A Short Table of Fourier Transforms

Description	Function	Transform
Definition	$x(t)=\int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$	$X(f)=\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$
Scaling	$x(t/T)$	$\lvert T\rvert .X(fT)$
Time shift	$x(t-T)$	$X(f).e^{-j2\pi fT}$
Frequency shift	$x(t).e^{j2\pi Ft}$	$X(f-F)$
Complex conjugate	$x^*(t)$	$X^*(-f)$
Temporal derivative	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n .X(f)$
Spectral derivative	$(-j2\pi t)^n .x(t)$	$\frac{d^n}{df^n}X(f)$
Reciprocity	$X(t)$	$x(-f)$
Linearity	$a.x(t)+b.y(t)$	$a.X(f)+b.Y(f)$
Multiplication	$x(t).y(t)$	$X(f) * Y(f)$
Convolution	$x(t) * y(t)$	$X(f).Y(f)$
Delta function	$\delta(t)$	1
Constant	1	$\delta(f)$
Rectangular function	$rect(t)$	$sinc(f)$
Sinc function	$sinc(t)$	$rect(f)$
Unit step function	$U(t)=$ 1 for t>0 0 for t<0	$\frac{1}{2}\delta(f)-\frac{j}{2\pi f}$
Signum function	$\mathrm{sgn}(t)=$ 1 for t>0 -1 for t<0	$\frac{-j}{\pi f}$
Two-sided decaying exponential function	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
One-sided decaying exponential function	$U(t).e^{-t}$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
Repeated function	$rep_T\{x(t)\}$ $=x(t) * rep_T\{\delta(t)\}$	$X(f).\left\lvert\frac{1}{T}\right\rvert rep_{\frac{1}{T}}\{\delta(f)\}$
Sampled function	$x(t).rep_T\{\delta(t)\}$	$X(f) * \left\lvert\frac{1}{T}\right\rvert rep_{\frac{1}{T}}\{\delta(f)\}$ $=\left\lvert\frac{1}{T}\right\rvert rep_{\frac{1}{T}}\{X(f)\}$

Question 1

i) Find the result of the convolution of signals $x(t) = e^{-\alpha t}u(t)$ and $y(t) = e^{-\beta t}u(t)$ with $\alpha > \beta > 0$ and $u(t)$ the unit step function.

[25%]

ii) Determine the closed-form Fourier transform, $X(f)$, of the signal $x(t) = 2\pi t \cdot \left[\mathrm{sinc}(t)\right]^2$, ignoring the component $X(0)$.

[25%]

iii) How many multiplications and additions are required for an *N*-point DFT? Justify your answer based on the DFT formula.

Note: N-point DFT of $x[n]$: $X(k) = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi nk/N}$, $k = 0, \ldots, N-1$

[25%]

ii) Use the example of a ***sinc(x)*** function to demonstrate quantitatively how:

a) scaling in the time domain;

b) scaling in amplitude;

c) shifting in frequency;

effects the frequency domain spectra of a signal.

[25%]

Question 2

A stereo analogue audio signal (two channels) is sampled at a rate of 48 KHz and each sample is quantised using a 16 bit A/D converter. If the range of the A/D converter is 10 Volt find:

i) The maximum quantisation error (assuming rounding and that the analogue signal is within the range of the converter) and the dynamic range of the A/D converter, defined as $20 \log_{10}(R/Q)$, where Q is the quantisation width.

[25%]

ii) Assuming that the dynamic range of human hearing is approximately 100 dB, explain why a 16-bit converter is normally adequate. Calculate how many Megabytes of hard disk space are required to store the stereo signal of three-minutes duration.

[25%]

iii) Describe *briefl*y (e.g. by means of a table) the stages of the successive-approximation A/D conversion process of an analogue voltage of 1.6 volts to its:

- offset-binary 4-bit representation, and
- two’s complement 4-bit representation

assuming that the full-scale range of the A/D converter is 8 volt.

[50%]