Programming Assignments 1 & 2 601.455 and 601/655 Fall 2021

Please also indicate which section(s) you are in (one of each is OK)

Score Sheet

Name 1	Sean Darcy
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Signature (required)	I (we) have followed the rules in completing this assignment

Grade Factor		
Program (40)		
Design and overall program structure	20	
Reusability and modularity	10	
Clarity of documentation and programming	10	
Results (20)		
Correctness and completeness	20	
Report (40)		
Description of formulation and algorithmic approach	15	
Overview of program	10	
Discussion of validation approach	5	
Discussion of results	10	
TOTAL	100	

Programming Assignment #2

Computer Integrated Surgery I
11/9/2021

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Description of formulation and algorithmic approach

PA 1

We decided to develop our program in Python. Specifically, we chose to develop a python notebook (.ipynb) in Google Collab so that we could code together simultaneously and organize our code by task with instructions displayed.

We began by developing a Cartesian math package for 3D points, rotations, and frame transformations. We imported and developed proficiency with Numpy, a standard numerical library with support for large, multi-dimensional arrays and matrices, as well as a collection of functions that operate on those arrays. We implemented 3-D points as 1x3 Numpy arrays. We defined several distinct methods to generate rotation matrices based on varying input parameters, described in the program overview section that follows. We implemented frames as python classes with corresponding rotation matrices R and translations/offsets p. F is represented homogeneously as a 4x4 vector. Key linear algebra operations were also implemented, like skew(), norm(), etc. (described in detail in the following section).

Next, we implemented two methods for 3D point set to 3D point set rigid registration. The first is based on Arun's method which involves SVD on the matrix H generated from the point sets a and b. Unfortunately, Arun's method has 2 failing cases. The first- when det(R) = -1- we were able to handle based on Arun's paper. Because of the second failing case, when det(R) = -1 and none of the singular values of H are 0, we decided to implement a second method for 3D point set to 3D point set rigid registration method based on the unit quaternion.

We then implemented pivot calibration based on sphere fitting, as described in https://mphy0026.readthedocs.io/en/latest/calibration/pivot_calibration.html. We chose to implement pivot calibration based on sphere fitting as it made sense to estimate the possible pivoting poses of the tracked pointers as spheres centered at the pivot dimple. Also, Ma et al. found that sphere fitting was a superior calibration method when data quality was good.

To perform steps 4, 5, and 6 in the assignment, we first had to develop several file input/output functions to read in the provided data, including calbody.txt, calreadings.txt, empivot.txt, optpivot.txt, and to output our results in output1.txt. To do so, we imported Pandas, which offers data structures and operations for manipulating numerical tables and time series in Python.

Next, we took a distortion calibration data set and computed the expected values C_exp for Ci using the give_C_exp() function. To do so, we first computed the transformation between optical tracker F_D and EM tracker coordinates, which involved computing a frame F_D such that $D_j = F_D * d_j$ using rigid registration. Next, we computed the transformation F_A between calibration object and optical tracker coordinates, which again involved computing a frame F_A such that $A_j = F_A * a_j$ using rigid registration. Finally, we were able to compute and output C_exp by $F_D^{-1}*F_A*cj$.

We were then able to read in the EM tracking data to perform pivot calibration for the EM probe. We used the first frame of the calibration data to define a local probe coordinate system as described in the assignment instructions. We then used our rigid registration method based on unit quaternions to determine Fg, the pose of the EM probe with respect to the EM tracker. We then were able to apply our pivot calibration method to determine the best estimate for ptip_g, the tip of the EM pointer with respect to its pose Fg.

We conducted a similar process to calibrate the optically tracked pointer, but first had to transform optical tracker beacon positions into EM coordinates by applying Fd^-. Next, we were able to again apply pivot calibration to solve for ptip_D, the tip of the optically tracked pointer with respect to its pose Fd.

