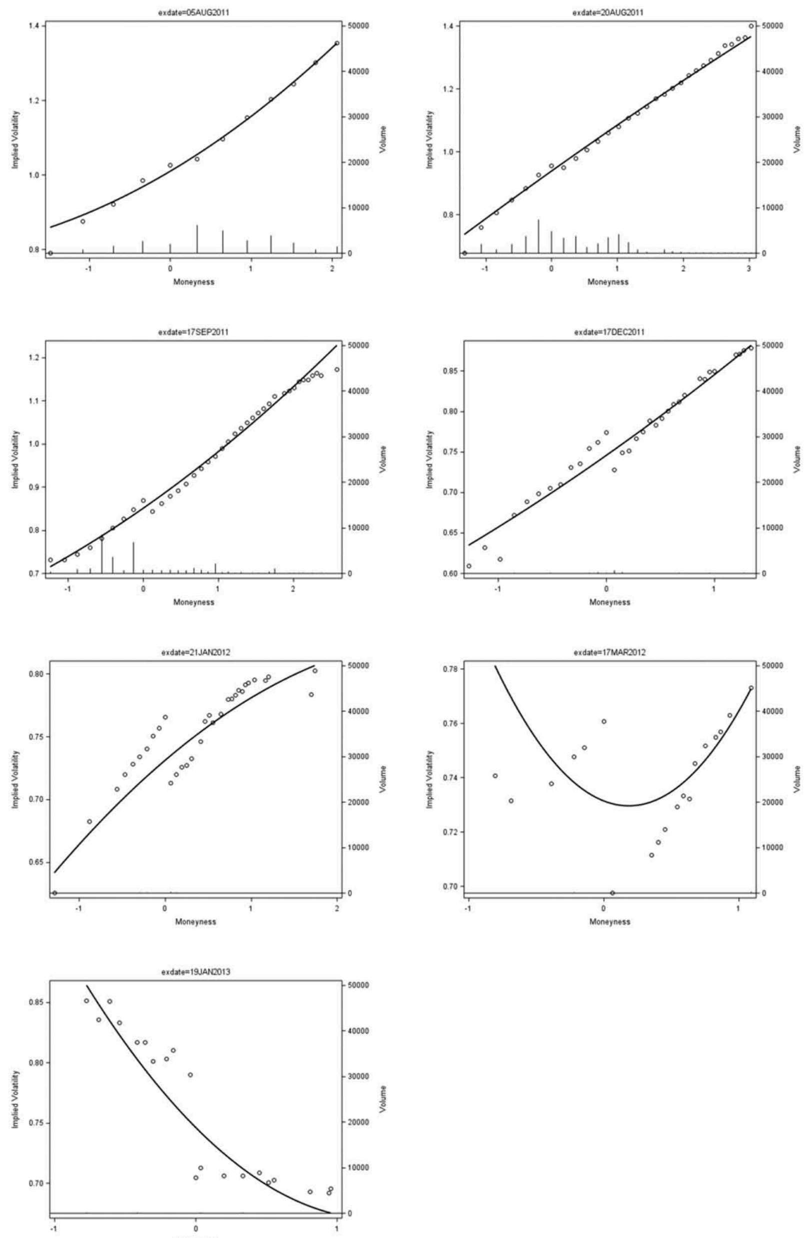

Figure 2. **IV against moneyness on 27 July 2011**. This figure plots the IV of VXX options against the moneyness for different maturities (9, 24, 52, 143, 178, 234 and 542 days to maturity) as at the close of 27 July 2011. The circles are computed from market prices of out-of-the-money calls and puts. The solid line is the fitted quantified IV curve. The bar chart is the trading volume for the corresponding options traded on 27 July 2011.

Constant maturity quantified IV curve

So far we have been examining the term structure of the VXX implied forward price and IV curves using maturity categories. However, within each maturity category there will often be multiple curves on a given day. To confirm the findings above, we create constant maturity implied forward prices and IV curve factors. This allows us to precisely study the term structure and time series of the variables covering the same horizon of traders' expectations.

To create constant maturity implied forward prices and IV curve factors, we interpolate/extrapolate them to several target maturities as follows:

$$F^{\tau} = F^{\tau_1} w_1 + F^{\tau_2}(1 - w_1), \tag{6}$$

$$\gamma_0^{\tau} = \gamma_0^{\tau_1} w_1 + \gamma_0^{\tau_2}(1 - w_1), \tag{7}$$

