Detection:
find $s(k)$ from $y(k)$

$|h(k)| \sim$ Rayleigh
$\angle h(k) \sim$ uniform

Pre-LAB – BER Simulation

± 1

time

AWGN

- A simple BPSK communication system can be written as

$$y(k) = \sqrt{P} \times h(k)\boxed{s(k)} + n(k), \quad \text{for time } k$$

$= \dfrac{P}{\sigma^2}$

Signal power

where $y(k)$ is the received signal
$h(k)$ is the fading channel
$s(k)$ is the BPSK symbol $\in \{-1, +1\}$
$n(k)$ is the additive noise

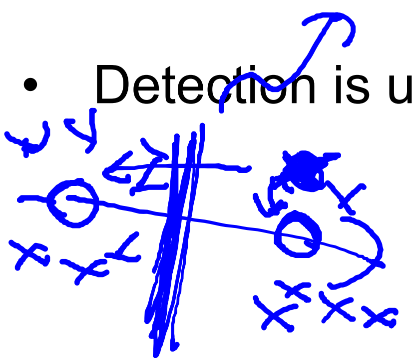
$\mathrm{SNR} = \dfrac{(\sqrt{P})^2 E[|h(k)|^2] \times E[s^2(k)]}{}$

$y(k) = \sqrt{P}\, h(k)\, s(k)$

$\dfrac{y(k)}{\sqrt{P}\, h(k)} = s(k)$ $\rightarrow E[|h(k)|^2]$

expectation

- Detection is usually done by



$$\tilde{s}(k) = \begin{cases} +1, & \text{if Re}\left\{\dfrac{y(k)}{\sqrt{P}\,h(k)}\right\} > 0, \\ -1, & \text{if Re}\left\{\dfrac{y(k)}{\sqrt{P}\,h(k)}\right\} \le 0, \end{cases}$$

Pre-LAB – BER Simulation

- An error occurs if $\tilde{s}(k) \neq s(k)$

$+1$ -1

- BER is the probability that an error for detecting a bit occurs
- BER can be estimated by simulating the detection for **many** bits and counting the number of errors, normalised by the total number of bits
- Let's go through the simulations together!

10 bits → 3 bits in error

1000000
10^6

$\mathrm{BER} = \frac{3}{10} = 0.3$

$y(k) = \sqrt{P}\, h(k)\, s(k) + n(k)$

Pre-LAB – BER Simulation in MATLAB

$k = 1, 2, 3, \dots, 10^6$ fixed P fixed $\sigma^2 = E[|h(k)|^2]$

$E[|h(k)|^2] = 1$ $P(1)(1)$

$\Rightarrow SNR = \frac{P(1)(1)}{\sigma^2}$

- Generate $s(k)$

rand

```
>> u=rand;
>> if u>.5, s(k)=1; else s(k)=-1;
```

s(k) is either 1 or -1 with equal probability

- Generate $h(k)$

randn mean = 0 variance = 1

```
>> h(k)=(randn+j*randn)/sqrt(2);
```

h(k) is complex Gaussian distributed, with 0 mean and 1 variance. Also, |h(k)| is Rayleigh distributed and ∠h(k) is uniformly distributed between 0 and 2π

$E[|h(k)|^2] = \frac{2}{(\sqrt{2})^2} = 1$

- Generate $n(k)$

```
>> n(k)=(randn+j*randn)/sqrt(2);
```

* sigma

n(k) is complex Gaussian distributed, with 0 mean and 1 variance, or the noise power is 1

$E[|h(k)|^2] = \sigma^2$ $\sigma^2 = 1$

fixed SNR fixed P

adjust σ

P=1

Pre-LAB – BER Simulation in MATLAB

- Generate $y(k)$

$k=1,2,3,\ldots,10^6$

```
⟩⟩ y(k)=P*h(k)*s(k)+n(k);
```

$\sqrt{\ }$

$Re\left\{\frac{y(k)}{\sqrt{P}\,h(k)}\right\} \overset{1}{\underset{-1}{\gtrless}} 0$

- Generate $\tilde{s}(k)$

s(k)

```
⟩⟩ stilde(k)=(real(y(k)/h(k))>0)*1+(real(y(k)/h(k))<=0)*(-1);
```

no_of_errors=0

- Count the number of errors

```
⟩⟩ if stilde(k)~=s(k), no_of_errors=no_of_errors+1;
```

"~=" means "≠"

fixed SNR

Pre-LAB – BER Simulation in MATLAB

fixed P

fixed σ

- Note that we need to run the above for many k, i.e.,

xplot

semilogy

```
》no_of_errors=0;
》for k=1:total_bits,
》      Simulate stilde(k) to update no_of_errors
》end
```

for different SNR

BPSK QPSK 16QAM BER(SNR) and

- Calculate the BER

SNR (dB)

```
》BER=no_of_errors/total_bits;
```

- Then, we have obtained the BER for SNR=10log10(P) dB **ONLY**!

$10\log_{10}\left(\frac{P}{\sigma^2}\right)$

More Efficient MATLAB Simulations

- FOR LOOP is SLOW in MATLAB
- BUT can be avoided by using VECTORS/MATRICES
- For the BER simulations, we can have

```
>> u=rand(total_bits,1);
>> s(1:total_bits,1)=1; s(find(u<0.5))=-1;
>> h=(randn(total_bits,1)+j*randn(total_bits,1))/sqrt(2);
>> n=(randn(total_bits,1)+j*randn(total_bits,1))/sqrt(2);
>> s_est=real((P*h.*s+n)./h);
>> stilde(find(s_est>0))=1; stilde(find(s_est<=0))=-1;
>> BER=length(find(stilde~=s))/total_bits;
```