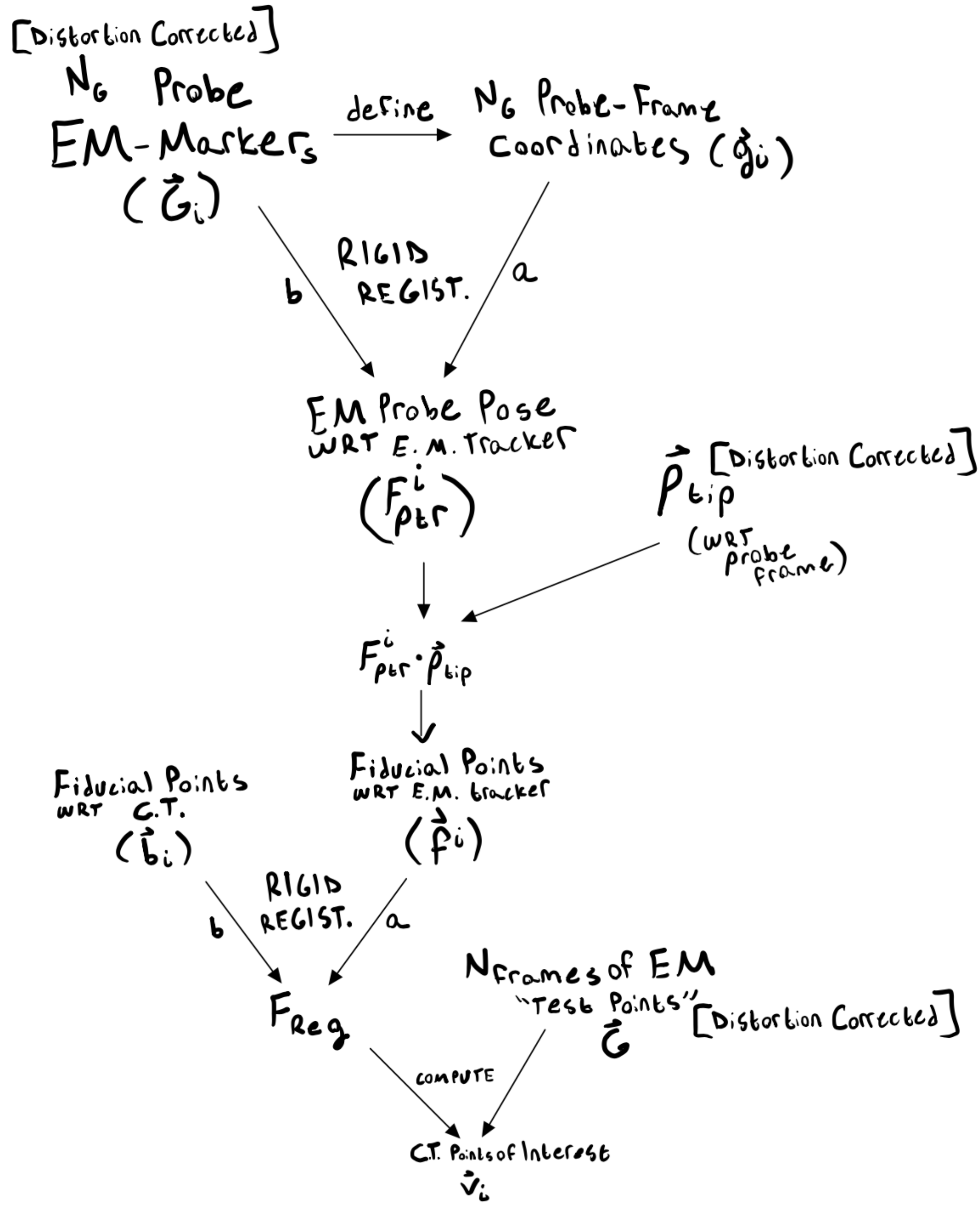
PA 2

We began part two of this process by calculating Ci,expected for each frame of calibration object data. The tools/methods to execute this were developed in the previous assignment. We first computed the transformation between optical tracker F_D and EM tracker coordinates, which involved computing a frame F_D such that $D_j = F_D * d_j$ using rigid registration. Next, we computed the transformation F_A between calibration object and optical tracker coordinates, which again involved computing a frame F_A such that $A_j = F_A * a_j$ using rigid registration. Finally, we were able to compute and output C_exp by $F_D^{-1}*F_A*cj$.

Next, we developed a distortion correction function. To do so, we began by developing a ScaleToBox method which scales input data to a range between [0,1]. Next, we developed a tensor form, 5th degree Bernstein interpolation polynomial using tools like binom() from the SciPy library. The reason for scaling our data in the first step is because these polynomials are optimized for the [0,1] range. We formulate and solve a least squares problem to determine the coefficients of our interpolation polynomial. Our C_exp data calculated in part 1 of the assignment is used as “ground truth” to fit these polynomials to our observed data and thus correct distortion.

In part 3, we re-applied our pivot calibration method developed in assignment 1. However, this time we applied our distortion correction function to the EM probe pivot data as a preliminary step to obtain a distortion-corrected value for ptip. In part 4 we define local, distortion corrected EM probe poses by defining a local probe coordinate system with the EM markers for each frame of EM fiducial landmark data and then computing a rigid registration between these two sets. We then apply these frames to our distortion corrected ptip to get the fiducial points with respect to the EM tracker. We’re then able to compute another rigid registration between these points and those given in CT coordinates to determine Fregistration between EM tracker and CT coordinates. We use Fregistration to determine vis in CT coordinates corresponding to various, distortion-corrected values of arbitrary EM probe navigation data. The figure below summarizes parts 4-6.