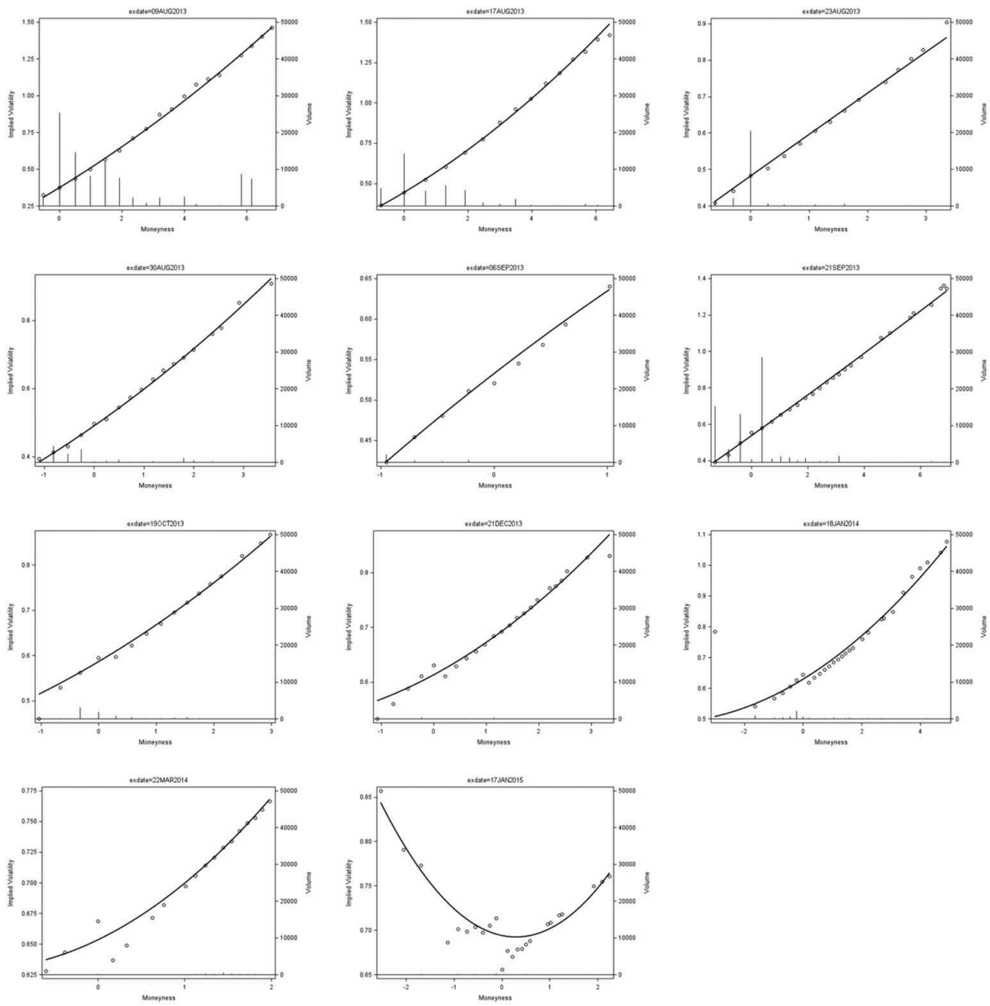

Figure 3. IV against moneyness on 2 August 2013. This figure plots the IV of VXX options against the moneyness for different maturities (7, 15, 21, 28, 35, 50, 78, 141, 169, 232 and 533 days to maturity)as at the close of 2 August 2013. The circles are computed from market prices of out-of-the-money calls and puts. The solid line is the fitted quantified IV curve. The bar chart is the trading volume for the corresponding options traded on 2 August 2013.

$$\gamma_1^{\tau} = \gamma_1^{\tau_1} w_1 + \gamma_1^{\tau_2}(1 - w_1), \tag{8}$$

$$\gamma_2^{\tau} = \gamma_2^{\tau_1} w_1 + \gamma_2^{\tau_2}(1 - w_1), \tag{9}$$

where

$$w_1 = \frac{\tau - \tau_2}{\tau_1 - \tau_2},$$

the superscript τ denotes the desired maturity, τ_1 is the closest (second closest) maturity to the target from below and τ_2 is the closest (closest) maturity to the target from above (below), when interpolating (extrapolating). We interpolate when there is a maturity either side of the target and extrapolate when the available maturities are all shorter than the target maturity.

Table 5 presents the mean and standard deviation of the interpolated implied VXX forward prices and level, slope and curvature coefficients and factors, for both the un-constrained and constrained estimations. These are also presented graphically in Figure 9. From the table we confirm the previous result that, on average, the implied VXX forward price is slightly decreasing as maturity increases. The average constant maturity forward price goes from 26.60 to 26.12, for the 30- and 360-day target maturity, respectively.

In Table 5 we can also see that the exact ATM IV (level factor) term structure is usually in contango. This is likely because the probability of a VXX volatility spike becomes larger as the maturity increases, during normal times. The variation in the exact

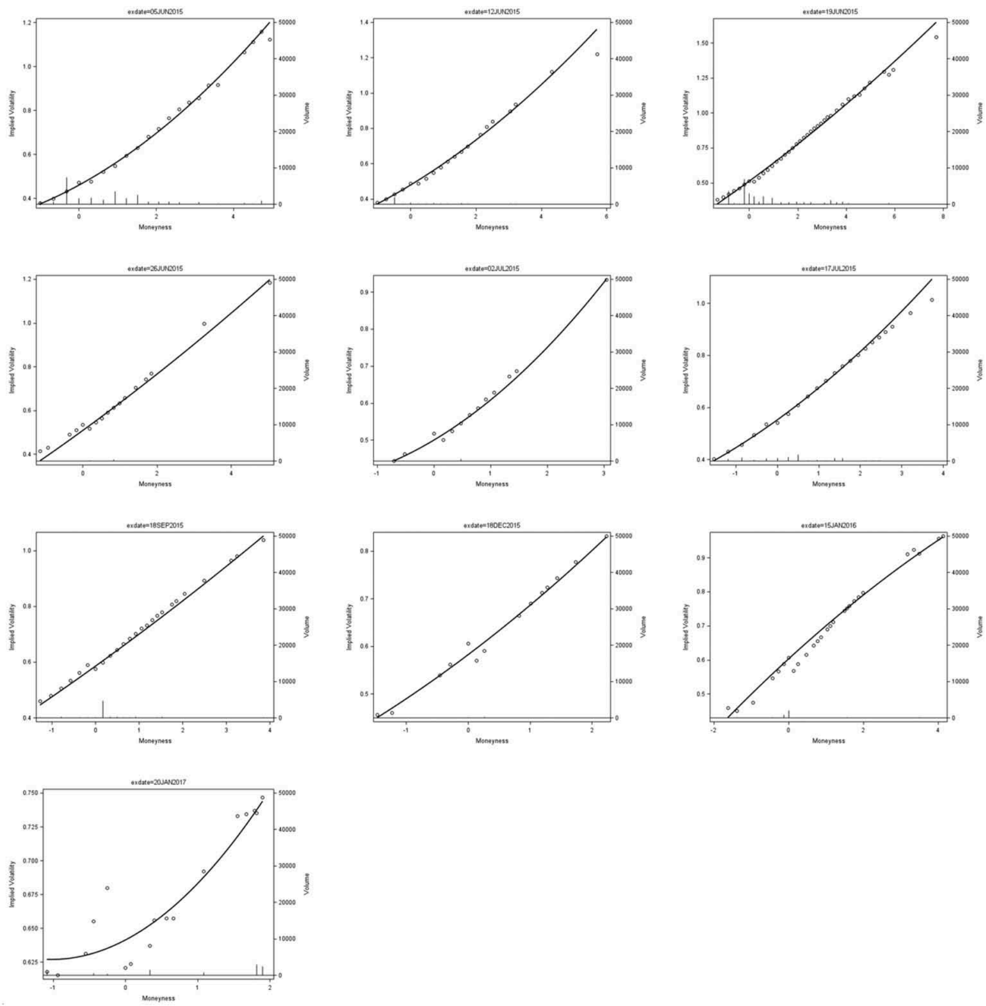

Figure 4. IV against moneyness on 27 May 2015. This figure plots the IV of VXX options against the moneyness for different maturities (9, 16, 23, 30, 36, 51, 114, 205, 233 and 604 days to maturity) as at the close of 27 May 2015. The circles are computed from market prices of out-of-the-money calls and puts. The solid line is the fitted quantified IV curve. The bar chart is the trading volume for the corresponding options traded on 27 May 2015.

