Introduction to Information Coding

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Examples of Codes

UCL

Input letter	Prob.	Code 1	Code 2	Code 3	Code 4
A	1/2	0	0	1	0
B	1/4	0	1	01	01
C	1/8	1	00	001	011
D	1/8	10	11	000	0111
L(C)		1.125	1.25	1.75	1.875

00 → AA

00 → C

Problems with these codes

- Code 1: Two input symbols have the same codeword
- Code 2: This problem is fixed. But suppose the decoder receives 00. What was the input? C or AA?
- Codes 3 and 4 look OK. Code 3 is shorter.
- Is that the best we can do?

More Examples of Codes

Input Letter	Singular	Non-sing *Not* U.D.	U.D. *Not* prefix	Prefix (Instant.)
A	0	0	10	0
B	0	010	00	10
C	0	01	11	110
D	0	10	110	111

Optimal Code

$$H(X) = \sum_{x_i \in X} p(x_i) \cdot \cancel{1} \log \underbrace{\frac{1}{p(x_i)}}_{p(x_i)^{-1}} = -\sum p(x_i) \log p(x_i)$$

$$H(X) = -\sum p(x_i) \log(x_i) \quad \underbrace{\phantom{p(x_i)^{-1}}}_{-\log p(x_i)}$$

$$L(X) = \sum_{x_i \in X} p(x_i)\, \ell(x_i)$$

$$\boxed{L(X) \geq H(X)}$$

$$H(X) \leq L^* < H(X) + 1$$

Optimal Code

Theorem 5.3.1 *The expected length L of any instantaneous D-ary code for a random variable X is greater than or equal to the entropy $H_D(X)$; that is,*

$$L \geq H_D(X), \tag{5.21}$$

with equality if and only if $D^{-l_i} = p_i$.

$$: -\sum p_i \log_D p_i = H_D(X)$$

Theorem 5.4.1 *Let $l_1^*, l_2^*, \ldots, l_m^*$ be optimal codeword lengths for a source distribution $\mathbf{p}$ and a D-ary alphabet, and let L^* be the associated expected length of an optimal code ($L^* = \sum p_i l_i^*$). Then*

$$H_D(X) \leq L^* < H_D(X) + 1. \tag{5.33}$$

Huffman Coding Example

- Example:

input symbols	Probabilities	Binary Codewords
A	3/4	1
B	3/16	01
C	1/16	00

- The entropy of the source is H = 1.012 bps
- Average length of this code is L = 1.25 bps
- The efficiency of the code is:

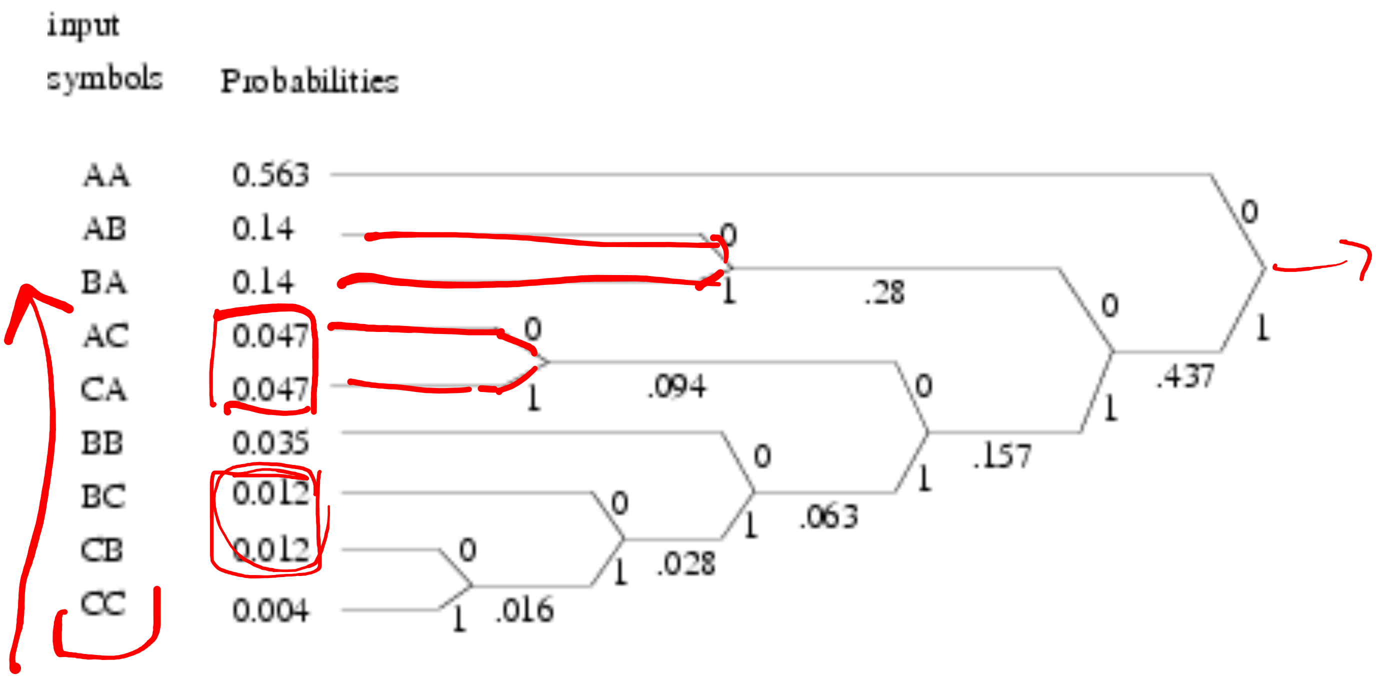
$$efficiency = \frac{H}{L} = \frac{1.012}{1.25} \approx 0.81$$

