Digital Transmission Principles

Prof. Cyril Renaud
c.renaud@ucl.ac.uk

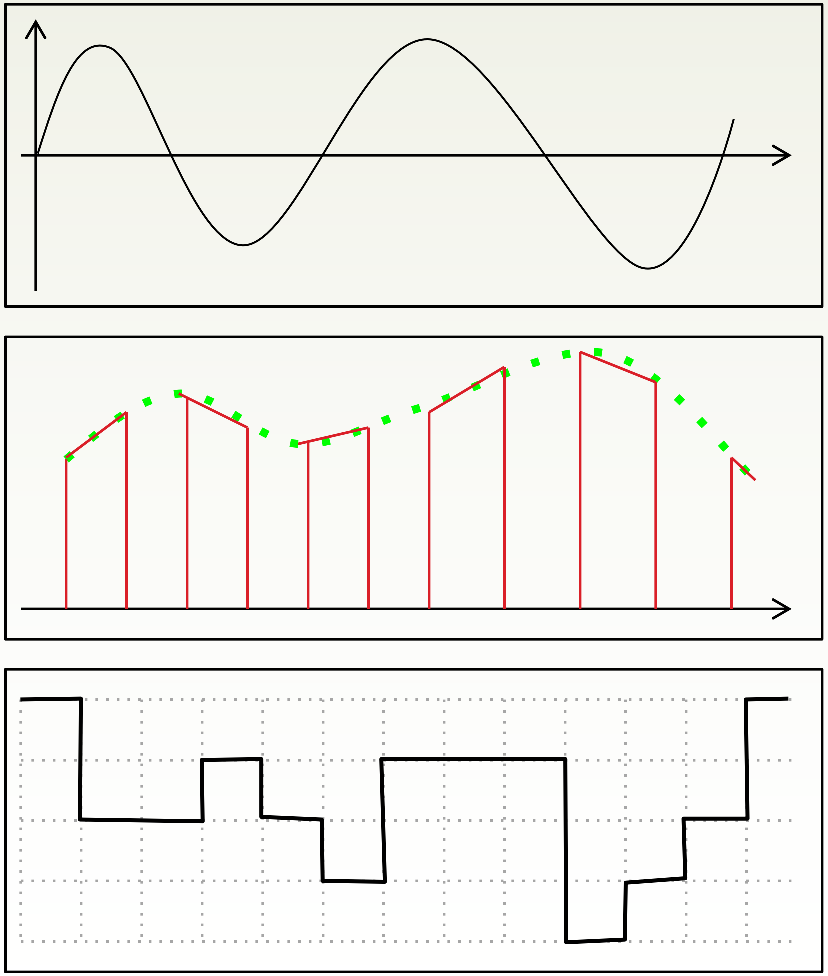
Digital Transmission

- Binary Transmission as a reference
- Additive white Gaussian noise channel
- Binary and ternary eye diagrams
- Eb/No and System SNR
- BER
- Multi-level signalling and choice of modulation

Signal types

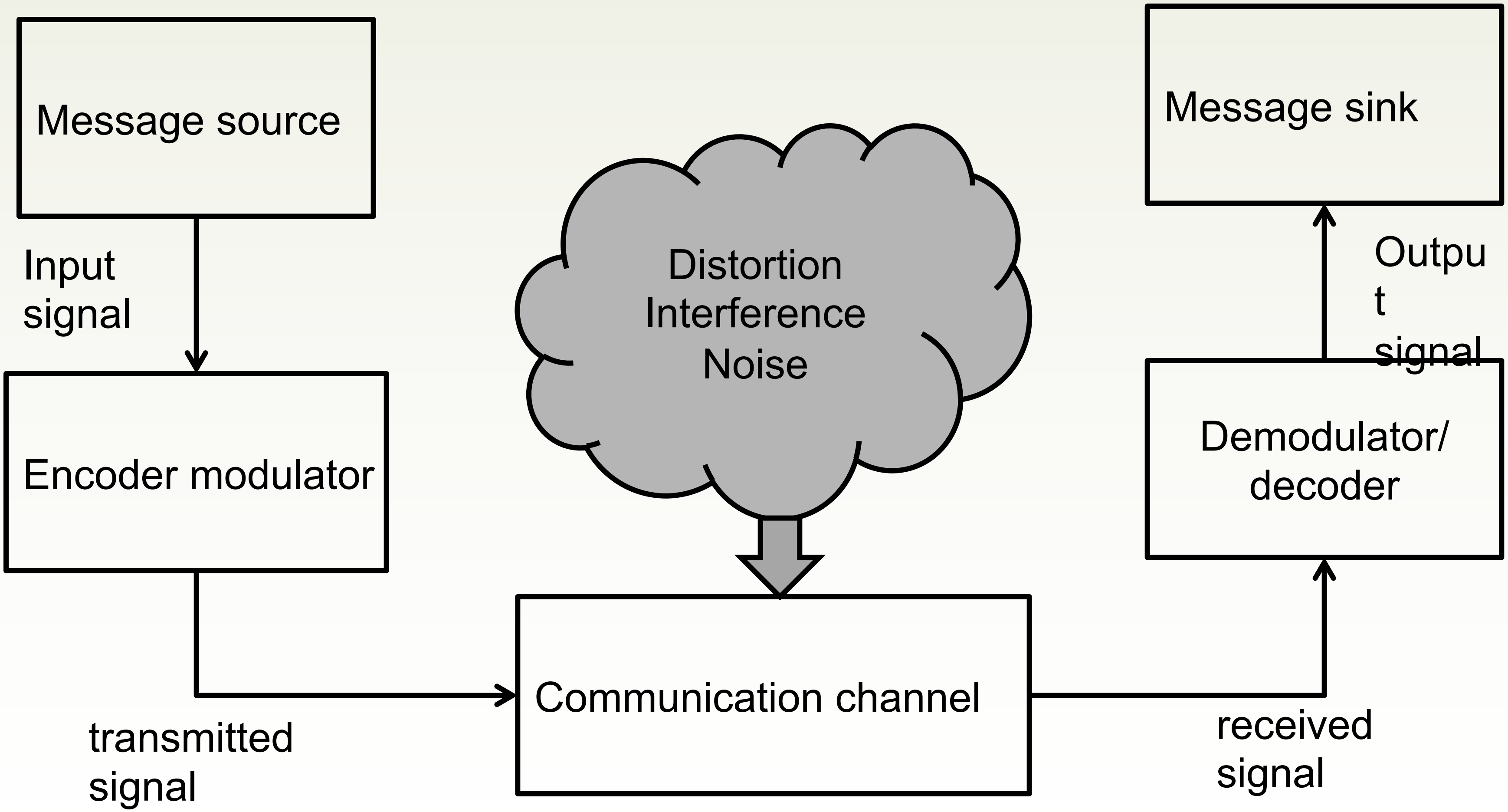
- Random – Deterministic
- Discrete Time – Continuous Time
- Discrete Amplitude – Continuous Amplitude
- Lowpass, Bandpass, Highpass
- Periodic or non Periodic

Discrete-Continuous



- Continuous time and amplitude
- Discrete time continuous amplitude – PAM signal
- Discrete time and amplitude, Multilevel signal.

Digital line system



Claude Shannon

- Shannon's Theorem predicts reliable digital communication in the presence of noise
  - *"Given a discrete, memoryless channel with capacity C, and a source with a positive rate R (R<C), there exist a code such that the output of the source can be transmitted over the channel with an arbitrarily small probability of error."*

B is the channel bandwidth in Hz and S/N is the signal to noise ratio

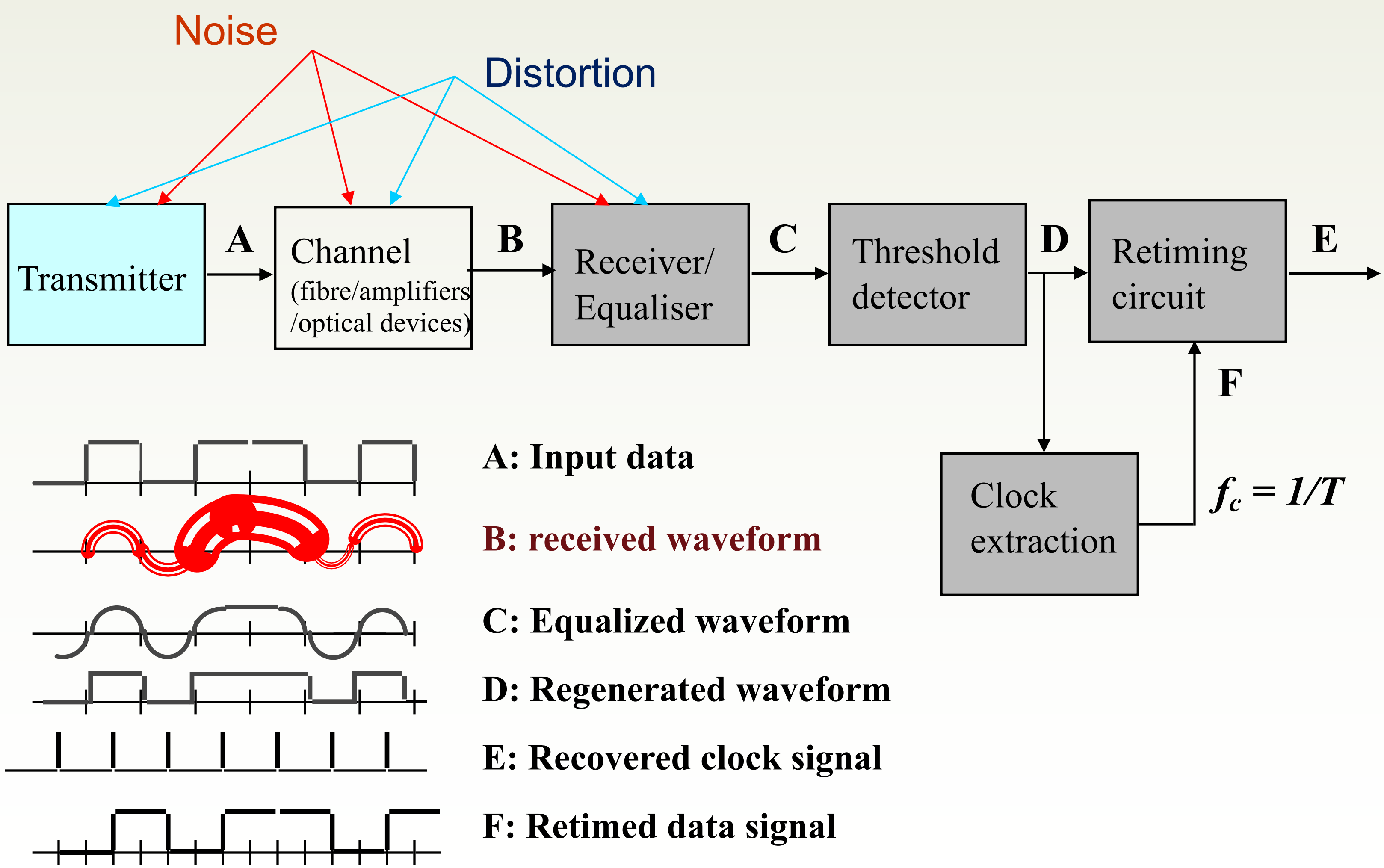
$$C_c = B\log_2\left(1+\frac{S}{N}\right)$$

For a "good" digital system;

- We aim to transmit a "clean" signal and receive an "identical" one
- "Identical" means;-

1- no added errors, all 1s received as 1s and all 0s received as 0s; <u>AND</u>
2- Received data has the same timing as transmitted data

Binary transmission system

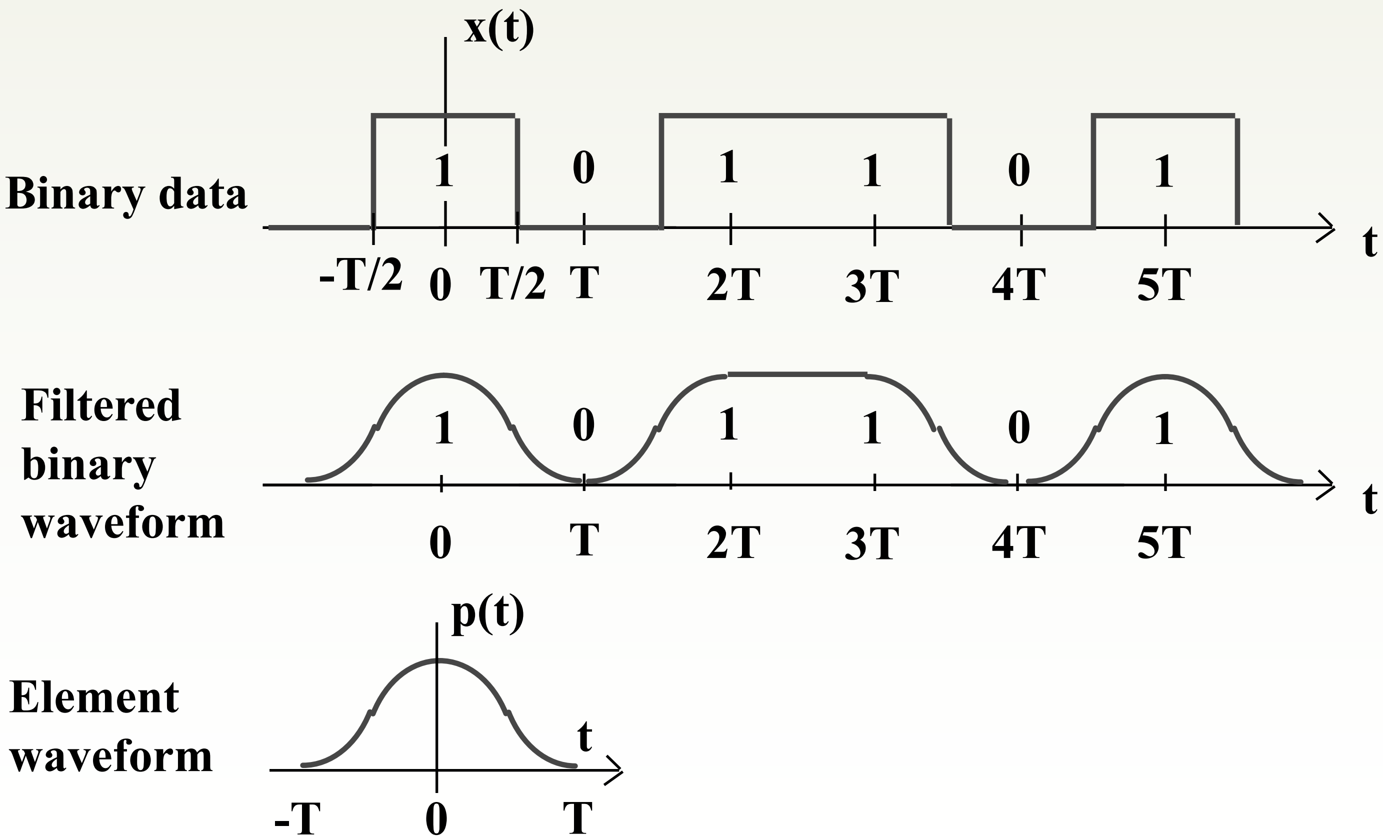


Effects of noise and Distortion

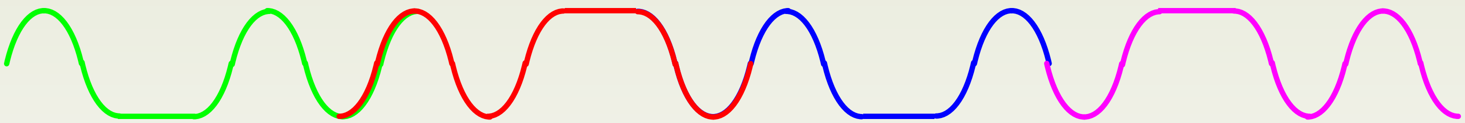
- Errors are introduced due to:
  - Noise on signal
  - Timing noise and jitter
  - Inter-Symbol-Interference (ISI)