ATM IV also decreases as time to maturity increases. The table also shows that the term structure of the slope factor is in backwardation and the variation in the slope factor is similar for all maturities. Lastly, we can see that the curvature factor's term structure is very flat around a value of zero with a very slight increase at longer maturities.

These results are consistent with our findings using the average values grouped by maturity categories in the previous section. However, we also want to study the time series of the ATM IV, forward prices and IV curve factors. We present the time series of 30- and 180-day constant maturity ATM IV and forward prices in Figure 10. Then we present the time series of the 30- and 180-day constant maturity level, slope and curvature factors in Figures 11 and 12 for the un-constrained and constrained estimations, respectively.[14]

From Figure 10 we can see that the ATM IV varies throughout time in a mean-reverting fashion. Referring to the difference between the 180- and 30-day ATM IV we can see that most of the time its term structure is in backwardation, although there are times when it is in contango. Examining the time series of the 30- and 180-day implied forward prices we can see that they vary significantly often spiking very quickly. We can also see that the term structure of implied forward

[14]For the time series investigation of the interpolated ATM IV, forward price and the curve factors we only use the 30 and 180 day interpolations because the data are often scarce and options illiquid, at longer maturities.

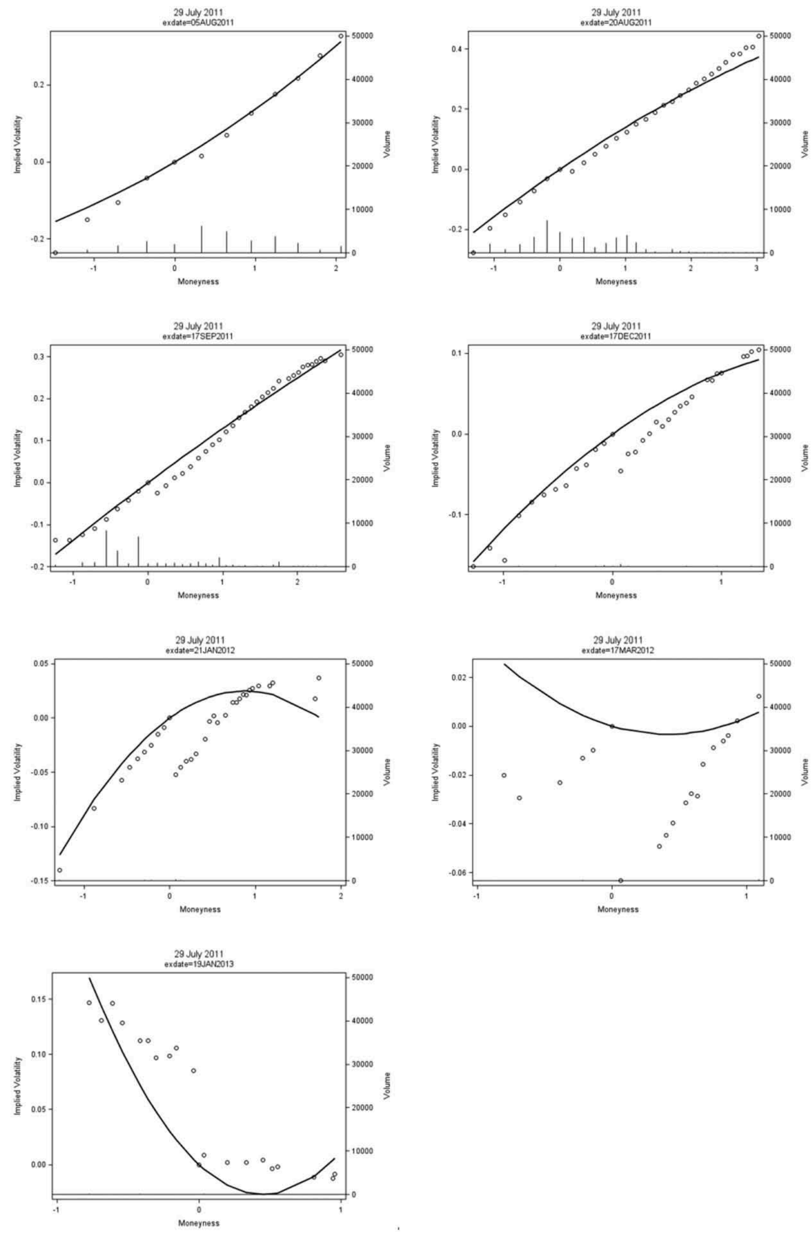

Figure 5. IV against moneyness on 27 July 2011: with restraint This figure plots the IV of VXX options against the moneyness for different maturities (9, 24, 52, 143, 178, 234 and 542 days to maturity) as at the close on 29 July 2011. The circles are computed from market prices of out-of-the-money calls and puts. The solid line is the fitted quantified IV curve. The bar chart is the trading volume for the corresponding options traded on 29 July 2011.

prices is usually almost flat, with some periods of strong contango and backwardation.

Turning to the time series of the IV curve factors in Figures 11 and 12, we can firstly see that the exact ATM IV (level factor) is also mean-reverting with a usually contango term structure, with brief times of backwardation. Secondly, we can see that the slope factor also seems to mean-revert with a usually backwarded term structure. Lastly, looking at the curvature factor we can see that it is usually very small in magnitude for the 30- or 180-day maturity. There are also days where the curvature becomes very negative or positive, resulting in abnormally concave or convex curves, respectively. Looking at the difference between the 30 and 180 day curvature factor, it is usually close to zero, indicating a flat term structure, with some spikes.

