Collatz Conjecture, Problem? Proofs.

by Sean Marius PEDERSEN from Hamburg, GERMANY - 13.11.2022

https://en.wikipedia.org/wiki/Collatz_conjecture

The Proofs

This document provides two proofs:

- One uses a “2D bijection of the operators” + structural induction
- The other uses “inverses of the Collatz Function”

Both show clearly that the conjecture by Lothar Collatz is correct for all natural numbers.

Definition

C(X) is defined in Python Code as the following 1D recursive function…

```
collatz(n):
  if n <= 1:
    return 0
 if n mod 2 == 0:
    return collatz(n/2)
  else:
    return collatz(3*n-1)
```

Conjecture

Every natural number n>0 fed into collatz(n) will end in the cycle of 4,2,1,..

Proven by me using “Geometric Induction” on 01.11.2022 .:.

Differentiation of the 1D Algebra (Operators)

Bijective mapping (projection) of the used Collatz operators from 1D to 2D:
1D: A+B, A-B => 2D: square-root(A+B), square-root(A-B)
1D: A*B, A:B => 2D: A+B, A%B (modulo)

The operators are transferred from the first dimension to the second dimension without loss in entropy (information density) by applying the square root to each operator. This happens without loss, i.e. bijectively.

This serves as a 2D projection of the equation or operator and thus the symbols (variables and operators) of the equation after being manipulated in 2D according to the existing rules of the existing algebra still represent the result losslessly (in a bijective manner).

The 2D equation can thus be returned to 1D with a simple bijective projection using the geometric algebra mapping introduced here.

Collatz Problem? The First Proof in 2D.

The Collatz function is fairly easy to define: Take any natural number X and apply the Collatz function:

> If X is even, divide X by 2.
> If X is odd, take X times 3 and add 1.

Now the Collatz function is called recursively until it ends in a cycle, which would be 4,2,1 as far as we believe so far. But can we prove it?

Thus the Collatz conjecture is**:**

every natural number N ends in the cycle 4,2,1 by recursively applying the Collatz function to its own current result.

2D Proof by Structural Induction

I show that the Geometric Transformation (Differentiation) of C(X) is a cycle and C(X+1) is also a cycle, thus for every X that is an element of the set of the Natural Numbers C(X) is proven (valid).

Case for Collatz(X) in 2D

The geometric derivation of the 1-dimensional Collatz function is for even X:

> **X:2 -> X-2**

and for odd X:

> **X*3+1 -> X+3+sqrt(1) = X+4**

It is immediately apparent that X+4 and X-2 form a cycle for any integer X since:

> **X+4-2-2 = X**
> **X-2+4-2 = X**
> **X-2-2+4 = X.**

Case For C(X+1) in 2D

The structural induction step is as follows.

Uneven X:
3*(X+1)+1 = 3X+3+1 = 3X+4 => 3+X+2 = X+5
OR
2*((2X+1)+2)+1 = 4X+2+2+1 = 4X+5 = 5*X

Even X:
The - operand becomes the modulo operator (i.e. - recursive) for X+1 (induction step).
(X+1):2 => sqrt(X mod 2 + 1 mod 2) = sqrt(X mod 2 + 1) = sqrt(X mod 2) + sqrt(1) = 0+1 = 1

Now, for X is odd: every odd number is even when 5 is added. So the Collatz function is a cycle in 2D also for X+1 since every even number becomes 1 (in 2D, as shown above).

Examples for uneven X: 1+5=6, 3+5=8 , … so in general:
    2*N+1+5=2*N+2*3=2*(N+3) is even.

So follows: X is always 1 as proven above for both uneven and even X.

Conclusion

We proved that every even number X becomes 1 in 2D using C(X).

We have also proved that every odd number Y becomes even, aka X using the Collatz function for odd numbers C(Y) in 2D: **Y + 5 equals X (an even integer)**.
Thus, for C(X+1) in 2D, the Collatz function is a cycle that ends in 1 for every integer.

Since the geometric derivation of the operators of the Collatz function is bijective, this proof also applies to the 1D bijective ur-function c(X), aka the original Collatz function. **Q.E.D.**

Somit ist die Collatz-Vermutung in 2D bewiesen, doch gilt dieser Beweis der Vermutung auch wirklich in 1D als für das bijektive Urbild? Die Antwort ist ja.