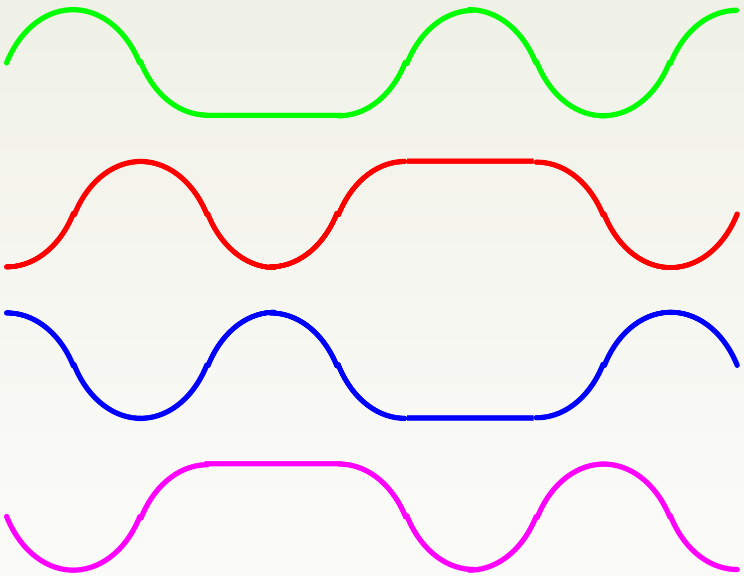
- Errors can be assessed by:
  - Considering the *eye diagram*
  - Measurement of Bit Error Rate (BER) or P_e

Binary signal waveforms

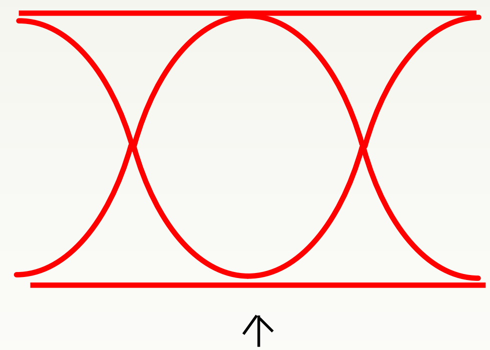


Eye diagrams





Illustrative signal segments



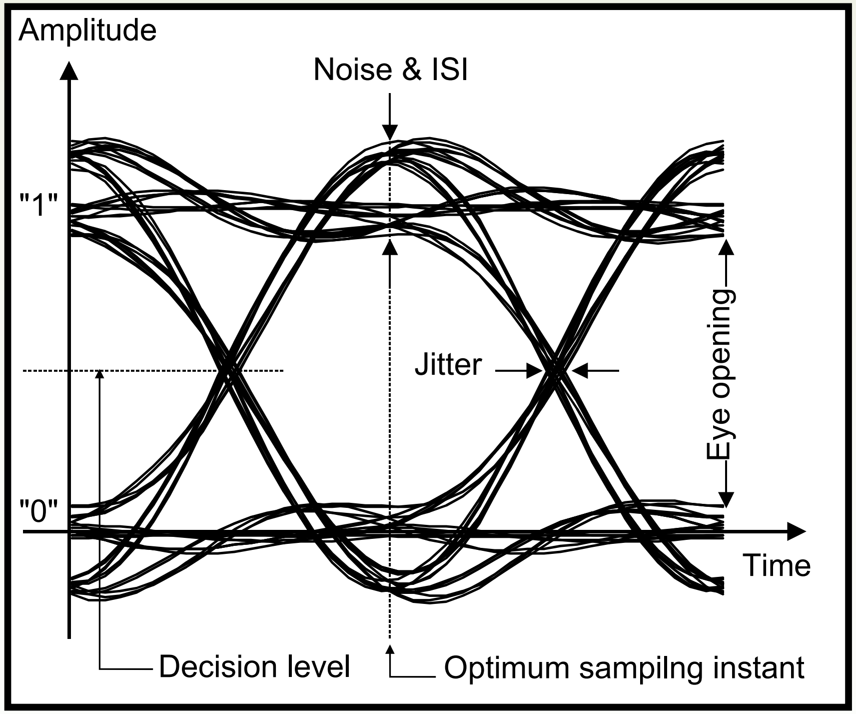
Decision threshold

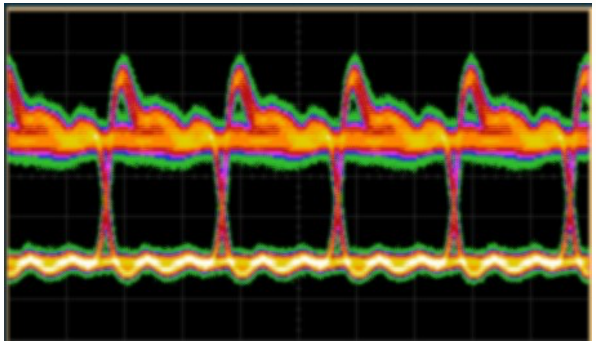
Sampling epoch

Eye diagram

Overlapped ensemble of possible waveforms

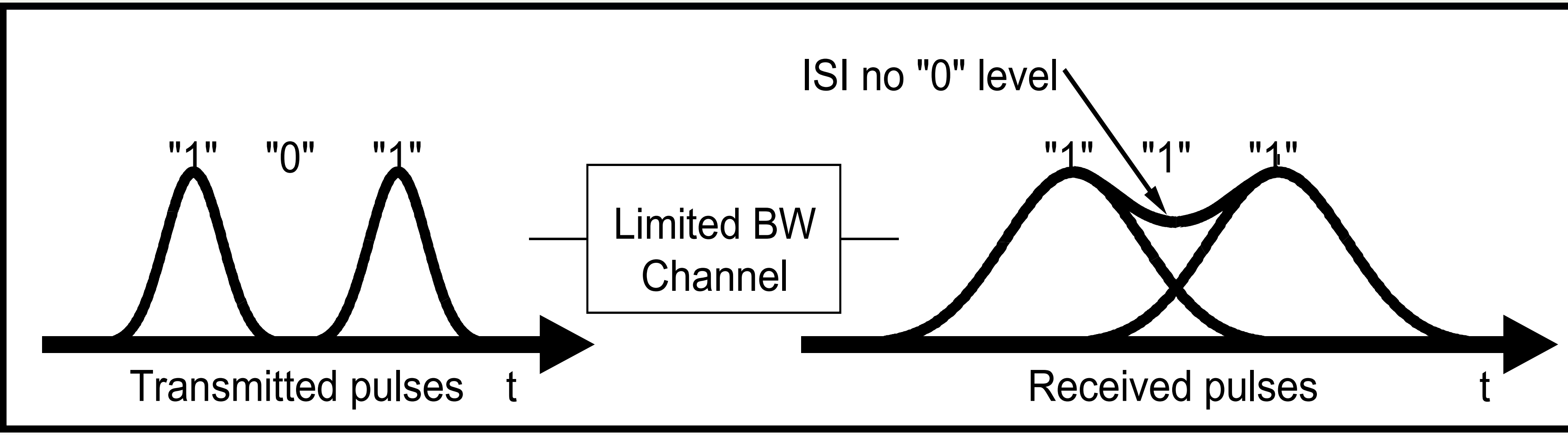
A more realistic eye…





- The eye diagram is a powerful diagnostic tool
  - Noise
  - Jitter
  - Telegraph Distortion
  - ISI
  - Patterning effects
  - Bit errors
- Normally displayed on a scope at the receiver

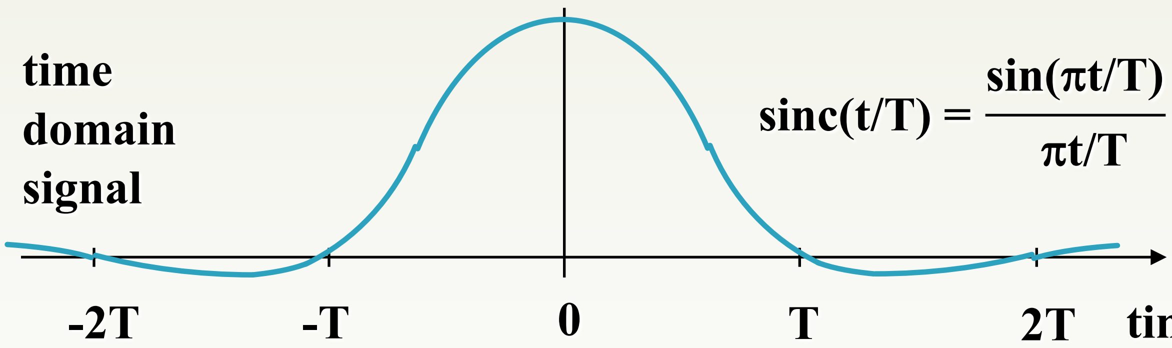
Concept of ISI in Binary signals



- The signals can be "designed" so as to have no ISI
- This is done at the signal shaping/equalisation stage in the receiver
- Requires knowledge of received signal shape
  (i.e transmitter, channel and receiver behaviour)

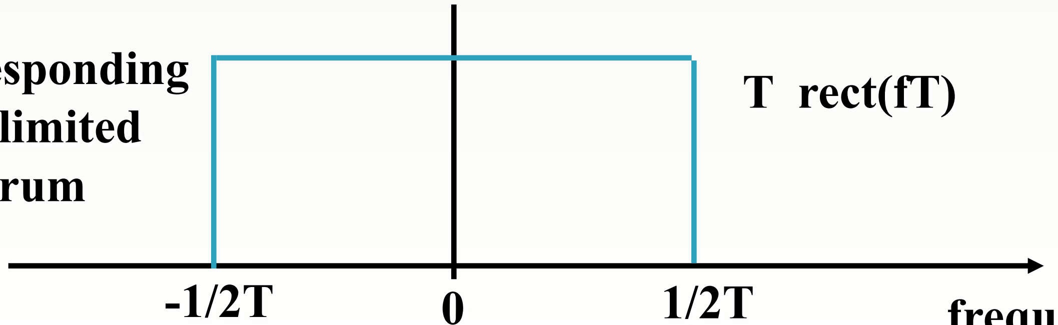
A zero isi signal

time
domain
signal



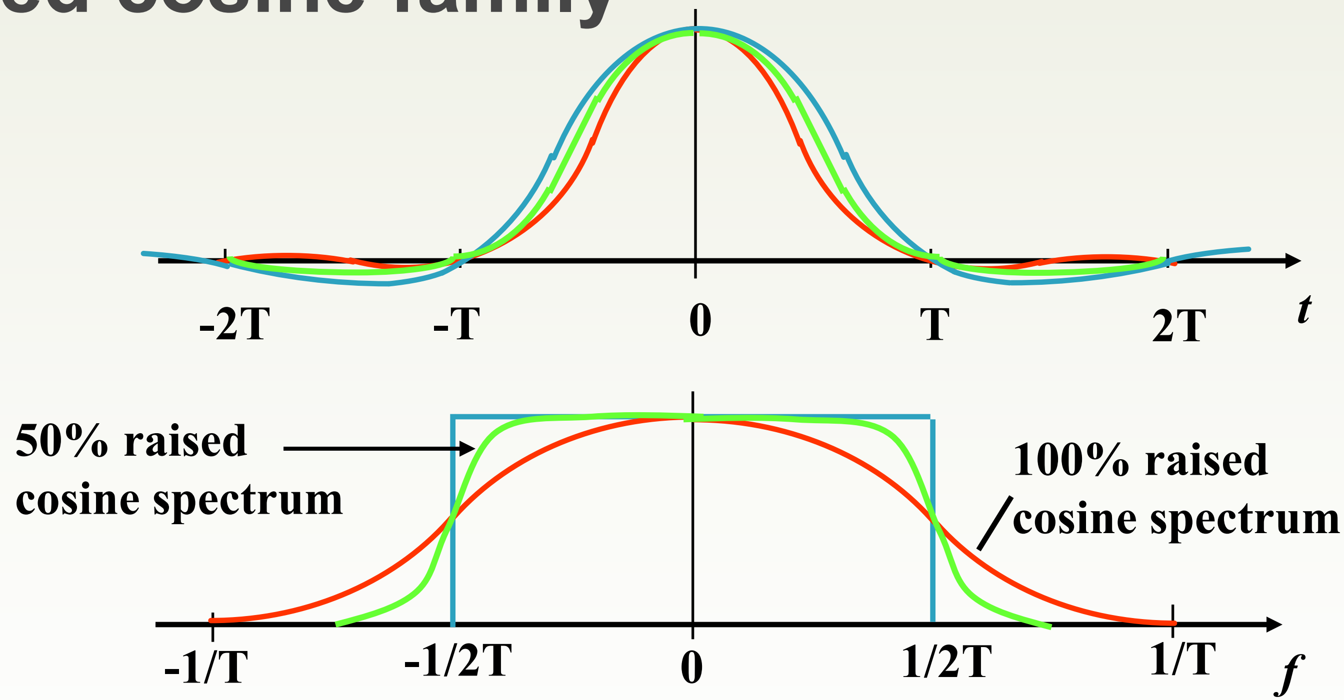
$$\mathrm{sinc}(t/T) = \frac{\sin(\pi t/T)}{\pi t/T}$$