These time series observations show us that a VXX option pricing model should produce risk

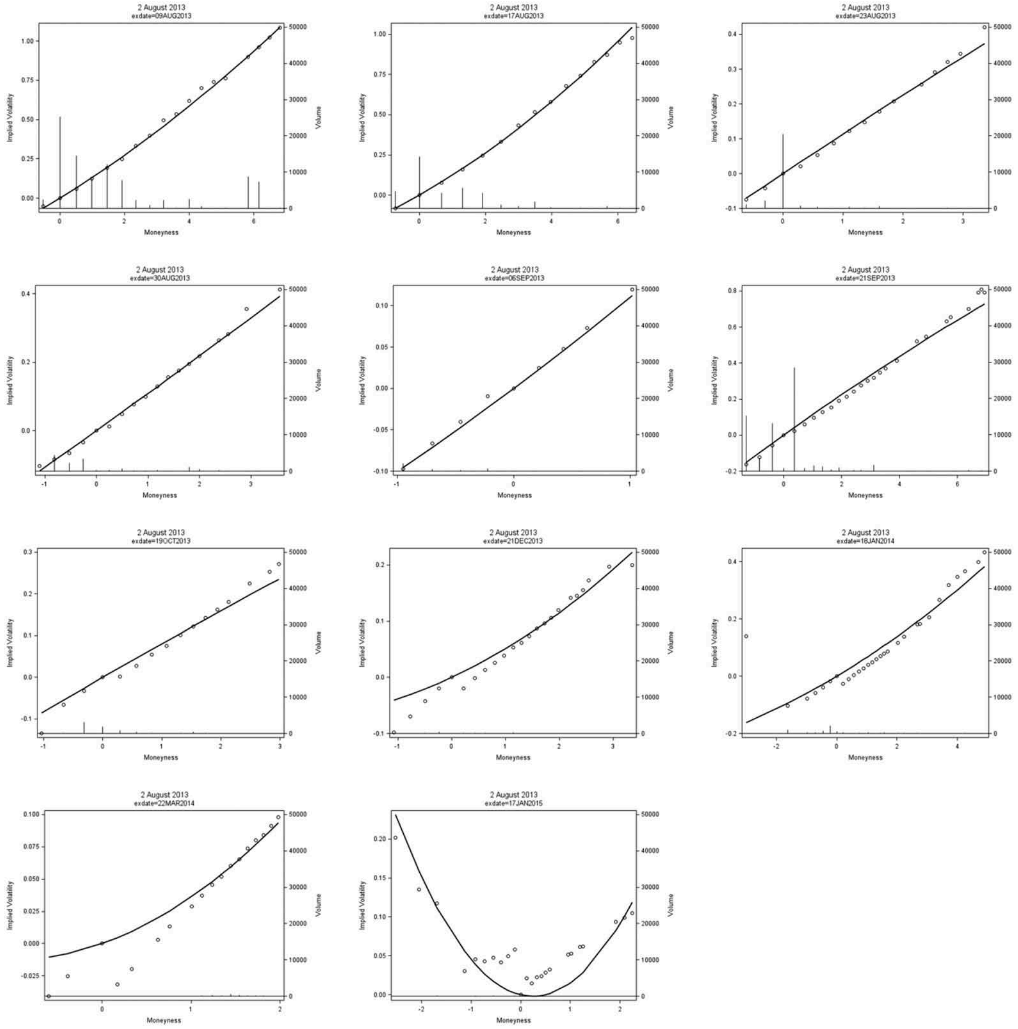

Figure 6. IV against moneyness on 2 August 2013: with restraint This figure plots the IV of VXX options against the moneyness (7, 15, 21, 28, 35, 50, 78, 141, 169, 232 and 533 days to maturity) for different maturities as at the close on 2 August 2013. The circles are computed from market prices of out-of-the-money calls and puts. The solid line is the fitted quantified IV curve. The bar chart is the trading volume for the corresponding options traded on 2 August 2013.

neutral volatility and skewness that are mean-reverting and can flip their term-structures during times of economic uncertainty. While the risk-neutral kurtosis can also become large (negative or positive) at these times.

Figure 13 shows the predicted IV curves using the mean of the interpolated factors. We can see a similar picture as in Figure 8; as maturity increases the slope decreases and the curves become more convex. The time series observations are consistent with what we found, on average, in prior discussions. Further studying what drives the time variation in the VXX's implied forward price and IV curve factors and their term structures is of interest for future research.

VXX option pricing model implications

Using the results from section 4.1 and 4.2 we can make recommendation for the dynamics of a VXX option pricing model. Using the conversion from IV curve factors to the moments of the risk-neutral distribution of the VXX, discussed in Zhang and Xiang (2008), we can say that the model must exhibit a volatility of around 69%, with a normally contango term-structure, which can flip briefly. The model must also exhibit