Die Erklärung basiert auf der präzisen mathematischen Überführung der in der Collatz-Funktion verwendeten 1D-Operatoren nach 2D und zurück nach 1D. Es ist zu zeigen, dass die geometrische Aufleitung von N Dim. nach N+1 Dim. der Operatorenmenge verlustfrei ist, also bijektiv. Also muss die geometrische Ableitung der Aufleitung eines beliebigen Operators oder Funktion verlustfrei (bijektiv) zurück zum Ursprung führen. Diese Bijektion der Operatoren habe ich im Abschnitt “Differenzierung der 1D Algebra (Operatoren)” demonstriert und sie enthält alle Operatoren, die in der Collatz-Funktion vertreten sind, damit ist der Beweis auch in 1D gültig.

Addendum: More Proofs

I show that any power of 2 is an exit point in the Collatz functions iteration, meaning that the rule: “if even divided by 2”, pulls all powers of 2s down to the 4,2,1 cycle.

Proof that any prime number P * 3 + 1 is a sum of power of 2s.
Show that after sqrt(P) 3*P+1 steps the divide by 2 rule hits the biggest summand of the above mentioned power of 2s composing P.

Proof by Kolmogorov Complexity + Entropy. Show that the function contains enough Kolmogorov Complexity to compute all natural numbers + the initial entropy is greater than 1. There are finite programs and infinite programs. Proof this one is not finite aka infinite. Define Kolmogorov Classes that map to infinities: natural, divisive, real and so aleph on. aka why is fun(x): return fun(x+1) infinite? Why is the inverse of collatz(x) infinite aka natural number complete?

fun(x): fun(x) -> Infinite LOOP aka ILOOP aka 1D with zero Entropy
fun(x): fun(x+1) -> 1D ILOOP plus 1 aka constant Entropy
fun(x): fun(x), fun(x) -> 2D ILOOP (TREE-2D-LOOPS) with ZERO ENTROPY
f(x): f(x)+f(x) -> 2D ILOOP + exponential Entropy
f(x): 2*f(x) -> 1D ILOOP + linear entropy
f(x): 2*f(x) OR (f(x)-1) / 3 -> 1D ILOOP + linear entropy

Entropy: Computational Complexity * Spatial Complexity
Kolmogorov Complexity: Recursive Execution Graph + Entropy

Show that Collatz Funk is ILOOP with linear entropy. Show the funk is growing Lippitzsch or minimally: always increasing.

Collatz Inverse Funktion Proof

Inverse of an N-Dim. hard IF condition is a soft N-Dimensional IF condition. This means the whole N-Dim. conditional quantum tree is computed as the inverse of the otherwise N-Dim. if conditional (decisional) tree.

Collatz Inverse Functions

Thus the inverse F function is:
Multiply X by 2 OR X minus 1 and divide by 3.

Other (outer) inversion G of Collatz Funktion:
1) If X is odd, divide X by 2.
2) If X is even, X times 3 and add 1.

Other (inner) inversion H of Collatz Funktion:
1) If X is even, multiply X by 2.
2) If X is odd, X minus 1 and divide by 3.

Outer inversion G times inner H is GxH defined as:

1) If X is odd or even:
    a) (A=1, multiply) OR (A=0, divide X by 2)
2) If X is even or odd:
    a) (if A is 1: X times 3 and add 1) OR (if A is 0: X minus 1 and divide by 3)

We see for GxH that once you multiply an even X by 2, you can not multiply an odd X by 3 and add 1 anymore. Proof that GxH(N) maps to all natural numbers and GxH(N+1) does too.

GxH Graph for X and X+1:

ALPHA: X,GxH1=2*X,GxH1=X
BETA: 2*X,GxH1=4*X,GxH2=(4*X-1)/3=4/3*X-⅓.