corresponding
bandlimited
spectrum



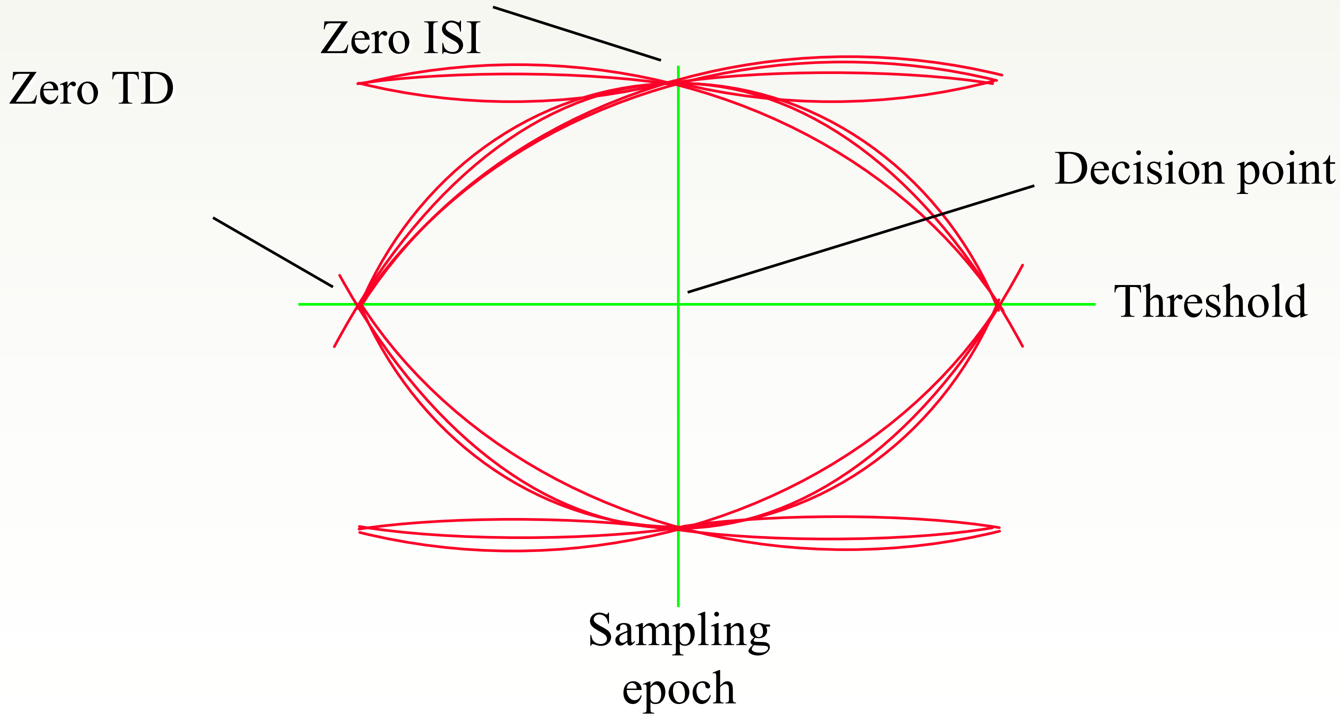
T rect(fT)

Raised cosine family



Eye diagram: 100% raised cosine signal

- The 100% raised cosine spectrum corresponds to the minimum bandwidth signal satisfying Nyquist Criteria
- This results in signals with zero ISI and zero TD

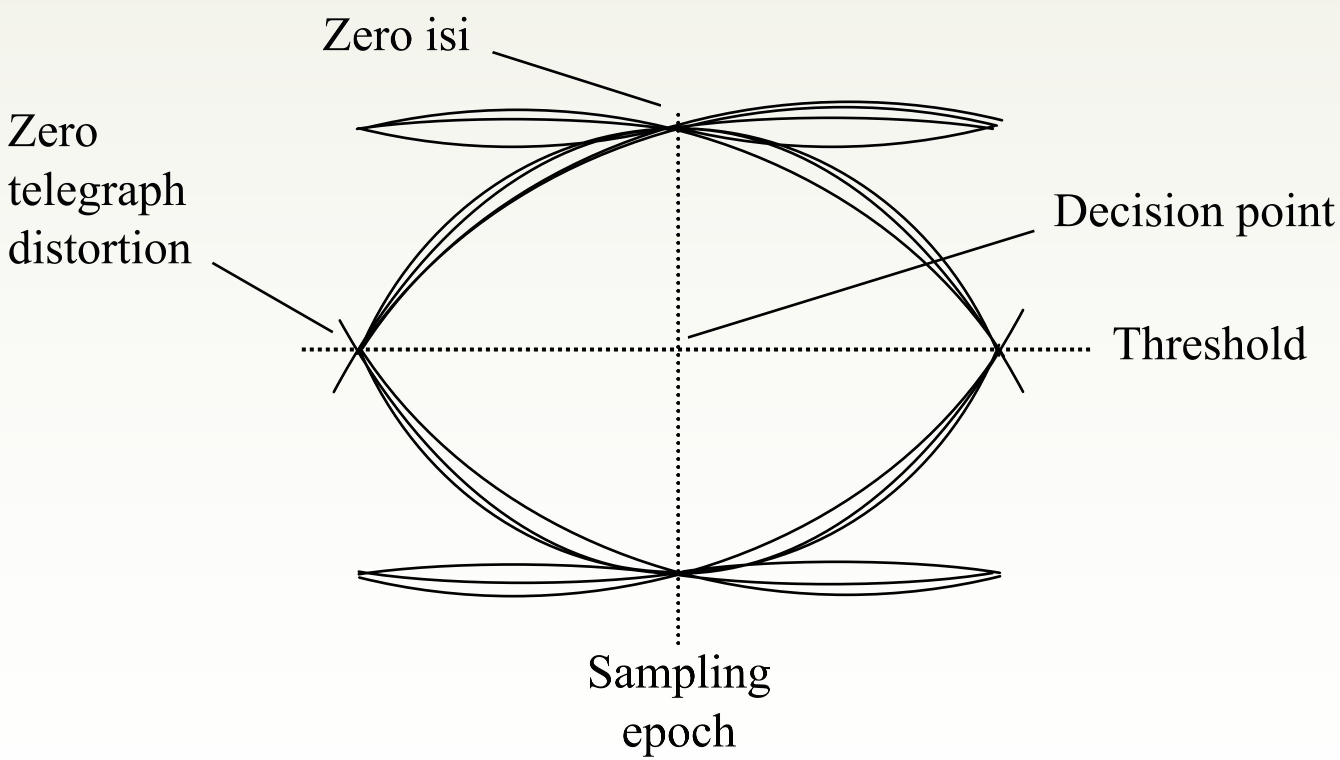


Raised Cosine Spectrum

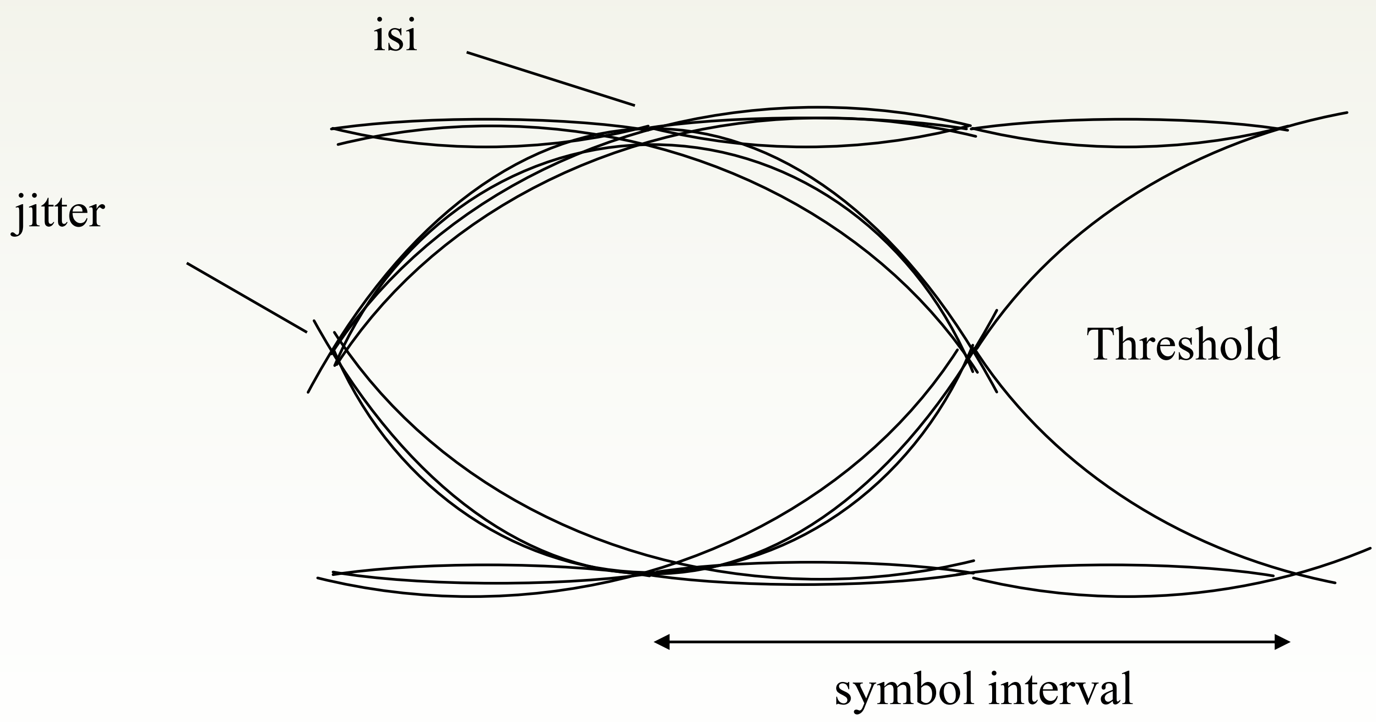
$$X_r(f) = \begin{cases} T & 0 \le |f| \le \dfrac{1-\beta}{2T} \\ \dfrac{T}{2}\left\{1+\cos\left[\dfrac{\pi T}{\beta}\left(|f| - \dfrac{1-\beta}{2T}\right)\right]\right\} & \dfrac{1-\beta}{2T} < |f| \le \dfrac{1+\beta}{2T} \\ 0 & f > \dfrac{1+\beta}{2T} \end{cases}$$

- **Where** β ***is the roll off factor 0<*** β***<1***
- ***Excess bandwidth (that above Nyquist) is usually referred to as a percentage of Nyquist***
  - β***=1/2 excess Bandwidth = 50%***
  - β***=1 excess Bandwidth = 100%***

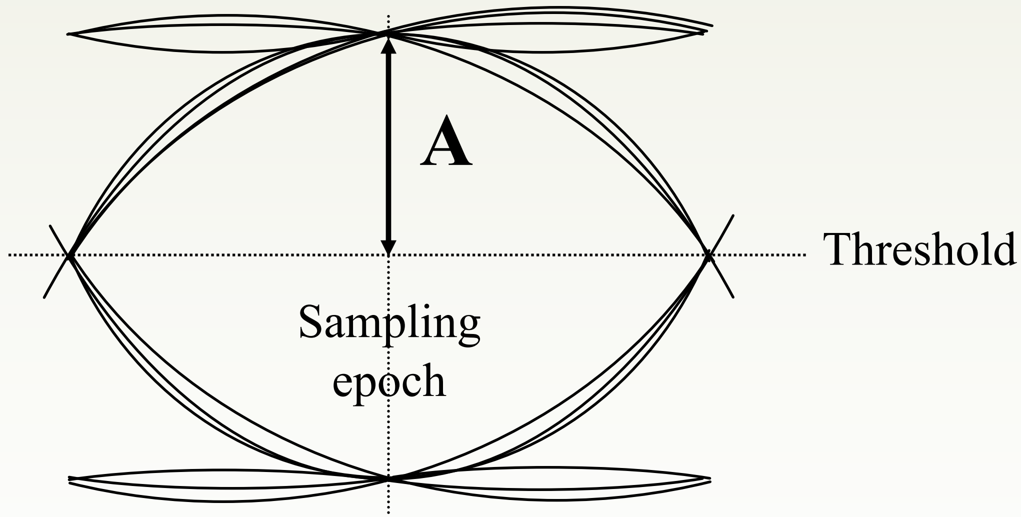
Eye diagram: 1



Eye diagram: 2



Signal to Noise Ratio (SNR)