H < L < H+1

1.012 1.25 2.012

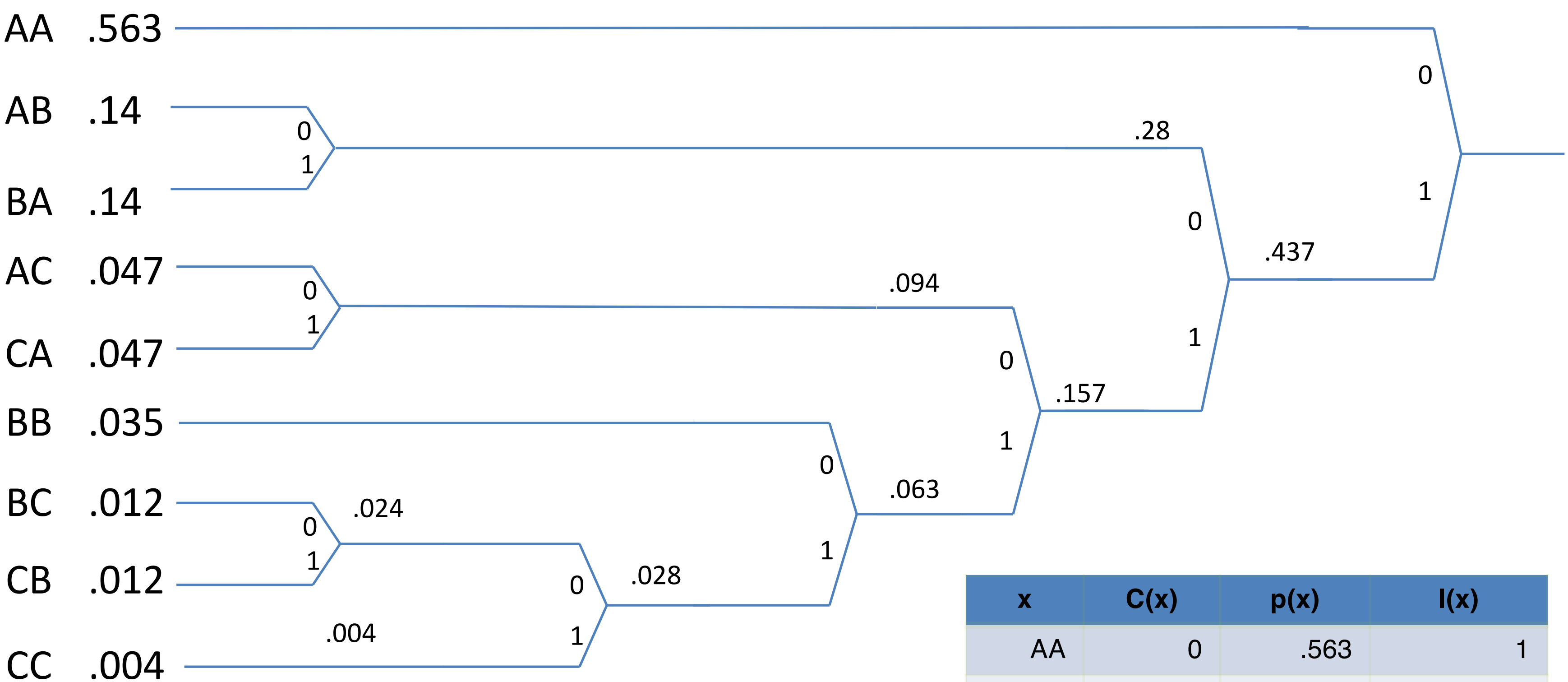
Huffman Coding Example Extended

- If we take pairs, get more efficient code:



- Expected length now 1.0375 bps
- Efficiency is up to 1.012/1.0375 = 0.97
- **Code/decode the message: AAABAAAAACBAAABA**

EXAMPLE



x	C(x)	p(x)	l(x)
AA	0	.563	1
