Baseband Digital Transmission Exercises

Exercise 1

Consider the digital signal given by:

$$x\left(t\right)=\sum_{k=-\infty}^{\infty}X_{k}p\left(t-kT_{s}\right) \tag{1}$$

where $p\left(t\right)=\frac{1}{\sqrt{T_s}}\cdot\Pi\left(\frac{t}{T_s}\right)$ is a unit-energy square pulse with duration T_s and T_s is the signalling interval. Consider also a binary transmission scheme where the symbols X_k are drawn from an alphabet $\mathcal{A}=\{A_1,A_2\}$ with equal probability where the symbol value A_1 encodes for the binary digit $'1'$ and the symbol value A_2 encodes for the binary digit $'0'$. Assume that $T_s=1\mu s$, $A_1=1$ and $A_2=-1$.

1. What is the baud rate (in symbols per second) associated with the digital signal?
2. What is the data rate (in bits per second) associated with the digital signal?
3. What is the energy conveyed per symbol and the energy conveyed per bit?
4. Assume that one wishes to convey the sequence of bits 00111001. Draw the waveform associated with the digital signal.

Exercise 2

Consider the digital signal given by:

$$x\left(t\right)=\sum_{k=-\infty}^{\infty}X_{k}p\left(t-kT_{s}\right) \tag{2}$$

where $p\left(t\right)=\frac{1}{\sqrt{T_s}}\cdot\Pi\left(\frac{t}{T_s}\right)$ is a unit-energy square pulse with duration T_s and T_s is the signalling interval. Consider now a quaternary transmission scheme where the symbols X_k are drawn from an alphabet $\mathcal{A}=\{A_1,A_2,A_3,A_4\}$ with equal probability where the symbol value A_1 encodes for the binary digits $'11'$, the symbol value A_2 encodes for the binary digits $'10'$, the symbol value A_3 encodes for the binary digits $'01'$, and the symbol value A_4 encodes for the binary digits $'00'$. Assume that $T_s=1\mu s$, $A_1=3$, $A_2=1$, $A_3=-1$ and $A_4=-3$.

1. What is the baud rate (in symbols per second) associated with the digital signal?
2. What is the data rate (in bits per second) associated with the digital signal?

3. What is the energy conveyed per symbol and the energy conveyed per bit?
4. Assume that one wishes to convey the sequence of bits 00111001. Draw the waveform associated with the digital signal.

Exercise 3

Consider the digital signal given by:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k p(t - kT_s) \tag{3}$$

where $p(t) = \frac{1}{\sqrt{T_s}} \cdot \Pi\left(\frac{t}{T_s}\right)$ is a unit-energy square pulse with duration T_s and T_s is the signalling interval. Consider also a binary transmission scheme where the symbols X_k are drawn from the alphabet $\mathcal{A} = \{A_1, A_2\}$ with equal probability. Assume that $T_s = 1\mu s$, $A_1 = 1$ and $A_2 = -1$.

1. Determine the power spectral density of the digital signal. Assume that the sequence of symbols $\{X_k\}$ is independent.
2. Determine the bandwidth of the digital signal.

Exercise 4

Consider the digital signal given by:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k p(t - kT_s) \tag{4}$$

where $p(t) = \sqrt{T_s} \cdot \mathsf{sinc}\left(\frac{t}{T_s}\right)$ and T_s is the signalling interval. Consider also a binary transmission scheme where the symbols X_k are drawn from the alphabet $\mathcal{A} = \{A_1, A_2\}$ with equal probability. Assume that $T_s = 1\mu s$, $A_1 = 1$ and $A_2 = -1$.

1. Determine the power spectral density of the digital signal. Assume that the sequence of symbols $\{X_k\}$ is independent.
2. Determine the bandwidth of the digital signal.

Exercise 5

Comment on power density spectra associated with Exercise 3 and Exercise 4. What scenarios would you envision for signalling using the signals associated with Exercise 3 or Exercise 4?

Exercise 6

Consider the digital signal given by:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k p(t - kT_s) \tag{5}$$

where $p(t) = \frac{1}{\sqrt{T_s}} \cdot \Pi\left(\frac{t}{T_s}\right)$ is a unit-energy square pulse with duration T_s and T_s is the signalling interval.

1. Assume that one wishes to convey a sequence of independent bits $\{b_k\}$ where b_k is either 1 or 0 with equal probability. The sequence of bits $\{b_k\}$ is mapped to the sequence of symbols $\{X_k\}$ where $X_k = 2 \cdot b_k - 1$. What is now the power density spectrum of the digital signal?

2. Assume that one also wishes to convey a sequence of independent bits $\{b_k\}$ where b_k is either 1 or 0 with equal probability. The sequence of bits $\{b_k\}$ is mapped to the sequence of symbols $\{X_k\}$ where $X_k = b_k - b_{k-1}$. What is now the power density spectrum of the digital signal?

3. What scenarios would you envision for signalling using the signals associated with the questions above?

Exercise 7

What are the advantages and disadvantages of binary digital signalling with respect to M-ary digital signalling? Justify.