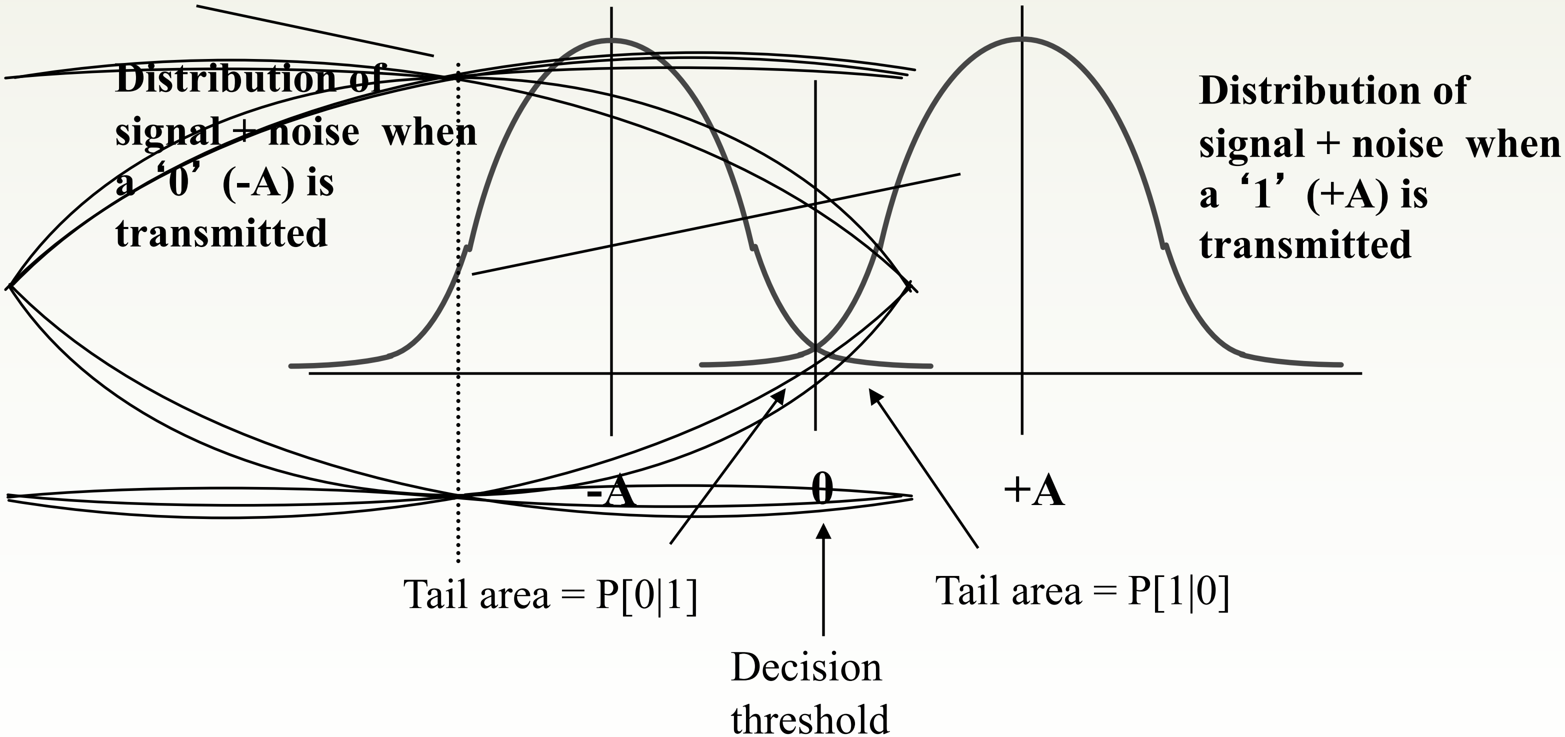


- SNR defined in terms of half eye opening and rms noise voltage, σ
- SNR=A/σ
- Adjusted for multi-level systems for No. levels

extinction ratio: In telecommunications, extinction ratio (re) is the ratio of two optical power levels of a digital signal generated by an optical source, e.g., a laser diode. The extinction ratio may be expressed as a fraction, in dB, or as a percentage.

https://en.wikipedia.org/wiki/Extinction_ratio

Bit error rate (BER)



Noise and errors

$$P_e = P(0)P[1|0] + P(1)P[0|1]$$

$$\text{with } P(0) = P(1) = \frac{1}{2} \text{ and}$$

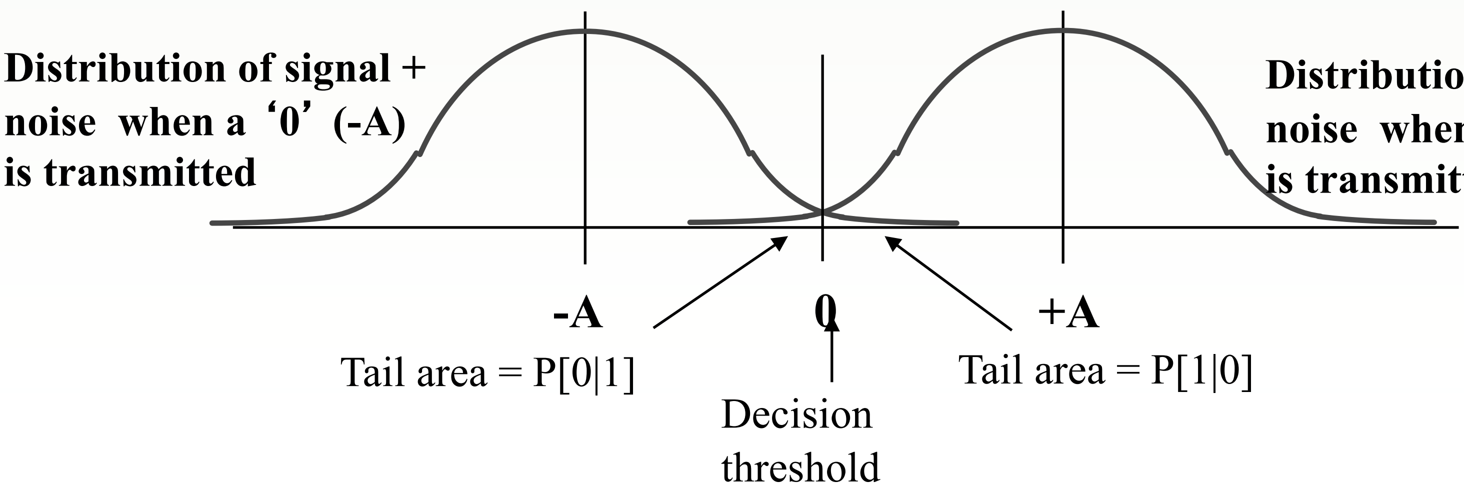
$$P[1|0] = P[0|1] \text{ then with } \alpha = A/\sigma$$

$$P_e = P[1|0] = P[0|1] \approx \frac{1}{\sqrt{2\pi}\alpha} e^{-\frac{\alpha^2}{2}}$$

Here α is the SNR and δ is the rms of noise

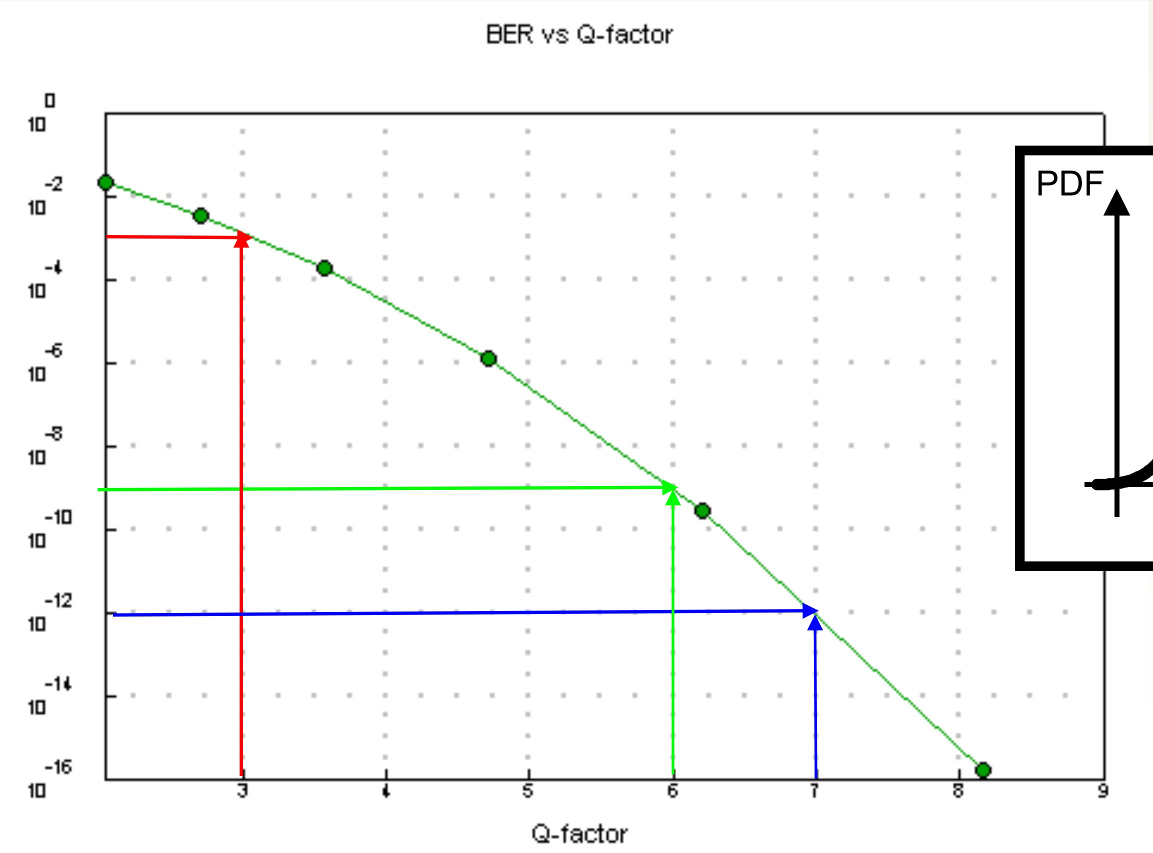
This applies for Gaussian noise and zero ISI

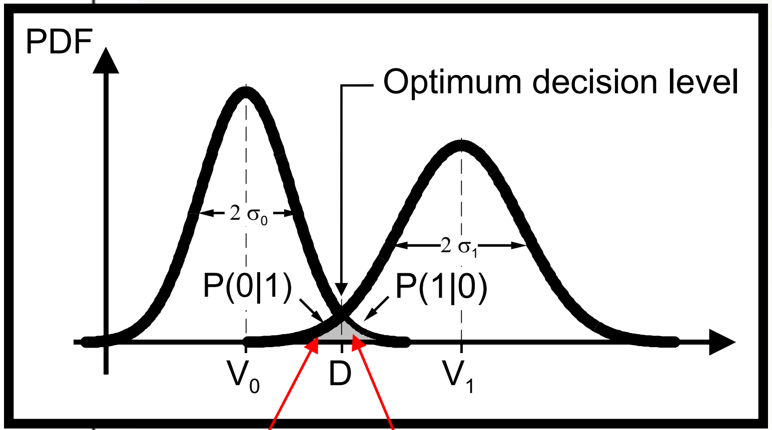
- Probability of error (Pe) or Bit Error Rate (BER) taken as ratio of number of bits received in error *to* the total number of bits, in a given transmission period.
- The primary measure of performance of a digital system
- For optical systems BER is < 10^{-9},
- This can be achieved for SNR value>6



Considerations for designers

1- The error rate deteriorates rapidly with SNR degradation (in the figure SNR is Q-Factor)
2- Noise may be different on the 1s and 0s. Different analysis and design will be needed





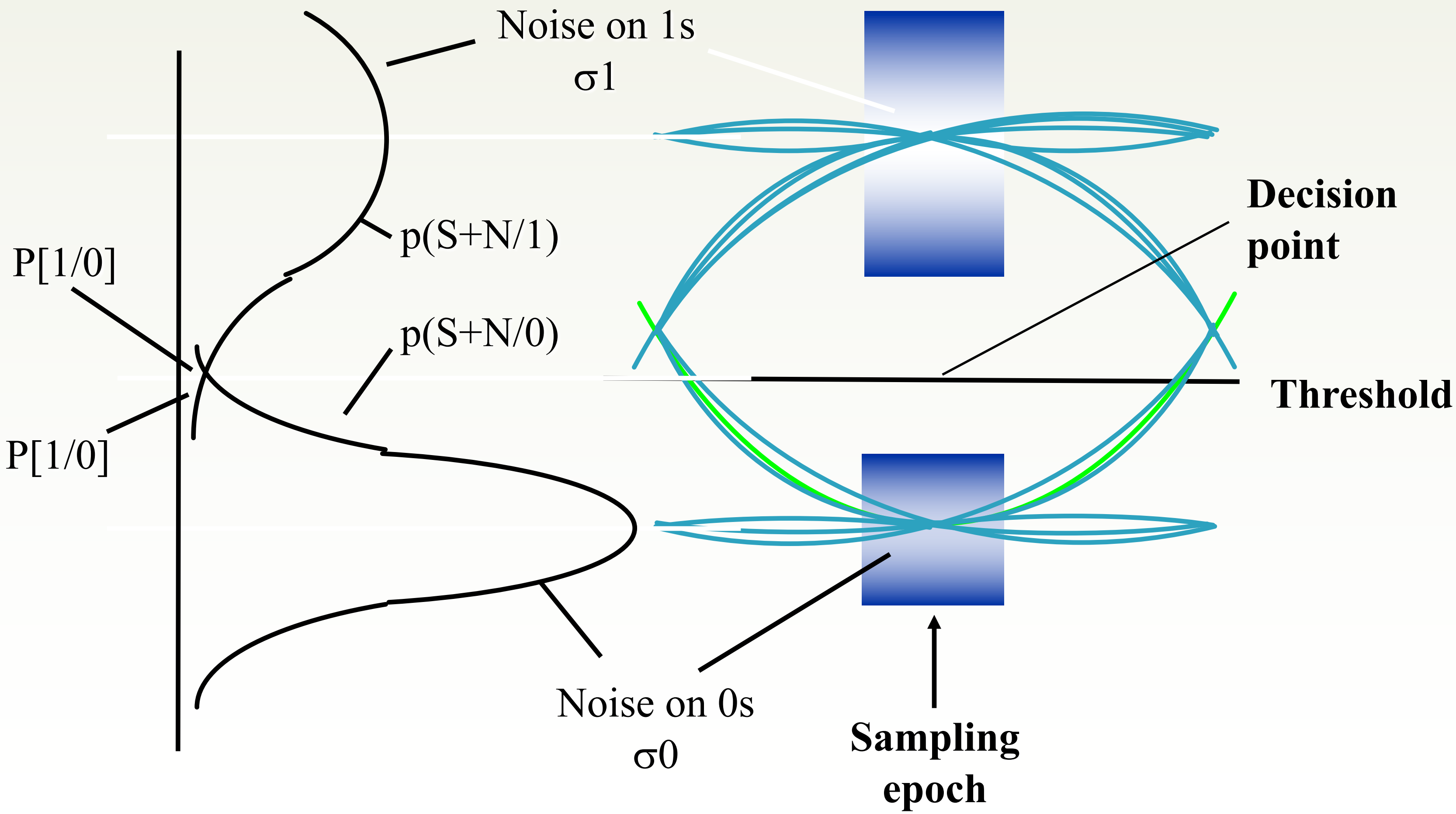
Different areas;

$P(0|1) \neq P(1|0)$

Q-based system design

- The operational system requirement is for a sufficiently low error rate at the end the system - which calls for a sufficiently large Q value at that point
- One can analyse a possible system arrangement and determine the signal and noise levels and hence the SNR
- From the SNR the corresponding Q can be found which allows the BER to be determined
- In design it is often convenient to work from a specified Q to the SNR
- Measuring Q can be effected by off-setting the threshold, necessary because systems are designed to operate error-free
- BER is measured and plotted versus threshold and then extrapolated to the region of practical interest

Noise on the Signal



Sources of Noise - 1

- Thermal Noise
  - Due to the random motion of electrons that is always present at a finite temperature

$$\sigma^2_{V\,thermal} = \frac{4\,k\,T\,B_e}{R}$$

  - R = Resistance (often 50Ω)
  - Temperature T
  - k_B = Boltzmann's Constant = 1.38×10^{-23} J/ ^oK
  - B_e= Electrical Bandwidth
- Signal Independent Noise