X+1,GxH1=2*(X+1),GxH1=4*(X+1)=5*X
5*X,GxH2=((4*X+4)-1)/3=4/3*X+1, INVOKE BETA: ((4*X+4)-1) : (3*(4/3*X-⅓)) =
(5*X-1) : (4*X-1) = X+1

We see that G * H is 1 (cyclic graph). The outer inversion times the inner inversion is 1 aka direct cycle (self-referencing graph / function).
So the Collatz function's inverse topology is outer-inner body symmetric. -> G * H must be by definition the inverse of the inverse Collatz function thus the Collatz function itself.
And the computation graph of GxH in alternating edges is a complete (infinite) tree spanning all natural numbers. Thus the Collatz function is powerful enough to construct all natural numbers beginning from 1,2,4.

Natural Number Construction Graph for GxH(X):

1,H1=2,H1=4
H2=1,H1=8,H1=16,H2=5,H1=10,-
>H2=3,H1=6,H1=12,H1=24,H1=48,H1=96
>H1=20,H1=40,H2=13,H1=26,H1=52
H2=17,H1=34
H2=11,H1=22
H2=7,H1=14,H1=28
H2=9,H1=18,H1=36,H1=72,H1=144,H1=288,H1=576

We observe that the graph of GxH(X) contains GxH(X+1).

Complete Natural Number Construction using GxH:

f(x+0): 2*(x+0) OR (x+0-1) / 3
f(x+1): 2*(x+1) OR (x+1-1) / 3 = f(x+0)+2 OR x+0 / 3
f(x+2): 2*(x+2) OR (x+2-1) / 3 = f(x+1)+2 OR x+1 / 3
f(x+3): 2*(x+3) OR (x+3-1) / 3 = f(x+2)+2 OR x+2 / 3

f(x+4): 2*(x+4) OR (x+4-1) / 3 = f(x+3)+2 OR x+3 / 3 = 2*f(x+4) OR x + 1.

The left part doubles X thus maps to all even natural numbers.
The right part maps to the set of all thirds which includes all odd natural numbers.

This proves that the inverse Collatz function maps to all natural integers. Q.E.D.

This proves the Collatz Conjecture, since the inverse of the inverse function is the Collatz function itself, thus the Collatz Conjecture is hereby proven to be true. Q.E.D.

EXAMPLES for GxH:

X:1 -> f(1+0)=f(1)=0*2+2 OR 0 / 3 = 0**2** OR **0**
X:2 -> f(1+1)=f(2)=1*2+2 OR 1 / 3 = 0**4** OR 1 / 3
X:3 -> f(2+1)=f(3)=2*2+2 OR 2 / 3 = 0**6** OR 2 / 3
X:4 -> f(3+1)=f(4)=3*2+2 OR 3 / 3 = 0**8** OR **1**
X:5 -> f(4+1)=f(5)=4*2+2 OR 4 / 3 = **10** OR 4 / 3
X:6 -> f(5+1)=f(6)=5*2+2 OR 5 / 3 = **12** OR 5 / 3
X:7 -> f(6+1)=f(7)=6*2+2 OR 6 / 3 = **14** OR **2**

References:

- https://www.youtube.com/watch?v=K0yMyUn--0s
- http://www.rzbt.haw-hamburg.de/dankert/spezmath/html/collatzproblem.html
- http://alienryderflex.com/collatz_analysis.shtml
- http://alienryderflex.com/collatz_2/
- http://www.oemg.ac.at/Mathe-Brief/fba2016/VWA_Oberkofler.pdf

Thanks to

- Mother Nature
- Deutschland
- Helmut Schmidt
- Albert Einstein
- Mama & Papa
- Familie
- Frau Engelmann
- Herr Stein
- Alexander
- Janis & Klaus & Christel
- Ron & Ute
- Frau Schröder
- Frau Meyer
- Max Planck
- Herr Satz

- Prof. Dr. Kai von Luck
- Kurt Gödel
- Uwe
- Alan Turing
- Darius Morawiec
- Prof. Dr. rer. nat. Christopher Klauck
- Andrei Nikolajewitsch Kolmogorov
- Dr. Andreas Jacob
- and the rest of the bunch
    - Stefanos
    - Demole
    - Lara
    - Cat
    - etc.

