

INTRODUCTION

The root mean square value of a quantity is the square root of the mean value of the squared values of the quantity taken over an interval. The RMS value of any function $y = f(t)$ over the range $t = a$ to $t = b$ can be defined as:

$$\text{RMS value} = \sqrt{\frac{1}{b-a} \int_a^b y^2 dt}$$

One of the principal applications of RMS values is with alternating currents and voltages.

ROOT MEAN SQUARE (RMS) VALUE

The value of an AC voltage is continually changing from zero up to the positive peak, through zero to the negative peak and back to zero again.

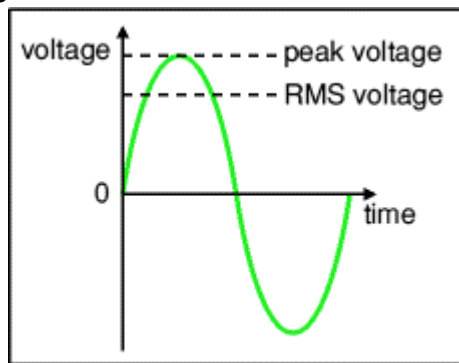


Figure-1: Difference between peak and RMS voltage

Clearly, for most of the time it is less than the peak voltage, so this is not a good measure of its real effect. Instead we use the *root mean square voltage* (V_{RMS}) which is $1/\sqrt{2} \approx 0.7$ of the *peak voltage* (V_{peak}):

$$V_{RMS} = 0.7 \times V_{peak} \text{ or } V_{peak} = 1.4 \times V_{RMS}$$

Similar equations also apply to the current. They are only true for sine waves (the most common type of AC) because the factors (here 0.7 and 1.4) take different values for other shapes. We can calculate these values as follows:

Take a sine wave representing either current or voltage with peak value A :

$$y(t) = A \sin \omega t \dots (1)$$

where $\omega = 2\pi f$ (rads⁻¹) and f = frequency (Hz). The time average $\bar{y}(T)$ over period T (seconds) of the signal $y(t)$ is given by:

$$\bar{y}(T) = \frac{1}{T} \int_0^T y(t) dt \dots (2)$$

So the RMS value y_{RMS} is given by:

$$y_{RMS} = \sqrt{\frac{1}{T} \int_0^T y^2(t) dt} = \sqrt{\frac{1}{T} \int_0^T A^2 \sin^2 \omega t dt} \dots (3)$$

We know that $\cos 2\theta = 1 - 2\sin^2 \theta$, hence:

$$\sin^2 \omega t = \frac{1}{2} (1 - \cos 2\omega t) \dots (4)$$

Substituting (4) into (3), we get, in sequence

$$y_{RMS} = \sqrt{\frac{1}{T} \int_0^T \frac{A^2}{2} (1 - \cos 2\omega t) dt}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T} \int_0^T (1 - \cos 2\omega t) dt}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T} \left[T - \frac{\sin 2\omega T}{2\omega} \right]}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2} - \frac{A^2 \sin(2\omega T)}{4\omega T}} \dots (5)$$

As $T \rightarrow \infty$, the second (oscillatory) term in equation (5) tends to zero. Hence, we obtain:

$$y_{RMS} = \frac{A}{\sqrt{2}}$$

$$\Rightarrow \text{RMS Value of } y = \frac{1}{\sqrt{2}} \times \text{Peak Value of } y$$

$$\therefore y_{RMS} = 0.7 \times y_{peak}$$

The *RMS value* is the effective value of a varying voltage or current. It is the equivalent steady DC (constant) value which gives the same effect. For example, a lamp connected to a 6V RMS AC supply will shine with the same brightness when connected to a steady 6V DC supply. However, the lamp will be dimmer if connected to a 6V *peak* AC supply because the RMS value of this is only 4.2V (it is equivalent to a steady 4.2V DC).

What do AC meters show? Is it the RMS or peak voltage?

AC voltmeters and ammeters show the *RMS value* of the voltage or current. DC meters also show the RMS value when connected to varying DC provided that the DC is varying quickly; if the frequency is less than about 10Hz you will see the meter reading fluctuating.

What does '6V AC' really mean? Is it the RMS or peak voltage?

If the peak value is meant it should be clearly stated, otherwise assume it is the *RMS value*. In everyday use, AC voltages (and currents) are always given as *RMS values* because this allows a sensible comparison to be made with steady DC voltages (and currents), such as from a battery. For example, a 6V AC *supply* means 6V RMS with the peak voltage about 8.6V. The UK mains supply is 230V AC; this means 230V RMS so the peak voltage of the mains is about 320V!

So what does root mean square (RMS) really mean?

First square all the values, then find the average (mean) of these square values over a complete cycle, and finally find the square root of this average. That is the RMS value. Confused? Then just accept that RMS values for voltage and current are much more useful quantities than peak values.

IMPORTANCE OF RMS VALUE

Since a digital voltmeter measures the RMS value of a fluctuating voltage, care must be taken when finding the RMS values of very low frequency signals otherwise inaccuracies can arise. The second term in equation (5) is an oscillation of maximum amplitude $\frac{A^2}{4\omega T}$ that decreases with

increasing period T . For a signal measured over a time period $T \ll \infty$, there is obvious uncertainty over how much the true RMS deviates from $\frac{A}{\sqrt{2}}$.

The maximum uncertainty is the maximum error over the assumed RMS and is given by:

$$\text{Maximum uncertainty} = \frac{\frac{A^2}{4\omega T}}{\frac{A^2}{2}} = \frac{1}{2\omega T} \dots (6)$$

So, at lower frequencies (smaller values of ω), longer averaging times are required to reduce the uncertainty of the measurements to an acceptable level.