Sources of Noise - 2

- Shot Noise
  - Representation of the random nature of the arrival of photons

$$\sigma^2_{V_{shot}} = 2\,e\,I\,B_e$$

  - e = Charge on an electron = 1.6x10^{-19} C
  - B_e= Electrical Bandwidth
  - $\hat{I}$ = Average photocurrent generated = $\Re P_{rx}/2$
  - $\Re$ = Responsivity
  - P_{rx} =Power at the Receiver
- Signal Dependent Noise

Sources of Noise - 3

- Signal Spontaneous Beat Noise
  - Due to ASE noise from an optical amplifier beating with the signal

$$\sigma^2_{V_{sig-spon}} = 4\,\Re\,G\,P_{opt}n_{sp}h\,f_c(G-1)B_e$$

- G = Amplifier Gain
- B_e= Electrical Bandwidth
- P = Power entering amplifier
- $\Re$ = Responsivity
- n_{sp} = spontaneous emission factor

f_c= optical frequency

h =Planck's Constant = 6.63×10^{-34} J/Hz

- Signal Dependent Noise

Signal Dependent Noise

- Signal-spontaneous noise is signal dependent
- Hence EDFA cascades and optical pre-amplifier based receivers exhibit substantial variation in noise level between 1s (light present) and 0s (light absent), which we then indicate as $\sigma 1, \sigma 0$
- It is appropriate to position the decision threshold somewhat below half way in the eye in order to minimise the total error rate, corresponding to errors occurring approximately equally for 1s and 0s: P[0/1]=P[1/0]

Defining Q:

systems without amplifiers

- Additive noise, independent of signal level
- Bit error rate (BER) corresponds to 'Gaussian Tail' or complementary error function, erfc:
- BER = erfc(Q) with Q the ratio of signal *s* to rms noise, i.e. $Q=s/\sigma$
- $Q(dB)=20\log_{10}Q$
- BER of 10^{-9} corresponds to Q=6
- System operating margin may be expressed as an electrical Q margin: $M_{elec}(dB)=20\log_{10}(Q/6)$

Decision Threshold

- The decision threshold is (nominally) set such that P[1/0] = P[0/1]
- If noise is equal on 1's and 0's then this will be in the middle – signal independent noise
- For signal dependent noise With perfect extinction for 0s and normalised decision threshold of this corresponds to equal 'margins' for 0s and 1s of D and (1-D) relative to the respective rms noise values σ_0, σ_1, such that:

$$\frac{D}{\sigma_0} = \frac{1-D}{\sigma_1} \Longrightarrow D = \frac{\sigma_0}{\sigma_0 + \sigma_1}$$

Defining Q:

systems with amplifiers

- Signal dependent noise requires a definition of Q which accommodates optimum threshold <0.5
- Peak signal level is 2s
- Threshold is optimally placed at $\sigma_0/(\sigma_0+\sigma_1)$

$$Q = \frac{2s}{\sigma_0 + \sigma_1} \;; \quad BER_{opt} = erfc\,(Q)$$

Q and SNR

- It is convenient to define a signal to noise ratio in terms of the mean number of signal photons per second, s, and the mean spontaneous noise in photons per second at any point in a system: SNR(dB)=10log(s/n)
- Note that optical dB calls for a factor of 10 multiplier while electrical Q uses a factor of 20
- In the beat noise limit SNR and Q are related by:

$$Q = \frac{2\,SNR}{1+\sqrt{1+4\,SNR}}\sqrt{\frac{\Delta\upsilon}{B_e}}, \Delta\upsilon \text{ being the optical bandwidth}$$

$$SNR = Q\left(1+Q\sqrt{\frac{B_e}{\Delta\upsilon}}\right)\sqrt{\frac{B_e}{\Delta\upsilon}}$$

Types of Coding

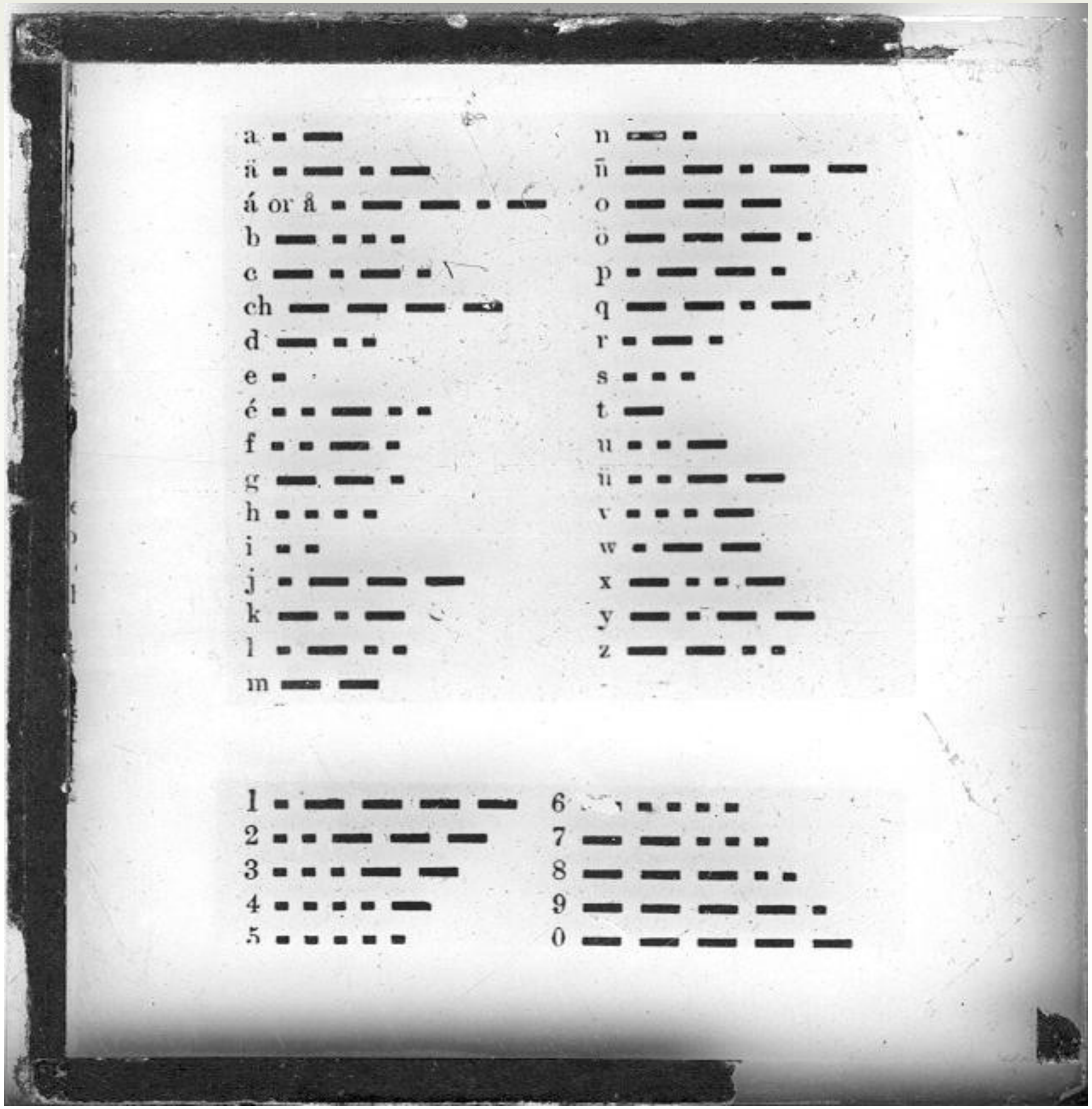
- Source Coding
  - Method of encoding the raw data
- Error Detection Coding
  - Detection of errors in the data sequence
- Error Correction Coding
  - Detection and Correction of Errors
- Line Coding
  - Formatting of the transmitted data stream to benefit transmission

Source Coding

- We have seen already how the digitised voice is encoded against a code word for each level.
- ASCII is a common source coding scheme for transmitting text.

!	0100001	A	1000001
/	0101111	Z	1011010
0	0110000	a	1100001
9	0111001	z	1111010
:	0111010	~	1111110
@	1000000		

Source Coding is not new



Error Detection/Correction

- We require a reliable channel, i.e. BER > x
- What can we do if this is not possible?
  1. Increase the power
  2. Diversity in space/frequency or time
  3. Full Duplex (echo information)
  4. Automatic repeat Request (ARQ)
  5. Forward Error Correction

All of these introduce a certain amount of redundancy

prediction/error correction is also not new…

- **cdnuolt blveiee taht I cluod aulaclty uesdnatnrd waht I was rdgnieg**

  THE PAOMNNEHAL PWEOR OF THE HMUAN MNID

  Aoccdrnig to a rscheearch at Cmabrigde Uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoatnt tihng is taht the frist and lsat ltteer be in the rghit pclae. The rset can be a taotl mses and you can sitll raed it wouthit porbelm. Tihs is bcuseae the huamn mnid deos not raed ervey lteter by istlef, but the wrod as a wlohe.

  Amzanig huh?

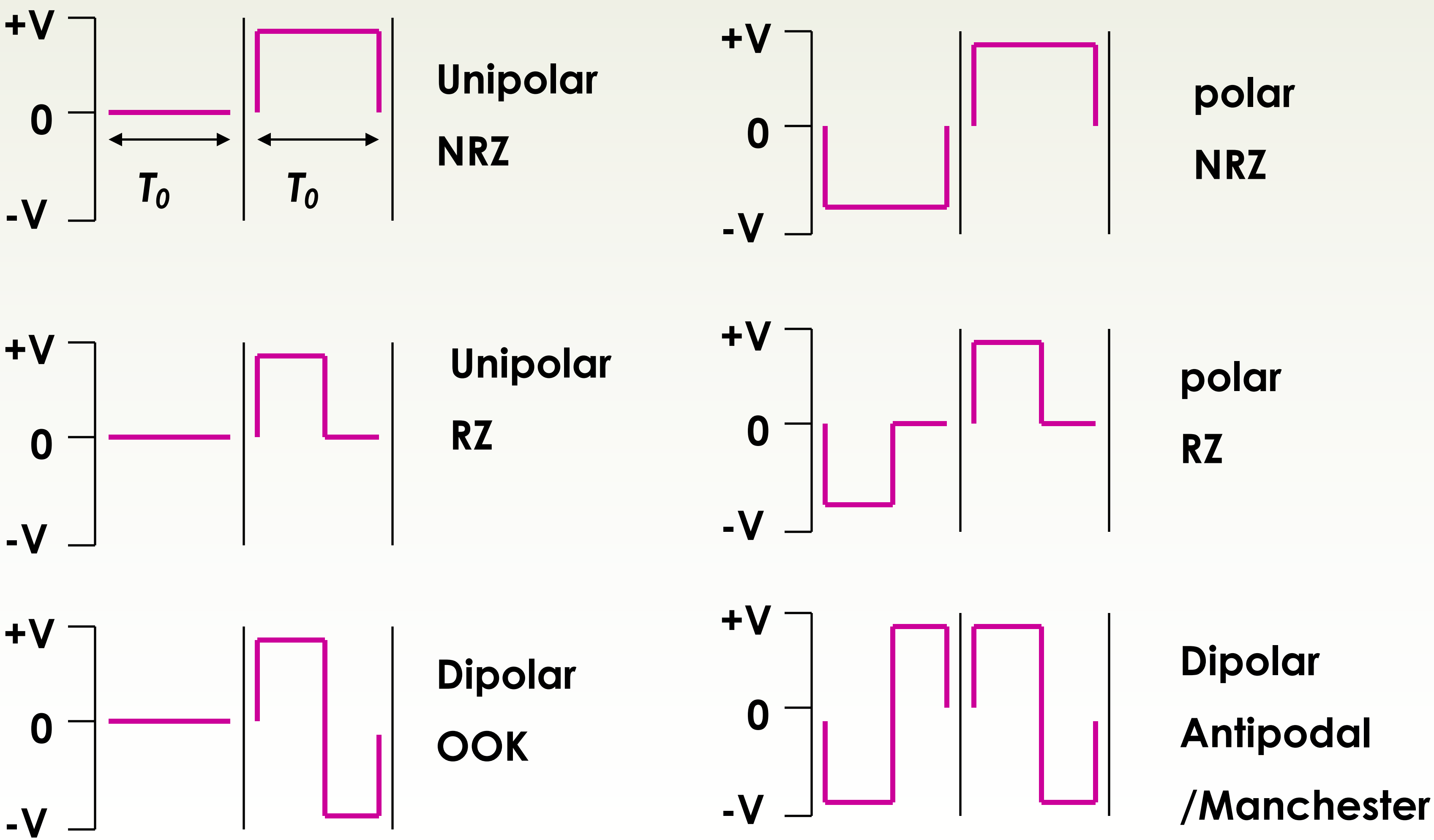
ARQ

- Three common ARQ techniques exist
  - **Stop-and-wait**: The sender(Tx) sends a frame and then waits for acknowledgement for the receiver (Rx).
  - **Go-back-n**: Sender sends frames in a sequence with acknowledgement from Rx. On error Rx discards current and further frames and notifies Tx of frame required. Tx starts again from this point
  - **Selective repeat**: Tx only repeats those frames for which negative acknowledgements are received.