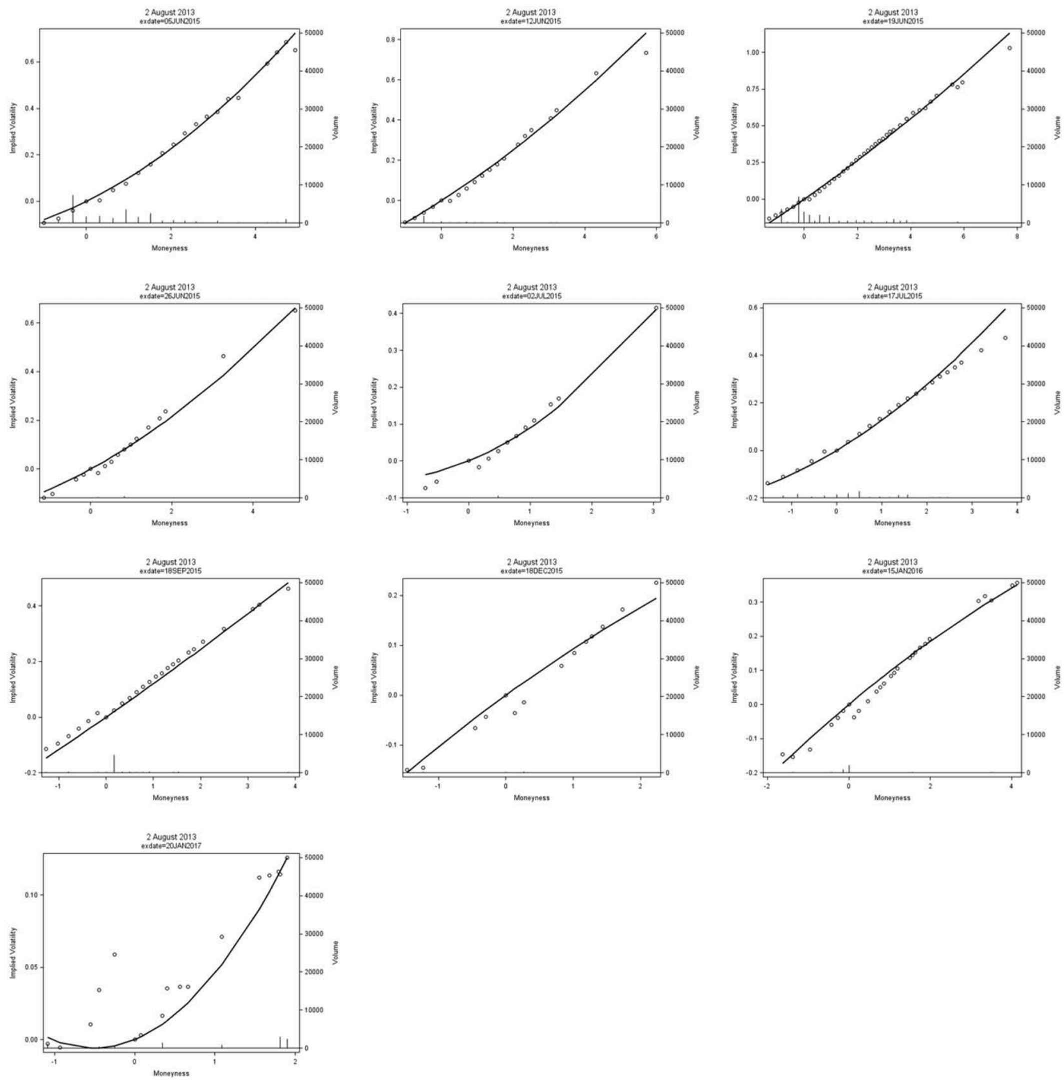

Figure 7. IV against moneyness on 27 May 2015: with restraint This figure plots the IV of VXX options against the moneyness (9, 16, 23, 30, 36, 51, 114, 205, 233 and 604 days to maturity) for different maturities as at the close on 27 May 2015. The circles are computed from market prices of out-of-the-money calls and puts. The solid line is the fitted quantified IV curve. The bar chart is the trading volume for the corresponding options traded on 27 May 2015.

positive risk-neutral skewness, with a backwarded term structure normally, that can also be in contango briefly. Lastly, the risk-neutral kurtosis of the VXX option pricing model is very small, unless the maturity is very long, with very short spikes to positive or negative kurtosis. A model that should be able to calibrate to these empirical characteristics is the best starting point for a VXX option pricing model. The quantified IV curve factors can be used to calibrate such models more quickly than calibrating to the entire IV-moneyness observations.

V. Conclusions

In this paper we document the empirical characteristics of the VXX options market as a starting place for developing a empirically grounded VXX option pricing model. We follow the methodology developed by Zhang and Xiang (2008) in order to quantify the IV curve of VXX options, through quadratic polynomial regressions. The IV curve is quantified through three factors – the level (exact ATM IV), slope and curvature – which we compute daily and for different maturities over

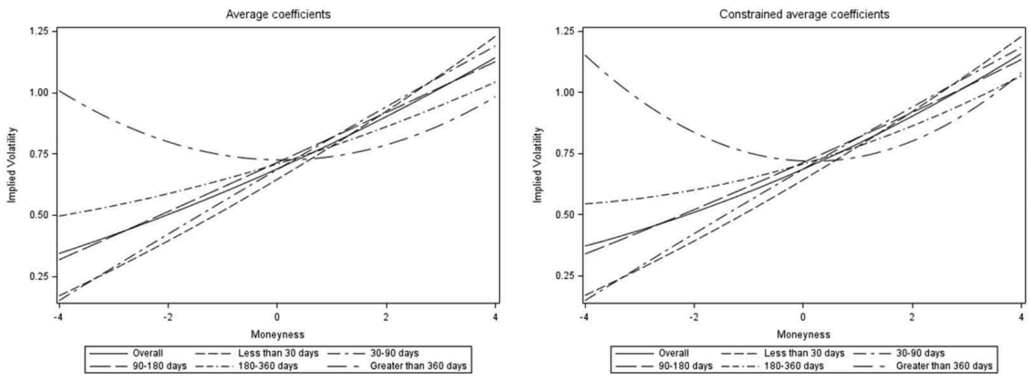

Figure 8. IV curves from mean factors. This figure shows the IV curves predicted by the mean factors (level, slope and curvature) for different maturity categories. The top plot shows this for the unconstrained while the bottom plot shows it for the constrained, regression.

Table 5. Interpolated Term structure. This table presents the mean and standard deviation of the interpolated implied forward price and the implied volatility curve factors by maturity. The statistics for the implied forward price, unconstrained and constrained IV curve factors are presented in panels A, B and C, respectively.

