Question 1

i)

$$x(t) = x_1 * x_2 = \int_{-\infty}^{\infty} e^{-\alpha\lambda} u(\lambda) e^{-\beta(t-\lambda)} u(t-\lambda)$$

$$u(\lambda)u(t-\lambda) = \begin{cases} 0 & \lambda < 0 \\ 1 & 0 < \lambda < t \\ 0 & \lambda > t \end{cases}$$

$$x(t) = \begin{cases} 0 & t < 0 \\ \int_0^t e^{-\beta t} e^{-(\alpha-\beta)\lambda} d\lambda = \frac{1}{\alpha-\beta}\left(e^{-\beta t} - e^{-\alpha t}\right) & t \geq 0 \end{cases}$$

[25%]

ii) From the properties: $\mathrm{sinc}(t) \xleftrightarrow{F} \mathrm{rect}(f)$. Time-domain multiplication of two $\mathrm{sinc}(t)$ functions in time is frequency-domain convolution of two $\mathrm{rect}(f)$ functions:

$$\mathrm{sinc}(t) \cdot \mathrm{sinc}(t) \xleftrightarrow{F} \mathrm{rect}(f) * \mathrm{rect}(f) = \begin{cases} f+1, -1 \leq f < 0 \\ 1-f \ \ , 0 < f < 1 \end{cases}$$

ignoring the DC component, i.e. $X(0)$.

In addition, $2\pi t \cdot x(t) \xleftrightarrow{F} -j\frac{d}{df}X(f)$ from the property of the spectral derivative.

Hence, $2\pi t \cdot \left[\mathrm{sinc}(t)\right]^2 \xleftrightarrow{F} \begin{cases} -j, -1 \leq f < 0 \\ j \ \ , 0 < f \leq 1 \end{cases}$.

[25%]

iii)

Computational Complexity

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

- N cpx multiplies + N-1 cpx adds per pt $\times$ N points ($k = 0..N$-1)
  - cpx mult: (a+jb)(c+jd) = ac - bd + j(ad+bc) = 4 real mults + 2 real adds
  - cpx add = 2 real adds
- Total: $4N^2$ real mults, $4N^2$-$2N$ real adds

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[25%]

iv) The answer should show that the frequency domain spectrum of a sinc(x) function is a rect(x) function. It should explain that as the time domain pulse narrows the frequency spectrum broadens and vice versa

[25%]

Question 2

Solution:

i) We have: $f_s = 48$ kHz, $B = 16$ bits and $R = 10$ Volts. The quantisation width is $Q = \frac{10}{2^{16}} = 0.1525$ mV. With rounding: $|\text{error}| \leq \frac{Q}{2} = 76.3$ μV. The dynamic range of the converter is then:

$$Dynamic\ Range = 20\log_{10}\left(\frac{R}{Q}\right) = 20\log_{10}\left(\frac{10}{1.526\times10^{-4}}\right) \cong 100 \text{ dB}$$

[25%]

ii) Given that the average dynamic range of the human ear is 100 dB and the range of the converter is adequate, no extra bits are needed in the converter.

The storage needed is :

$$\frac{(3\times60\text{sec})\times(2\text{channels})\times\left(\frac{48000\text{samples}}{\text{sec}}\right)\times\left(\frac{16\text{bits}}{\text{sample}}\right)}{\left(\frac{8\text{bits}}{\text{byte}}\right)\times\left(\frac{1024^2\text{bytes}}{\text{Mbyte}}\right)} \cong 33 \text{ Mbytes.}$$

[25%]

iii) Here $R = 8$ Volts, $B = 4$ and hence $Q = \frac{8}{2^4} = 0.5$ Volts. Thus, the full scale of the bipolar converter is between -4 to 3.5 Volts. The conversion table is found below :

offset bin.	offset bin. x_Q	2's comp.	2's comp. x_Q
1111	3.5	0111	3.5
1110	3.0	0110	3.0
1101	2.5	0101	2.5
1100	2.0	0100	2.0
1011	1.5	0011	1.5
1010	1.0	0010	1.0
1001	0.5	0001	0.5
1000	0.0	0000	0.0
0111	-0.5	1000	-0.5
0110	-1.0	1110	-1.0
0101	-1.5	1101	-1.5
0100	-2.0	1100	-2.0
0011	-2.5	1011	-2.5
0010	-3.0	1010	-3.0
0001	-3.5	1001	-3.5
0000	-4.0	1000	-4.0

To convert $x = 1.6$ Volts first shift it by $\frac{Q}{2} = 0.25$ Volts, so that $y = x + \frac{Q}{2} = 1.85$. The following table summarizes the successive approximation offset-binary conversion :

Test bit:	$b_1b_2b_3b_4$	x_Q (Volts)	Is $y = 1.85 > x_Q$?
b_1	1000	0.0	Yes
b_2	1100	2.0	No
b_3	1010	1.0	Yes
b_4	1011	1.5	Yes
	1011	1.5	

(i.e. 1.6 Volts are rounded to 1.5).

The two's complement conversion is obtained by setting the MSB to zero (as the number is positive) and applying the test for the remaining bits – in this case we get 0011, which is also 1.5 Volts.

[50%]