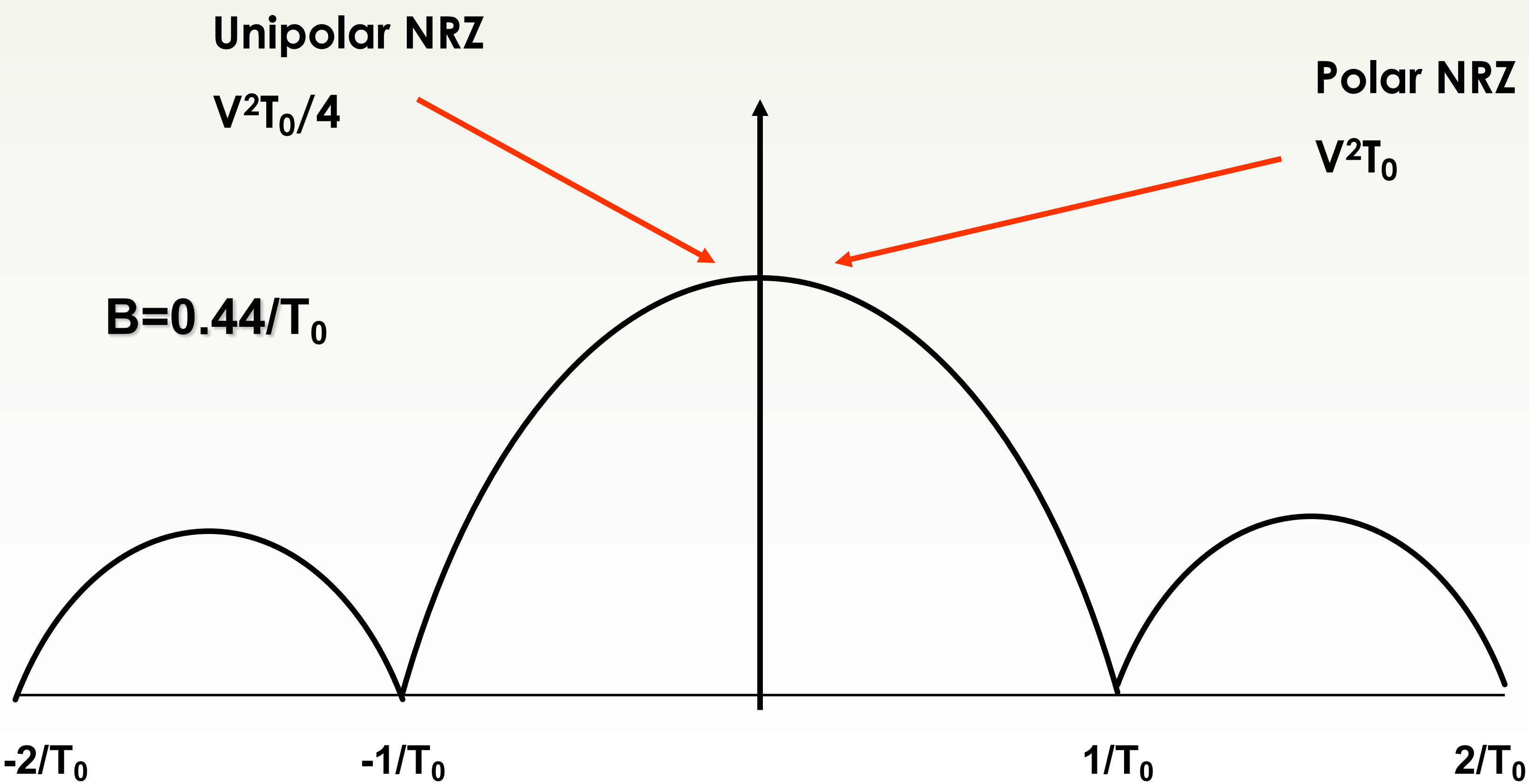
Line Coding

- We have considered so far only unipolar pulses to represent the binary bits in our system
- However, other pulse types can be used to represent these bits
- This is line coding, use for one or more of the following reasons
  - Presence or absence of a DC level
  - To shape the power spectral density
  - Bandwidth
  - Noise immunity
  - Ease of clock recovery

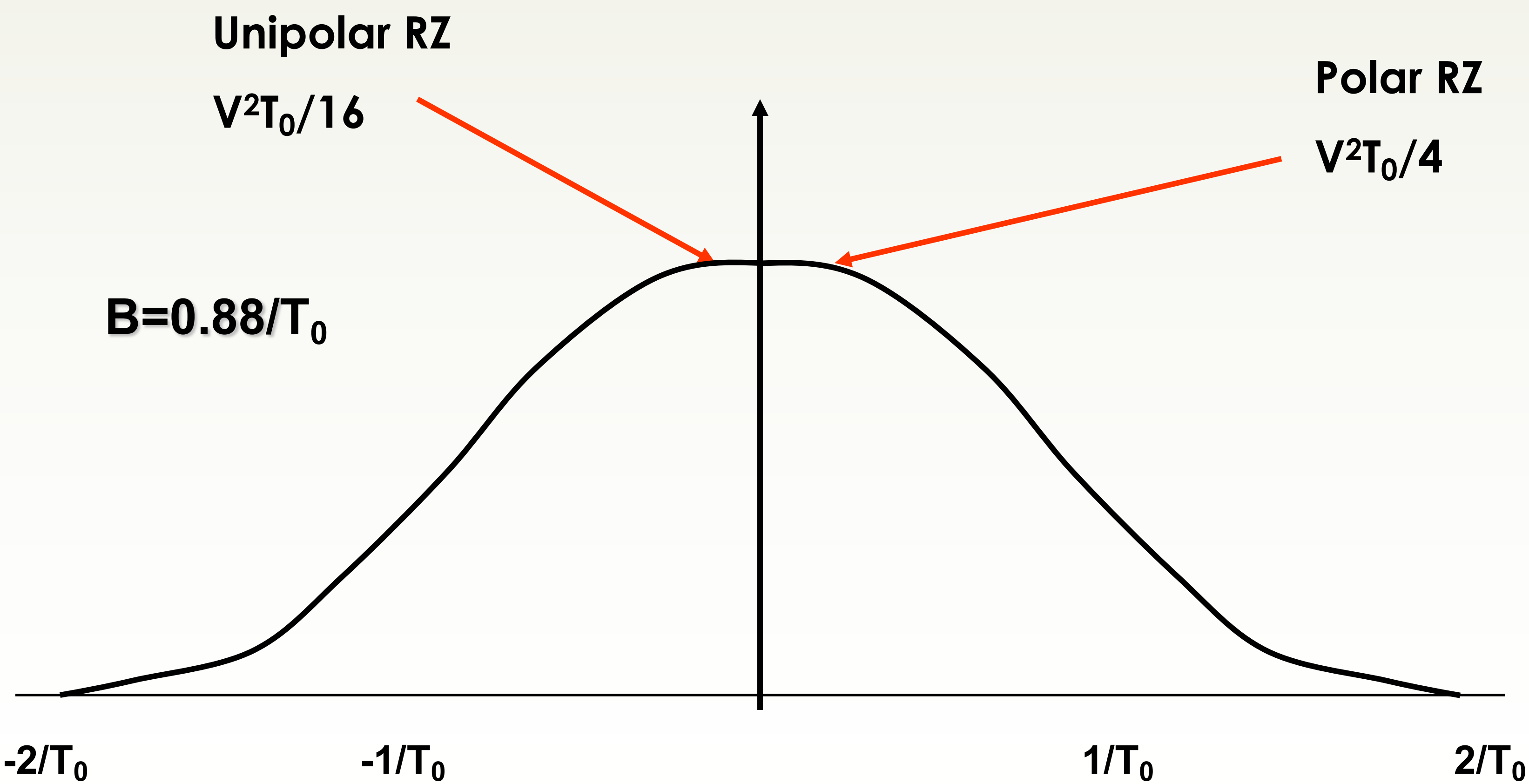
Example Line Codes



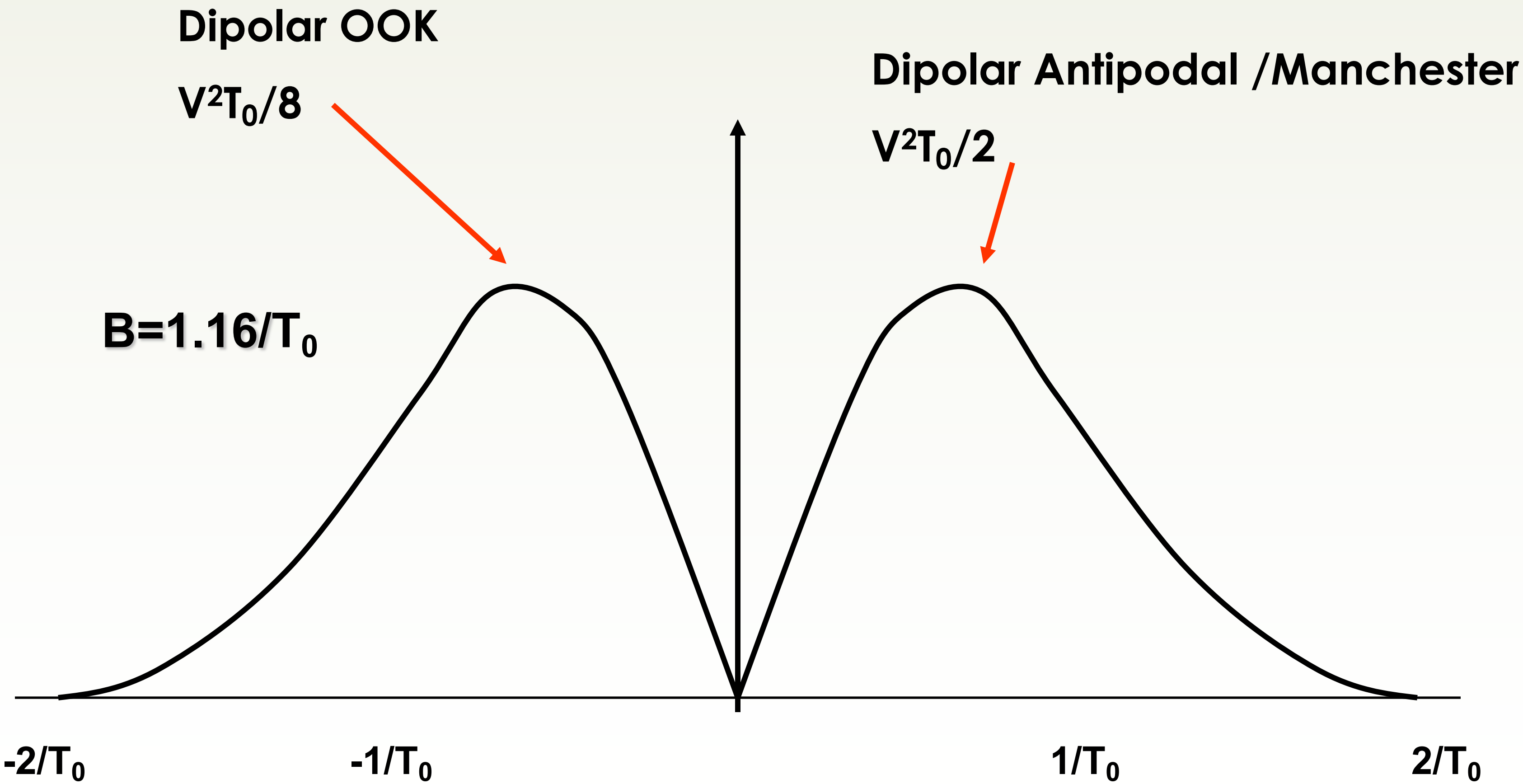
Line Code Spectrum



Line Code Spectrum



Line Code Spectrum



Example Line Codes

Coding Type	Comment
Unipolar NRZ	Usual format, DC term
Unipolar RZ	Simple timing extraction, DC term
Polar NRZ	Constant envelope, DC term
Polar RZ	Timing extraction by rectifying, DC term
Dipolar OOK	Simple timing extraction, no DC term
Manchester	Zero DC and Constant envelope
CMI	Simple timing extraction, no DC term
*n*B*m*T	Timing extraction by rectifying, no DC term

8B10B

- Maps 8-bit symbols to 10-bit symbols
  - The low 5 bit of data are encoded into a 6-bit group (the 5b/6b portion)
  - The top 3 bits are encoded into a 4-bit group (the 3b/4b portion).
- DC-balance
  - Difference between number of1s and 0s in a string of **at least** 20 bits is no more than 2
- enough state changes to allow clock recovery
- run-length limit of 5 consecutive equal bits
- Firewire, SerialATA, HDMI, USB3.0, some GE

Multi-level signalling

- For the same data rate the symbol rate (Baud rate) is reduced, which implies less noise at the output of the receiver filter
- For the same average transmit power the amplitude discrimination between adjacent signal levels is reduced, which implies reduced tolerance against noise
- Overall the BER/power performance is worse for m-ary than for binary - but the reduced bandwidth requirements may justify it use

m-ary baseband signalling

- Increasing the number of levels for the same symbol rate provides for increased data rate over a given bandwidth channel
- With the same average power this implies a reduction in margin against noise - or to maintain the noise margin and hence BER there must be an appropriate increase in received power.
- Alternatively, for a given information rate requirement increasing the number of levels can allow the symbol rate (Baud rate) to be reduced - which reduces the bandwidth and hence the noise - and so tends to ameliorate the increase compared with the binary case

Modulation

- Amplitude Modulation
  - Envelope AM
  - Double sideband suppressed carrier modulation, DSB-SC
  - Quadrature amplitude modulation (QAM)
- Angle Modulation
  - Frequency modulation (FM)
  - Phase Modulation (PM)
- Combined amplitude and phase modulation

AM schemes - analytic form

$$x_c(t) = a(t)\cos(\omega_c t) + b(t)\sin(\omega_c t)$$

Envelope AM: $a(t) = 1 + km(t), b(t) = 0$

DSBSC: $a(t) = m(t), b(t) = 0$

SSBSC: $a(t) = m(t), b(t) = \hat{m}(t)$

VSB: $a(t) \equiv 1 + km(t), b \equiv \hat{m}_{HF}(t)$

- Message bandwidth W
- Envelope AM, DSBSC and QAM require bandwidth of 2W
- SSBSC requires bandwidth of W
- VSB requires W<bandwidth<2W
- SSB and QAM are power and bandwidth efficient

How to choose a digital modulation scheme

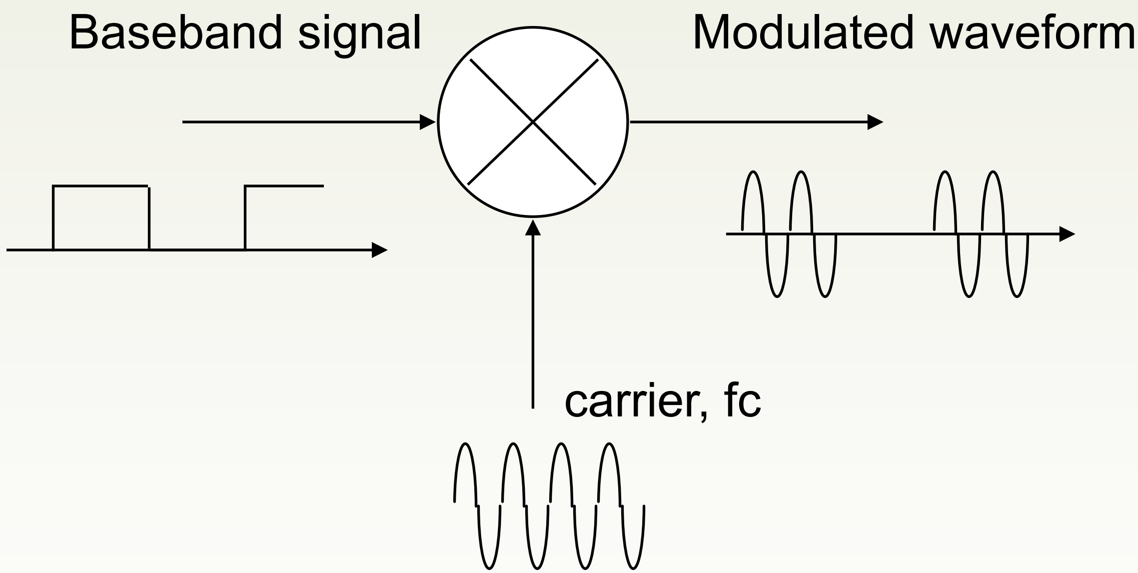
- Power efficiency (E_b/N_o for a given BER)
- Bandwidth efficiency (bits/s/Hz)
- Performance in the presence of non-idealities (fading, noise, distortion, jitter, non-linearity)
- Cost and complexity

Not all of these can be satisfied simultaneously!!