	Maturity (days)							
	30	60	90	120	150	180	270	360
Panel A: Implied Forward Price								
Mean								
F^τ	26.5956	26.5433	26.5135	26.4586	26.4046	26.3488	26.3689	26.1209
Standard Deviation								
F^τ	9.7797	9.7615	9.7592	9.7457	9.7470	9.8518	10.3749	10.1653
Panel B: IV regression								
Mean								
α_0^τ	0.6768	0.6954	0.7042	0.7096	0.7130	0.7148	0.7182	0.7194
α_1^τ	0.1285	0.1212	0.1119	0.1028	0.0928	0.0823	0.0600	0.0417
α_2^τ	0.0013	−0.0011	−0.0005	0.0002	0.0010	0.0020	0.0040	0.0059
γ_0^τ	0.6768	0.6954	0.7042	0.7096	0.7130	0.7148	0.7182	0.7194
γ_1^τ	0.1971	0.1785	0.1623	0.1479	0.1329	0.1177	0.0858	0.0602
γ_2^τ	0.0017	−0.0018	−0.0008	0.0001	0.0015	0.0030	0.0059	0.0085
Standard Deviation								
α_0^τ	0.1475	0.1199	0.1047	0.0955	0.0899	0.0853	0.0773	0.0744
α_1^τ	0.0379	0.0388	0.0378	0.0380	0.0392	0.0406	0.0446	0.0501
α_2^τ	0.0115	0.0115	0.0113	0.0127	0.0155	0.0202	0.0341	0.0512
γ_0^τ	0.1475	0.1199	0.1047	0.0955	0.0899	0.0853	0.0773	0.0744
γ_1^τ	0.0645	0.0597	0.0572	0.0569	0.0580	0.0601	0.0640	0.0702
γ_2^τ	0.0166	0.0158	0.0152	0.0166	0.0206	0.0263	0.0441	0.0664
Panel C: Constrained IV regression								
Mean								
α_0^τ	0.6761	0.6953	0.7036	0.7089	0.7121	0.7138	0.7161	0.7159
α_1^τ	0.1286	0.1208	0.1109	0.1013	0.0909	0.0798	0.0568	0.0377
α_2^τ	0.0009	−0.0014	−0.0002	0.0011	0.0024	0.0038	0.0068	0.0099
γ_0^τ	0.6761	0.6953	0.7036	0.7089	0.7121	0.7138	0.7161	0.7159
γ_1^τ	0.1973	0.1779	0.1611	0.1460	0.1304	0.1143	0.0816	0.0549
γ_2^τ	0.0014	−0.0019	−0.0002	0.0016	0.0037	0.0059	0.0102	0.0146
Standard Deviation								
α_0^τ	0.1483	0.1211	0.1061	0.0966	0.0910	0.0865	0.0797	0.0784
α_1^τ	0.0387	0.0393	0.0387	0.0393	0.0409	0.0430	0.0500	0.0601
α_2^τ	0.0137	0.0138	0.0147	0.0181	0.0241	0.0332	0.0565	0.0842
γ_0^τ	0.1483	0.1211	0.1061	0.0966	0.0910	0.0865	0.0797	0.0784
γ_1^τ	0.0658	0.0605	0.0585	0.0588	0.0602	0.0629	0.0706	0.0831
γ_2^τ	0.0200	0.0189	0.0196	0.0230	0.0314	0.0428	0.0717	0.1069

a six year sample. We extend the methodology of Zhang and Xiang (2008) by estimating constant maturity factors, which allows us to study the time-series and term structure dynamics of the VXX IV more concisely. We quantify the IV curves with and without a constraint that the

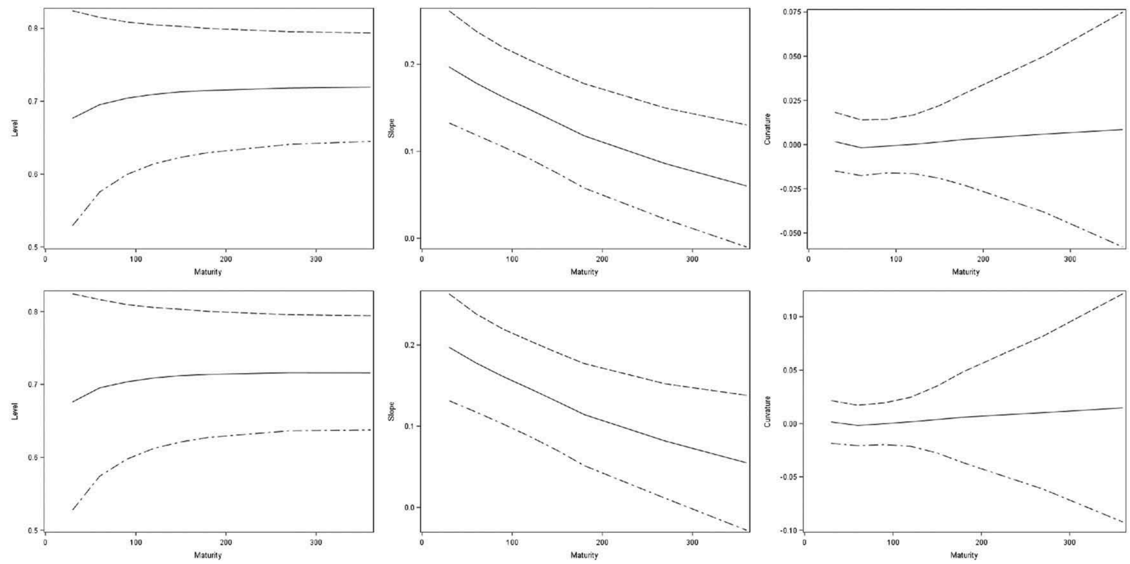

Figure 9. Term structure of mean interpolated factors. This figure shows the term structure of the mean interpolated factors (level, slope and curvature) and their one standard deviation bands. The unconstrained and constrained regression results are shown in the top and bottom row of plots, respectively.

