Digital Modulation Principles

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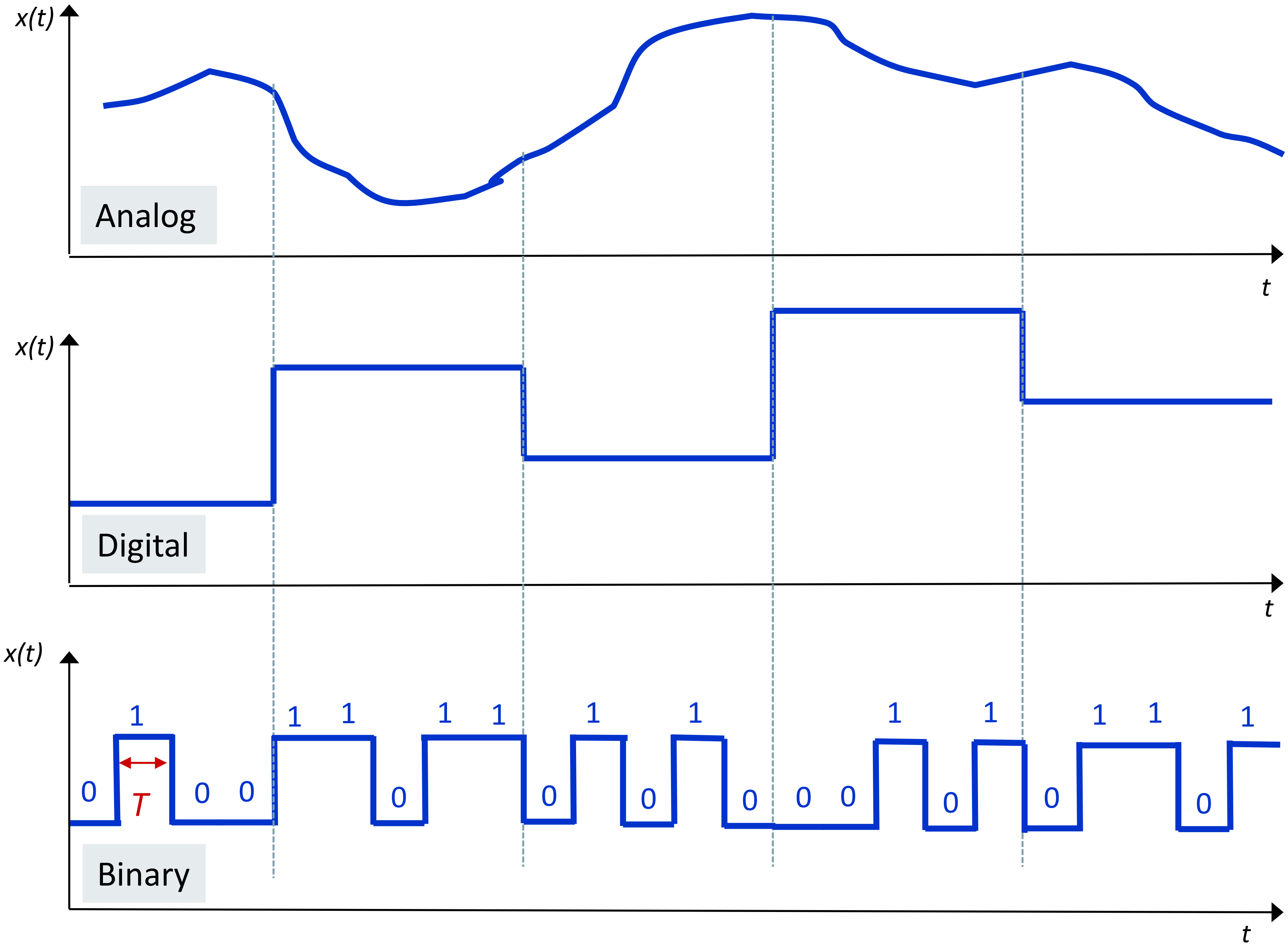
Lecture outline

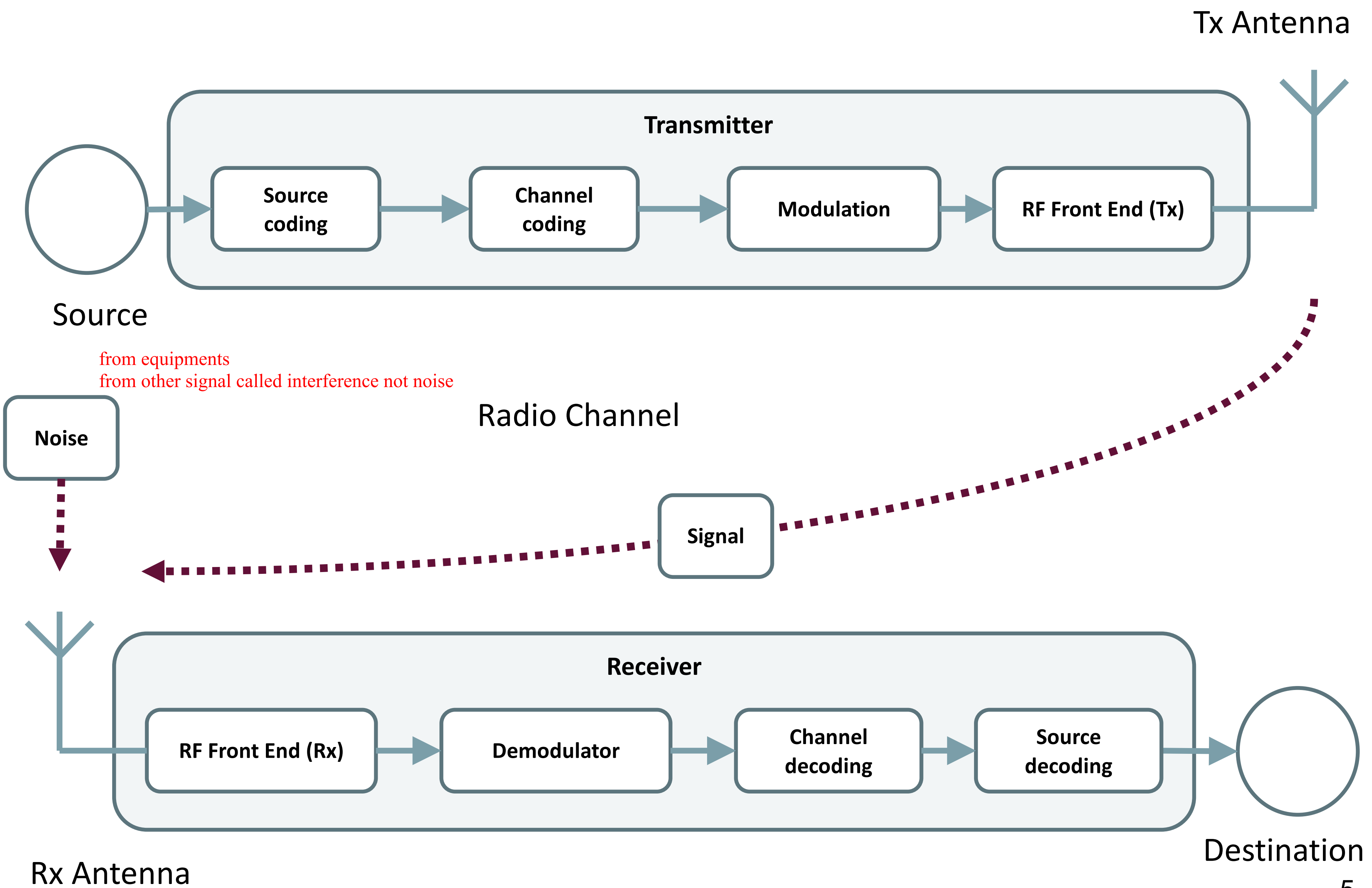
- **Part 1 : Systems, signals and noise**
  - Generic radio communications system.
  - Introduction to signals and noise.
- **Part 2 : Basic Modulation Schemes**
  - The need for modulation.
  - Single carrier digital modulation (ASK, FSK, PSK).
- **Part 3 : Higher order Modulation Schemes**
  - Quadrature amplitude modulation (QAM).
  - Multi carrier modulation schemes.

Part 1 : Systems, signals and noise

Analog versus Digital Communications



Generic digital radio communications system

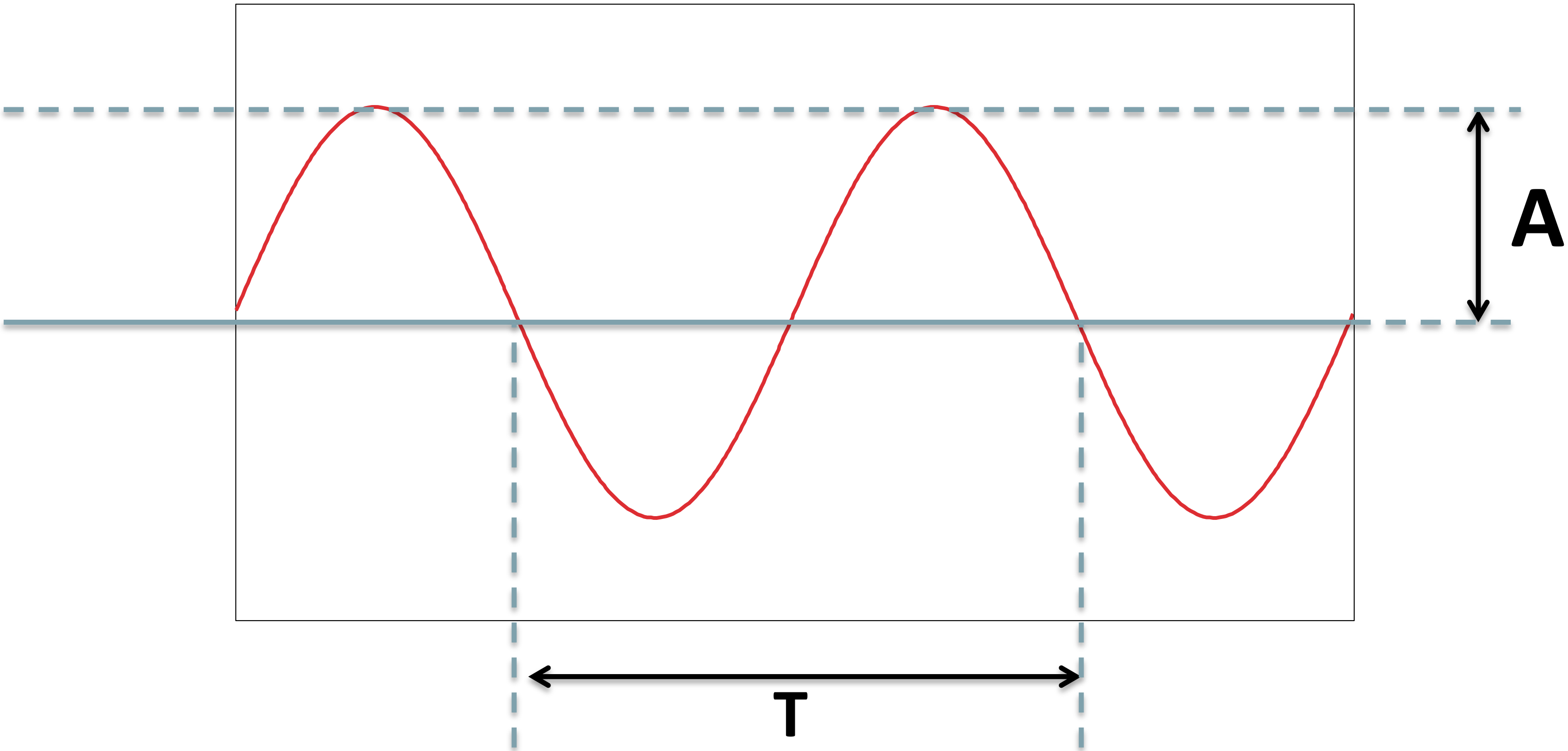


Signals

change any of three parameters will be called modulation

$$v(t) = A\cos(\omega t + \phi)$$

Amplitude Frequency Phase



Typical Thermal Noise waveform

noise voltage: $$V_n = \sqrt{4k_B TRB}$$

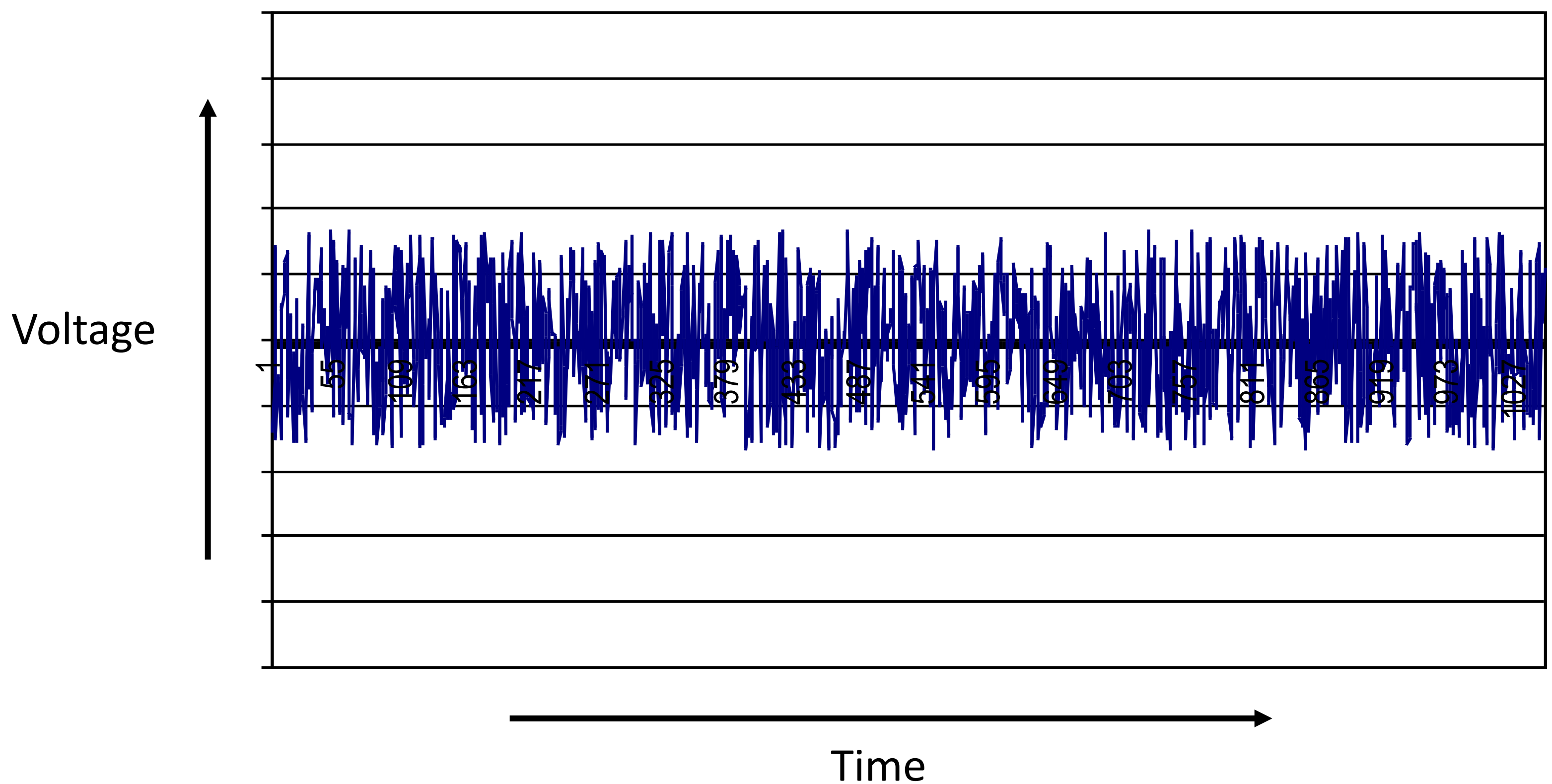
relationships between parameters

- k_B Boltzmann's constant 1.3807×10^{-23} [J/K]
- T is absolute temperature [K]
- R is the resistance of the circuit [Ω]
- B is the measurement bandwidth [Hz]

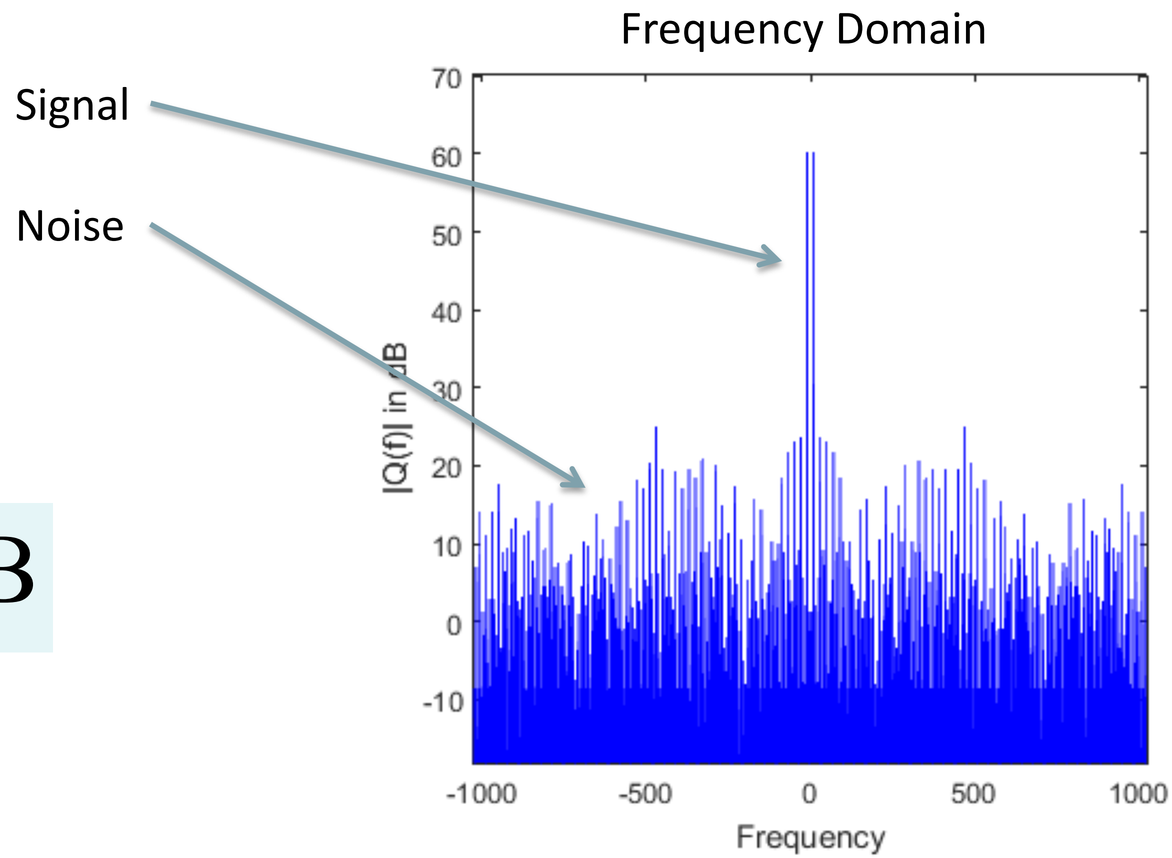
noise power: $$N_o = k_B TB$$ (V)