PROGRAM STRUCTURE



Program overview

PA1

rotFromComponents(Rx, Ry, Rz)

This function generates a 3D rotation matrix from 3 input component 3D rotation matrices. This is derived from the fact that any rotation can be described by consecutive rotations about three primary axes x, y, and z.

Rxyz(alpha, beta, gamma) = R(x, alpha)*R(y, beta)*R(z, gamma)

rotFromAngles(alpha, beta, gamma)

This function generates a 3D rotation matrix from 3 input rotation angles. 3 rotation matrices Rx, Ry, and Rz are generated from alpha, beta, and gamma. The method rotFromComponents(Rx, Ry, Rz) is then called to return the final rotation matrix R.

identity()

This function returns a 3x3 Numpy array representing the identity matrix:

[1, 0, 0]
[0, 1, 0]
[0, 0, 1]

norm(v)

This function returns the norm of the vector v, computed as v / sqrt(v*v). The Numpy functions np.dot()- which computes the dot product of two input parameters v and v- and np.sqrt()- which computes the square root of this result- are called.

skew(v)

This function generates a skew matrix from the input, which is a normalized (via call to norm() within the method) 1x3 Numpy array:

[0,   -Vz, Vy]
[Vz,   0,   -Vx] = skew(v)
[-Vy, Vx,   0]]

smallAngleR(v, theta)

This function returns a small angle approximation for R ≅ [I + skew(norm(v)*theta)]

homogeneousVector(v, scale)

This function generates a 4-D homogeneous representation of 3-D vector v, with scaling factor scale.

scale * [Vx, Vy, Vz, 1]

class Frame

This class represents a frame, or pose, consisting of a Rotation matrix R and offset/translation vector p.

Methods:

getFrame(self)

Method defines frame with rotation R and translation as parameters and returns F

appFrame(self, v)

Method applies frame transformation F to input vector v, F*v = R*v + p.

appInvFrame(self, v)

Method applies frame transformation F^- to input vector v, F^- * v = R^- * v - R^- * p. R^- is computed via Numpy R.T, representing the transpose of R, R^T. Rotation matrices have the property that R^T = R^-.

Next, we developed 3D point set to 3D point set registration algorithms. One based on Arun’s method, and another based on unit quaternions.

rigid_registration_Arun(a, b)

This function takes two point clouds, a and b, as parameters. First, the means of a and b are calculated using np.mean() and subtracted from each respective point in a and b, and stored in A and B. Next, the matrix H is computed via np.dot(A, B.T). The SVD of H is computed via np.linalg.svd(H), with output matrices U, S, V^T. Next, R is computed by np.dot(V^T.T, U.T) = np.dot(V, U.T). There are two potential failing cases. If det(R) = -1, then R = Vprime*U.T, where the third column of Vprime is the product of the third column of V multiplied by -1, and the 1st and second columns are equivalent. The second failing case is when none of the singular values of H are 0. We handled the first failing case as described, but for the second case chose to implement a different flavor of 3D point set to 3D point set registration. The function returns a frame corresponding to the transformation from a to b.

rigid_registration(a, b)

This function, based on the unit quaternion, takes two point clouds, a and b, as parameters. Similar to Arun's method, the means of a and b are first calculated using np.mean() and subtracted from each respective point in a and b, and stored in A and B. Next, the matrix H is computed via np.dot(A, B.T). Next, the matrix G is computed
G =
[trace(H), Δ^T]
[Δ, H + H^T - trace(H)I]

where Δ^T = [$H_{2,3}$ - $H_{3,2}$, $H_{3,1}$ - $H_{1,3}$, $H_{1,2}$ - $H_{2,1}$]

trace(H) was computed via np.trace(H). Note, I in this matrix was computed via np.eye(3) for numpy compatibility.

Next, the eigenvalue decomposition of G is computed via np.linalg.eig(G), with outputs l and Q corresponding to the eigenvalues and eigenvectors of G, respectively. The index qi of the largest eigenvalue of G in l is then computed via np.argmax(l), and then the largest eigenvector in Q is found via q = Q[:, qi]. This largest eigenvalue q is a unit quaternion corresponding to the rotation matrix. The rotation matrix can be computed via the components of q. The method returns a frame corresponding to the transformation from a to b.

pivotCalibration(frames)

Function used to perform sphere-fitting pivot calibration. Takes parameter frames, the N X 4 X 4 ndarray of frames used to perform the calibration. Function returns a list of the following:
ptr_o- 3-Dimensional pointer offset 1 X 3 ndarray
ptr_p- coordinates of the pivot point 1 X 3 ndarray
err- RMS error about centroid of pivot

read_calbody(name)