Angle Modulation

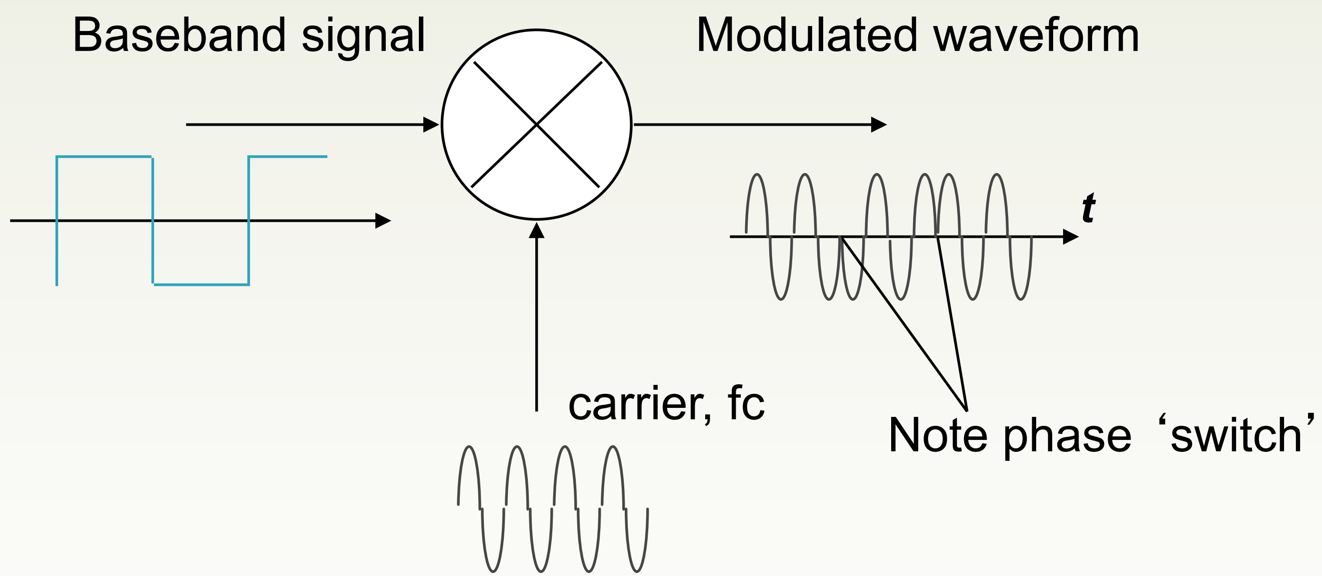
- Frequency and phase modulation are closely related - frequency corresponds to rate of change of phase
- FM requires a bandwidth of W=2Fm(1+β) - Carson's rule - β is the modulation index or peak deviation ratio
- FM and PM are tolerant to amplitude nonlinearity and so are compatible with high power amplifiers.
- FM offers an 'improvement factor' compared with AM of the order of $3\beta^2$
- FM systems have noise proportional to f^2; pre-emphasis is employed to allow for this

ASK schemes 1:



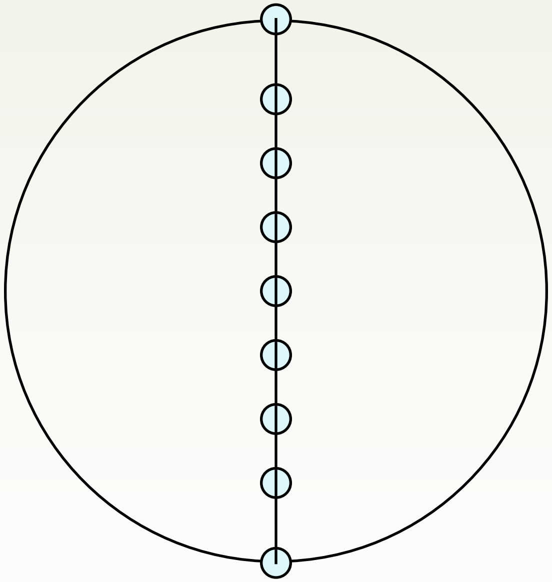
- The carrier is switched on and off by the unipolar binary signal, hence this form of ASK is sometimes referred to as on-off keying (OOK)
- In terms of analogue AM this is equivalent to binary envelope modulation with 100% modulation depth

ASK Schemes 2:



- Note that this arrangement is equivalent to binary phase shift keying (BPSK) with 0 and π as the two phases
- In terms of analogue AM it is the equivalent of DSB-SC
- With an m-ary baseband input the modulated signal is multi-level ASK (m-ary ASK)

Amplitude Modulation

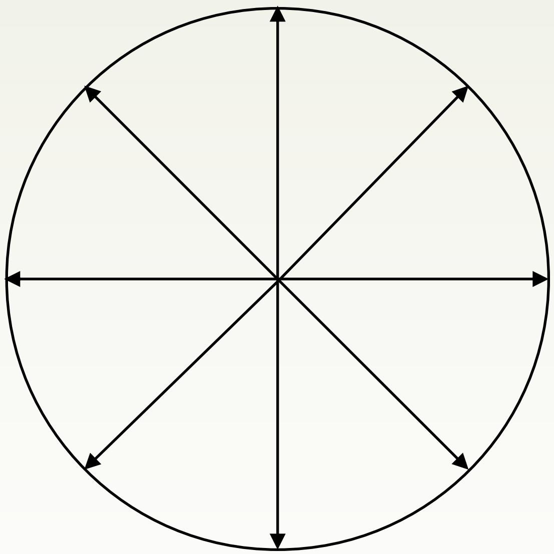


$$x(t) = A_m(t)\cos(\omega_c t)$$

$A_m(t)$ conveys the data

- Can carry n-bits/symbol with 2^n amplitudes
- Envelope varies with the data - sensitive to amplitude distortion

Phase Modulation

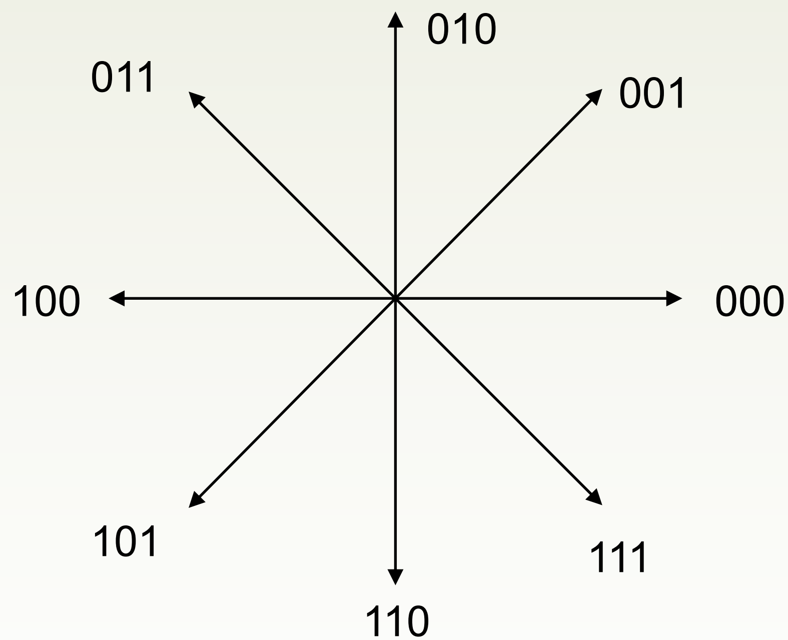


$$x(t) = \cos(\omega_c t + \phi_m(t))$$

$\phi_m(t)$ conveys the data

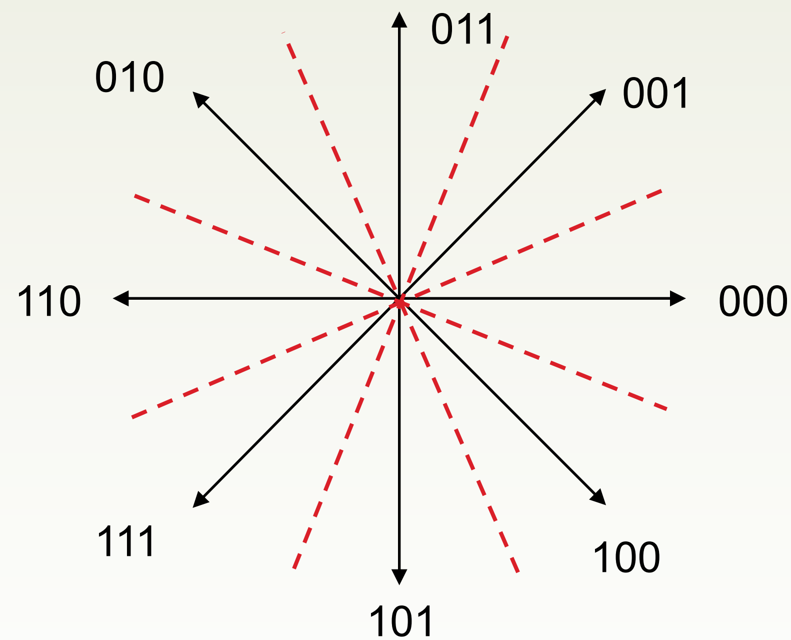
- Can carry n-bits/symbol with 2^n phases
- Constant envelope provides tolerance to amplitude distortion

Phase-shift keying



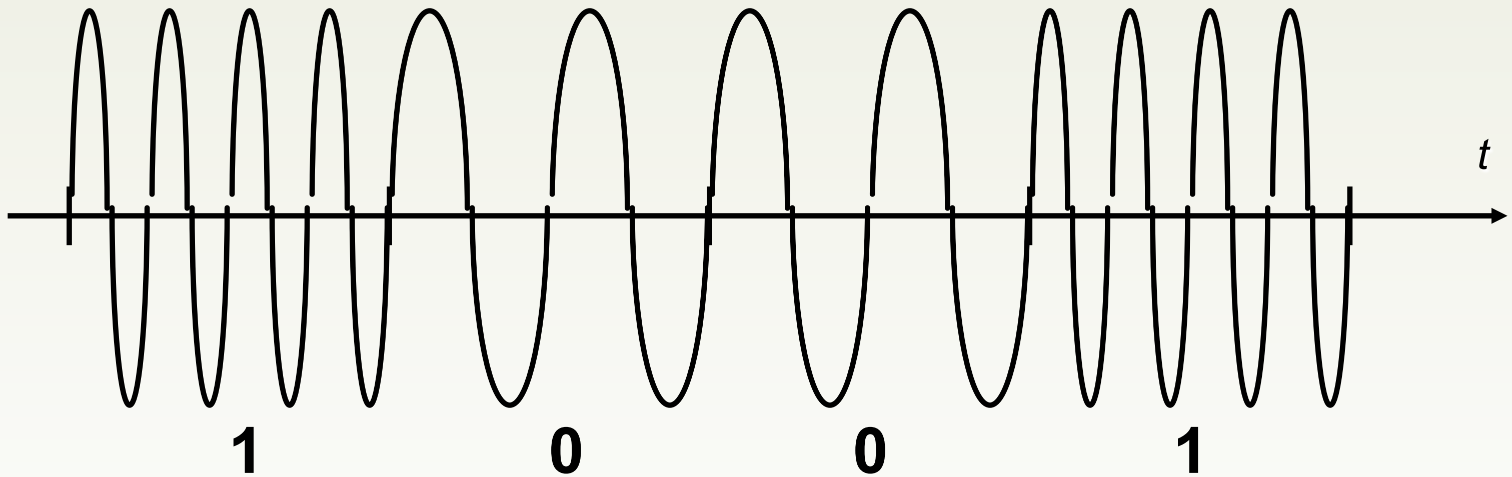
- Possible signal phases for 8-phase PSK
- With $8=2^3$ phases there are 3 bits per symbol
- The phase coding shown is illustrative - not necessarily the best choice!

