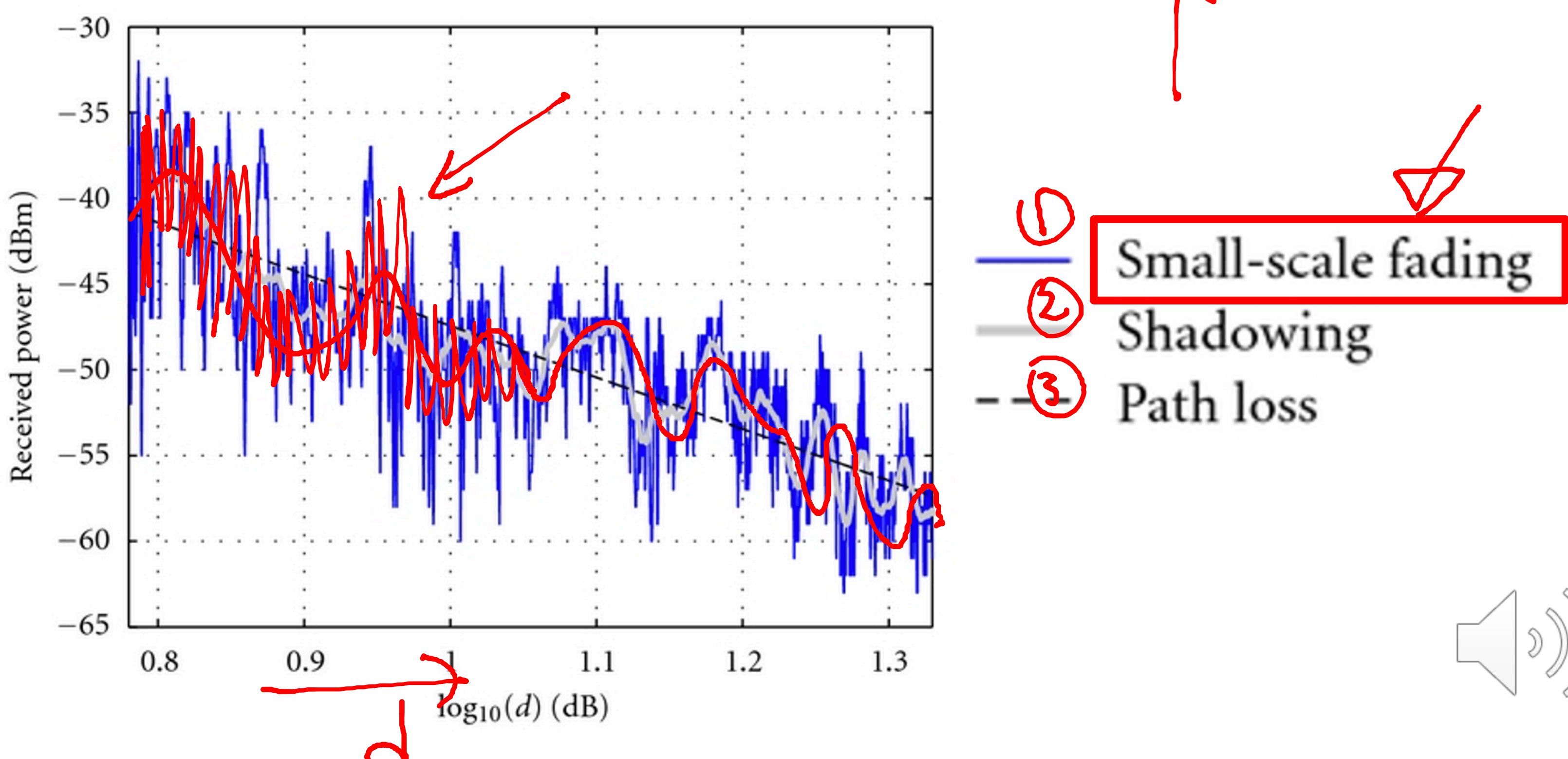
Channel Modelling

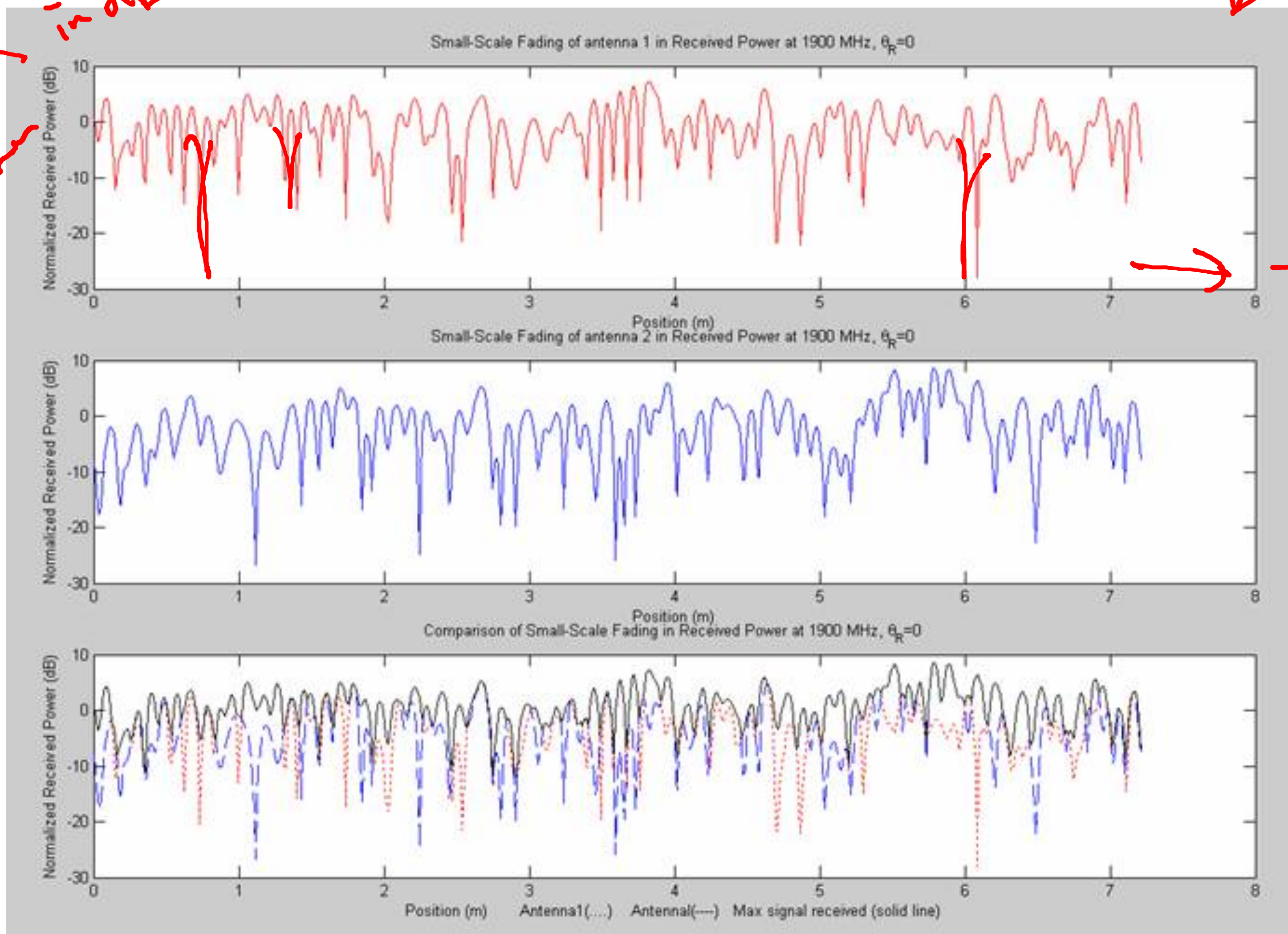
General Model

- A general model for wireless channels is difficult to get
- Use the PATHLOSS + SHADOWING + SMALL-SCALE FADING model