noise features: 1. random not deterministic
2. can be described statistically

sources of noise: equipment e.g. resistor, capacitor



Noise power spectral density



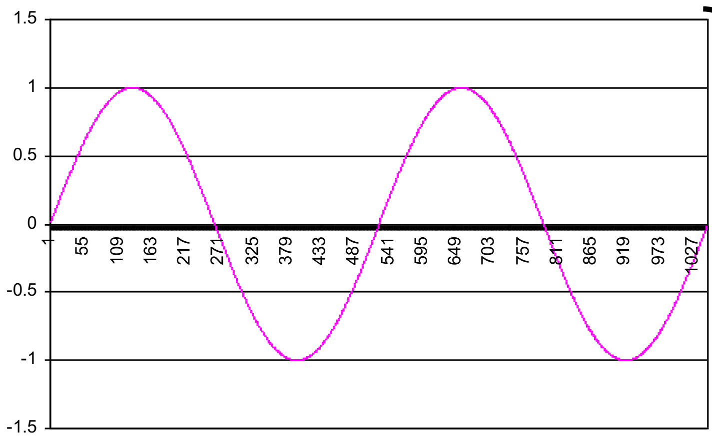
$$N_o = k_B TB$$

equal value in all frequency

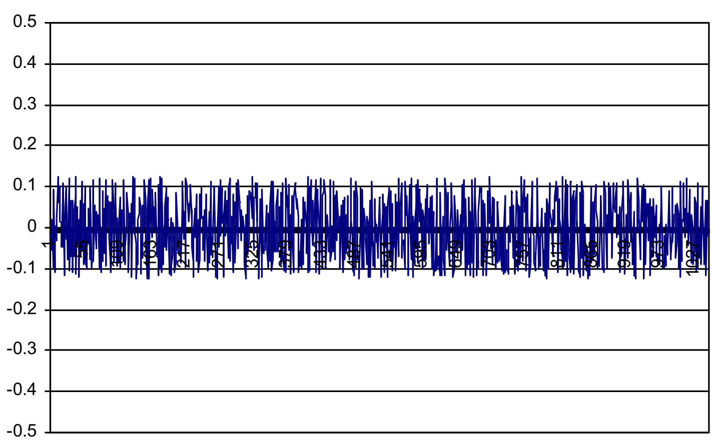
$$N_{o(dB)} = 10\log(k_B) + 10\log(T) + 10\log(B)$$

$$= 10\log(T) + 10\log(B) - 228.6\ \text{dBW}$$

Signal plus noise

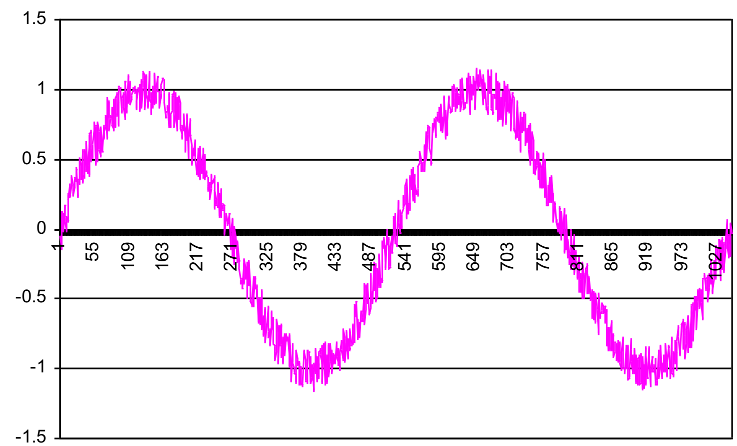


Signal Without Noise



Noise

+



Signal plus Noise

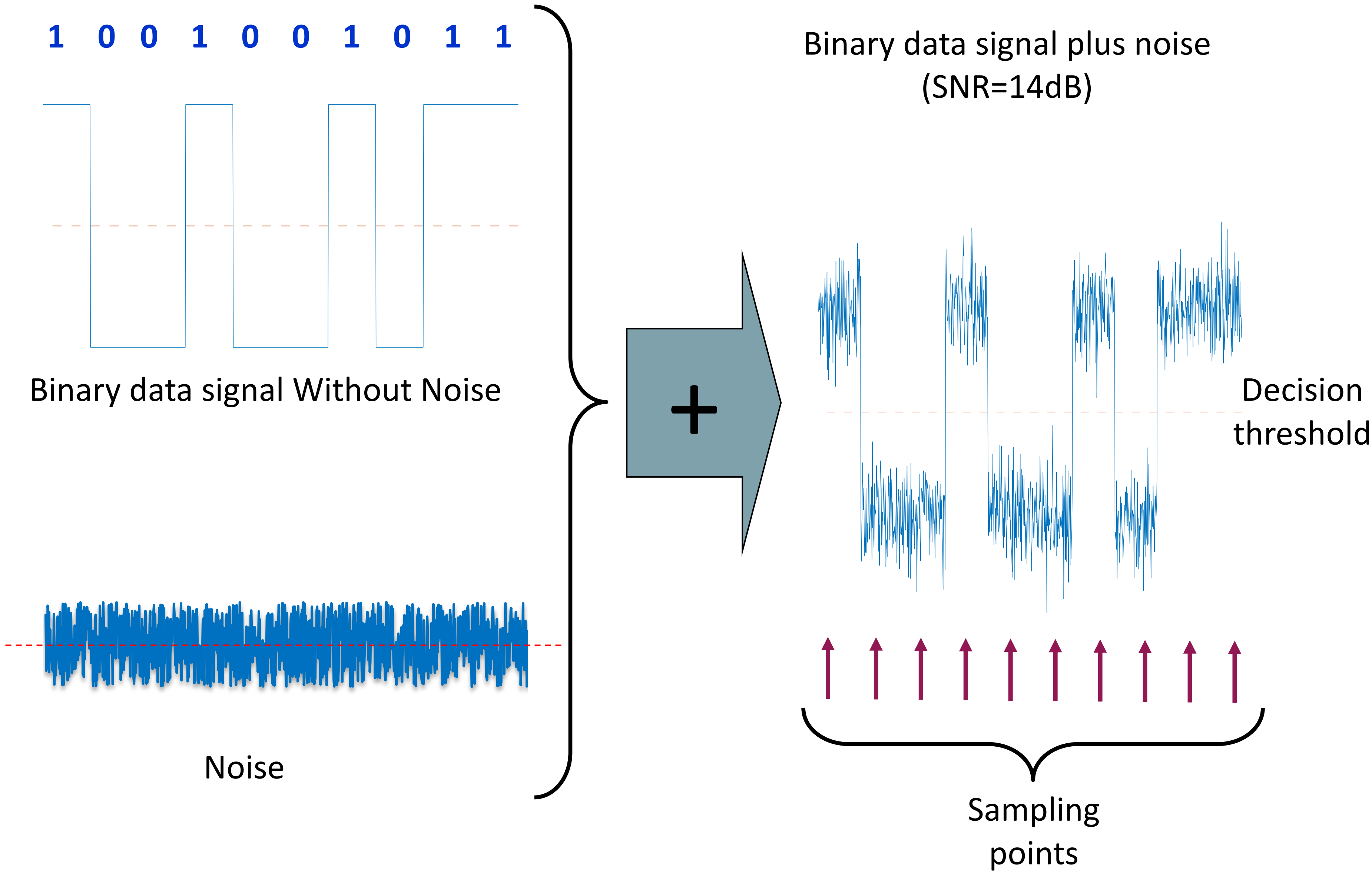
(Time domain)

Signal to Noise ratio

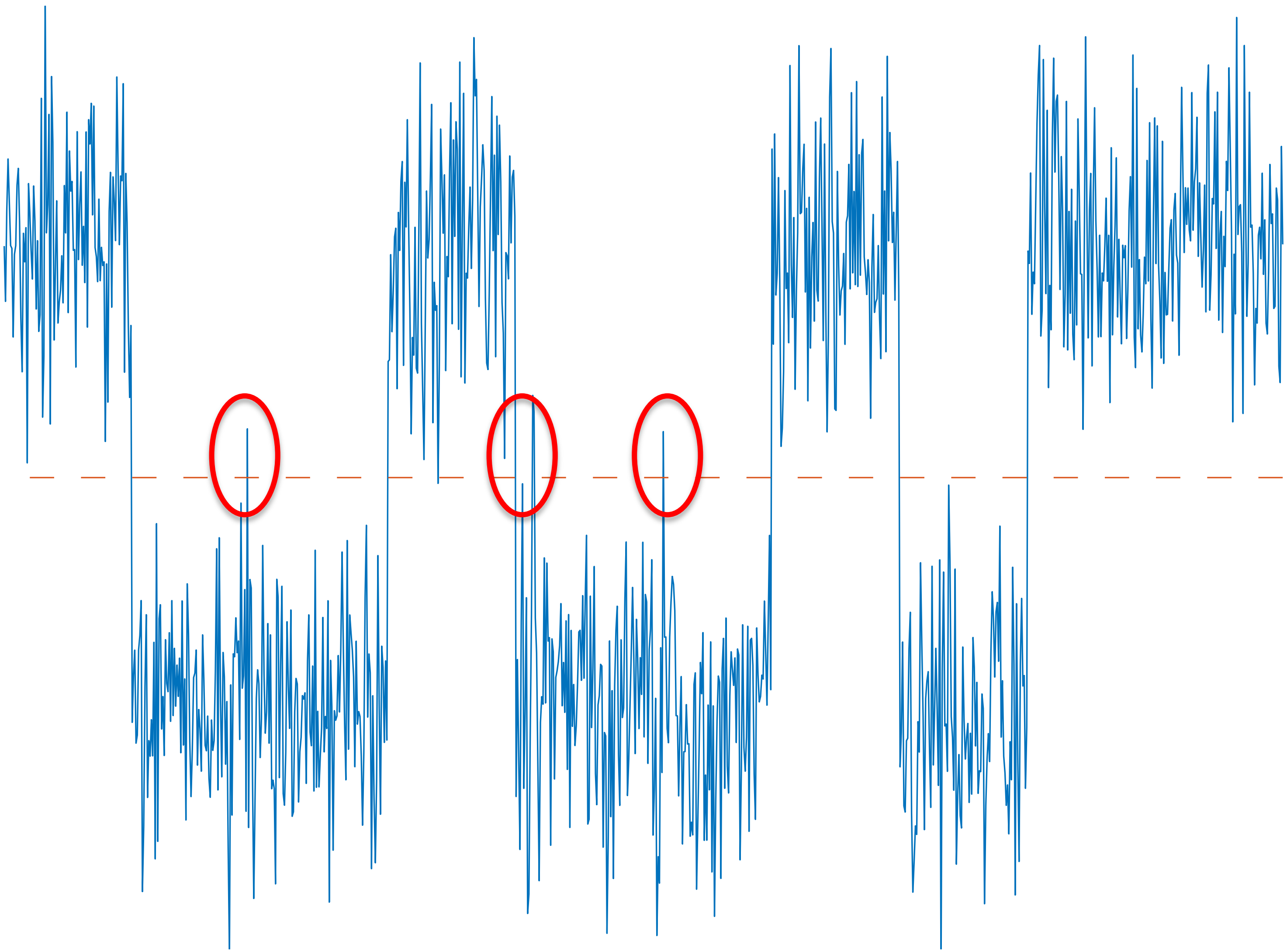
- We need a way to quantify the amount of noise that we are dealing with, by relating the noise power to the signal power.
- The commonly used metric is the signal-to-noise ratio, or SNR, which is a ratio of 'signal (wanted) power'-to-'noise (unwanted) power' expressed in decibels.
- The formula for calculating SNR is shown here, where Ps is the signal power and Pn is the Noise power. The lower the signal to noise ratio the more 'noisy' our signal will be and the more difficult it will be to recover the information from it.

$$SNR_{dB} = 10 \log\left(\frac{P_s}{P_n}\right)$$

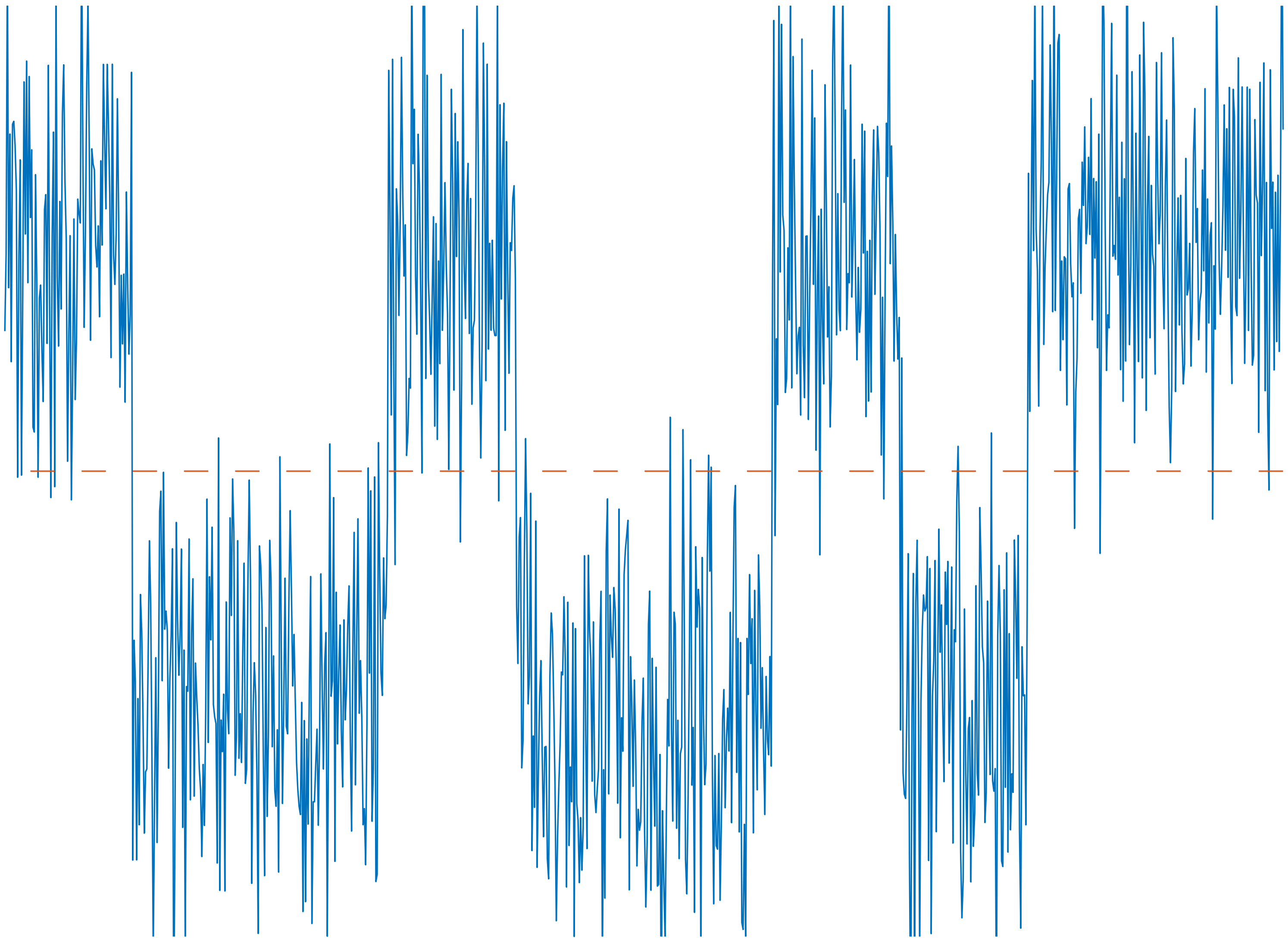
Noise in data signals



Data signal with noise (SNR=11dB)



Data signal with noise (SNR=8dB)

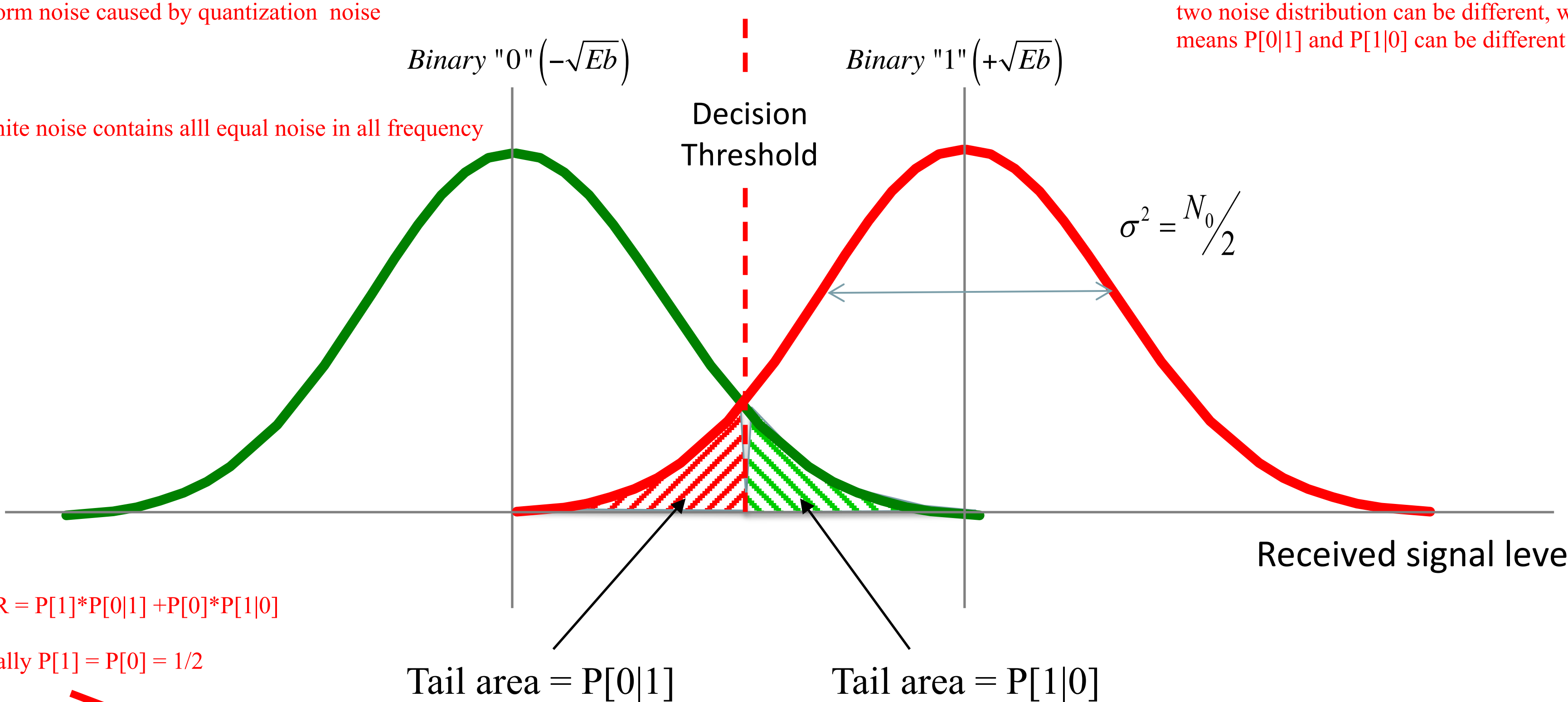


Bit Error Rate (BER)

uniform noise caused by quantization noise

two noise distribution can be different, which means P[0|1] and P[1|0] can be different

white noise contains alll equal noise in all frequency



BER = P[1]*P[0|1] +P[0]*P[1|0]

usually P[1] = P[0] = 1/2

$$BER = \frac{(P[0\,|\,1] + P[1\,|\,0])}{2}$$

relative to signal e.g. amplitude SNR

$$P_b = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Where :

P[0|1] is the probability of a 0 being erroneously detected as a 1.

P[1|0] is the probability of a 1 being erroneously detected as a 0.