Phase-shift keying – Grey Code



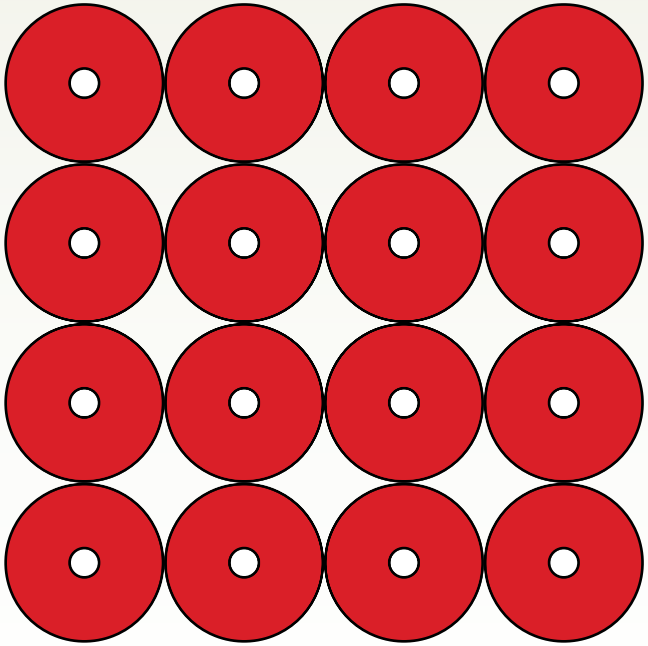
- Grey Coding means that there will only be one bit in error if a decision is made in the wrong sector
- This form of coding is used with most angle modulation schemes

Frequency shift keying



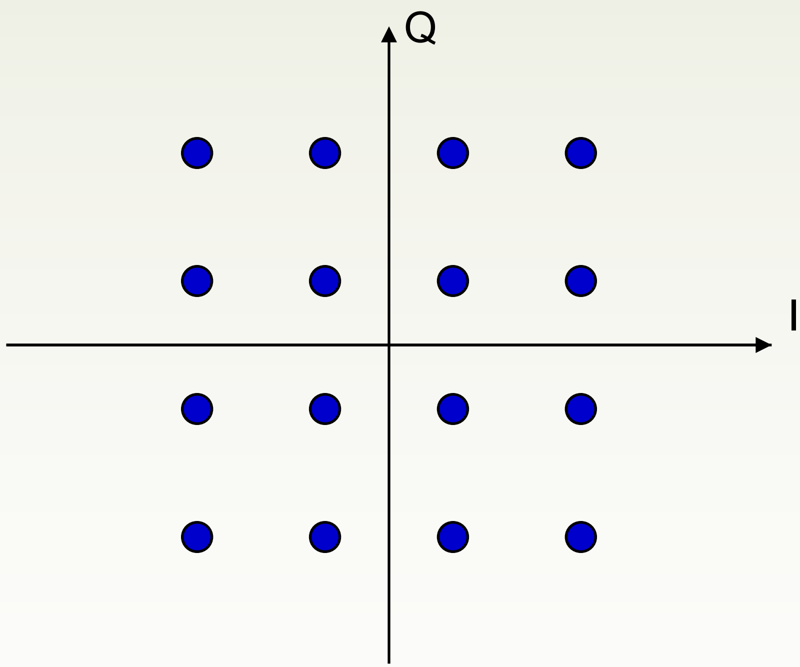
- Binary FSK involves gating between two oscillators of different frequencies - with an obvious extension to m-ary FSK
- An alternative is to apply the baseband signal to drive a voltage controlled oscillator, which avoids phase discontinuities
- Again, the m-ary extension is straightforward

QAM Signals



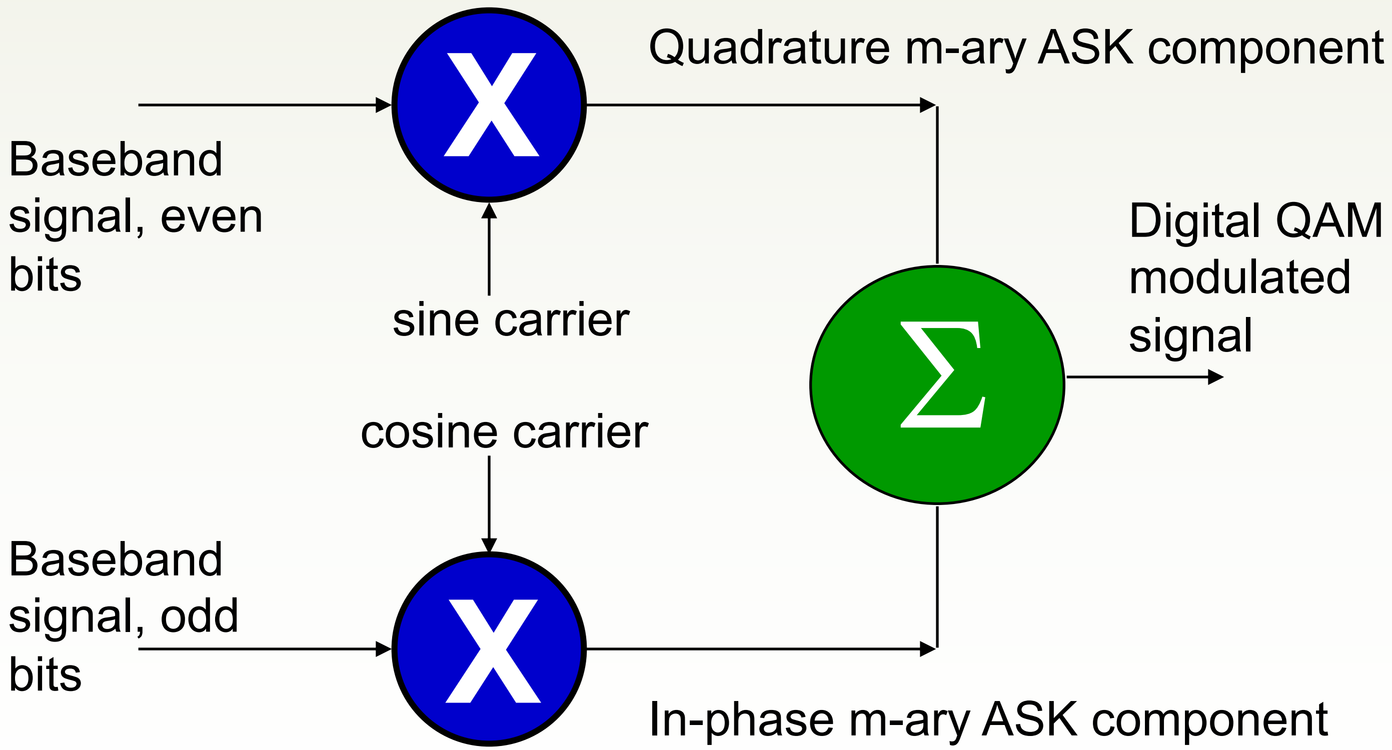
- Signal space diagram or constellation
- 16 state QAM signal
- 4 bits per symbol
- Amplitude depends on state (data) being conveyed

Quadrature amplitude modulation (QAM)



- 16-ary QAM has 4 in-phase and 4 quadrature signal levels, giving a total of 16 possible points in the signal constellation
- The capacity of this arrangement is 4 bits per symbol - 2 each for the in-phase and quadrature components
- The above is only ONE example. There are many others…

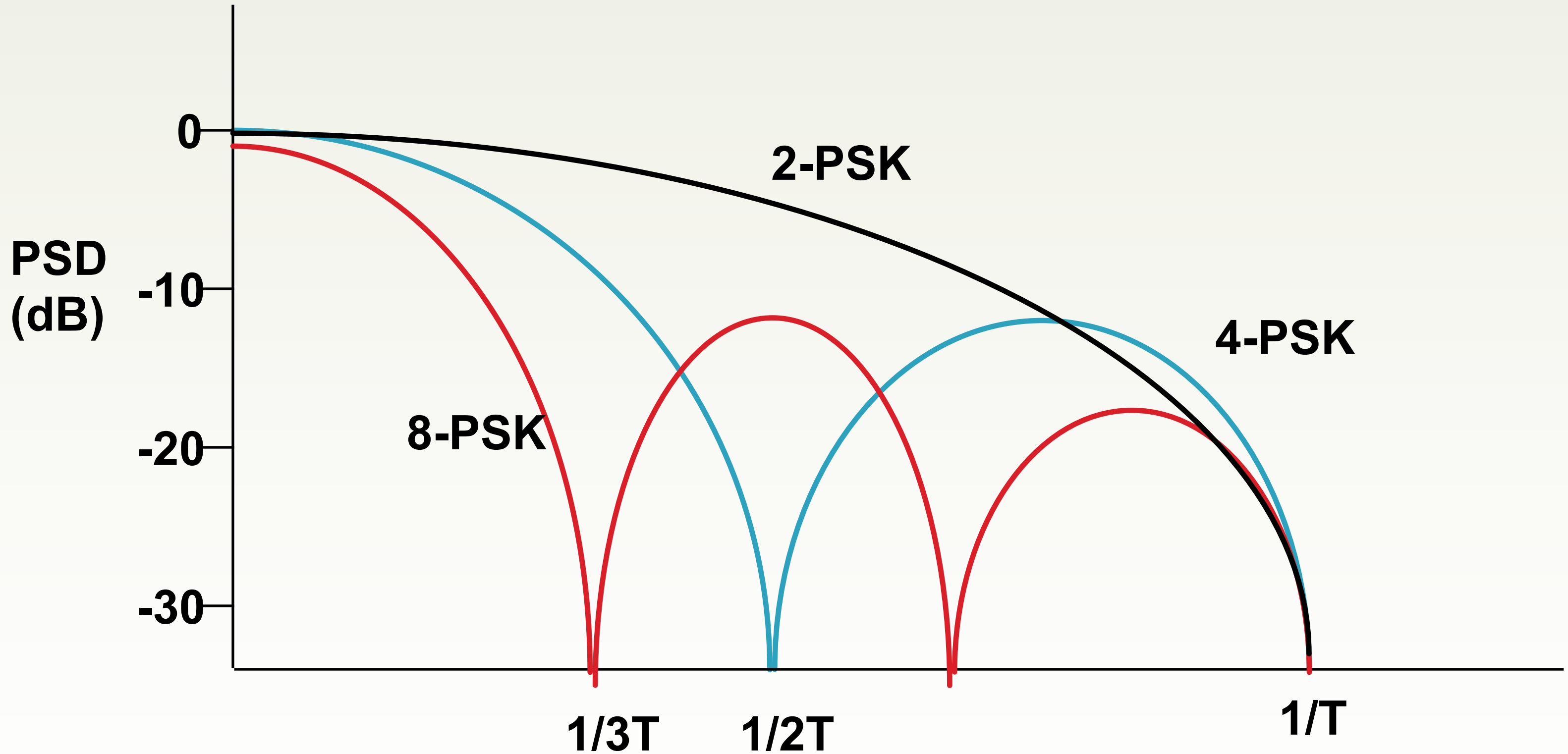
QAM signal generation



Phase Shift Keying

- From analogue modulation we know that DSB-SC provides the same SNR as does baseband transmission
- Hence 2 ϕ PSK (which corresponds to polar binary ASK) similarly provides the same SNR - and hence BER - as baseband binary transmission
- 4-f PSK(equivalent to 4 state QAM) similarly achieves the same performance as baseband transmission for the same data rate - or equivalently allows a factor of 2 increase in data rate over a given bandwidth channel at the expense of only a factor of 2 (3dB) increase in required received power
- m-ary PSK provides further increase in data rate for a given channel bandwidth at the expense of increased received power requirements for a given BER

Unfiltered PSK Power Spectra



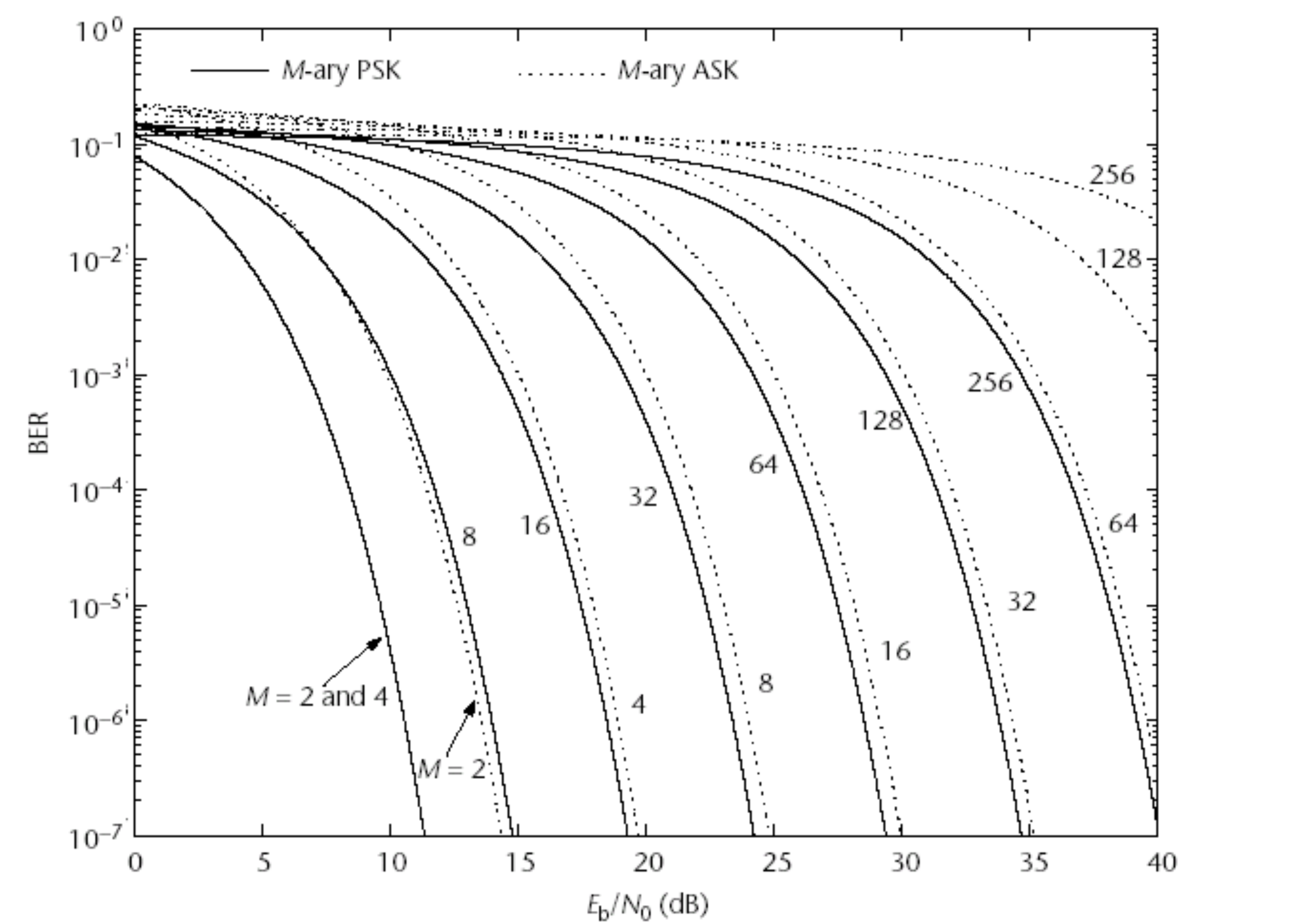
- Based on uncorrelated data and rectangular NRZ signalling

MSK - minimum shift keying

- A form of binary FSK (Continuous Phase FSK)
- Frequencies are such that exactly $\pi/2$ phase shift exists between the two signals in one symbol interval. This is achieved by fixing the frequency separation between carriers to be equal to the symbol rate (or its integer multiples)
- MSK produces the *maximum* phase difference using the *minimum* difference in signalling frequencies
- For MSK $\Delta\omega=\pi/T$ and $m_2(t)$ is a symmetric binary NRZ signal
- The result is bandwidth limitation

And there is Gaussian MSK (GMSK) used in GSM systems

Performance



Taken from *Communication Engineering Principles*, © Ifiok Otung, published 2001 by Palgrave

Summary

- Binary signalling
- Eye diagrams
- ISI and jitter considerations
- Noise and system performance
- Modulation and multi-level signalling