Small Scale Fading

received power in dB

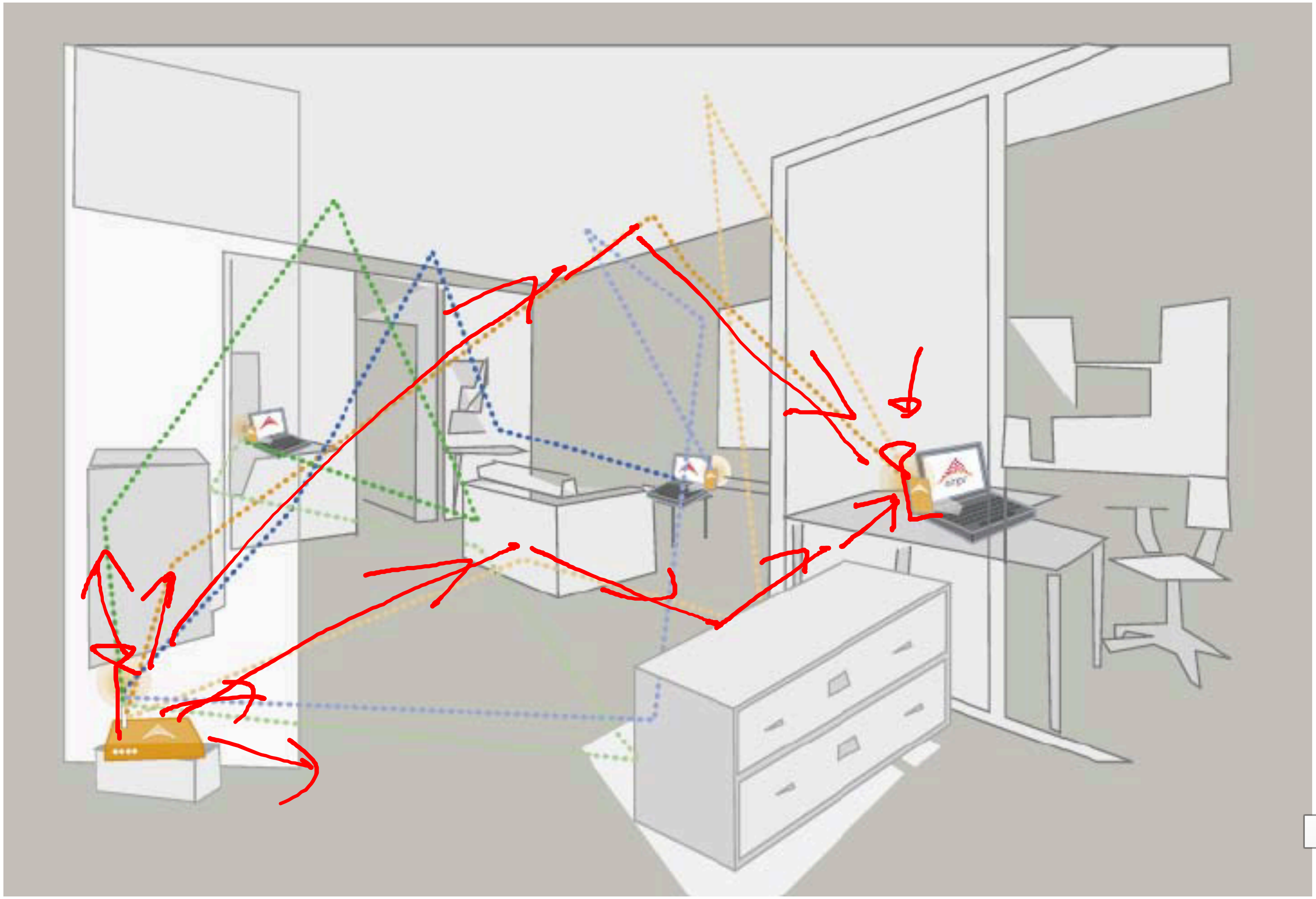


5dB

Time

-30dB

Fading due to Multipath Propagation

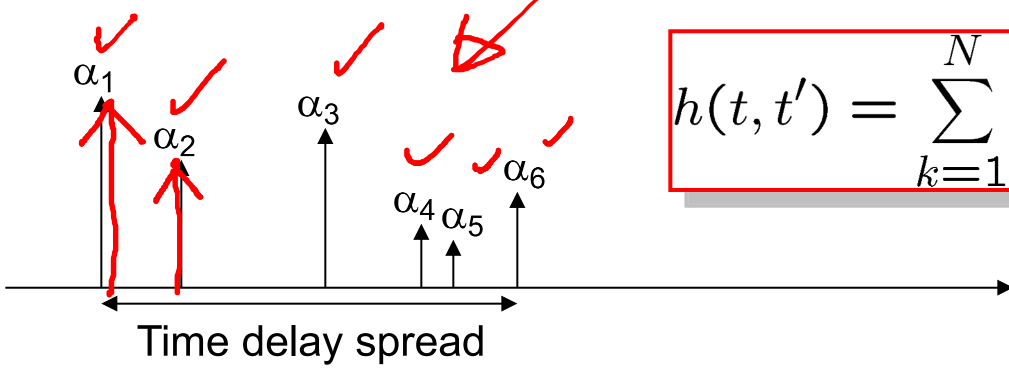


Multipath Fading

chan

IR

- Variations over a few wavelengths
  - Interference between multiple path with different path lengths (in λ)
  - Movement of the mobile of environment makes this effect time varying
- A snapshot of the channel response may be



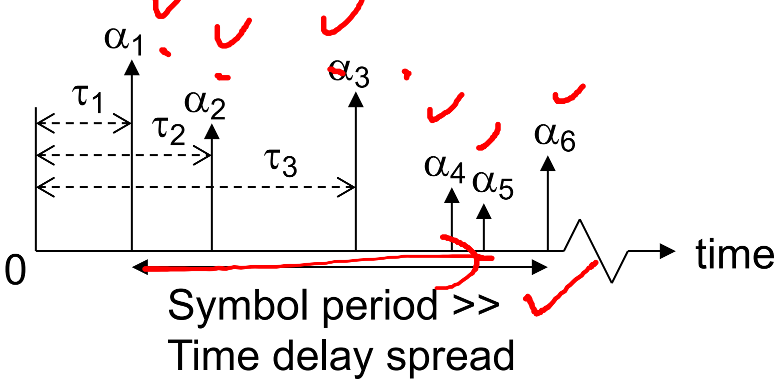
$$\boxed{h(t,t') = \sum_{k=1}^{N} \alpha_k(t')\delta(t-\tau_k(t'))}$$

  - α_n captures the reflections, attenuation, phase shift, etc. for a particular path
  - Many paths arriving almost in all time within the time delay spread

Rayleigh Flat Fading

$\alpha_1+\alpha_2+\dots+\alpha_N$

- If the symbol period is much greater than the time delay spread,



$$\boxed{\begin{aligned} h(t) &= \sum_k \alpha_k \delta(t-\tau_k) \\ H(f) &= \sum_k \alpha_k e^{-j2\pi f \tau_k} \approx \sum_k \alpha_k \end{aligned}}$$