BER curves

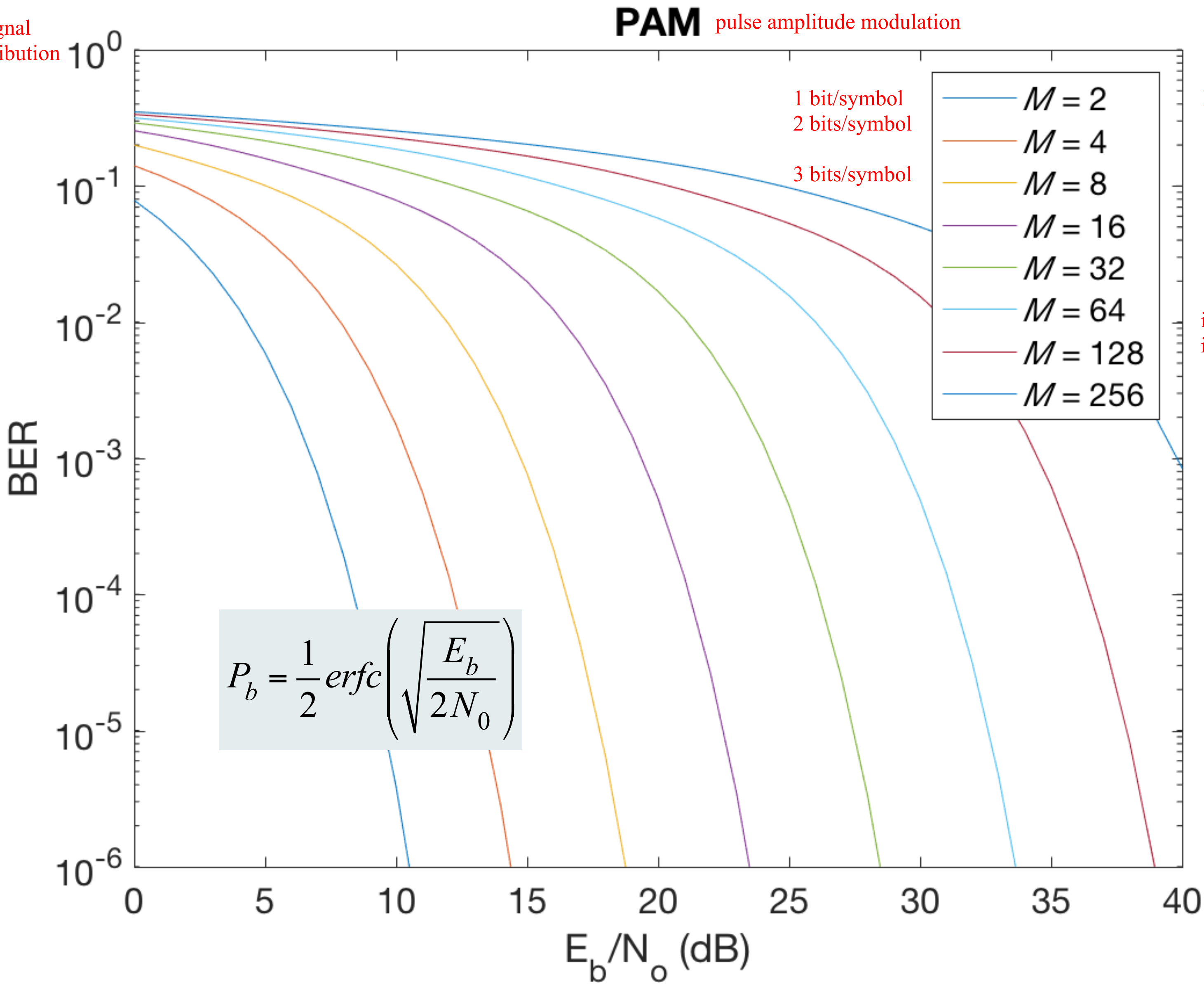
additive white Gaussian noise(AWGN)

1. adds to the signal
2. gaussian distribution
3. random noise
4. all frequency

pink noise

blue noise

colored noise

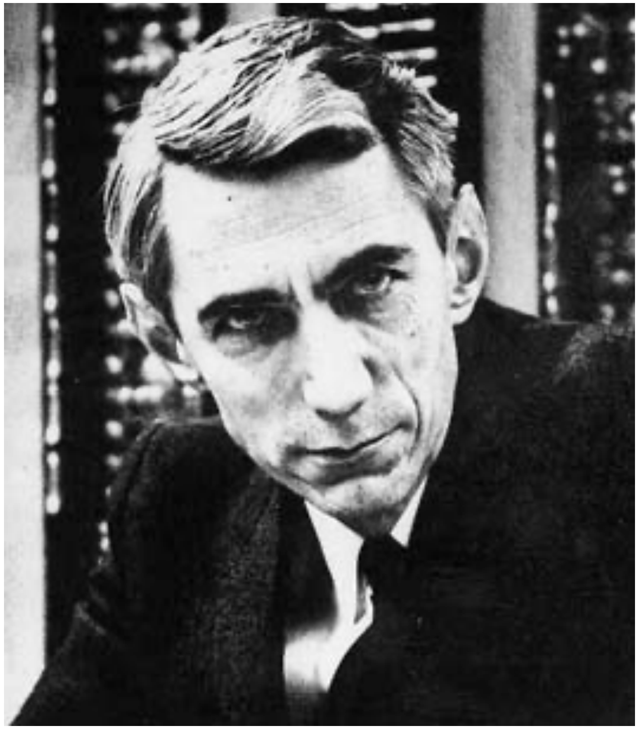


$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

M: levels

increase the bits, increase the BER

Shannon’s 1948 Theorem predicts reliable communication in the presence of noise and sets the fundamental limit of channel capacity.



Given a discrete, memoryless channel with capacity C, and a source with a positive rate R (R<C), there exist a code such that the output of the source can be transmitted over the channel with an arbitrarily small probability of error.

We define the channel capacity Cc in bits/s as a function of B (the channel bandwidth in Hz) and S/N is the signal to noise ratio

$$C_c = B \log_2 \left(1 + \frac{S}{N} \right)$$

Shannon’s capacity concept

- The channel capacity gives us the maximum data rate that can be conveyed per unit of bandwidth.
- The capacity of a Gaussian noisy channel (channel with Additive White Gaussian Noise (AWGN) chan

$$C_c = B \cdot log_2\left(1 + \frac{S}{N}\right) =$$

$$B \cdot log_2\left(1 + \frac{S}{BN_0}\right)$$

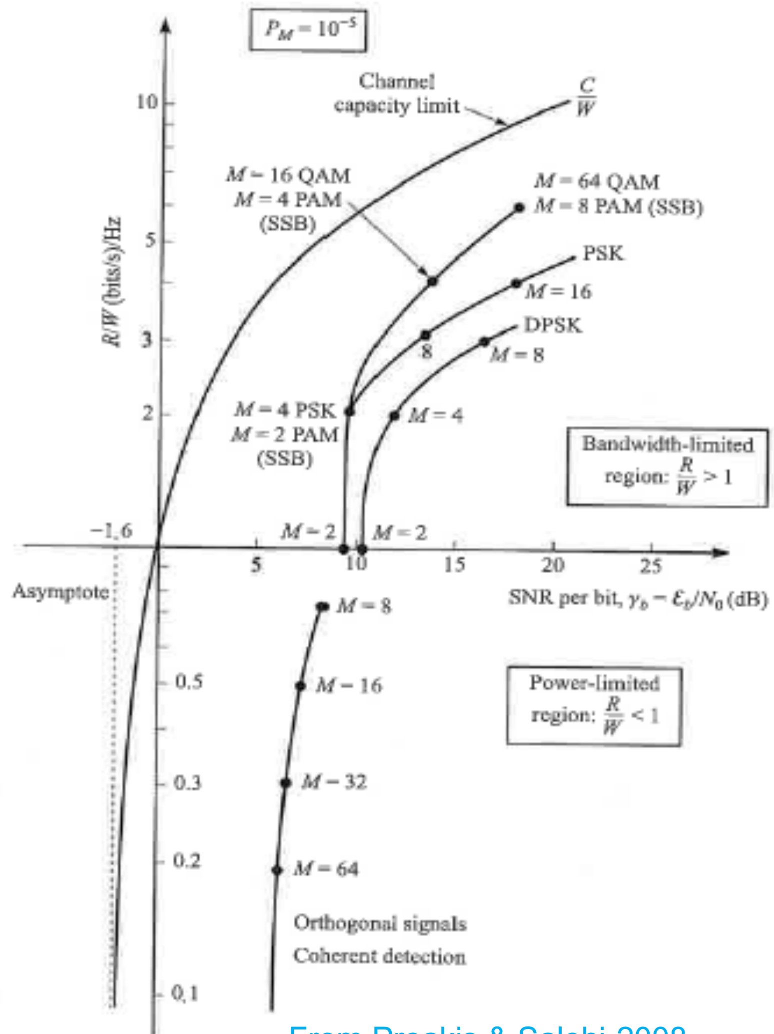
spectral efficiency is:

$SE = C_c/B$

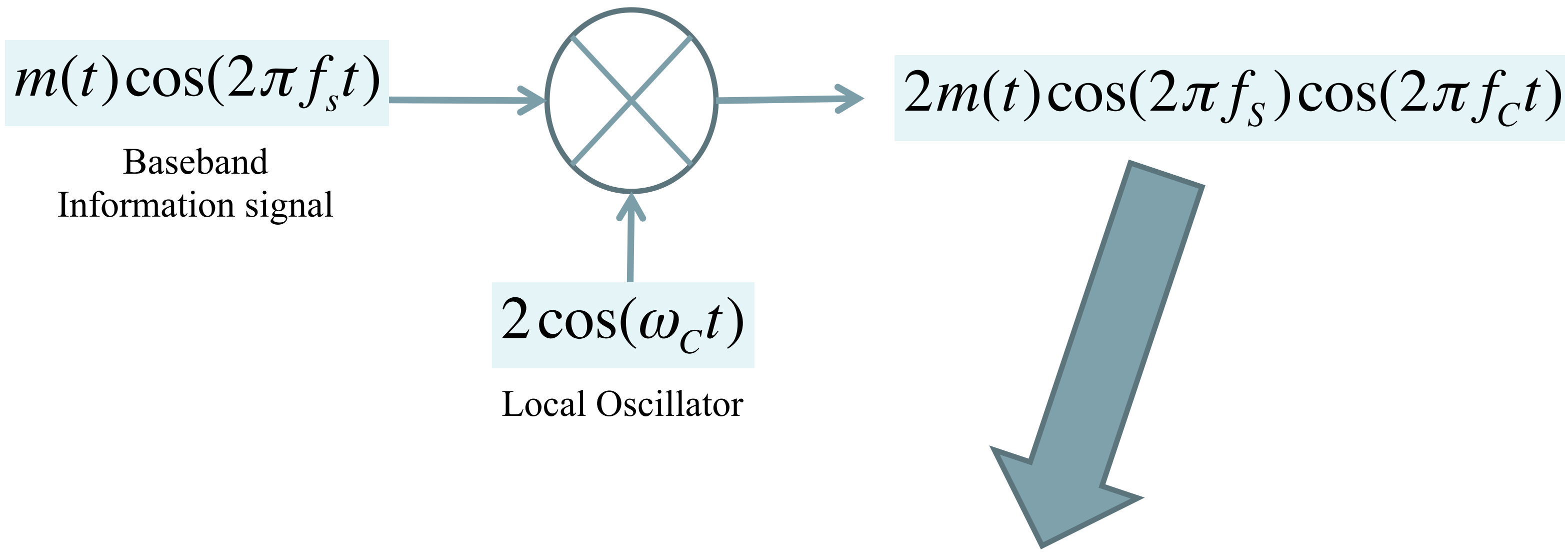
units in bits/s/Hz

shannon capacity curve

no system can be better than shannon capacity curve



From Proakis & Salehi-2008



$$2m(t)\cos(2\pi f_S)\cos(2\pi f_C t) = m(t)\cos[2\pi(f_C - f_S)t] + m(t)\cos[2\pi(f_C + f_S)t]$$

Lower sideband

Upper sideband

Channel Capacity

The Shannon's capacity limit tells us how many bits can be transmitted per second without errors over a channel of bandwidth B Hz, when the signal power is limited to S Watts and is exposed to N Watts of Gaussian White (uncorrelated) Noise that is added to the signal.

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

$$C = 3.32\, B \log_{10} \left(1 + \frac{S}{N}\right)$$

where :

C = Channel information capacity (bps)

B = bandwidth (hertz)

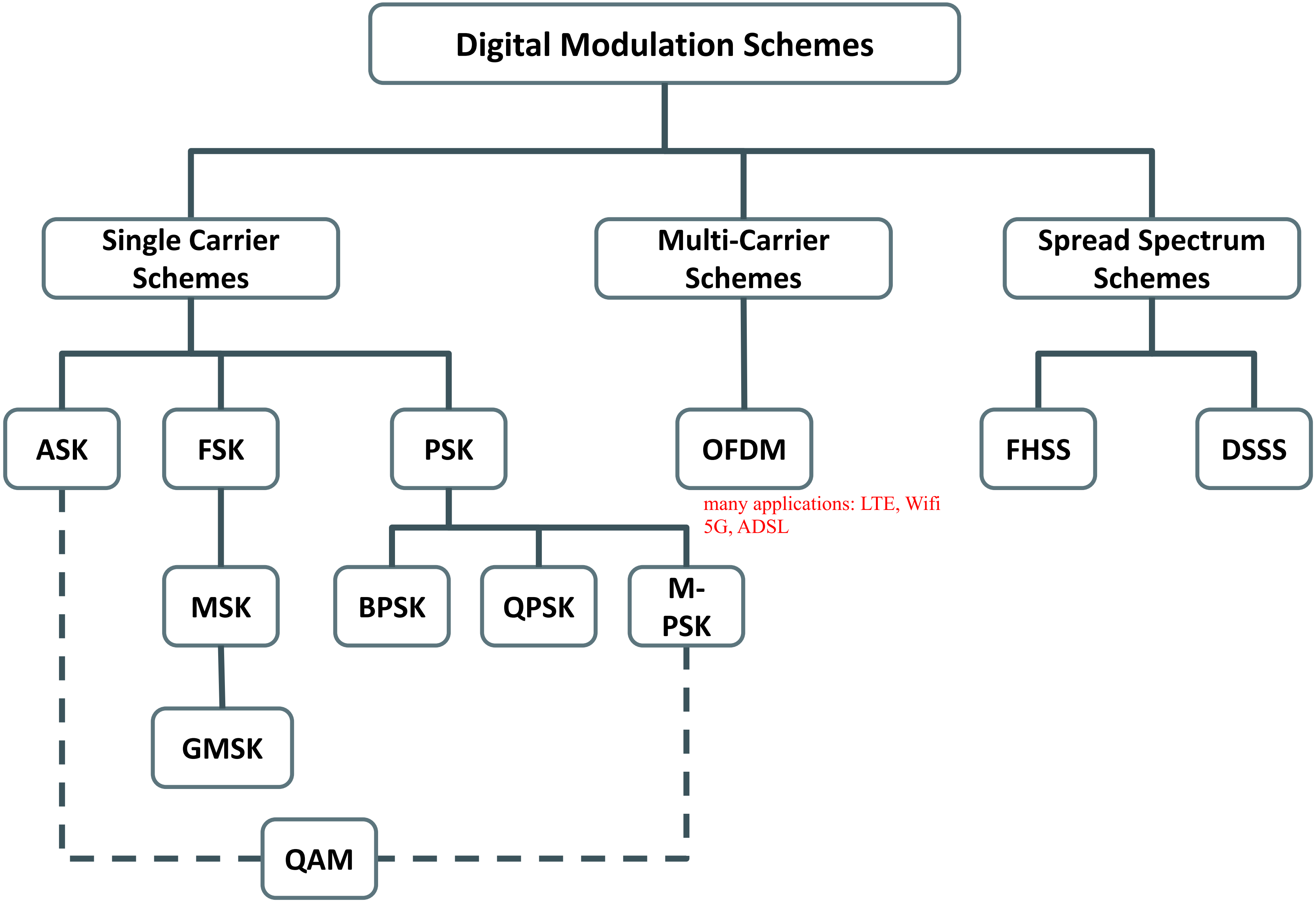
S /N = signal-to-noise power ratio (numerical, not dB)

Part 2 : Basic Modulation Schemes

Modulation Schemes

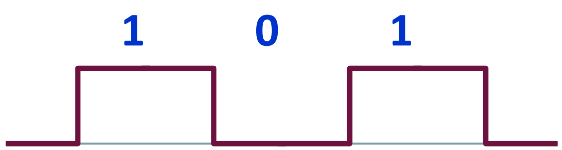
- **Analogue modulation schemes**
  - Amplitude Modulation (AM)
  - Frequency Modulation (FM)
  - Phase Modulation (PM)
- **Digital modulation schemes**
  - Amplitude Shift Keying (ASK)
  - Phase Shift Keying (PSK)
  - Frequency Shift Keying (FSK)
  - Gaussian Minimum Shift Keying (GMSK)
  - Quadrature Phase Shift Keying (QPSK)
  - Quadrature Amplitude Modulation (QAM)
  - Orthogonal Frequency Division Modulation (OFDM)
  - Spread spectrum schemes

Digital Modulation Scheme Hierarchy



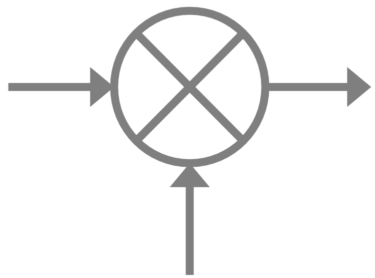
Amplitude Shift Keying (ASK)