Collatz Steps - Computations

StepCode: AB,AB,AB - A: encodes max val(2,1,0), B: encodes even or odd (0,1)

C(000):01,01,01
C(001):01,01,01
C(002):01,01,01
C(003):01,01,01
C(004):21,01,01---
C(005):10,21,01
C(006):11,00,21
C(007):20,11,00
C(008):20,10,01
C(009):01,20,10
C(010):21,01,10
C(011):10,21,01
C(012):10,00,21
C(013):11,20,00
C(014):01,01,20
C(015):21,01,01---
C(016):21,21,01
C(017):00,21,21
C(018):10,00,21
C(019):20,10,00
C(020):20,20,00
C(021):01,20,20
C(022):01,01,20
C(023):21,01,01---
C(024):21,21,01
C(025):00,21,21

C(026):21,00,11
C(027):00,21,00
C(028):21,00,11
C(029):10,21,00
C(030):00,00,21
C(031):00,00,00
C(032):20,00,00
C(033):01,20,10
C(034):10,01,20
C(035):11,20,01
C(036):01,01,20
C(037):21,01,01---
C(038):21,21,01
C(039):01,01,01
C(040):20,01,01
C(041):00,20,11
C(042):21,00,10
C(043):00,21,00
C(044):11,00,21
C(045):10,21,00
C(046):00,00,21
C(047):00,00,00
C(048):20,00,00
C(049):01,20,10
C(050):10,01,20
C(051):20,20,01
C(052):00,00,00
C(053):01,20,20
C(054):01,01,20
C(055):20,01,01
C(056):20,20,01
C(057):01,20,20
C(058):10,01,20
C(059):01,20,01
C(060):20,01,20
C(061):01,20,01
C(062):01,01,20
C(063):21,01,01---
C(064):21,21,01
C(065):00,21,21
C(066):11,00,21
C(067):21,21,00
C(068):01,01,01
C(069):00,21,21
C(070):00,00,21
C(071):00,00,00

C(072):20,00,00
C(073):10,20,00
C(074):21,00,10
C(075):00,21,00
C(076):00,10,21
C(077):20,00,20
C(078):20,20,00
C(079):21,00,00
C(080):21,21,00
C(081):01,21,21
C(082):10,01,21
C(083):20,10,01
C(084):20,20,00
C(085):01,20,20
C(086):01,01,20
C(087):20,01,01
C(088):20,20,01
C(089):01,20,20
C(090):20,01,20
C(091):01,20,01
C(092):20,01,10
C(093):01,20,01
C(094):01,01,20
C(095):21,01,01---
C(096):21,21,01
C(097):00,21,21
C(098):20,00,11
C(099):11,20,00
C(100):01,01,20
C(101):01,01,01
C(102):01,01,01
C(103):01,01,01
C(104):21,01,01---
C(105):00,21,11
C(106):10,00,21
C(107):00,20,00
C(108):20,00,10
C(109):21,10,00
C(110):21,21,00
C(111):01,01,01
C(112):01,21,21
C(113):00,11,21
C(114):00,10,21
C(115):21,00,10
C(116):21,21,00
C(117):00,21,21

C(118):00,00,21
C(119):21,00,00
C(120):21,21,00
C(121):00,21,21
C(122):21,00,11
C(123):00,21,00
C(124):10,00,21
C(125):20,10,00
C(126):20,20,00
C(127):00,00,00
C(128):00,20,20
C(129):01,10,20
C(130):21,01,10
C(131):10,21,01
C(132):00,00,21
C(133):00,00,00
C(134):00,00,00
C(135):00,00,00
C(136):21,00,00
C(137):01,21,10
C(138):20,01,11
C(139):01,20,01
C(140):11,01,20
C(141):01,21,01
C(142):01,01,21
C(143):21,01,01---
C(144):21,21,01
C(145):01,21,21
C(146):20,01,11
C(147):20,20,01
C(148):00,00,00
C(149):01,20,20
C(150):01,01,20
C(151):01,21,21
C(152):01,01,21
C(153):21,01,01---
C(154):20,11,01
C(155):01,20,01
C(156):21,01,10
C(157):10,21,01
C(158):00,00,21
C(159):00,00,00
C(160):20,00,00
C(161):00,20,10
C(162):20,00,10
C(163):11,20,00