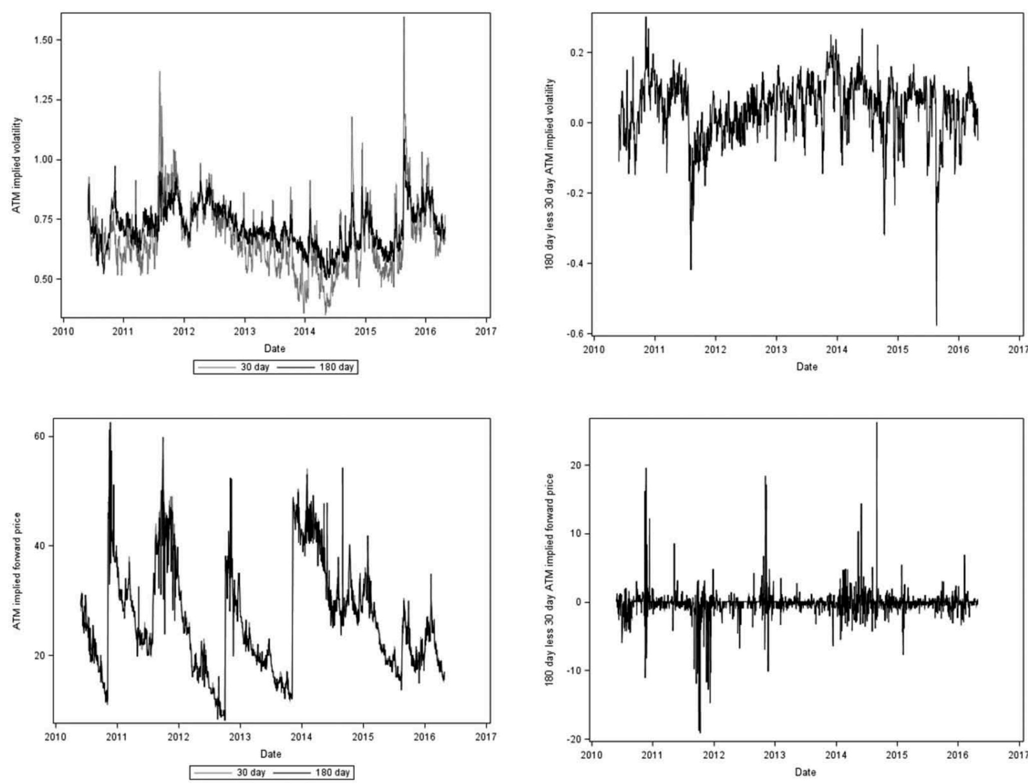

Figure 10. Time series of interpolated ATM IV and forward prices. This figure presents the time series of the interpolated ATM IV and forward prices for the 30 day and 180 day maturities (left) and their differences (right).

curve has to pass through the ATM IV, resulting in a very similar average shape.

We find that the implied VXX forward price term structure is usually in backwardation. We also show that the average exact ATM IV (level factor) increases with maturity and estimates become less variable with longer maturities. Which could be explained by traders expecting VXX ATM volatility to mean-revert,

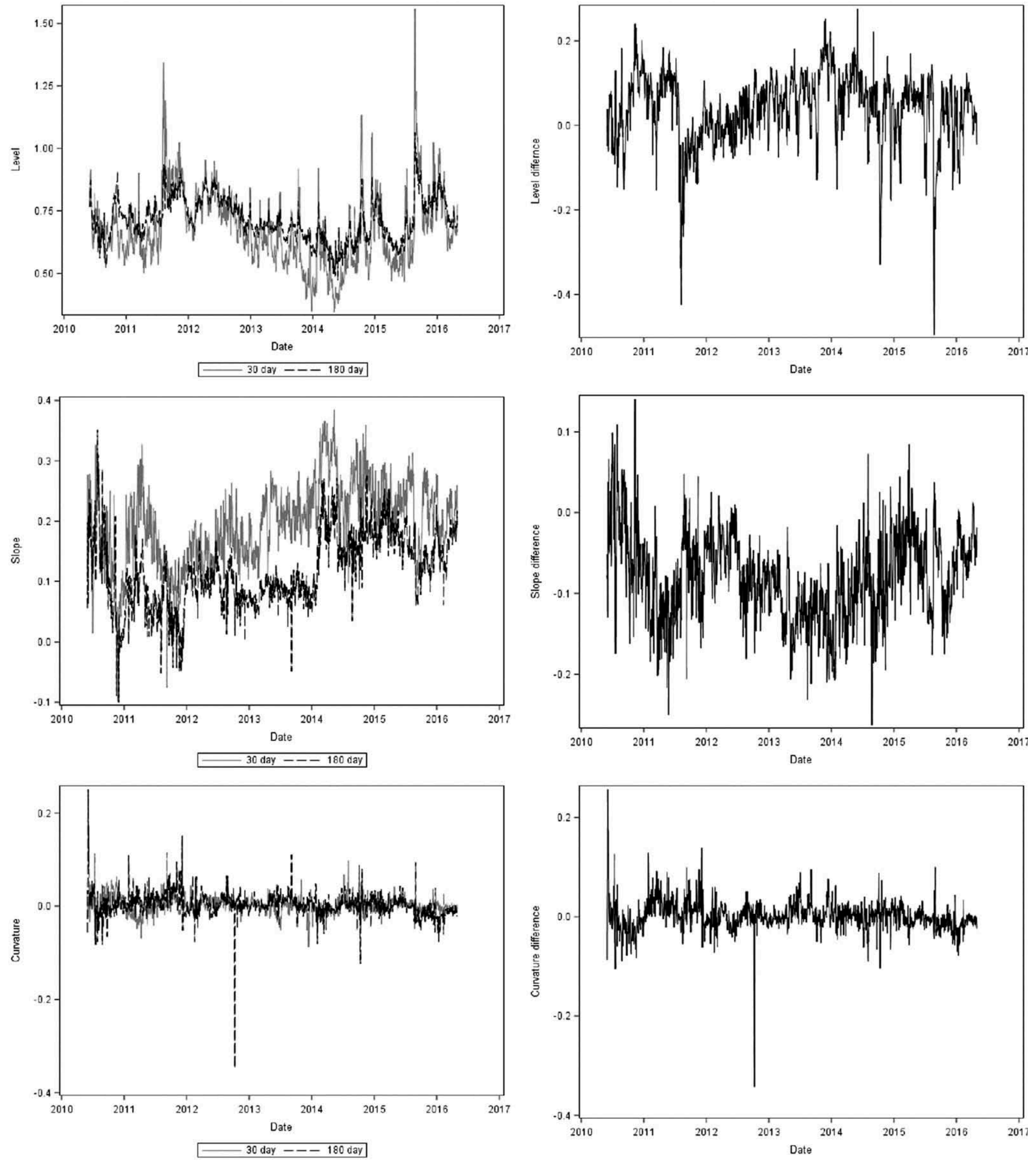

Figure 11. Time series of interpolated IV factors. This figure shows the time series of the 30 and 180 day interpolated constant maturity level, slope and curvature (left) factors and the difference between the 180 day and 30 day (right).

which we show it does. The IV curves are also usually significantly upward sloping, although as the maturity increases they become flatter and even downward sloping for the longest maturities. The IV curves are slightly convex, on average, and become more convex as the maturity increases.

Our quantification of the VXX IV summarizes all the information contained in VXX option prices and should therefore be used when developing a VXX option pricing model. The model needs to be able to calibrate to a volatility of around 69%, positive skewness and very little kurotosis in the risk-neutral distribution of VXX returns. Although during market turmoil it has to be able to produce brief periods of negative skewness and larger magnitude kurtosis and short periods of flipped term-structures. Very importantly, the risk-neutral volatility and skewness need to mean-revert.