$m(t)$

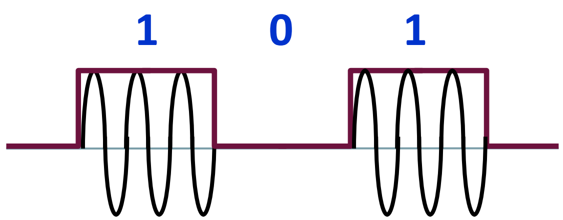


Unipolar binary data

Mixer



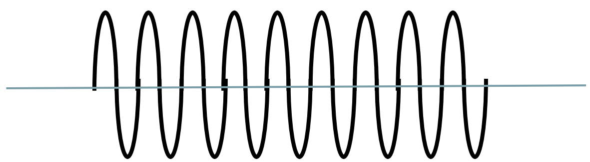
$v(t) = m(t)\, A_c \cos(2\pi f_c t)$



Binary ASK signal

Better BER compared to PSK

$A_c \cos(2\pi f_c t)$

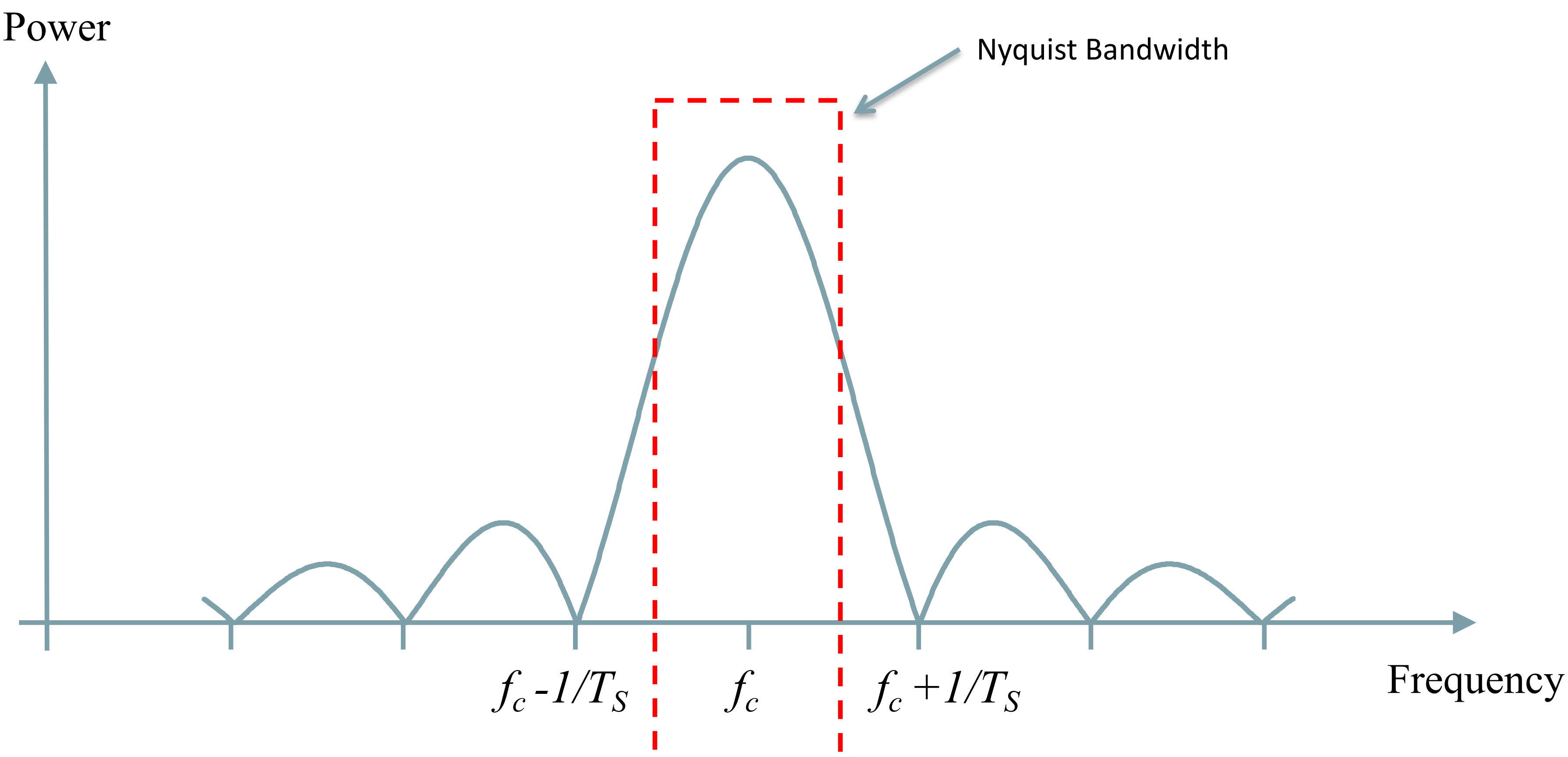


Carrier

Constellation: ASK

$$v(t) = \begin{cases} A_c \cos(2\pi f_c t) & symbol\ "1" \\ 0 & symbol\ "0" \end{cases}$$

Bandwidth of ASK



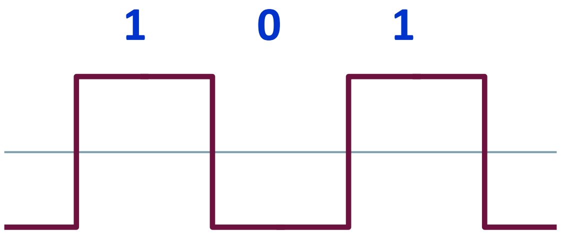
$$Nyquist\ Bandwidth\ (ASK) = \frac{1}{T_S}$$

minimum bandwidth to transfer the signal is Nyquist bandwidth

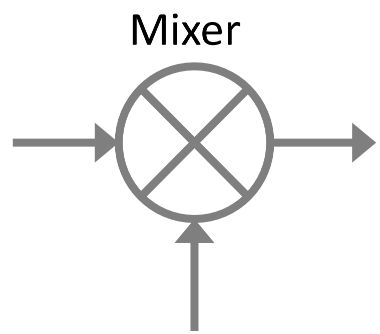
mainloe is from fc-1/Ts to fc+1/Ts

Binary Phase Shift Keying (BPSK)

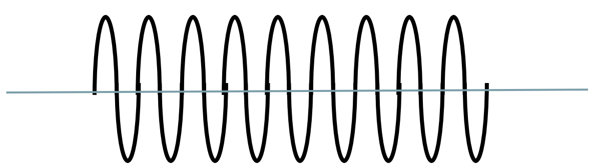
$m(t)$



Bipolar binary data

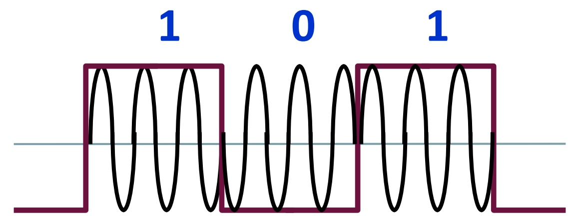


$$A_c \cos(2\pi f_c t)$$



Carrier

$$v(t) = m(t)\, A_c \cos(2\pi f_c t)$$



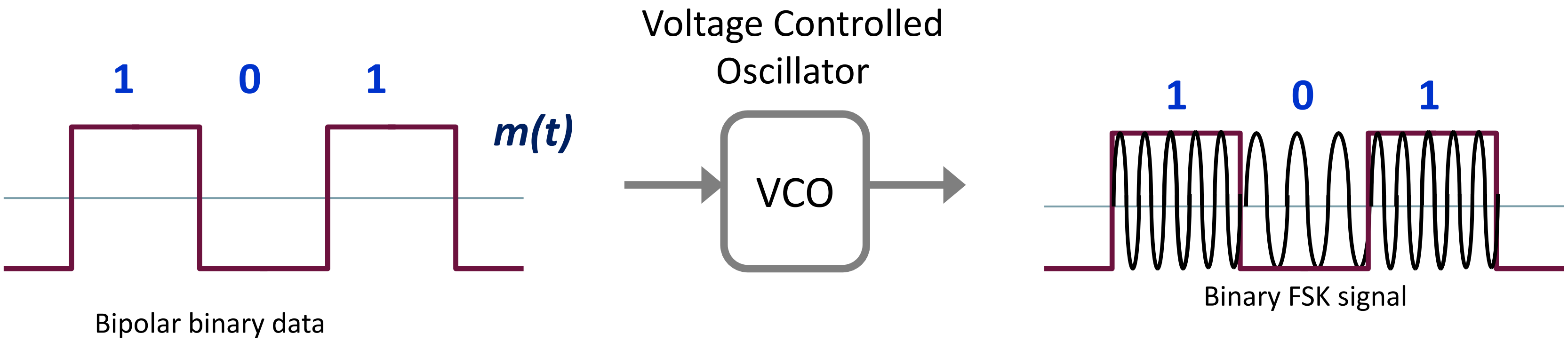
Binary PSK signal

same amplitude but different in phase

Constellation: BPSK

$$v(t) = \begin{cases} A_c \cos(2\pi f_c t) & symbol\ "1" \\ A_c \cos(2\pi f_c t + \pi) & symbol\ "0" \end{cases}$$

Frequency Shift Keying (FSK)

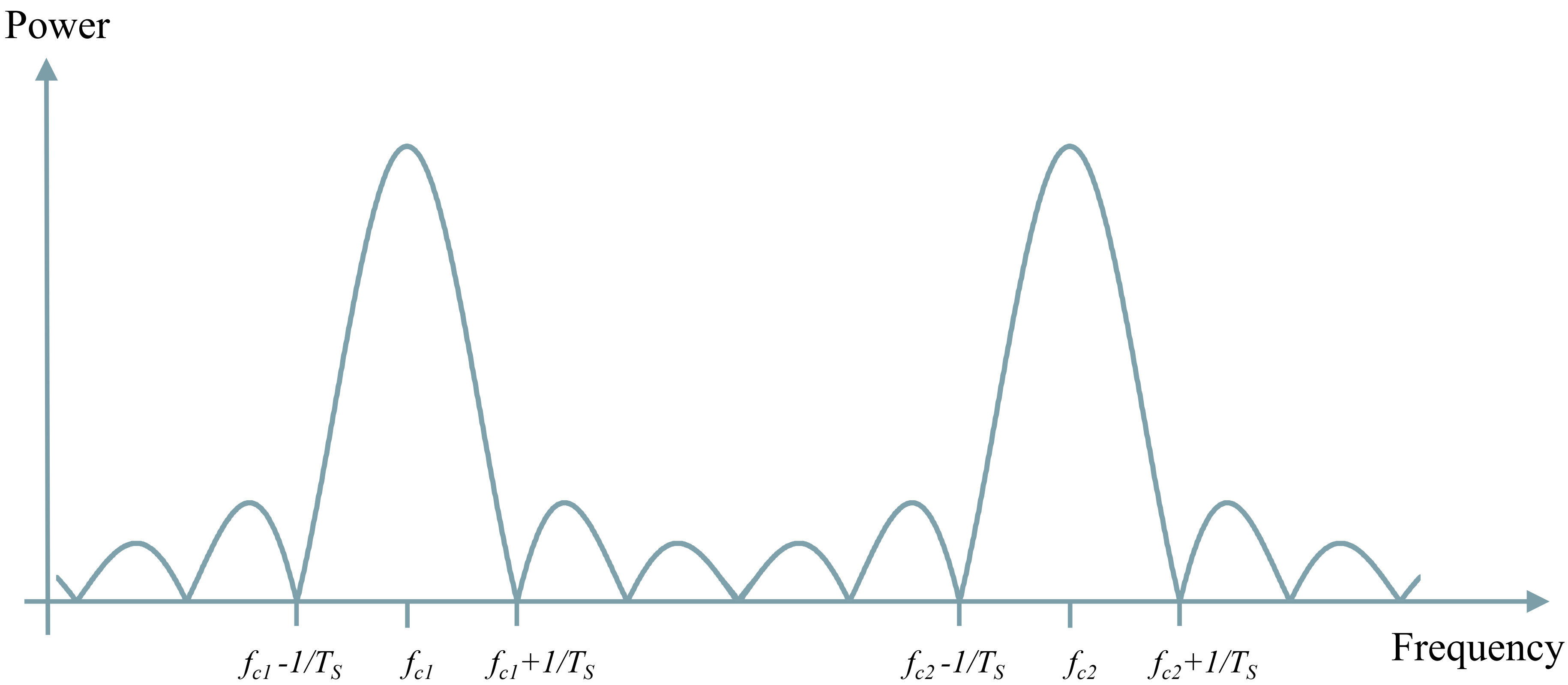


use different frequency

$$v(t) = \begin{cases} A_c \cos(2\pi f_{c1} t) & symbol\ "1" \\ A_c \cos(2\pi f_{c2} t) & symbol\ "0" \end{cases}$$

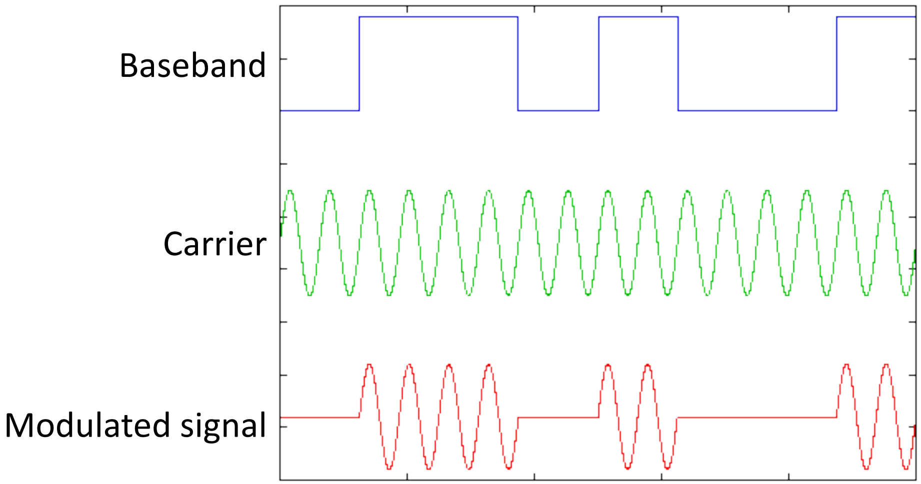
constellation can only be used when frequency is the same

Binary FSK Spectrum

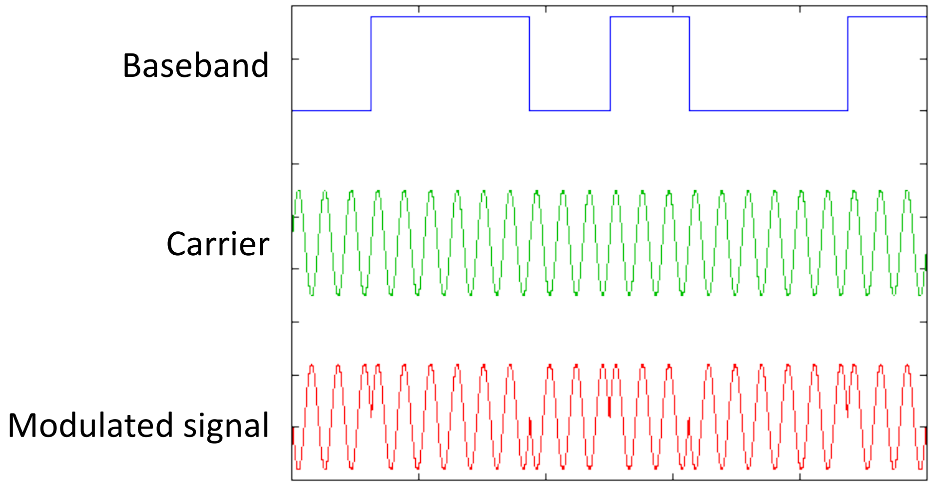


Summary of Basic Modulation Schemes

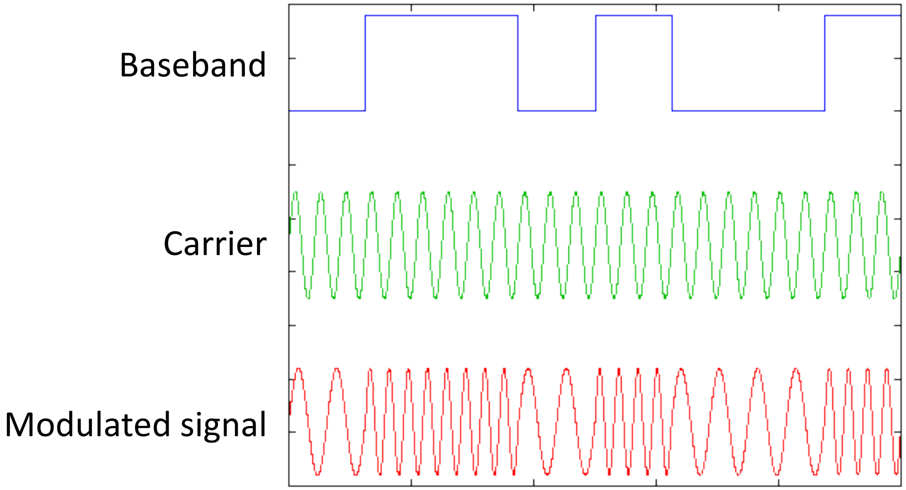
Amplitude Shift Keying (ASK)



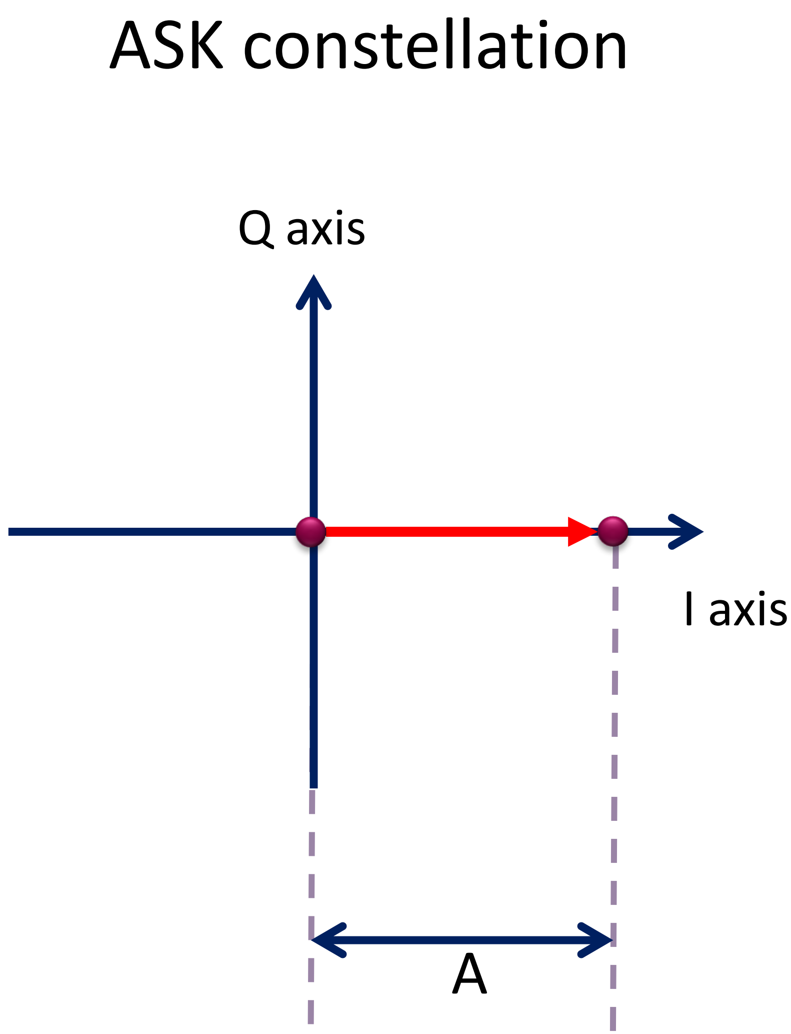
Phase Shift Keying (PSK)

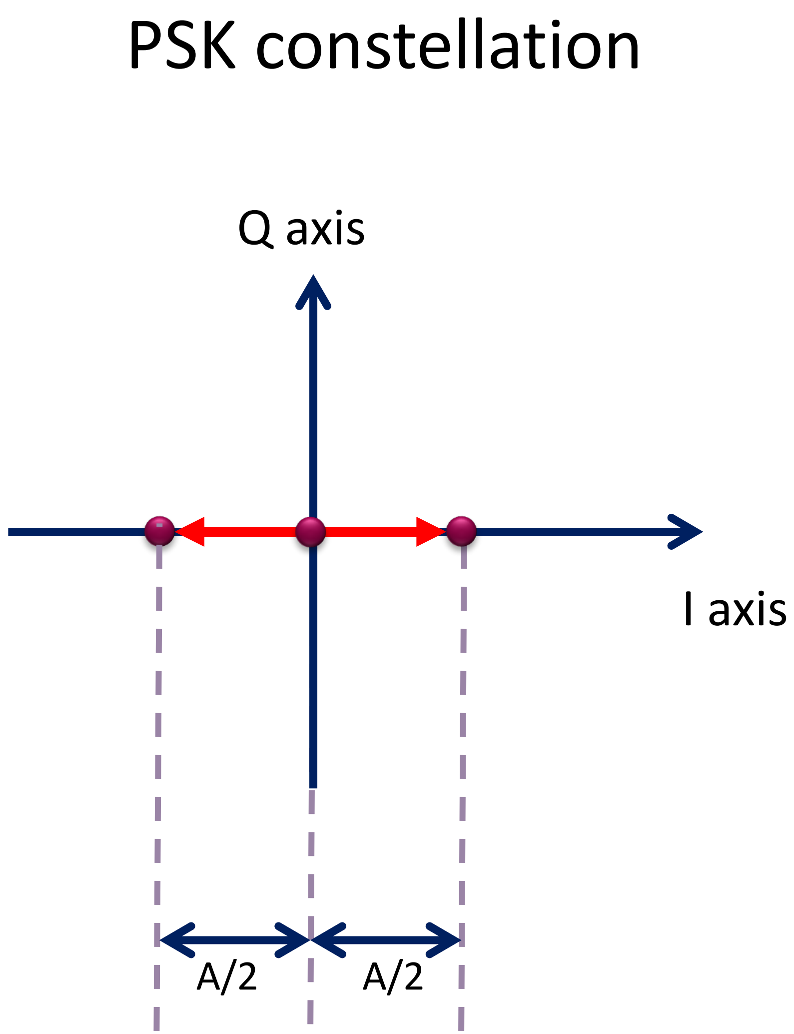


Frequency Shift Keying (FSK)



Constellation Diagrams



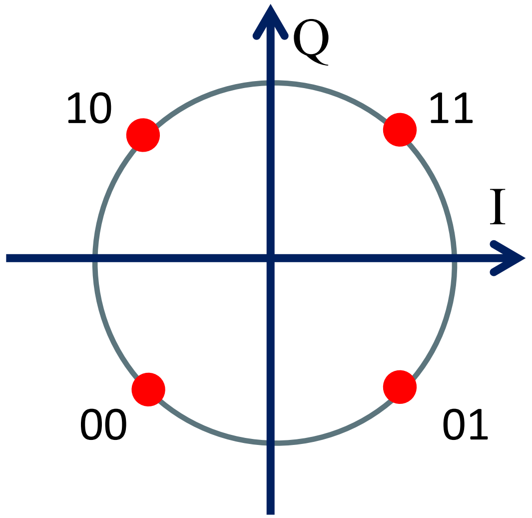


Note: These two constellation are for signals with the same average power

When M=4 : QPSK

- Consider the case when we have four symbols, represented by four different carrier phases all having the same amplitude. This is referred to as **Quadrature Phase-Shift Keying**, or **QPSK**.
- We can now encode two bits per **symbol**, which gives us twice the bit rate as BPSK for the same channel symbol rate. This means that QPSK is twice the **bandwidth efficiency** of BPSK. Each two-bit combination is referred to as a **dibit,** and the carrier phase used to represent each dibit is shown in the table here.