Function to read the data file for registration point locations.Takes parameter name corresponding to the name of the data set being read.Function return a list consisting of the following:
d_coords - coordinates of d N_D X 3 ndarray
a_coords - coordinates of a N_A X 3 ndarray
c_coords - coordinates of c N_C X 3 ndarray

read_calreadings(name)

Function to read the data file for calibration readings.Takes parameter name corresponding to the name of the data set being read. Function return a list consisting of the following:
D_coords - coordinates of D, F X N_D X 3 ndarray
A_coords - coordinates of A, F X N_A X 3 ndarray
C_coords - coordinates of C, F X N_C X 3 ndarray

read_empivot(name)

Function to read the data file for EM probe. Takes parameter name corresponding to the name of the data set being read. Function return a list consisting of the following:
G_coords - coordinates of G, F X N X 3 ndarray.

read_optpivot(name)

Function to read the data file for optical probe calibration. Takes parameter name corresponding to the name of the data set being read. Function return a list consisting of the following:
D_coords - coordinates of D, F X N X 3 ndarray
H_coords - coordinates of H, F X N X 3 ndarray

read_output(name)

Function to read the debug data set. Takes parameter name corresponding to the name of the data set being read. Function return a list consisting of the following:
em_pt - coordinates of the EM probe, 1 X 3 ndarray
opt_pt - coordinates of the optical probe, 1 X 3 ndarray
c_exp - coordinates of the expected values of C, F X N X 3 ndarray

write_output(name, em_pt, opt_pt, C_exp)

Function to write all output data to a text file. Takes parameters name, em_pt, opt_pt, and c_exp, corresponding to the name of the file, coordinates of the EM probe 1 X 3 ndarray, the coordinates of the optical probe (1 X 3 ndarray), and the coordinates of the expected values of C (F X N X 3 ndarray),respectively.

give_C_exp(name, save=False, m_pt=np.array([0,0,0]), opt_pt=np.array([0,0,0]))

Function used to compute the expected values of C. Parameters include name, save, em_pt, opt_pt, which correspond to the name of the data set being used, whether or not the output file is to be saved, coordinates of the EM probe, and coordinates of the optical probe, respectively. The function returns c_exp, the coordinates of the expected values of C.

em_pivot_calibration(name)

Function to apply EM tracking data to perform a pivot calibration for the EM tracking probe. Takes parameter name, the name of data set being used and returns em_pts - coordinates of EM probe wr/ EM tracker

op_pivot_calibration(name)

Function to apply optical tracking data to perform a pivot calibration for the EM tracking probe. Takes parameter name, the name of data set being used and returns op_pts - coordinates of optical probe wrt optical tracker.

PA2

Part 2

ScaleToBox(q, qmin, qmax)

Function to scale values in q to a bounding box. Takes parameter q - an ndarray of dimensions n x 3, and corresponding maximum and minimum values- and returns u - a ndarray of dimensions n x 3 with values scaled from [0, 1].

B(v, k, N=5)

Function to generate the pmf of a binomial distribution, which will be used to generate a 5th degree Bernstein polynomial. Takes parameters v (the argument for computation), k (which will take a basis i, j, k), and N = 5, the degree of the polynomial. Returns the pmf of a binomial distribution.

compute_distortion(p, q, qmin, qmax, N=5)

Function takes parameters q, qmin, and qmax and calls ScaleToBox(), described above. The function then concatenates B()s with outputs of ScaleToBox() and corresponding ijks from 0 to 5 as parameters. Least squares is then performed to estimate the coefficients ci of the B.P. These coefficients are returned.

correct_distortion(x, name, N=5)

Function takes Cexp “ground truth” computed in part 1 as well as distorted calibration data and applies the Bernstein Polynomial-based distortion correction described above to correct for the distortion in the distorted data. Returns the “cleaned,” undistorted data.

Part 3

em_pivot_calib_raw(G, nf, ng, ptip = True)

Function begins by taking the first "frame" of pivot calibration data to define a local "probe" coordinate system, which is then used to compute corresponding points gi defined in that same probe coordinate system. Transformations Fg are computed between these two point sets using the rigid_registration() method designed in PA1. The method then calls our pivot_calibration() method defined earlier to return a value for ptip.

em_pivot_calibration_distortion_correction(name)

This function takes an input distorted data set as a parameter, and applies correct_distortion() to account for distortion before calling em_pivot_calib_raw() to ultimately return an optimized, undistorted value for ptip.

Part 4

locate_fiducials_em(name)

This function takes name as a parameter corresponding to a particular dataset. For the data set, the fiducial points in CT coordinates and corresponding EM probe markers are read. First, the EM data is dewarped. Next, the ptip value is obtained from the dewarped data using the em_pivot_calibration_distortion_correction() function. For each fiducial landmark, the EM probe pose is determined and applied to ptip. The resulting points correspond to the known CT fiducial points vi. The output of the function is the set of fiducial points in EM tracker coordinates (Bi/fi) corresponding to the CT fiducial points.

