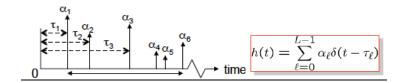
MSc Module: Communications Systems Modelling Lab Session 1 – Simulating OFDM Communications Systems in Frequency Selective Fading Channels

This lab sheet has to be read in conjunction with the lecture notes on OFDM!

1.1 Simulation of L-Ray Equal-Power Rayleigh Fading Channels



- 1.1.1 Generate a Poisson random variable L to decide the number of rays of the channel.
- 1.1.2 Generate L complex fading coefficients, each has a Rayleigh distributed amplitude and a uniformly distributed phase (α_1 , α_2 , α_3 , ... α_L)
- 1.1.3 Using uniform random variables from 0 to 2 to model the time delays for the rays. Generate L delays (τ_1 , τ_2 , τ_3 , ... τ_L)
- 1.1.4 Plot the channel impulse response (hint: use "stem" instead of "plot"). Make sure the *average* power of the channel is the same (e.g., unity) regardless of the number of rays in the channel. This means that we want

$$E[|\alpha_1|^2] + E[|\alpha_2|^2] + \cdots + E[|\alpha_L|^2] = 1$$

If the average power of each ray is assumed to be the same, then you can set $E[|\alpha_i|^2]=1/L$ to achieve this.

1.1.5 Repeat the above to generate a number of independent realisations of the channel impulse response.

1.2 Simulation of QPSK symbols

- 1.2.1 Generate a sequence of N independent QPSK symbols (s)
- 1.3 Simulation of the Input-Output Relationship
- 1.3.1 Assuming f is a rectangular function, obtain $g_{m,n}$

$$y_{m} = \sum_{n=0}^{N-1} s_{n} \begin{bmatrix} \sum_{\ell=0}^{L-1} \alpha_{\ell} f((m-n)T - \tau_{\ell}) \end{bmatrix} + n_{m} = \sum_{n=0}^{N-1} s_{n} g_{m,n} + n_{m}$$

$$\Rightarrow \begin{bmatrix} y_{0} \\ y_{1} \\ y_{2} \\ \vdots \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} g_{0} & 0 & 0 & \cdots & 0 & 0 \\ g_{1} & g_{0} & 0 & 0 & \cdots & 0 & 0 \\ g_{2} & g_{1} & g_{0} & 0 & \cdots & 0 & 0 \\ g_{3} & g_{2} & g_{1} & g_{0} & 0 & \cdots & 0 \\ 0 & g_{3} & g_{2} & g_{1} & g_{0} & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{N-1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_{0} \\ g_{1} & g_{0} & 0 & 0 & \cdots & 0 \\ g_{2} & g_{1} & g_{0} & 0 & \cdots & 0 \\ 0 & g_{3} & g_{2} & g_{1} & g_{0} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{N-1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_{0} \\ g_{1} & g_{0} & 0 & \cdots & 0 \\ g_{2} & g_{1} & g_{0} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{N-1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_{0} \\ g_{1} & g_{0} & 0 & \cdots & 0 \\ g_{3} & g_{2} & g_{1} & g_{0} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{N-1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_{0} \\ g_{1} & g_{0} & 0 & \cdots & 0 \\ g_{3} & g_{2} & g_{1} & g_{0} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n_{N-1} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \text{ Toeplitz channel matrix} \end{bmatrix}$$

- 1.3.2 Generate a sequence of N independent noise samples (mean 0, variance σ^2) (n)
- 1.3.3 Generate the sequence of y_m

1.3.4 Write a MATLAB code that obtains the symbol error rate (SER) for the system. Note that in order to perform detection, one must first remove the perturbation of the channel carrying the message. Therefore, we should perform detection based on the following signal

$$\tilde{s}_m = \frac{y_m}{g_{m,m}} = \frac{y_m}{g_0}$$

Also, it is important to note that SER results should be averaged over the randomness of the channel h

1.3.5 Plot the SER results against $10\log_{10}(1/\sigma^2)$. You can try to change some parameters to see how the results are affected, e.g., the number of rays, the delay spread, the pulse shape, etc.

1.4 Simulation of the OFDM System

1.4.1 Obtain the circulant channel matrix

$$\begin{bmatrix} g_0 & 0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 \\ g_1 & g_0 & 0 & 0 & \cdots & 0 & g_3 & g_2 \\ g_2 & g_1 & g_0 & 0 & \cdots & \cdots & 0 & g_3 \\ g_3 & g_2 & g_1 & g_0 & 0 & \cdots & & 0 \\ 0 & g_3 & g_2 & g_1 & g_0 & 0 & & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ & & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 & g_0 \end{bmatrix}$$

- 1.4.2 Generate the IFFT of s
- 1.4.3 Obtain the sequence of y (y = G*IFFT(s) + n)
- 1.4.4 Obtain the FFT of y
- 1.4.5 Write a MATLAB code that obtains the SER for the system assuming the channels are known
- 1.4.6 Plot the SER results against $10\log_{10}(1/\sigma^2)$

Bonus Tasks Channel Estimation and Error Correction Coding

- B1 Repeat the simulations of 1.4 but assuming that the channels need to be estimated. Hint: channels h can be estimated by first sending known symbols (called pilots) and then the estimated channels can be used for the detection of unknown symbols.
- B2 Repeat the simulations of 1.4 with 1.5.1 and add repetition coding with rate 1/3. Note that your SER results show steeper decline if coding is used. Also, use the simulation results to estimate the diversity order of the system.
- B3 In OFDM systems, the SER will be dominated by the worst subcarriers. To improve the SER of the system, it is possible to disable the worst subcarriers but transmit at higher rates for those good subcarriers. Design a system to take advantage of this observation and study its performance using simulations.

Lab report Submission

- 1. A lab report needs to be submitted online on Moodle on or before 4PM 18 JAN 2021 (FRIDAY)
- 2. The lab report will be marked and the marks will contribute 50% of your overall marks for CSM
- 3. In the lab report, you will need to include the following (but not limited to):
 - An introduction section that describes the lab (only LAB SESSION 1)
 - A discussion section that discusses the assumptions of the simulations specific to the lab, and the results obtained from the simulations
 - Some extra simulations that are related but not done in the lab
 - A conclusion section
 - Include your MATLAB code as appendices