DiBit	Phase offset
00	-135^o
01	-45^o
11	45^o
10	135^o



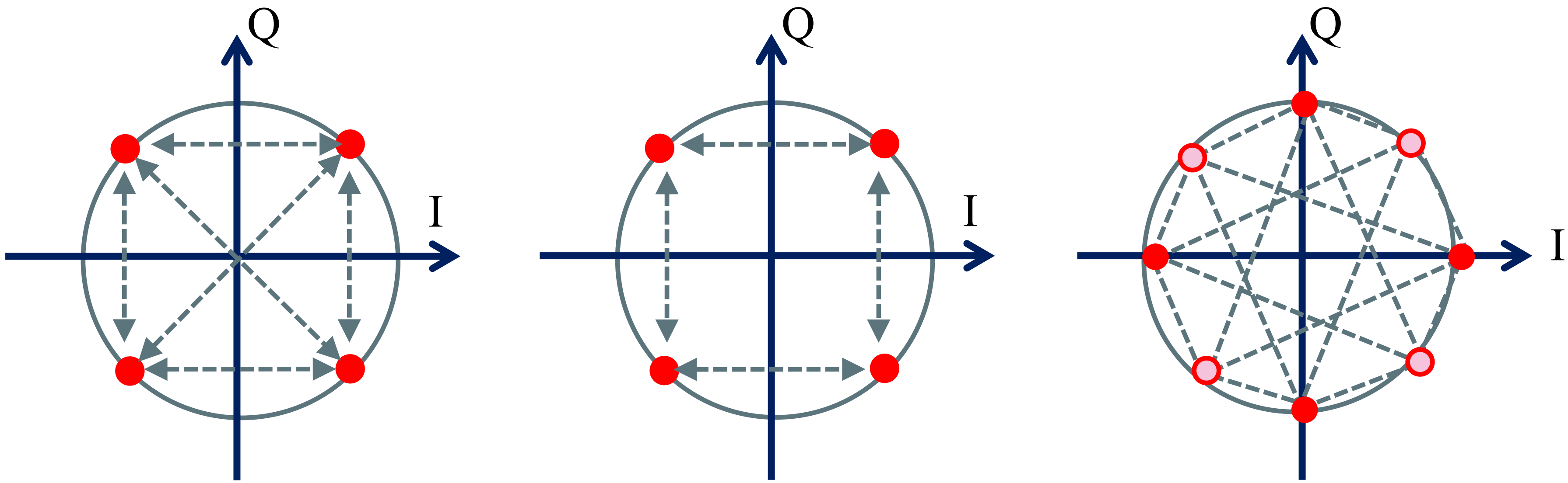
QPSK constellation

Varieties of QPSK

- In conventional QPSK the transition from one constellation point to the other passes through the origin of the constellation diagram. This means that there will be a 180 degree phase transition in the time domain waveform: a problem in practical communication system design as it requires expensive linear amplification to prevent distortion of the signal.

- In **offset QPSK** the phase transitions are limited to 90 degrees by offsetting the timing of the odd and even bits by one bit-period, or half a symbol-period. This ensures that the in-phase and quadrature components will never change at the same time.

- In **π/4 QPSK** two identical constellations are rotated by 45 degrees, or π by 4 radians, with respect to one another. This has the effect of reducing the phase transitions between symbols to a maximum of 135 degrees. Note that the phase transitions never pass through the origin, meaning that there are never any 180 degree phase transitions in the π/4 QPSK waveform.



Conventional QPSK

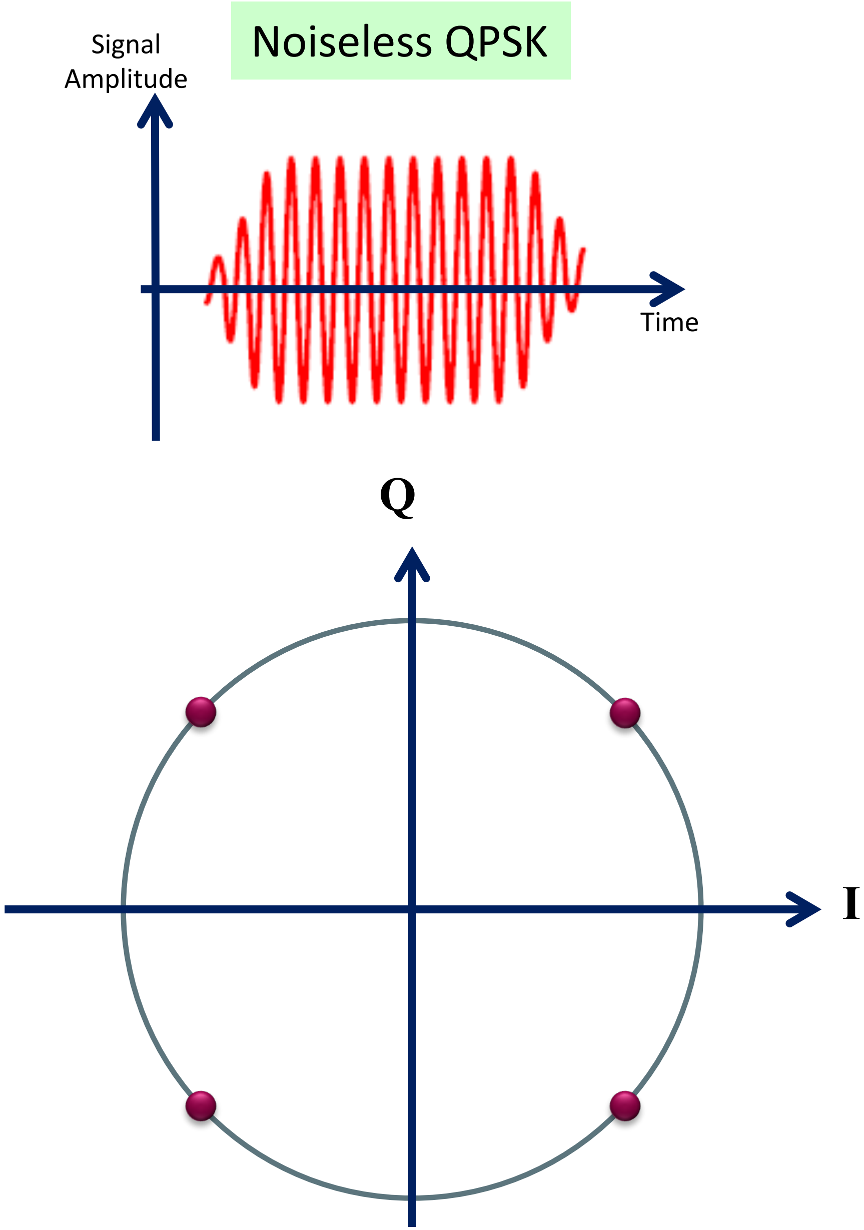
Offset QPSK

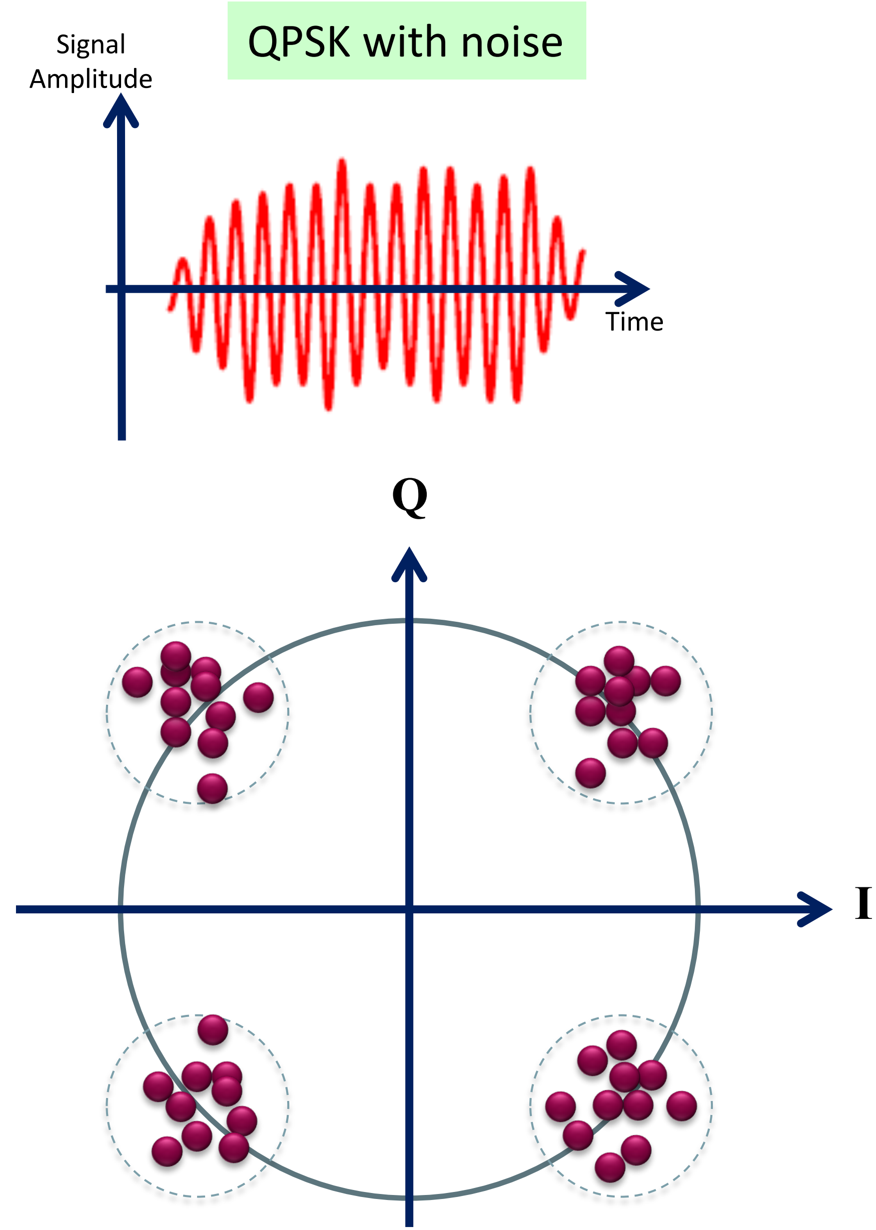
π/4 QPSK

PAM signals

Constellation: M-ary Signaling (M=4) PAM 4

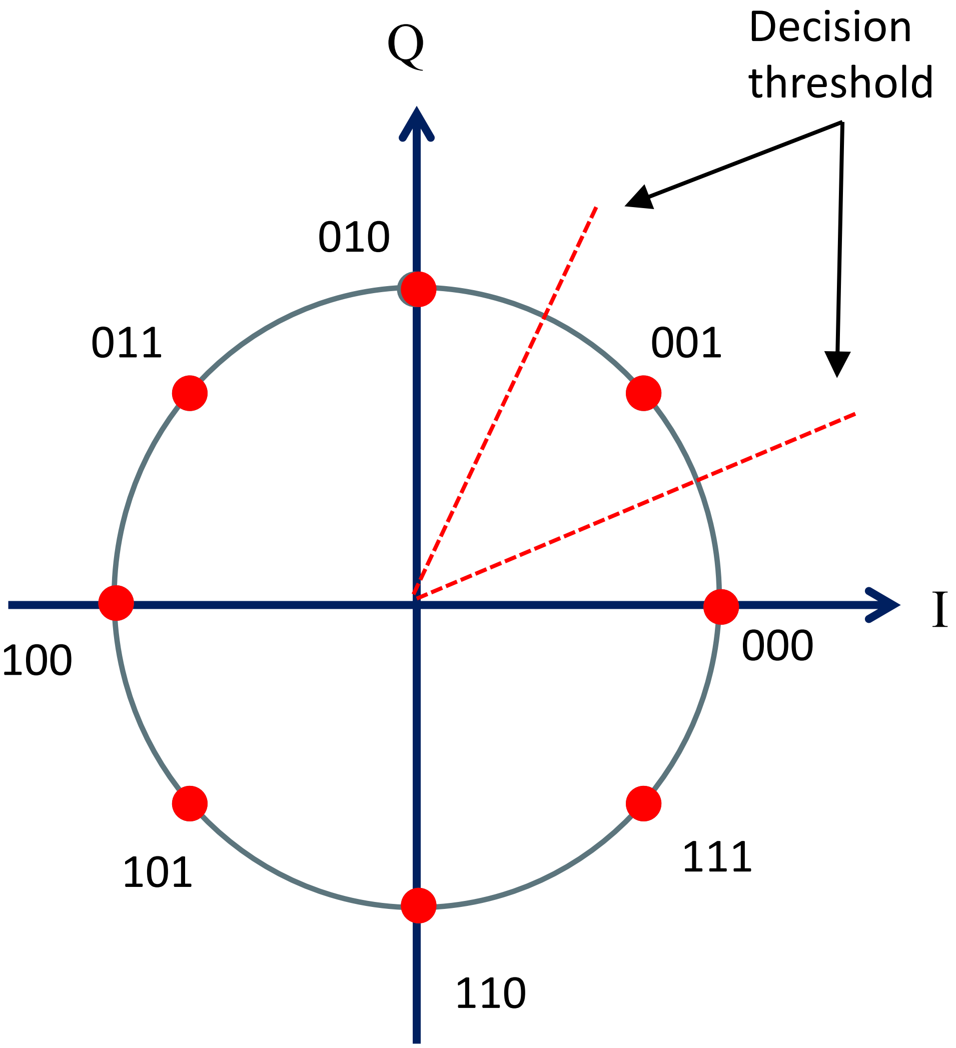
Effect of noise on QPSK





8-PSK Constellation

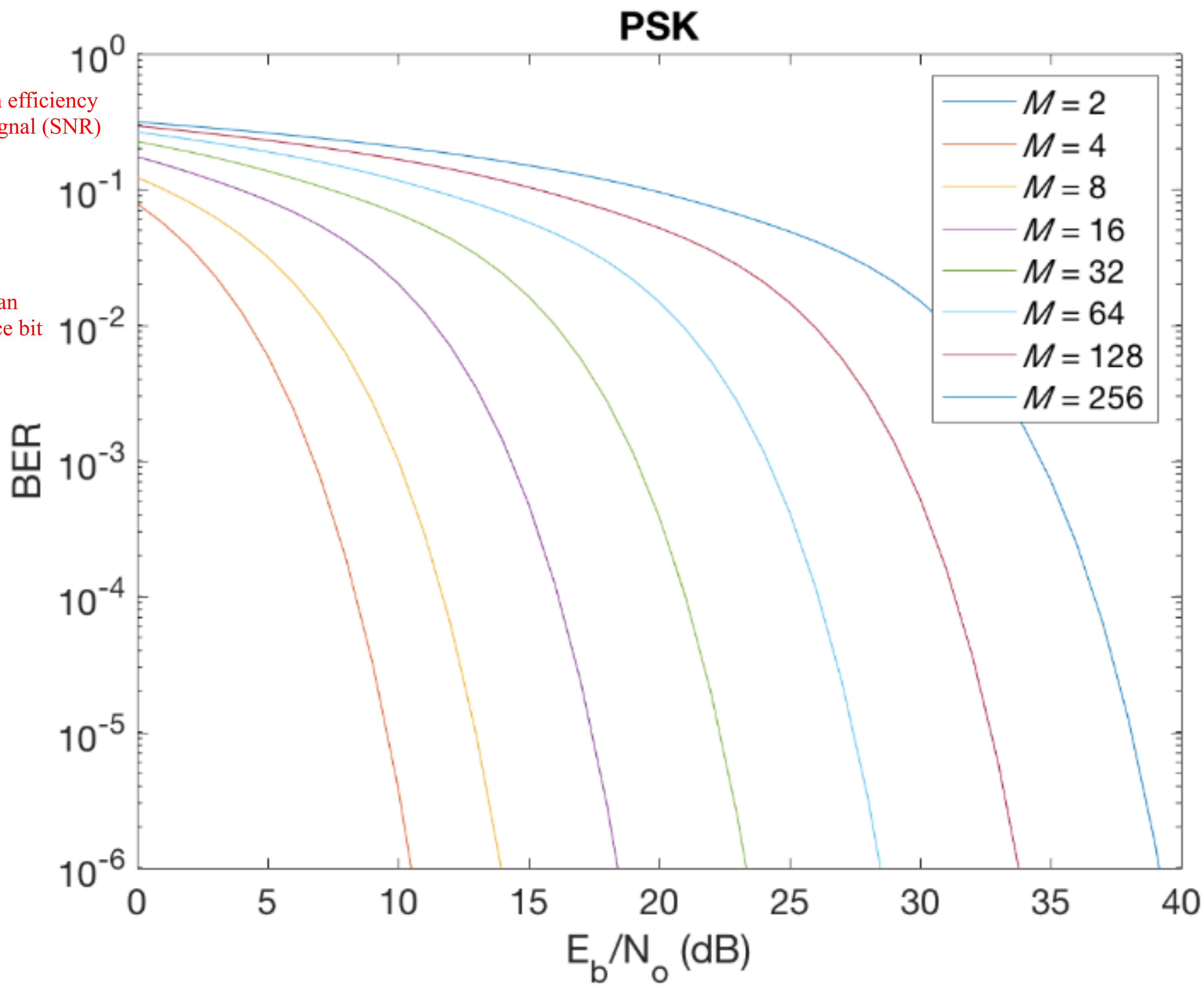
TriBit	Phase offset
000	0^o
001	45^o
010	90^o
011	135^o
100	135^o
101	-135^o
110	-45^o
111	45^o



BER performance of PSK

improve bandwidth efficiency
higher quality of signal (SNR)

send signal slowly can
send far away (reduce bit
rate)