Part 5

comp_F_reg(name)

This function computes Fregistration between CT and EM tracker spaces via simple registration between CT fiducial points bi and the corresponding EM tracker fiducial points computed in part 4 via locate_fiducials_em(). Fregistration is returned as a frame object.

Part 6

nav_to_CT_coords(name)

This function uses Fregistration to determine and return a set of vi's defined in CT coordinates corresponding to various, distortion-corrected values of EM probe navigation data.

Validation approach

We implemented and conducted unit testing for each major component of our program—these included the correct implementation of the distortion correction, the correct identification of the fiducial landmarks in EM tracker coordinates, and the correct conversion of the EM nav points to the CT coordinate frame using the frame found in part 5. We chose to organize our unit tests in code cells directly above the driver for the entire PA2 assignment in order to allow for the compilation of the necessary testing suite (test2_similarity), which computes the accuracy of the computed CT locations of the EM nav points. These unit tests include:

PA1

-Rotation test
-Transform test
-Transformation is 90 degree rot over x and +1 translation on z test
-Test module for 3D registration with two sets of points to check 3D-3D registration

-pivotCalibration

We ultimately tested our pivotCalibration method by plotting the error rate between generated calibration output and expected output for various pre-specified coordinate threshold values between 0 and 3.5 coordinate units, determining at which of these thresholds does the accuracy of our pivot calibration exceed 99.9%.

-test_Similarity(true, comp, error)

Function to test similarity between a given output file and computed output values. Takes parameters true, comp, and err. Parameter true is a list of EM and optical coordinates and respective C_exp values for all frames and markers from given output. Parameter comp is a list of EM and optical coordinates as well as C_exp values computed for all frames and markers and outputted by our program. Parameter error is the threshold representing the maximum difference between any coordinate true value and our computed value for computed output to be considered correct. The function returns the % accuracy of correct.

-resultsTables(debug)

Function generates content described below and takes debug data as input.

PA2

Test for Distortion Correction

The test for distortion correction was centered around specific debug sets, namely those with either no distortion, or only distortion for the EM tracker. For the sets with no distortion, the pivot calibration for EM was performed for the coordinates of the dimple with and without the application of distortion correction to verify that correction of no distortion yields the same results as the original pivot calibration, which was shown to be correct in the previous assignment. Moreover, the debug sets with only an EM distortion and no other noise were tested to verify that performing the pivot calibration with distortion corrected yielded results that were closer to the expected output than the pivot calibration for PA1. Upon inspecting printed values and verifying that the distortion correction appeared to improve accuracy, this test was passed.

Testing Identification of Fiducial Landmarks in EM Tracker Coordinates

This test was rather informal, and performed by using a series of print statements that can be found in the code for part 5. These print statements would verify that the CT fiducial coordinates were equivalent to those same coordinates but found by applying the EM to CT transformation to the EM fiducial coordinates. This testing would effectively both test the computation of the transformation between EM to CT coordinates (rigid registration) and the actual computation of fiducial landmarks in EM coordinates. However, since we have already shown our rigid registration method to be correct in PA1, we were confident that these print statements served only to test the computation of fiducial landmarks in EM coordinates. When the printed values agreed, we knew that the landmarks were computed correctly.

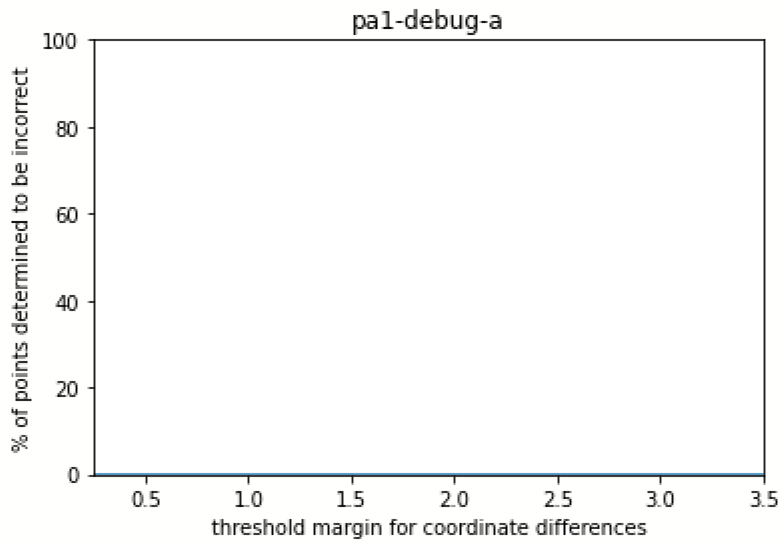
Testing Identification of Test Points in CT Coordinate Frame

The second module in the testing module section tests the correct transformation of EM Nav points to CT coordinates using F_{reg}. This is carried out by assuming that the distortion correction and identification of fiducial landmarks in EM tracker coordinates were computed correctly (since they have been tested already), and comparing the resulting CT-frame locations of the test points with the expected output. Upon a high accuracy, this test is passed.