- By Central Limit Theorem

$$\boxed{\alpha} = \sum_k \alpha_k = \mathrm{Re}\left\{\sum_k \alpha_k\right\} + j\,\mathrm{Im}\left\{\sum_k \alpha_k\right\}$$

$$= x + jy$$

Independent Gaussian random variables

$\boxed{h} = x + jy = re^{j\theta}$

Phase and Magnitude Distributions

θ uniform

r Rayleigh

- Therefore, if $x + jy = re^{j\theta}$ then the joint PDF is

$$f_{R,\Theta}(r,\theta) = f_\Theta(\theta) f_R(r) = \frac{1}{2\pi}\frac{r}{\sigma_R^2} e^{-\frac{r^2}{2\sigma_R^2}}$$

- **Phase** is uniform and **Magnitude** is Rayleigh

$$f_R(r) = \begin{cases} \dfrac{r}{\sigma_R^2} e^{-\frac{r^2}{2\sigma_R^2}} & \text{if } r \geq 0, \\ 0 & \text{if } r < 0, \end{cases}$$

$$f_\Theta(\theta) = \begin{cases} \dfrac{1}{2\pi} & \text{if } 0 \leq \theta \leq 2\pi, \\ 0 & \text{otherwise}, \end{cases}$$

$$\mathrm{E}[r^2] = 2\sigma_R^2$$

$r(t) = h(t)s(t) + n(t)$

Signal Model for Flat Fading

- The received signal at time t is modelled as

$$r(t) = h(t)s(t) + n(t) + i(t)$$

Received signal

Channel fading coefficient
e.g., complex,
Rayleigh distributed amplitude
Uniform distributed phase

Modulated symbol
e.g., QPSK, QAM …

Noise signal

Possible interference

expectation

- The signal power

$$\boxed{S} = \mathrm{E}[|s(t)|^2]$$

- The noise power

average

$$N = \mathrm{E}[|n(t)|^2] = kTB$$

$r(t) = h\,s(t) + n(t) + i(t)$

Performance Metrics

- Signal-to-Interference Plus Noise Radio (SINR)

$$\boxed{\mathrm{SINR} = \frac{S}{I+N}} = \frac{|h|^2 E[|s(t)|^2]}{E[|n(t)|^2]}$$