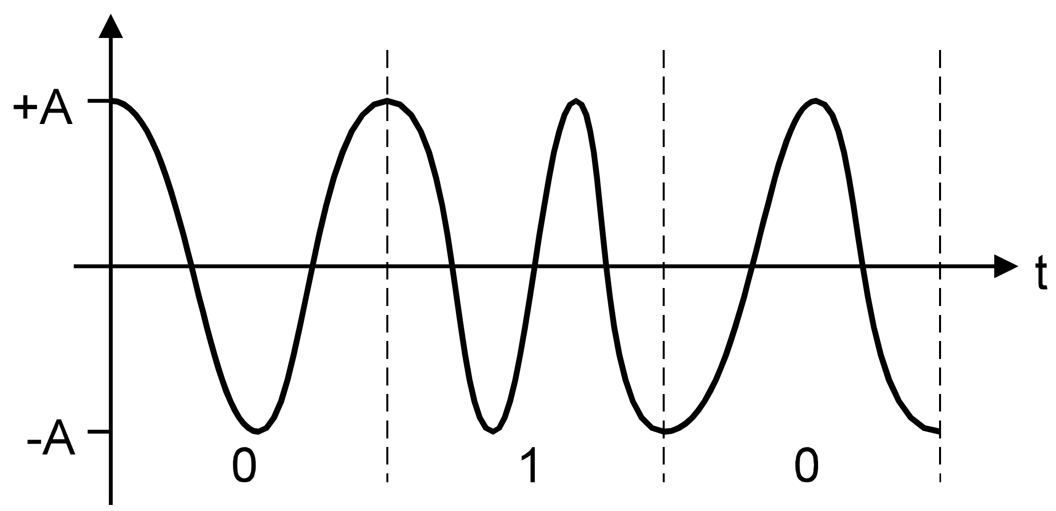


Exercise:

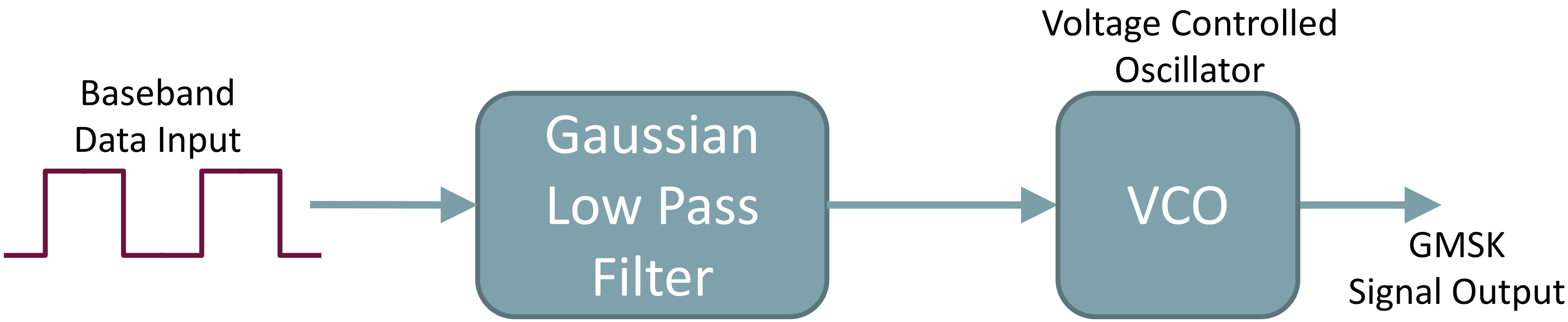
- If the required binary bit rate is 1,500 bits per second, what will be the **symbol rate** if we use **QPSK**?
- What would it be if we use **8-PSK**?

Minimum Shift Keying (MSK)

- MSK is a type of continuous-phase frequency-shift keying that was designed to limit the excessive bandwidth of conventional FSK.
- When looking at a plot of a signal using MSK modulation, it can be seen that the modulating data signal changes the frequency of the signal and there are no phase discontinuities.
- In MSK the frequency separation between the two tones is $\Delta f=1/(2T_b)$, which is the minimum frequency separation necessary to ensure orthogonality between the two tones over the signalling interval of length T_b.
- The power efficiency of MSK is similar to the power efficiency of BPSK, but the bandwidth efficiency of MSK is twice the bandwidth efficiency of BPSK.

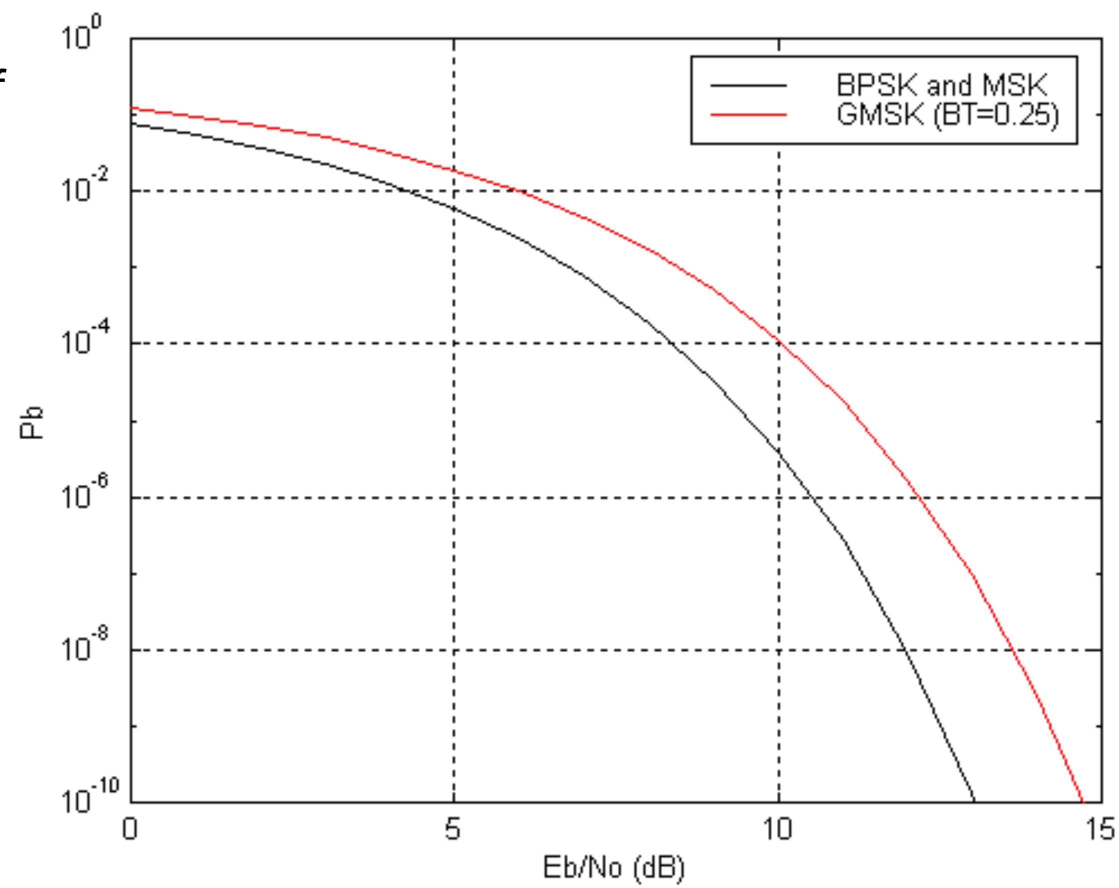


Gaussian Minimum Shift Keying (GMSK)



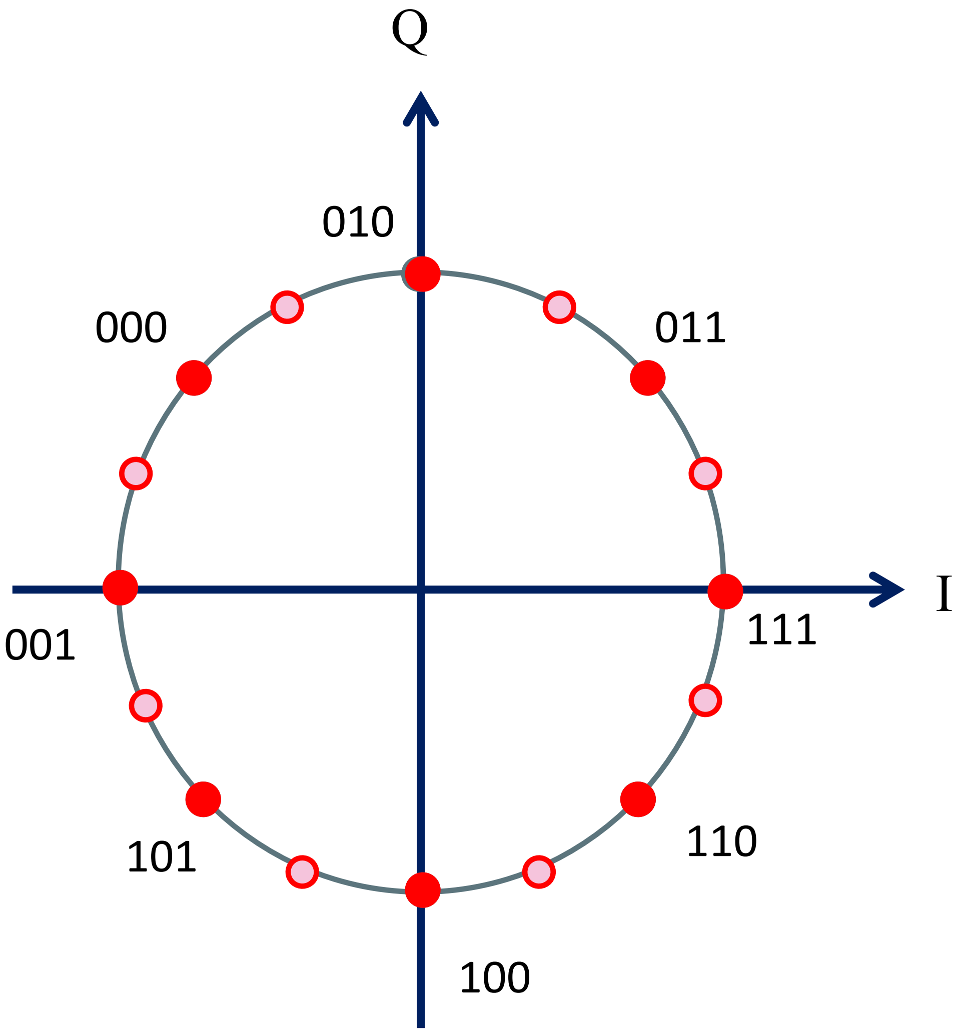
To further reduce the sidelobes of MSK is to constrain the rate of change of frequency by shaping the baseband data waveform. **Gaussian Minimum Shift Keying**, or **GMSK** uses a Gaussian filter to further increase spectral efficiency.

It is for this reason that GMSK was chosen as the modulation scheme for use in the **GSM** system.



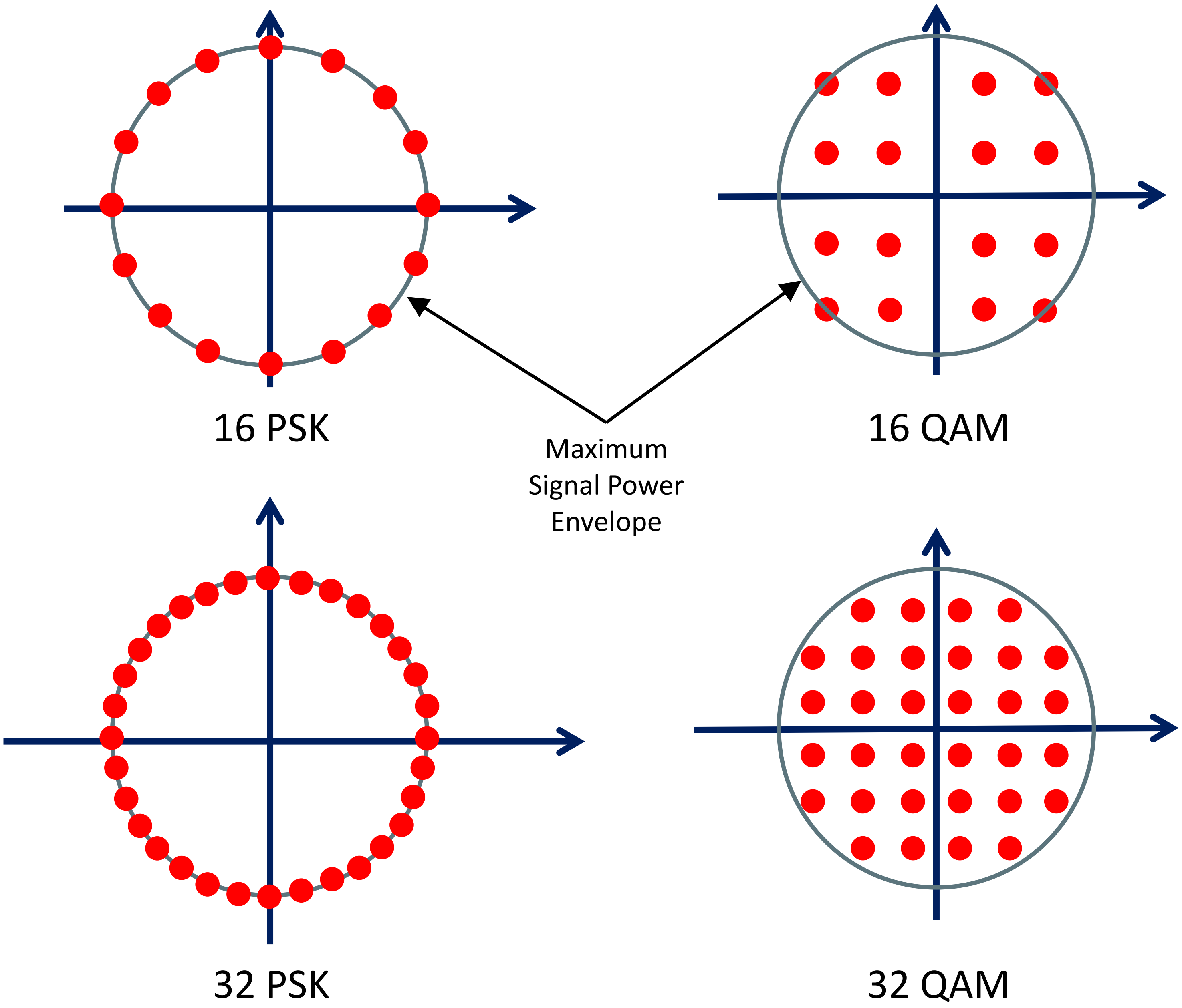
Practical application of GMSK and PSK : EDGE

- EDGE: “Enhanced Data rates for GSM Evolution”.
- EDGE employs both GMSK as well as 8-PSK modulation to achieve a theoretical maximum data rate of 59.2kbps per timeslot, which equates to **473.6 kbit/s** for eight timeslots.
- Although the 8 PSK has three times the spectral efficiency of GMSK, The frequency spectrum occupied by 8-PSK and GMSK in EDGE is the same, illustrating the advantage of multi-level modulation.
- EDGE 8-PSK uses a 3 pi/8 version of PSK, in other words two constellation sets offset by 3 pi/8 in order to limit phase transitions, following the same principle as pi/4 PSK.

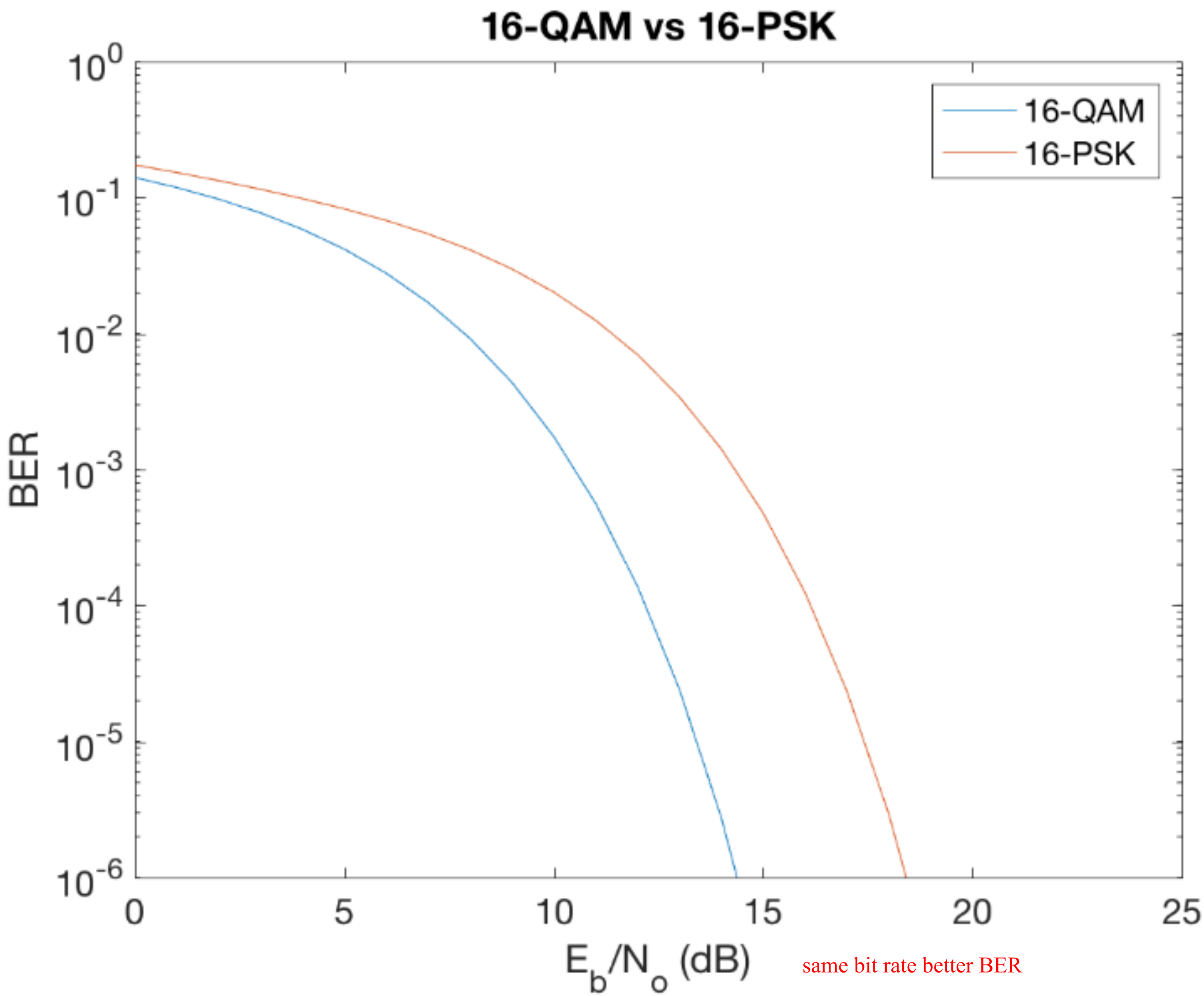


Part 3 : Higher Modulation Schemes

Phase plus Amplitude Modulation



BER for 16-QAM versus 16-PSK

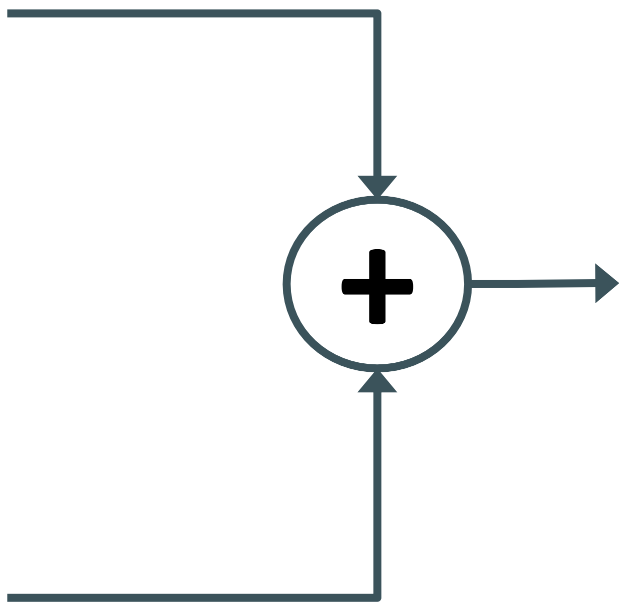


same bit rate better BER

Generation of M-ary Digital Signals

In-phase carrier :