We study the time series of the short end of the term structure (less than 180 days). We show that the level and slope factors seem to mean-revert through time, while the curvature does not follow an easily observed pattern. Although the level's (slope's) term structure is usually in contango (backwardation), there are times when it goes into backwardation (contango). The shorter maturity end of the curvature's term structure is usually almost flat,

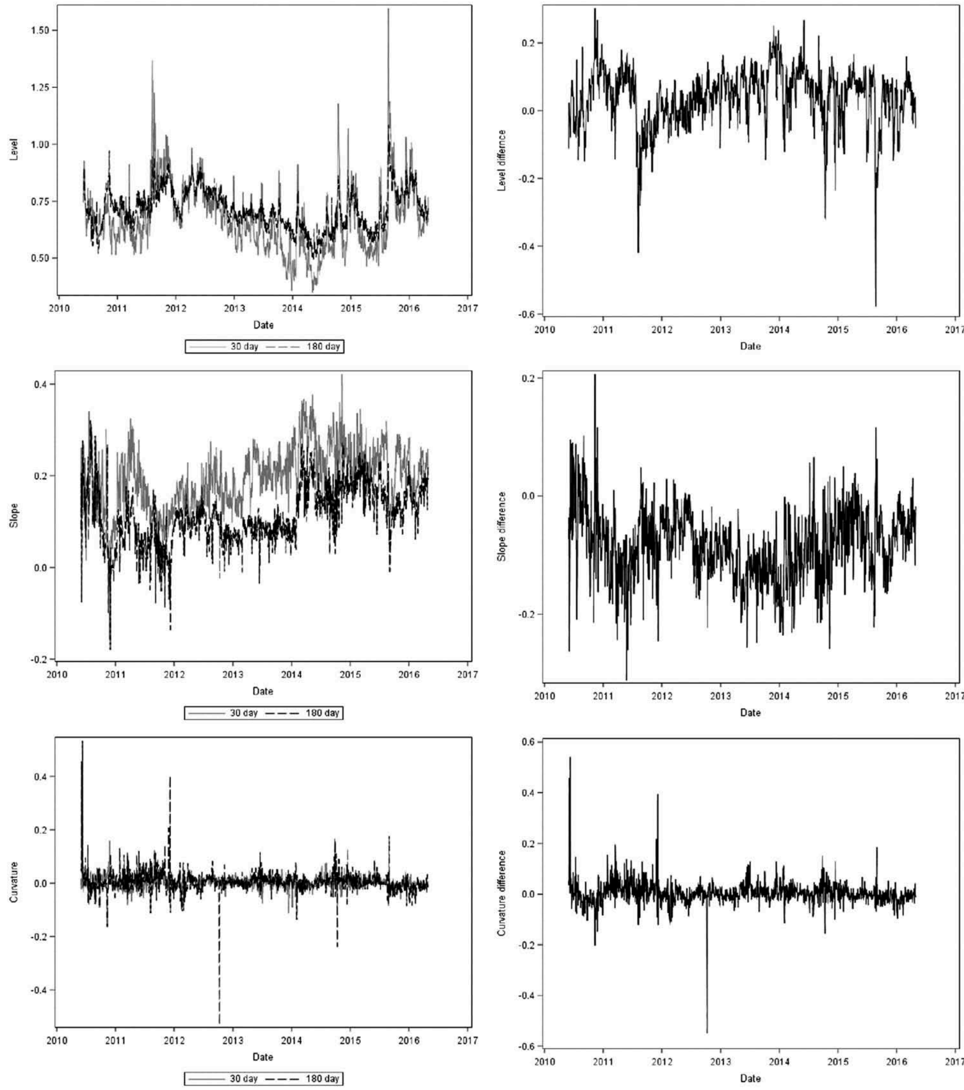

Figure 12. Time series of interpolated coefficients: constrained. This figure shows the time series of the 30 and 180 day interpolated constant maturity level, slope and curvature (left) factors and the difference between the 180 day and 30 day (right), when the fitted curve is forced to cross the point of ATM IV.

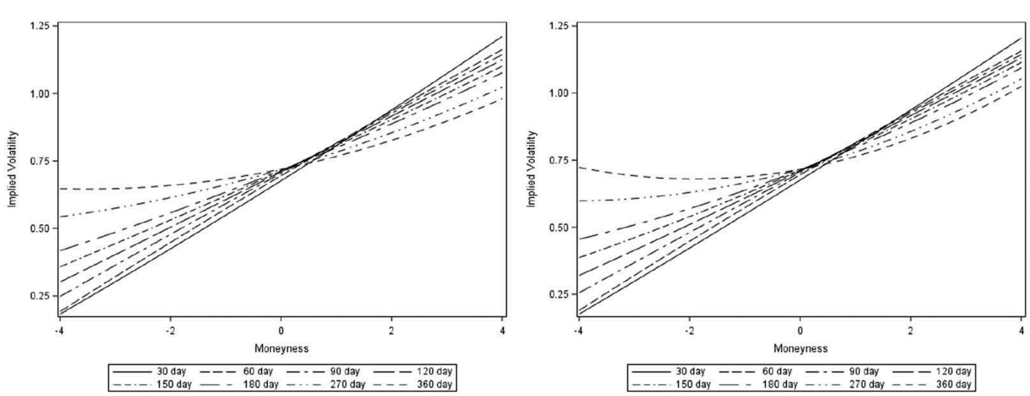

Figure 13. IV curves from mean interpolated coefficients. This figure shows the IV curves predicted by the mean interpolated factors (level, slope and curvature) for different maturities. The top plot shows this for the unconstrained while the bottom plot shows it for the constrained regression.

with some short-lived moments of backwardation or contango.

Studying the drivers of the periodic shifts in the factors and their term structures is left for future research. The quantified VXX IV factors could also be converted to estimates of the VXX's risk-neutral moments, which can then be used to calibrate new VXX option pricing models. We could potentially use the quantified VXX IV factors to to predict the VXX returns or VXX option returns. The relationships between the SPX, VIX and VXX option implied volatility curves is also a topic of interest as understanding these would allow for more direction on developing a comprehensive option pricing model for all three markets. These extensions of the current work are left for future research.

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