Discussion of Results

PA1

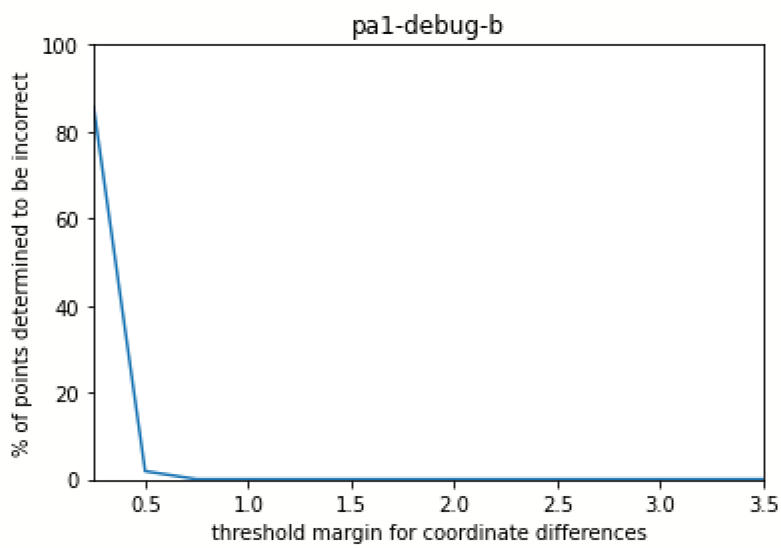
We were able to verify that individual components of our program work as expected via unit testing, as discussed previously. To evaluate the overall performance of our program, we ran a suite of test_Similarity() tests for every debugging data set provided for programming assignment 1, for multiple values of the error parameter, which represents some threshold value for computed coordinates to be considered accurate. This test_Similarity suite can be found in Main Module, which essentially executes EM and Optical probe pivot calibration on the data, computes C_expected from the calibrated data via give_C_exp(), generates an output file, reads the output file, and runs test_Similarity() between each debugging output set and the computed output file. In addition, Main Module also generates output for the unknown data sets, but this output has no reference to test similarity. Below are the results for each debug set for varying values of error threshold. Each debug set has varying levels of EM noise, EM distortion, and OT ‘jiggle’.