$A_I \cos(2\pi f_c t)$



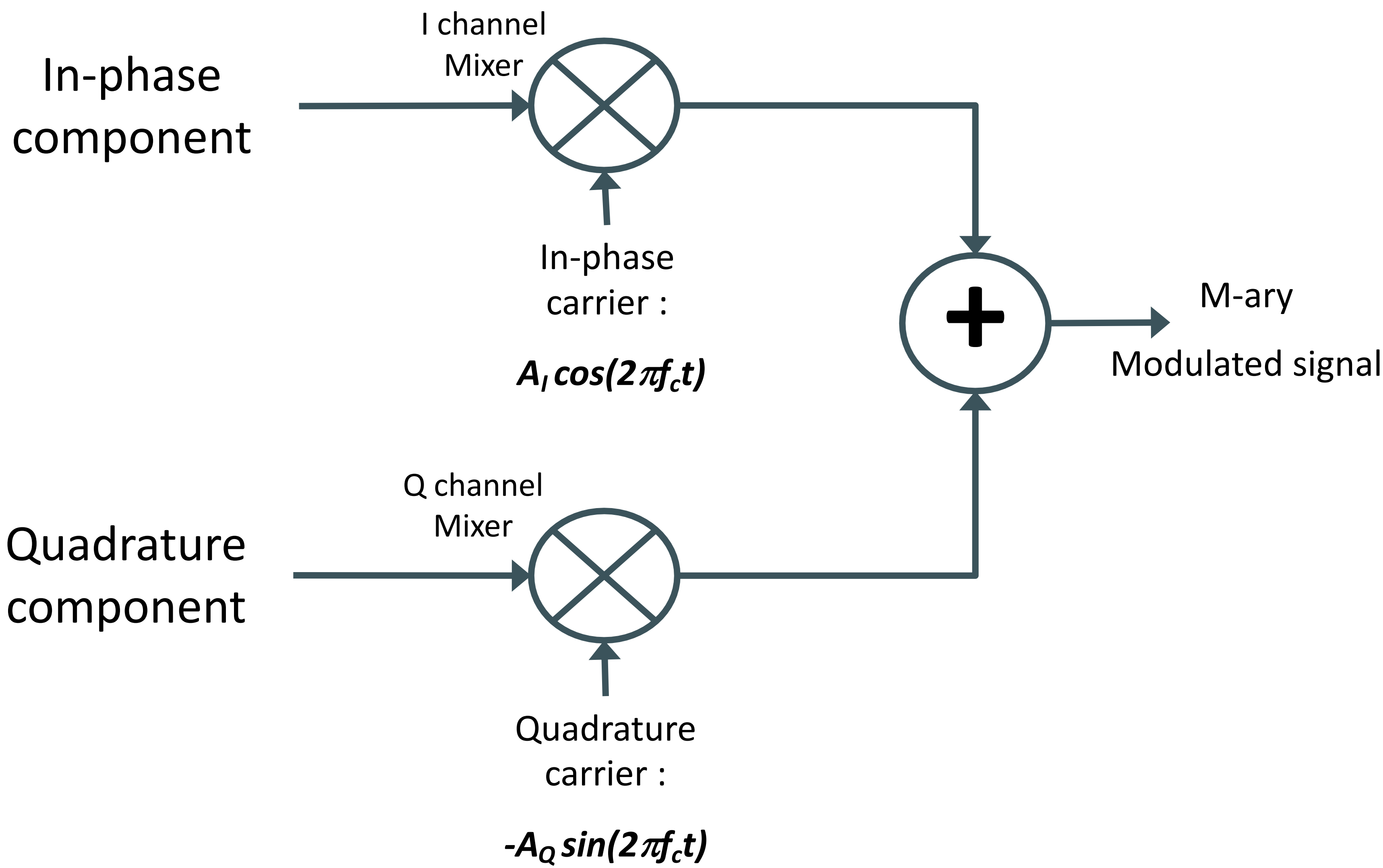
Quadrature carrier :

$-A_Q \sin(2\pi f_c t)$

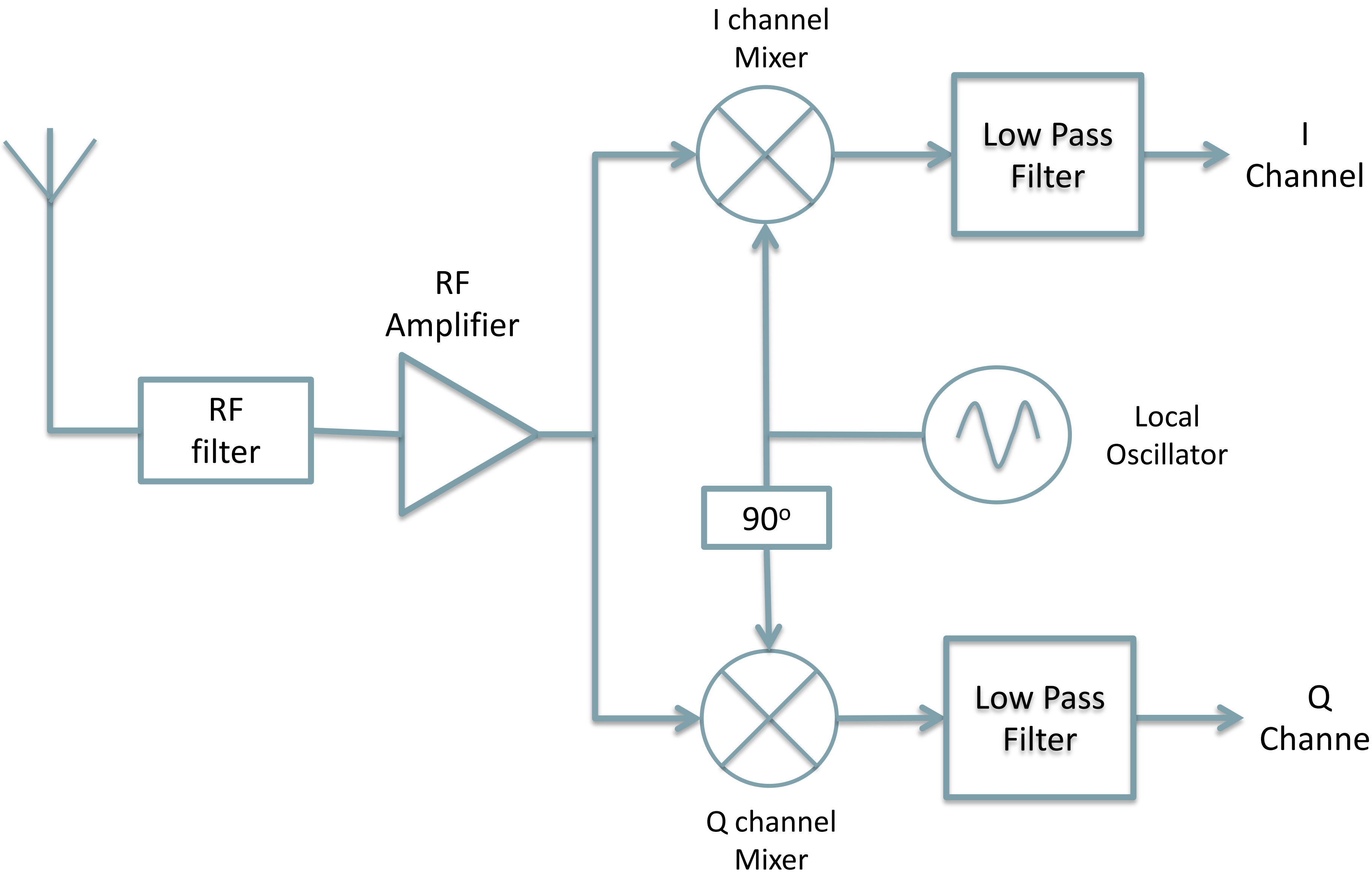
$$A_I \cos(2\pi f_c t) - A_Q \sin(2\pi f_c t)$$

$$= A_C \cos(2\pi f_c t + \phi)$$

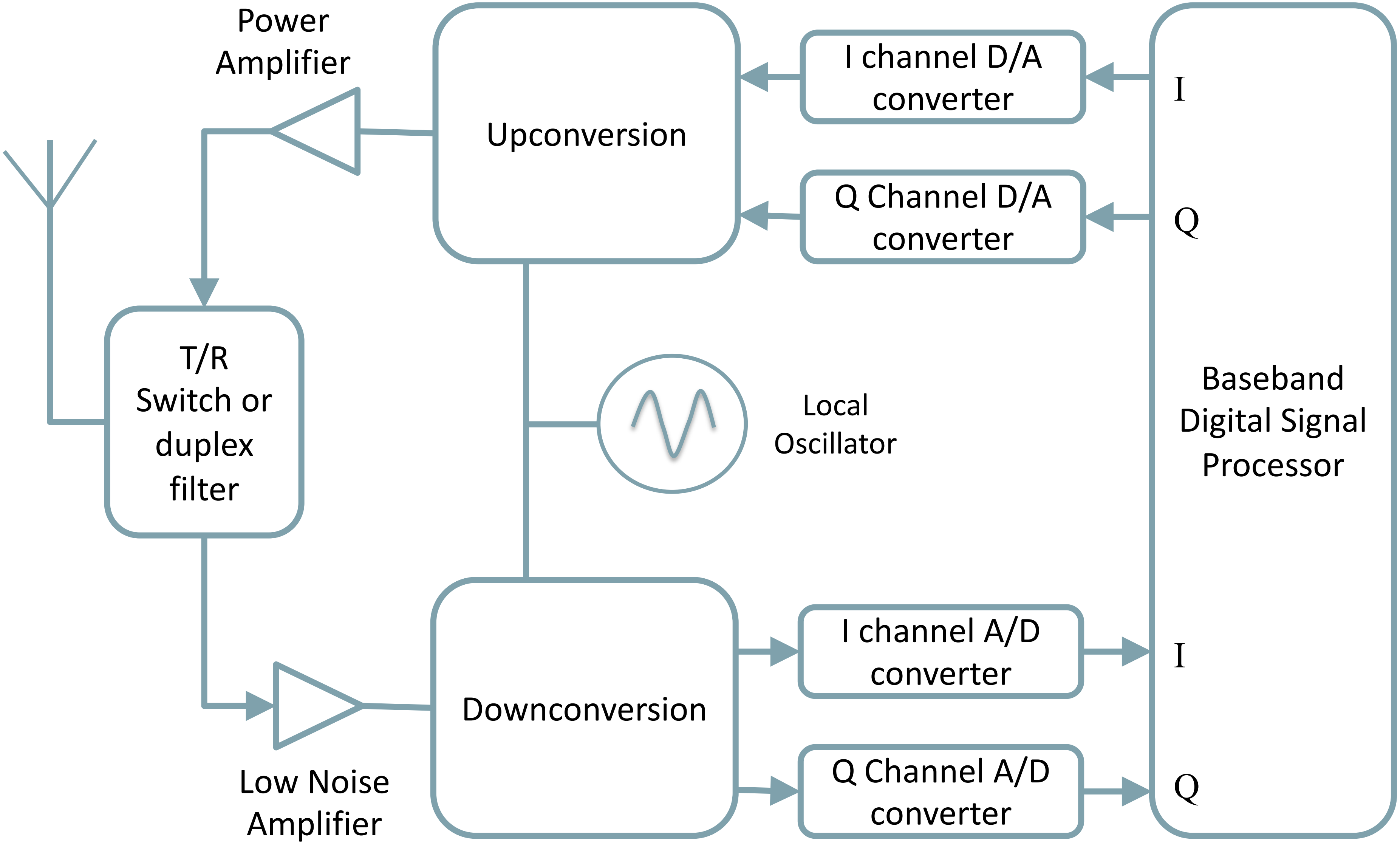
Generation of M-ary Digital Signals



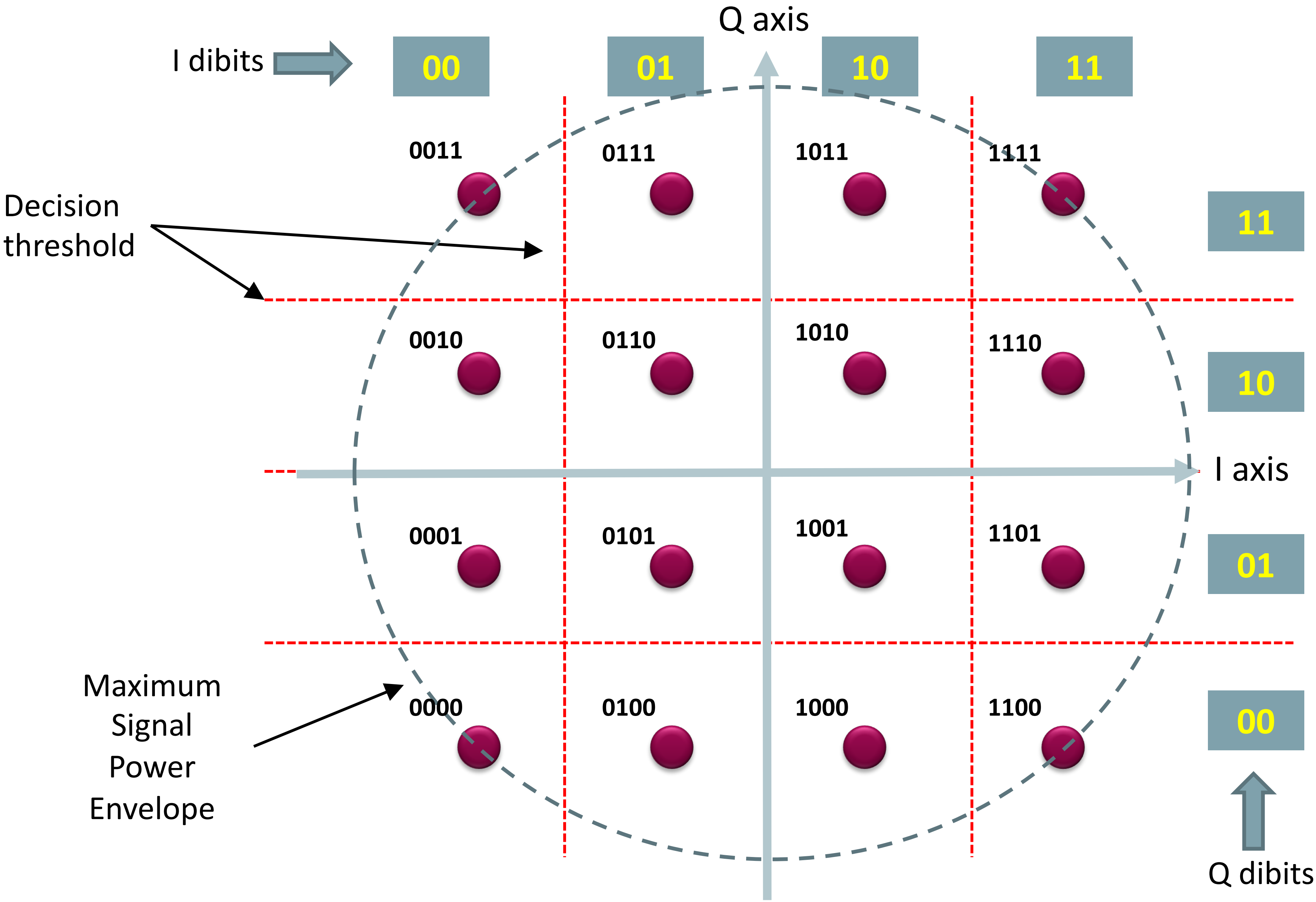
Direct Conversion (Zero IF) architecture