AB	100	.14	3
BA	101	.14	3
AC	1101	.047	4
CA	1101	.047	4
BB	1110	.035	4
BC	111101	.012	6
CB	111101	.012	6
CC	11111	0.004	5

$L(x)=2.0830$

Outline

Introduction to Information compression

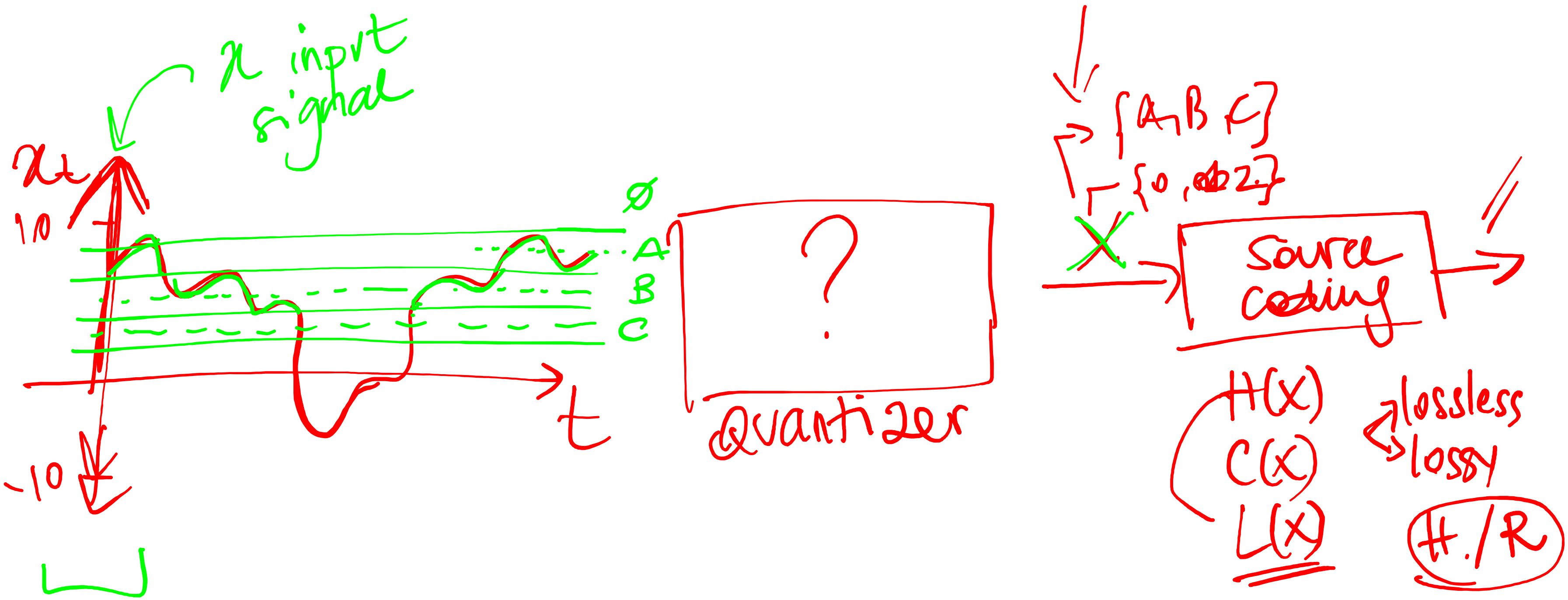
- Source Coding
- Information and Entropy
- Variable length coding
- Quantization