EM noise = False

EM distortion = False

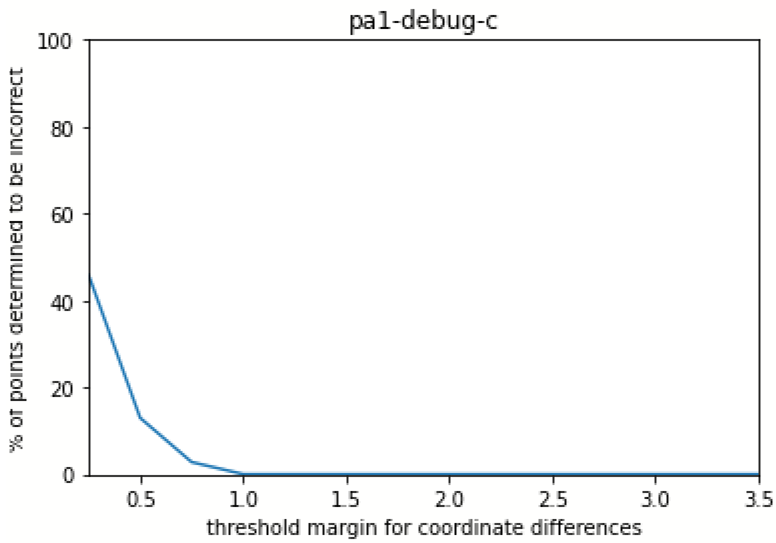
OT ‘jiggle’ = False



EM noise = True

EM distortion = False

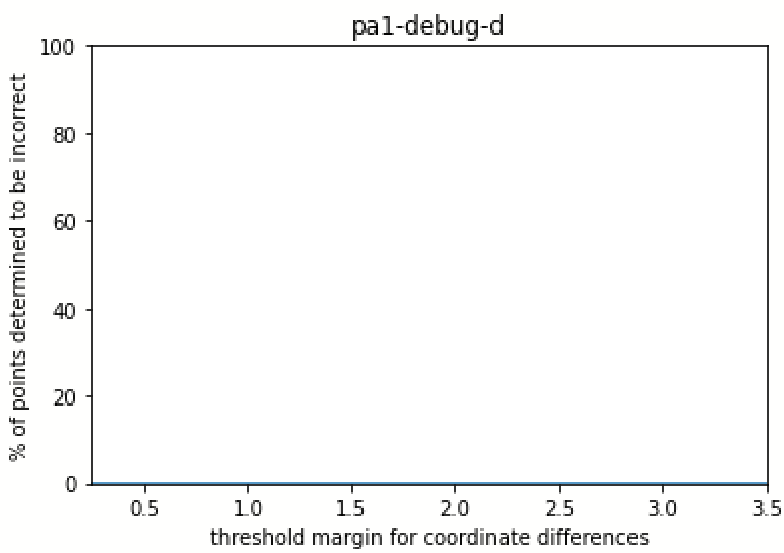
OT ‘jiggle’ = False



EM noise = False

EM distortion = True

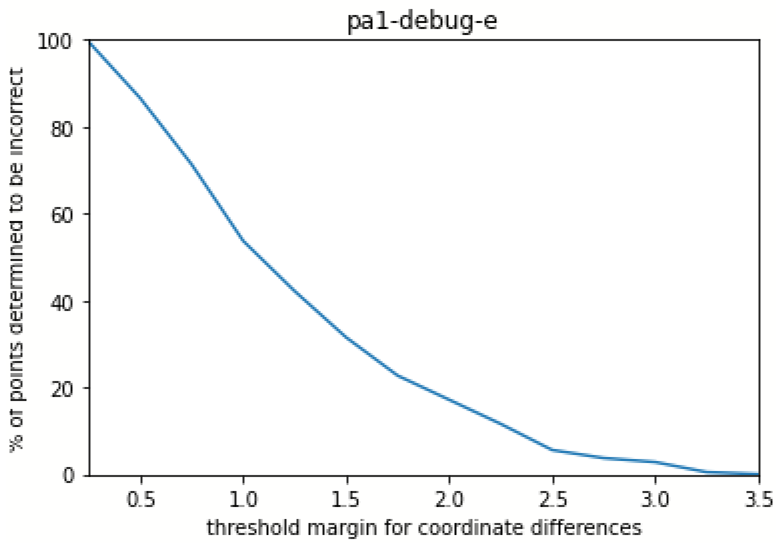
OT ‘jiggle’ = False



EM noise = False

EM distortion = False

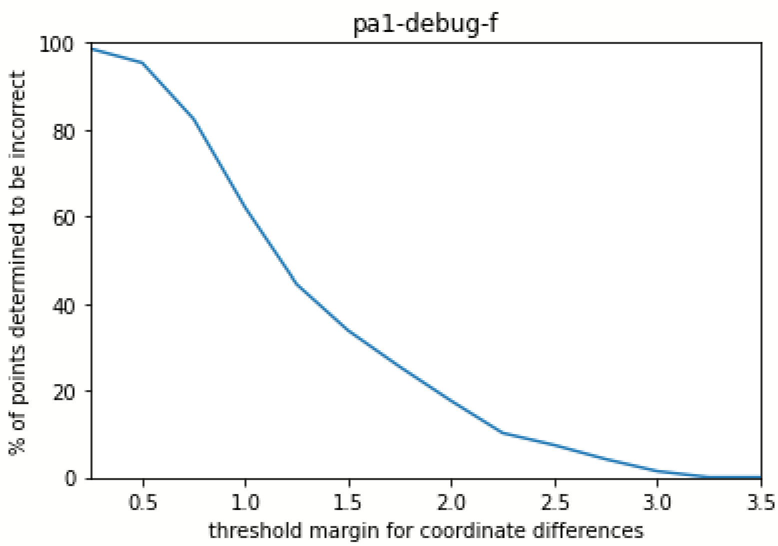
OT ‘jiggle’ =True



EM noise = False

EM distortion = True

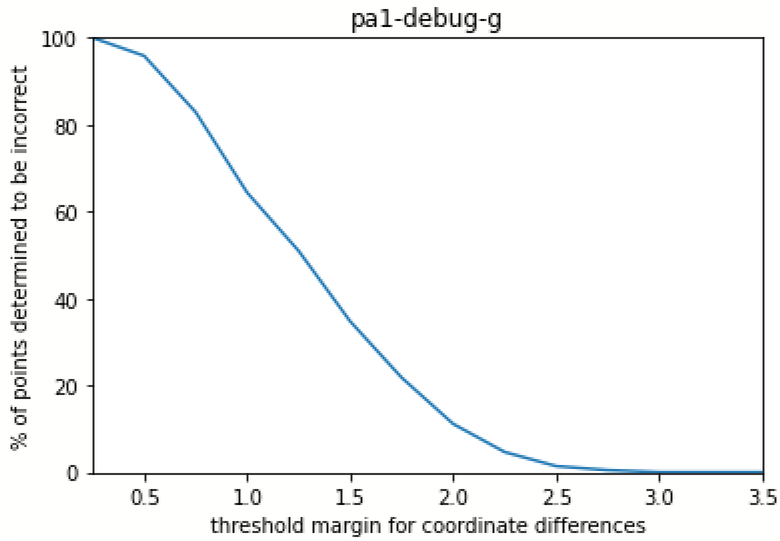
OT ‘jiggle’ = True



EM noise = True

EM distortion = True

OT ‘jiggle’ =True



EM noise = True
EM distortion = True
OT ‘jiggle’ = True

We wanted to find a threshold margin such that each set of debugging data had an accuracy rate greater than 99.9%, or an error rate less than 0.1% to determine the optimal error margin of error for our data.

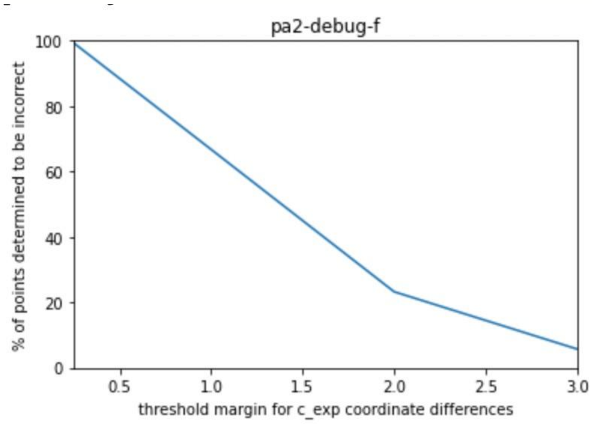
The optimal values for each debug set are below:

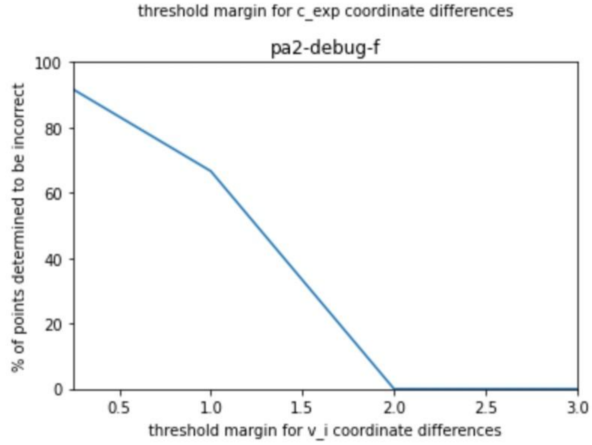
pa1-debug-a < 0.25
pa1-debug-b 0.75
pa1-debug-c 1
pa1-debug-d < 0.25
pa1-debug-e 3.25
pa1-debug-f 3.25
pa1-debug-g 2.75

Thus a margin of 3.25 coordinate units is the accepted error in our calibration that returns an accuracy greater than 99.9% for all combinations of EM noise, EM distortion, and OT “jiggle”.

PA2

We were able to verify that individual components of our program work as expected via unit testing, as discussed in the validation section. Like in programming assignment 1, to evaluate the overall performance of our program, we ran a suite of test2_Similarity2() tests for each debugging data set provided