Equivalent uncertainties may be derived for other non-sinusoidal waveforms.

WORKED EXAMPLE – 1:

A sinusoidal voltage has a maximum value of 100V. Calculate its RMS value.

Solution: A sinusoidal voltage v having a maximum value of 100V may be written as:

$$v(\theta) = 100 \sin \theta$$

Over the range $\theta = 0$ to $\theta = 2\pi$ (a complete cycle), the RMS value is given by:

$$\begin{aligned} \text{RMS value} &= \sqrt{\frac{1}{2\pi - 0} \int_0^{2\pi} v^2 d\theta} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (100 \sin \theta)^2 d\theta} \\ &= \sqrt{\frac{10000}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta} \end{aligned}$$

Using equation (4), we can write:

$$\begin{aligned} \text{RMS value} &= \sqrt{\frac{10000}{2\pi} \int_0^{2\pi} \frac{1}{2} (1 - \cos 2\theta) d\theta} \\ &= \sqrt{\frac{10000}{2\pi} \frac{1}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi}} \\ &= \sqrt{\frac{10000}{2\pi} \frac{1}{2} \left[\left(2\pi - \frac{\sin 4\pi}{2} \right) - \left(0 - \frac{\sin 0}{2} \right) \right]} \\ &= \sqrt{\frac{10000}{2\pi} \frac{1}{2} [2\pi]} \\ &= \frac{100}{\sqrt{2}} = 70.71V \end{aligned}$$

Please note: Being a sinusoidal wave, we can calculate the RMS value for this voltage using the relation derived on page-1, i.e.

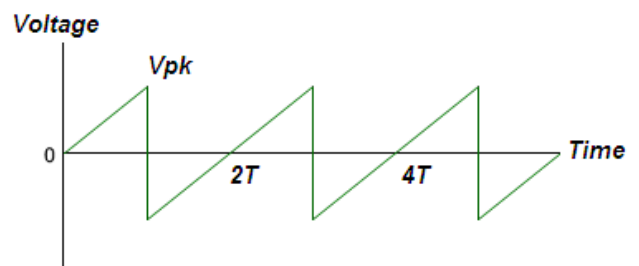
$$V_{RMS} = 0.7 \times V_{peak}$$

$$\Rightarrow V_{RMS} = 0.7 \times 100 \approx 70.71V$$

But it is not so straight-forward in case of a non-sinusoidal function as shown in the next worked example.

WORKED EXAMPLE – 2:

Find the RMS value for a voltage defined as the following saw-tooth function over an interval $[0, 2T]$:



$$V(t) = \begin{cases} \frac{V_{pk}}{T} t, & \text{for } 0 < t < T \\ -V_{pk} + \frac{V_{pk}}{T} (t - T), & \text{for } T < t < 2T \end{cases}$$

Solution: Since the function is symmetrical, we can find the required RMS value by finding it only in the interval $[0, T]$ as follows:

$$\begin{aligned}
 V_{\text{RMS}} &= \sqrt{\frac{1}{T} \int_0^T \left(\frac{V_{pk}}{T} t \right)^2 dt} \\
 &= \sqrt{\frac{1}{T} \frac{V_{pk}^2}{T^2} \int_0^T t^2 dt} \\
 &= \sqrt{\frac{V_{pk}^2}{T^3} \left[\frac{t^3}{3} \right]_0^T} \\
 &= \sqrt{\frac{V_{pk}^2}{T^3} \frac{T^3}{3}} \\
 &= \frac{V_{pk}}{\sqrt{3}}
 \end{aligned}$$

Hence, if the peak value of this voltage is 200V, then its RMS value will be approximately 115V.

EXTENSION ACTIVITY – 1:

A current, $i(t) = 30 \sin 100\pi t$ amperes is applied across an electric circuit. Determine its mean and RMS values, each correct to 4 significant figures, over the range $t = 0$ to $t = 10$ milliseconds.

(Answers: 19.10 A, 21.21 A)

EXTENSION ACTIVITY – 2:

A company responsible for drying the timber used in engineering applications has found that the percentage moisture in timber, y , is related to the number of days of drying, x , needed to become dry by the equation $y(x) = -x^{0.05} - 0.25x + 100$. Determine, correct up to 3 significant figures, the RMS value of moisture present in the timber between days $x = 0$ to $x = 100$. Also plot a graph of this function using any available software and estimate after how many days, the timber will be completely dry, i.e. $y = 0$.

(Answer: 86.6%, Approximately 392 days)

EXTENSION ACTIVITY – 3:

The tidal height H at a particular location varies with time t over a fortnight as:

$$H = (4 + 3.5 \cos \frac{\pi t}{7}) \cos 4\pi t$$

where t is measured in days. To assess the location's potential as a site for generating electricity through tidal power, an engineer wants to know the RMS value of H over a fortnight, i.e. for $0 \leq t \leq 14$ days. Plot H as a function of t and find its RMS value over this period.

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. <http://www.kpsec.freeuk.com/acdc.htm>

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- What RMS value is
- Trigonometrical identities
- Integration
- Plotting graphs of any function using ICT

TOPICS COVERED FROM “MATHEMATICS FOR ENGINEERING”

- Topic 1: Mathematical Models in Engineering
- Topic 3: Models of Oscillations
- Topic 4: Functions
- Topic 5: Geometry
- Topic 6: Differentiation and Integration

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 3.2: Relate trigonometrical expressions to situations involving oscillations
- AC 4.1: Identify and describe functions and their graphs
- AC 4.2: Analyse functions represented by polynomial equations
- AC 6.3: Find definite and indefinite integrals of functions
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science