Multimedia Systems

- Image and Lossy Compression
  - Transforms
  - JPEG Quantization
  - JPEG Lossless Compression

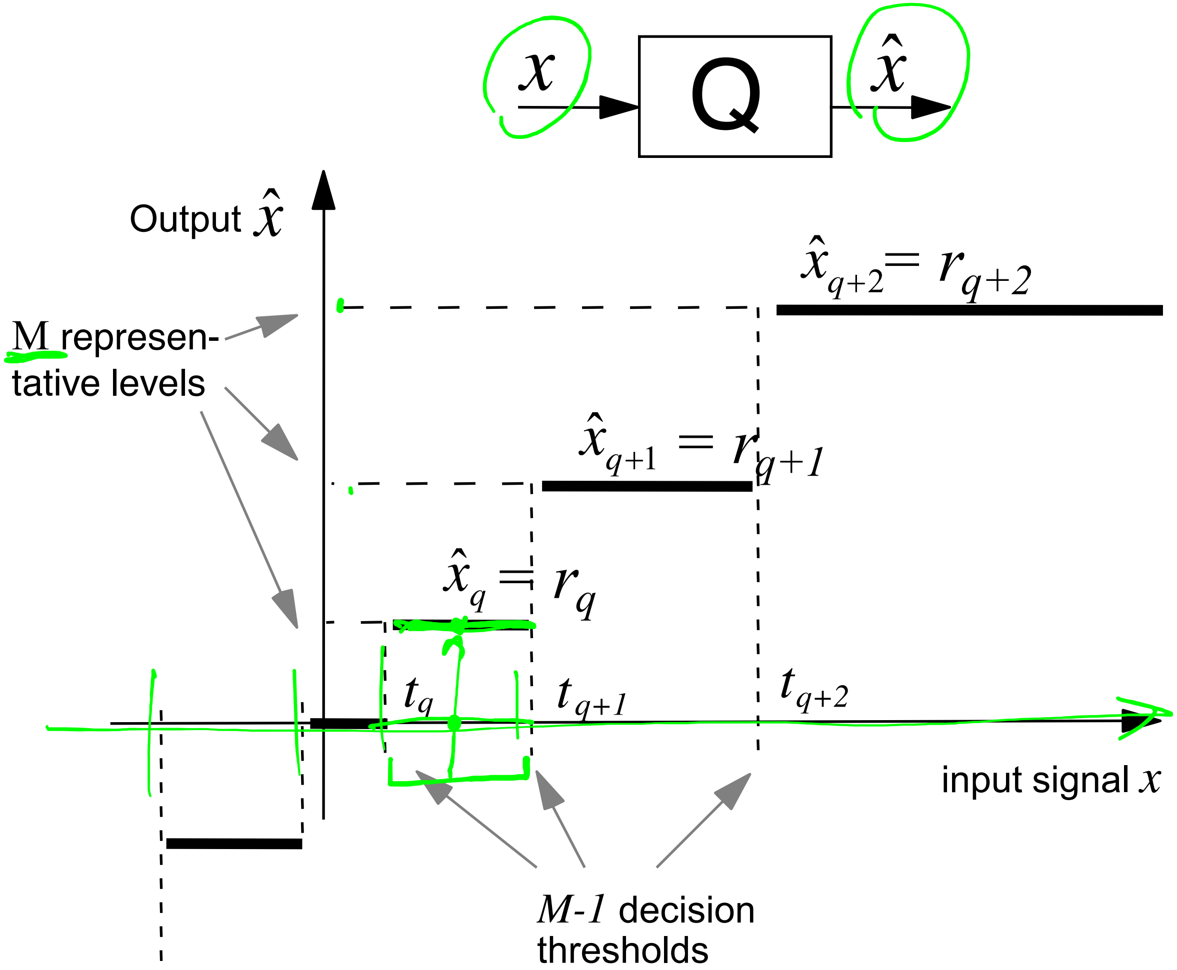
- Video Compression
  - Motion Compensation

Why do we need quantization?