Generic mobile transceiver

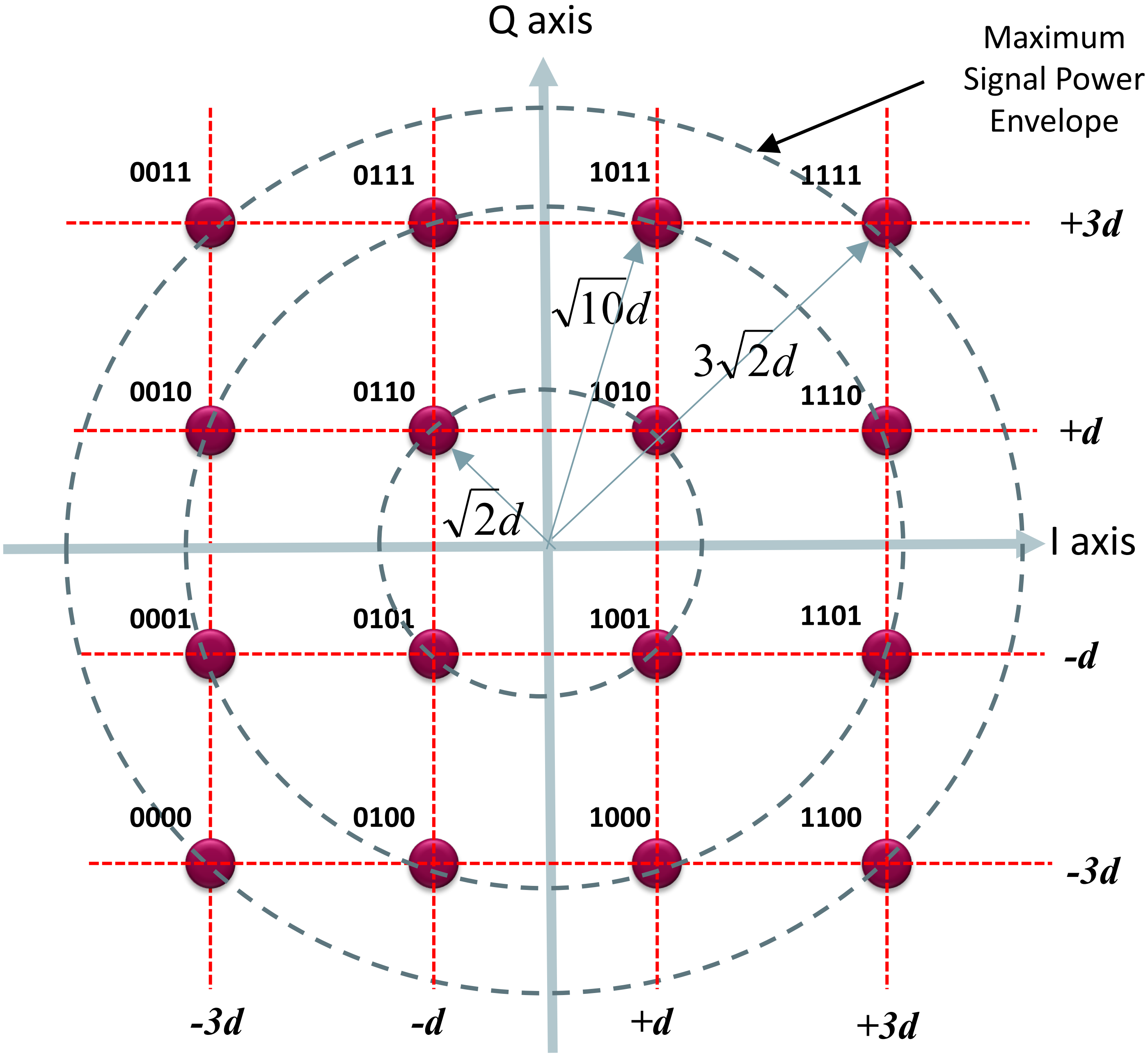


16-QAM constellation diagram



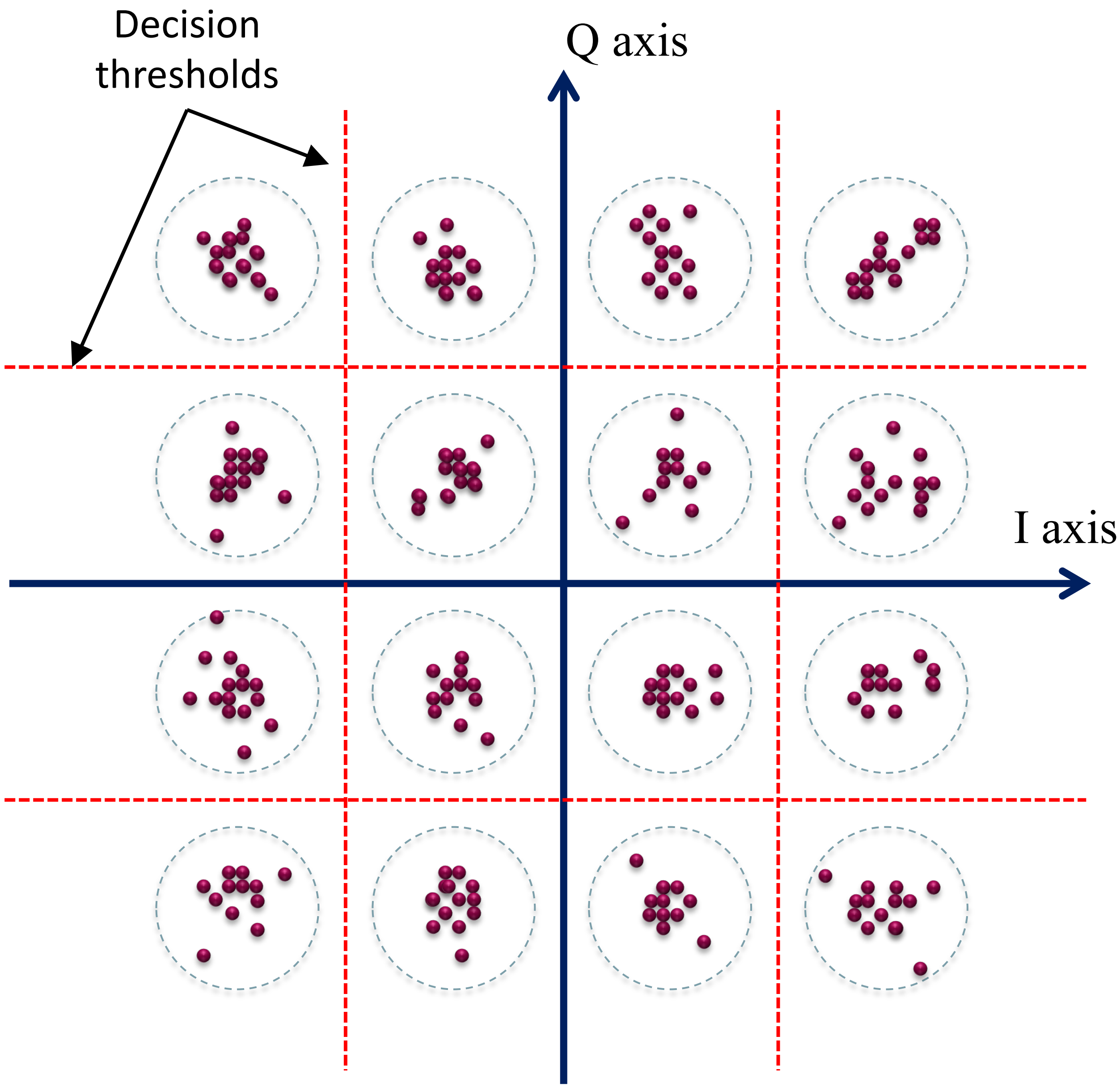
16-QAM average signal power

Symbol	r
0000	$3\sqrt{2}d$
0001	$\sqrt{10}d$
0010	$\sqrt{10}d$
0011	$3\sqrt{2}d$
0100	$\sqrt{10}d$
0101	$\sqrt{2}d$
0110	$\sqrt{2}d$
0111	$\sqrt{10}d$
1000	$\sqrt{10}d$
1001	$\sqrt{2}d$
1010	$\sqrt{2}d$
1011	$\sqrt{10}d$
1100	$3\sqrt{2}d$
1101	$\sqrt{10}d$
1110	$\sqrt{10}d$
1111	$3\sqrt{2}d$



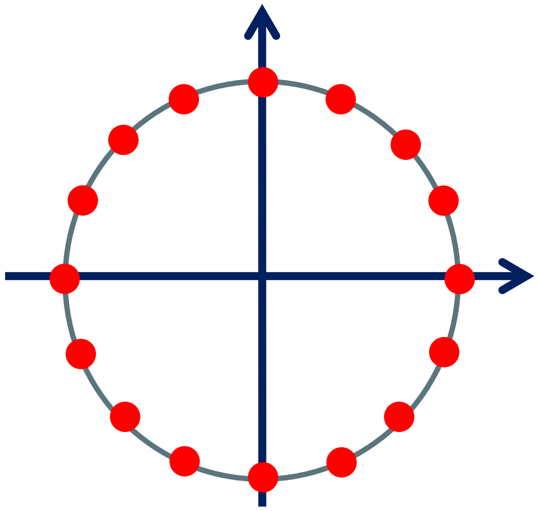
Effect of noise in QAM

- Noise will cause the constellation point for a given received symbol to be different from the one transmitted.
- Errors will occur when the received constellation point is closer to a different constellation point than to the point that was transmitted.
- The symbol error rate can be approximated by computing the probability that the noise voltage exceeds the distance to the decision threshold.

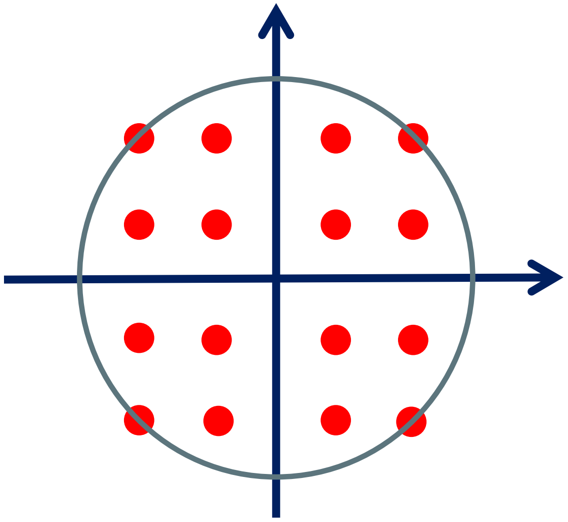


Application of QAM : Evolved EDGE

Modulation and Coding Scheme (MCS)	EDGE / Evolved EDGE	
	Type of modulation	Throughput (4 timeslots)
MCS-1	GMSK	35.2 kbps
MCS-2	GMSK	44.8 kbps
MCS-3	GMSK	59.2 kbps
MCS-4	GMSK	70.4 kbps
MCS-5	8-PSK	89.6 kbps
MCS-6	8-PSK	118.4 kbps
MCS-7	8-PSK	179.2 kbps
MCS-8	8-PSK	217.6 kbps
MCS-9	8-PSK	236.8 kbps
MCS-10	16-QAM with turbo codes	268.8 kbps
MCS-11	16-QAM uncoded	326.4 kbps

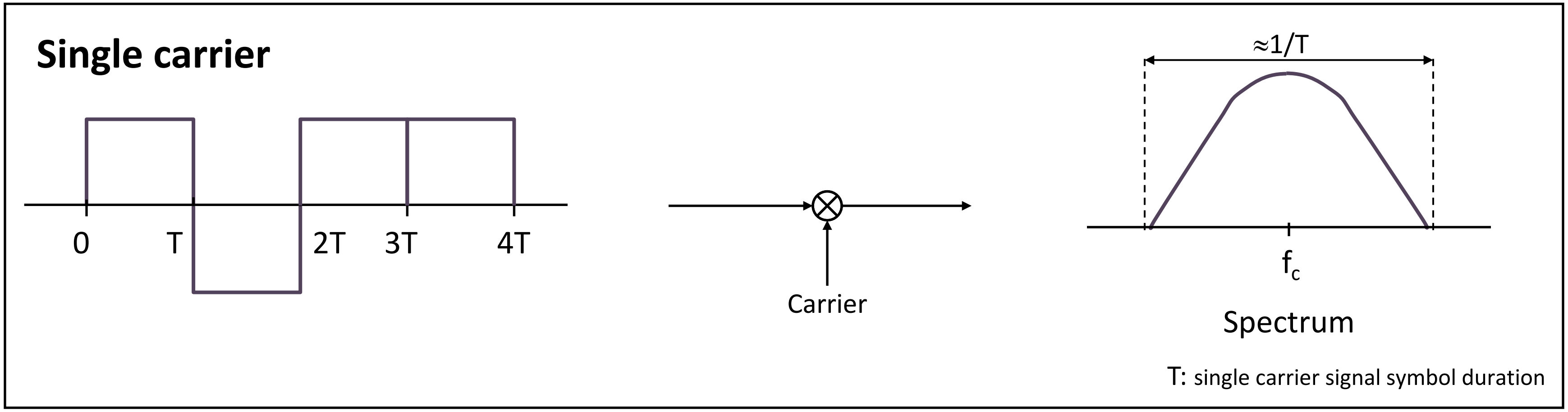


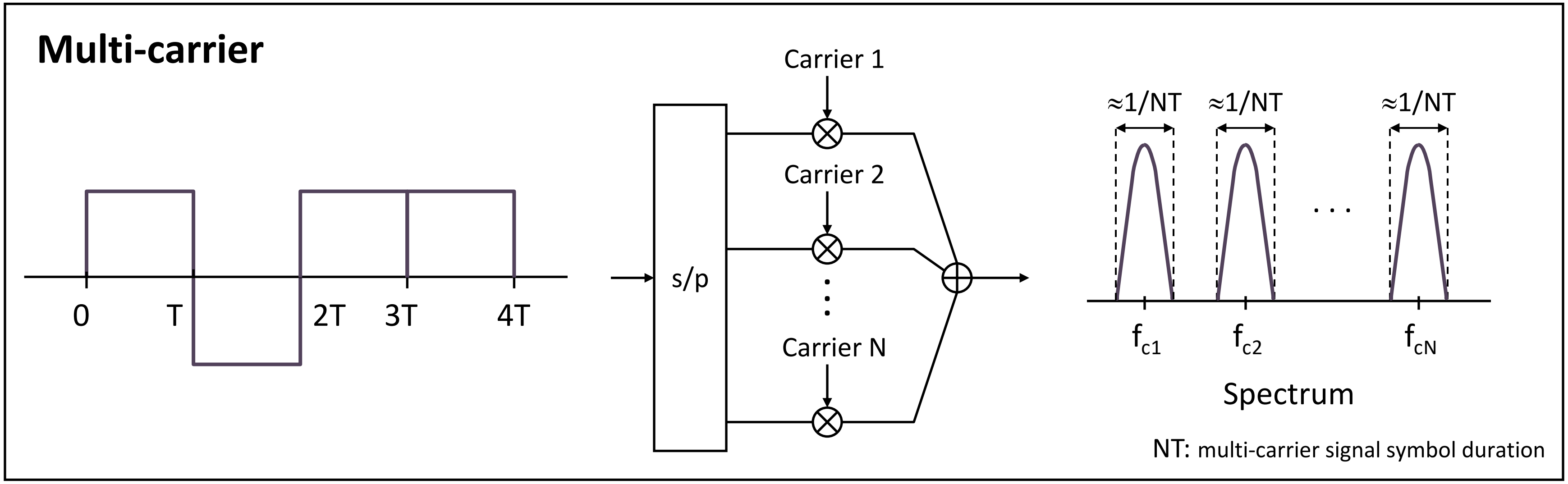
16 PSK



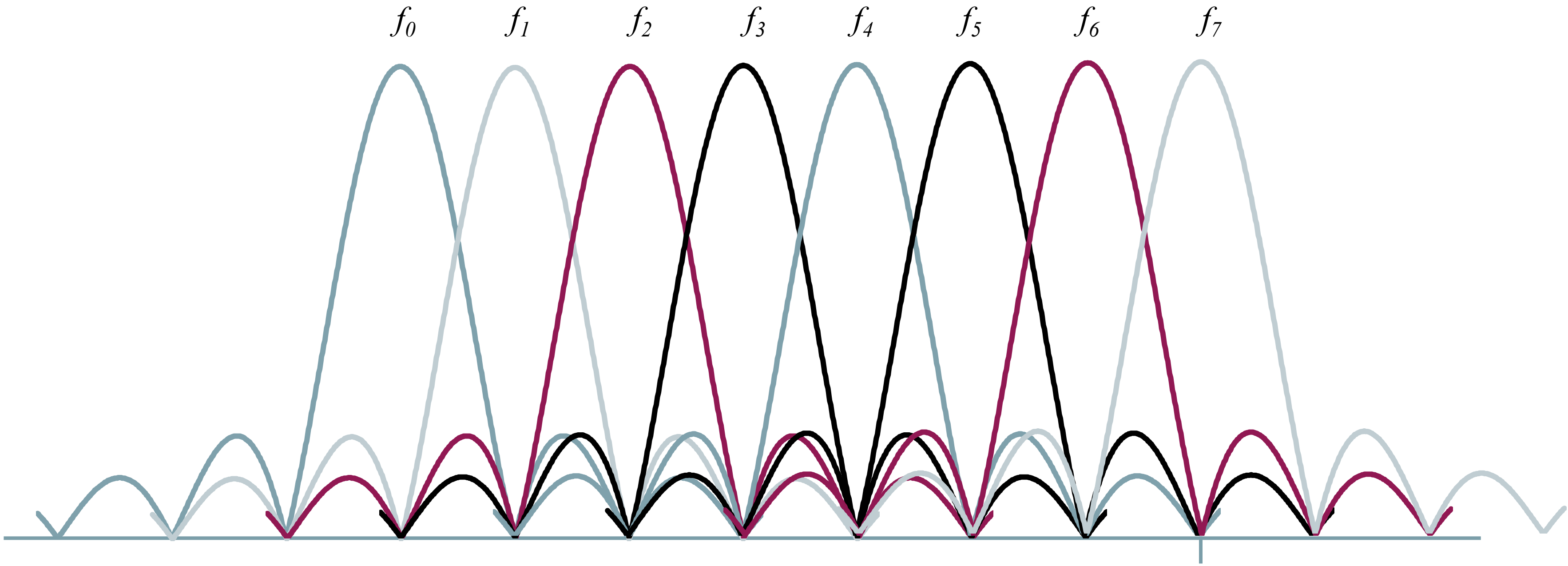
16 QAM

Multi-carrier Modulation



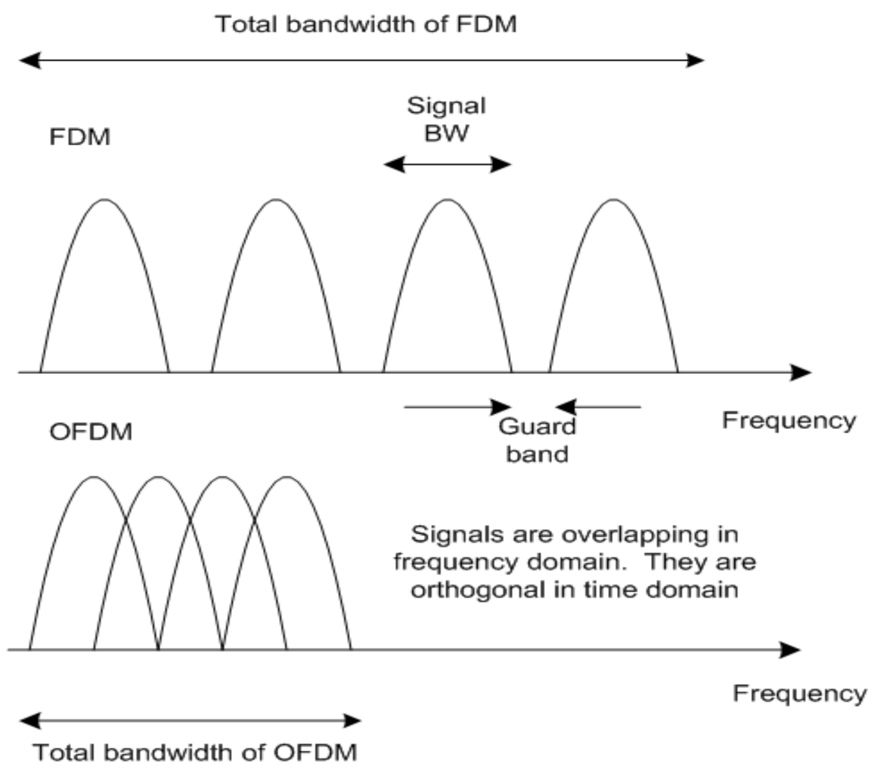


Orthogonal Frequency Division Multiplexing (OFDM)

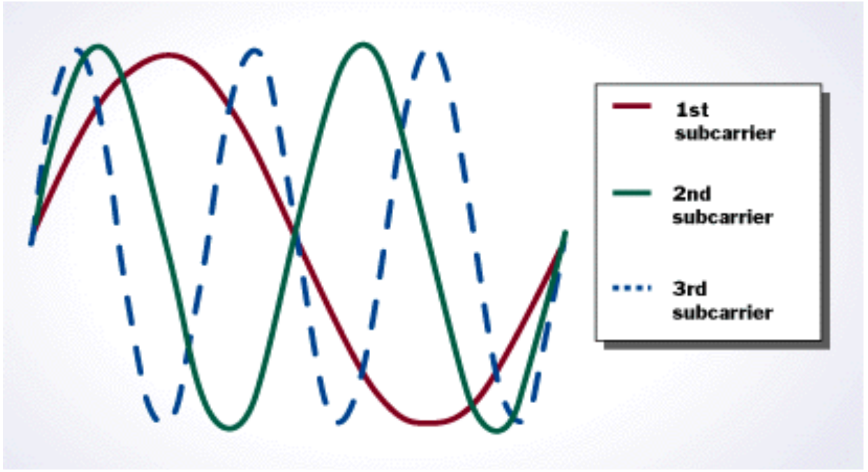


FDM versus OFDM

Frequency Domain



Time Domain



OFDM Example - 802.11 a/g

- Uses OFDM with 52 sub-carrier (4 of which are pilot tones) in a 20MHz bandwidth to provide a maximum of 54MBit/s
- Each tone is modulated with up to 64-QAM
- Forward error correction is also used which reduces the total bandwidth available (this will be covered later)

Data Rate	Modulation	Coding
6	BPSK	½
9	BPSK	¾
12	QPSK	½
18	QPSK	¾
24	16-QAM	½
36	16QAM	¾
48	64-QAM	2/3
54	64-QAM	½

Application of OFDM : LTE and LTE-Advanced

Channel Bandwidth	Subcarriers (downlink)	Subcarriers (uplink)
1.4 MHz	73	72
3 MHz	181	180
5 MHz	301	300
10 MHz	601	600
15 MHz	901	900
20 MHz	1201	1200