- Outage Probability

target

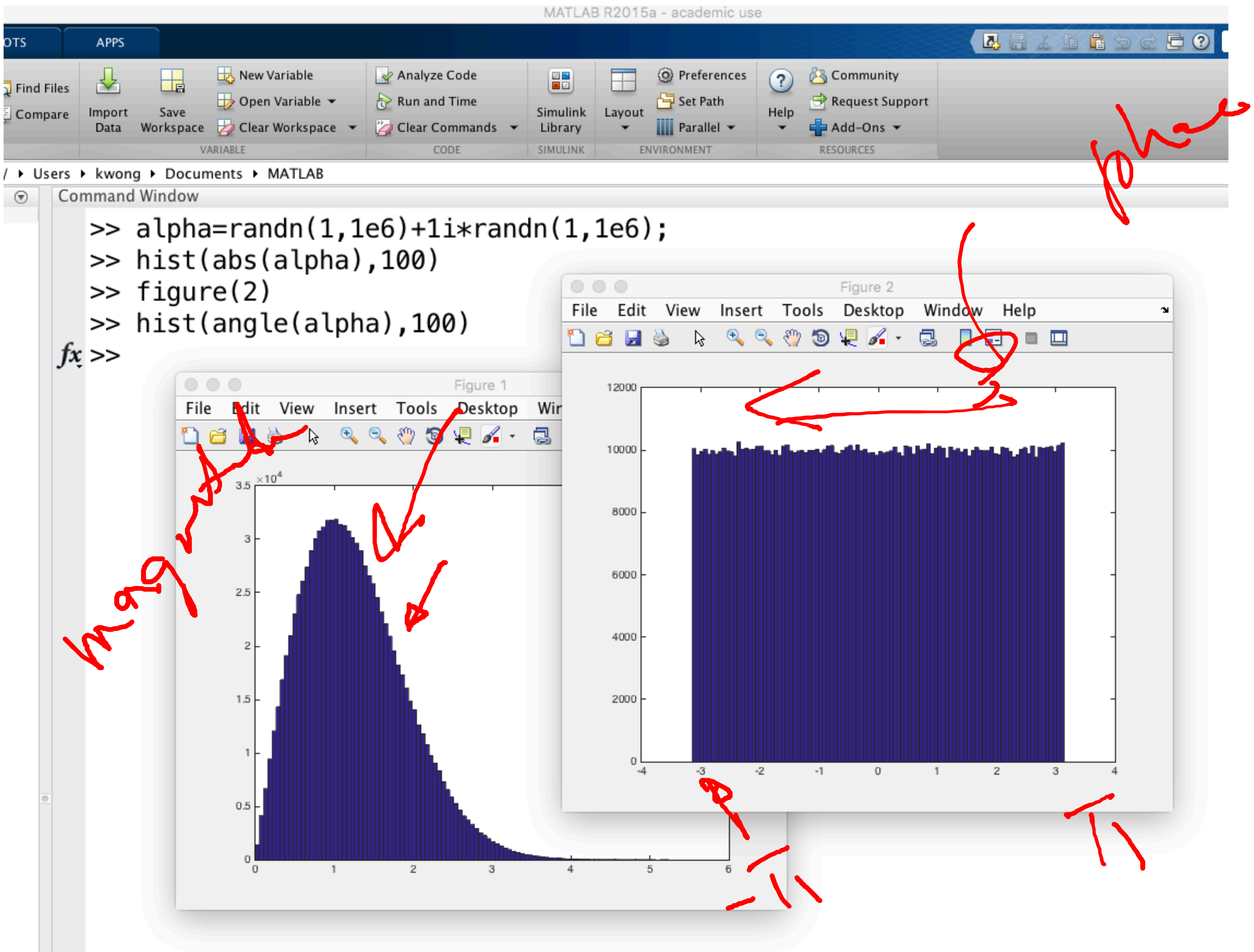
$$\boxed{\mathrm{OP} = \mathrm{Prob}(\boxed{\mathrm{SINR}} \leq \Gamma)}$$

- Capacity

$$\boxed{\mathrm{C} = W \log_2(1 + \mathrm{SINR})}$$

Bandwidth

MATLAB Simulations for Fading Channels



h(t) α1 α2 α3 t

Rayleigh Frequency-Selective Fading

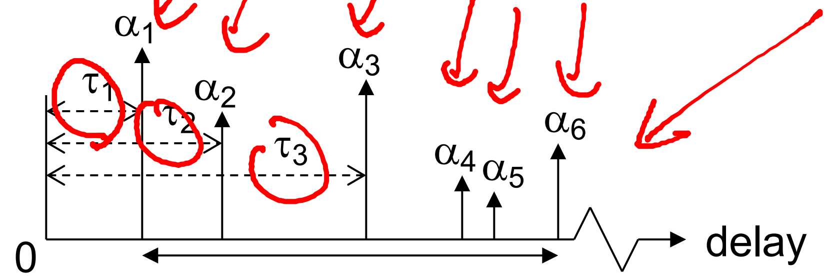
- When delay spread is more significant, the multiple paths cause **inter-symbol interference (ISI)**, i.e., the delay copies of a symbol are jamming the other transmitted symbols

s(t)

x(t) → IR → y(t)

h(t)

- Usually, it is modelled as the multi-ray model



Time delay spread is in the order of the symbol durations

= h(t) * x(t)

  - All $\{|\alpha_k|\}$ are independent and Rayleigh distributed
  - All inter-arrival times $\{\tau_{k+1}-\tau_k\}$ are exponentially distributed
  - Number of rays (or paths) is Poisson distributed
  - $E[|\alpha_k|^2]$ usually follows an exponential power profile