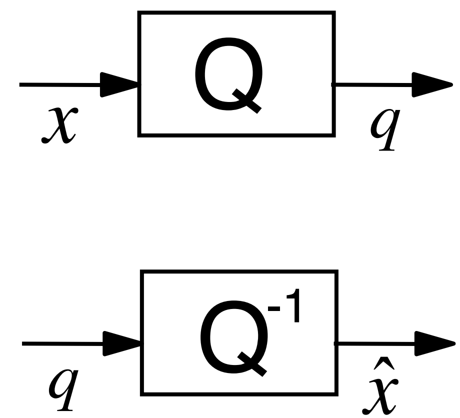


Scalar Quantization (SQ)

Input-output characteristic of a scalar quantizer

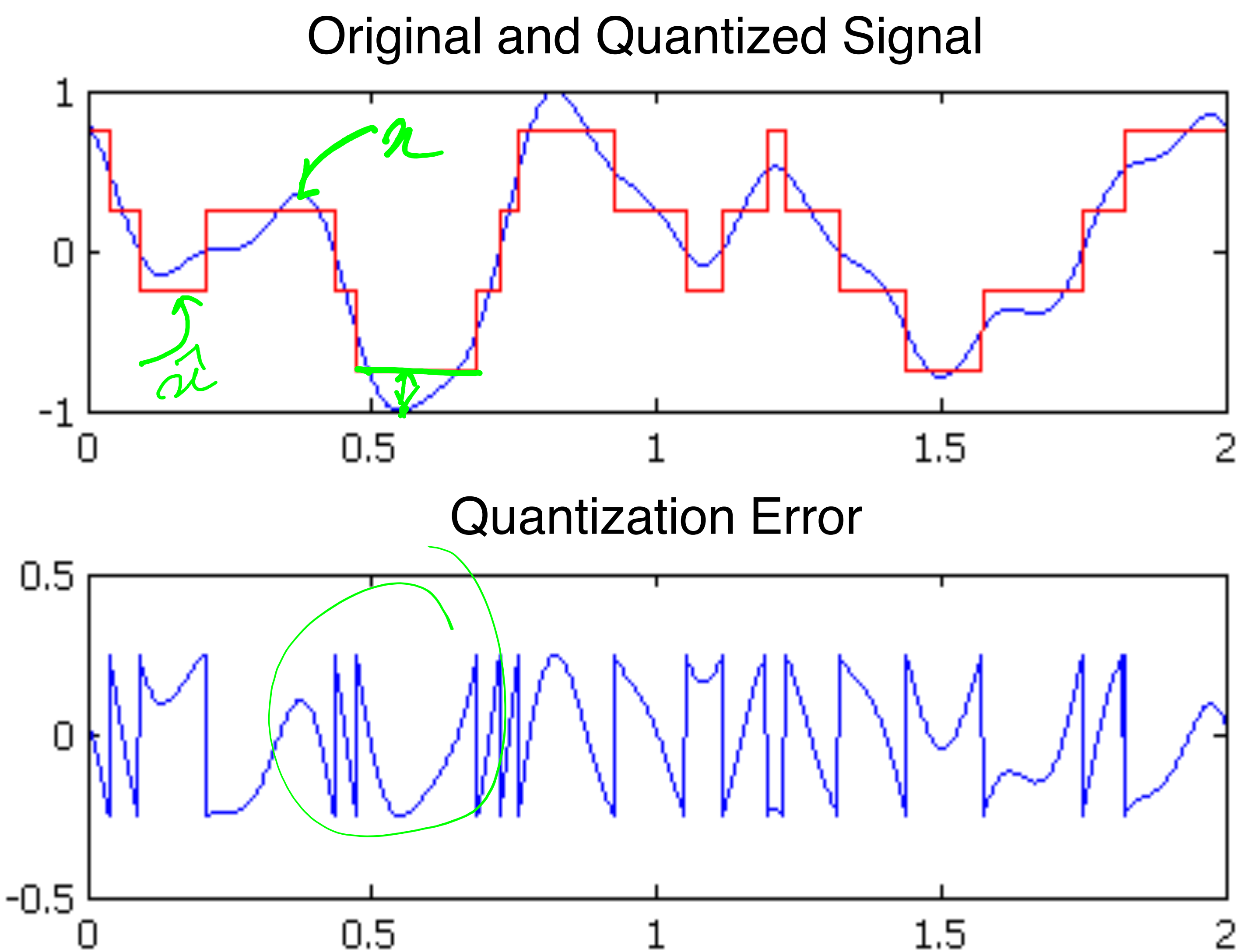


Sometimes, this convention is used:

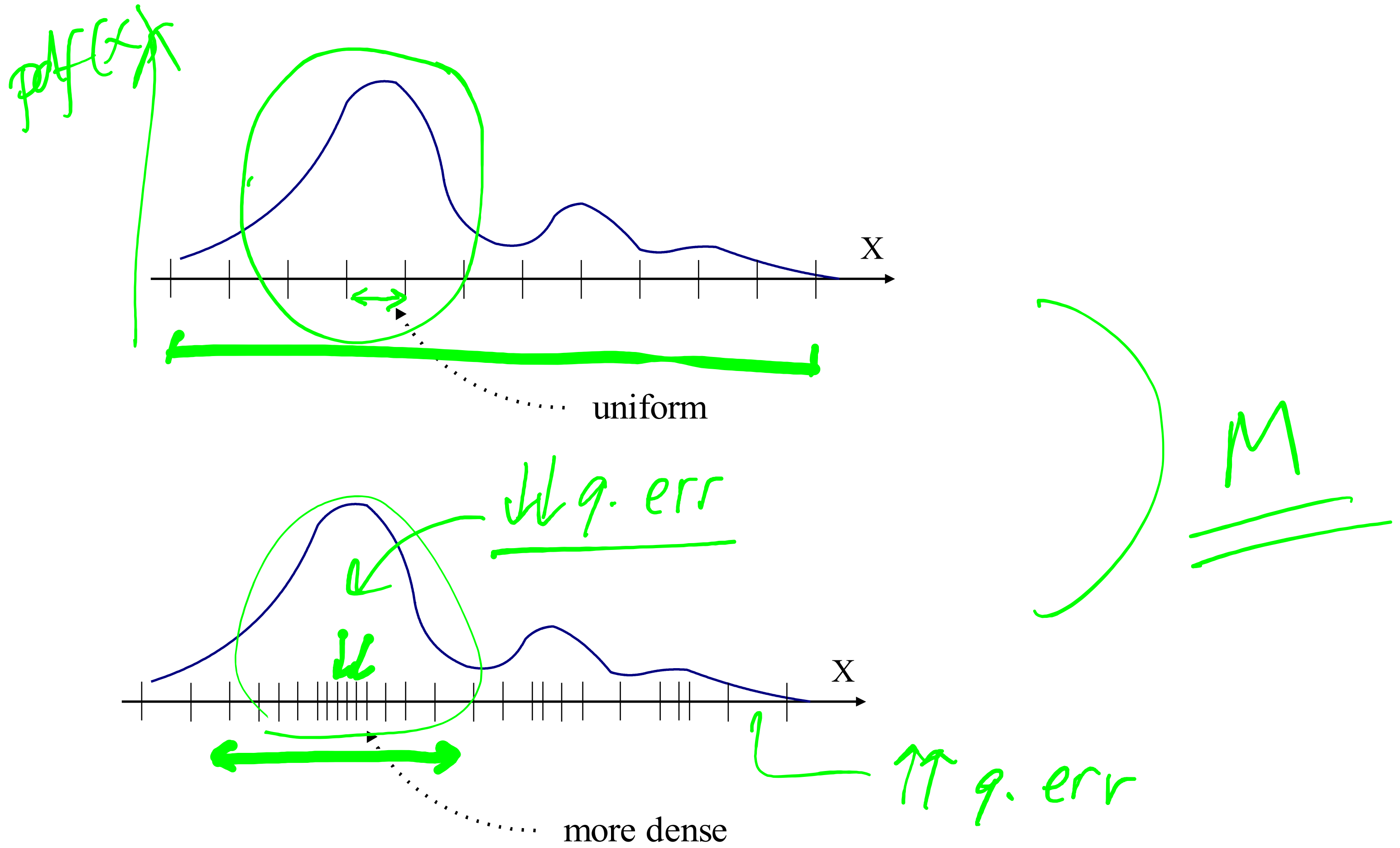


→ M↑↑, lower quantiz err, ↑↑ |X| C(X) → RM

Example of Quantized Waveform



Scalar Quantization



- Uniform quantization is not always the best

Thank You