for programming assignment 2 for multiple values of an error parameter. This error parameter represents some threshold value for computed coordinates to be considered accurate. This test2_Similarity2 suite can be found in our Main Module, which essentially runs parts 1 through 6 of programming assignment 2, generates an output file, reads the output file, and runs test2_Similarity2() between each debugging output set and the computed output file. In addition, Main Module also generates output for the unknown data sets, but this output has no reference to test similarity. Below are the results for one of the “noisiest” (in terms of distortion, noise, and jiggleO) debug sets for varying values of the error threshold.





EM noise = True

EM distortion = True
OT ‘jiggle’ =True

One should note that the threshold margin has changed due to our dewarping method. Previously a margin of 3.25 coordinate units was determined to be the accepted error in calibration in order to return an accuracy greater than 99.9% for all combinations of EM noise, EM distortion, and OT “jiggle”. Thanks to our dewarping method, that threshold value has converged to 2.0 coordinate units. There is still OT jiggle and random noise.

References

Ma, B., Banihaveb, N., Choi, J., Chen, E. C., & Simpson, A. L. (2017, March). Is pose-based pivot calibration superior to sphere fitting?. In *Medical Imaging 2017: Image-Guided Procedures, Robotic Interventions, and Modeling* (Vol. 10135, p. 101351U). International Society for Optics and Photonics.

K. Arun, et. al., IEEE PAMI, Vol 9, no 5, pp 698-700, Sept 1987