C(164):01,01,20
C(165):21,01,01---
C(166):21,21,01
C(167):01,01,01
C(168):01,21,21
C(169):00,11,21
C(170):11,00,21
C(171):00,21,00
C(172):20,00,11
C(173):11,20,00
C(174):01,01,20
C(175):01,01,01
C(176):20,01,01
C(177):00,20,11
C(178):11,00,20
C(179):21,21,00
C(180):01,01,01
C(181):00,21,21
C(182):00,00,21
C(183):21,00,00
C(184):21,21,00
C(185):00,21,21
C(186):10,00,21
C(187):00,20,00
C(188):20,00,20
C(189):20,10,00
C(190):20,20,00
C(191):00,00,00
C(192):00,20,20
C(193):01,10,20
C(194):21,01,10
C(195):21,21,01
C(196):01,01,01
C(197):00,21,21
C(198):00,00,21
C(199):00,00,00
C(200):21,00,00
C(201):00,21,00
C(202):00,10,21
C(203):20,00,20
C(204):21,10,00
C(205):00,21,00
C(206):00,00,21
C(207):20,00,00
C(208):20,20,00
C(209):01,20,20

C(210):11,01,20
C(211):21,21,01
C(212):01,01,01
C(213):01,21,21
C(214):01,01,21
C(215):21,01,01---
C(216):21,21,01
C(217):20,01,01
C(218):00,20,11
C(219):20,00,20
C(220):10,20,00
C(221):20,00,20
C(222):20,20,00
C(223):00,20,20
C(224):00,00,20
C(225):01,20,20
C(226):10,01,20
C(227):01,20,11
C(228):01,01,20
C(229):20,01,01
C(230):20,20,01
C(231):00,00,00
C(232):21,00,00
C(233):01,21,10
C(234):11,01,21
C(235):01,21,01
C(236):21,01,11
C(237):10,21,01
C(238):00,00,21
C(239):00,00,00
C(240):20,00,00
C(241):01,20,10
C(242):01,01,20
C(243):20,01,01
C(244):20,20,01
C(245):01,20,20
C(246):01,01,20
C(247):21,01,01---
C(248):21,21,01
C(249):21,01,01---
C(250):01,21,01
C(251):21,01,21
C(252):11,21,01
C(253):21,01,21
C(254):21,21,01
C(255):01,21,21

C(256):01,01,21
C(257):00,21,21
C(258):20,00,11
C(259):20,20,00
C(260):00,00,00
C(261):01,20,20
C(262):01,01,20
C(263):01,01,01
C(264):20,01,01
C(265):01,20,01
C(266):20,01,10
C(267):01,20,01
C(268):01,11,20
C(269):21,01,21
C(270):21,21,01
C(271):20,01,01
C(272):20,20,01
C(273):00,20,20
C(274):11,00,20
C(275):21,11,00
C(276):21,21,01
C(277):00,21,21
C(278):00,00,21
C(279):20,00,00
C(280):20,20,00
C(281):00,20,20
C(282):20,00,20
C(283):00,20,00
C(284):20,00,10
C(285):20,10,00
C(286):20,20,00
C(287):00,00,00
C(288):00,20,20
C(289):00,10,20
C(290):11,00,20
C(291):21,11,00
C(292):21,21,01
C(293):01,01,01
C(294):01,01,01
C(295):01,01,01
C(296):01,21,21
C(297):00,11,21
C(298):21,00,11
C(299):00,21,00
C(300):21,00,11
C(301):00,21,10

C(302):00,00,21
C(303):00,00,00
C(304):20,00,00
C(305):10,20,00
C(306):11,00,20
C(307):21,21,00
C(308):01,01,01
C(309):00,21,21
C(310):00,00,21
C(311):20,00,00
C(312):20,20,00
C(313):01,20,20
C(314):20,01,10
C(315):01,20,01
C(316):01,01,20
C(317):01,01,01
C(318):01,01,01
C(319):21,01,01---
C(320):21,21,01
C(321):01,21,21
C(322):10,01,21
C(323):21,10,01
C(324):21,21,00
C(325):00,21,21
C(326):00,00,21
C(327):00,00,00
C(328):21,00,00
C(329):10,21,00
C(330):00,10,21
C(331):20,00,20
C(332):00,20,10
C(333):20,00,20
C(334):20,20,00
C(335):00,20,20