ELEC101P – Introduction to Electronic and Electrical Engineering

Signals & Systems pre-reading

Lecture 2 – Introduction to communications and Fourier series

By the end of this lecture, you should be able to:

- Describe how engineers have built modern communication over centuries
- Understand and identify the use of bandwidth and filters in everyday technologies
- Understand the mathematics behind a real sound wave.

You don't need to know anything about engineering to know what communication is. We all start communicating with unintelligible noises, and we eventually learn to form words (some people on TV, though, haven't yet). The result is the same: we communicate to convey an idea.

If you're having a conversation with a friend, you'd understand them only if you heard their words. That's simple enough, but if there was another pair next to you talking as loudly, you would hear both your friend's words and their conversation. Inevitably it would be harder for you to understand your conversation. That's because both conversations are taking place on the same range of frequencies, that of human speech. This issue is much like that in radio broadcasting.

You probably know that radio shows take up different frequencies, but this isn't intuitive – wouldn't two talk shows both use the frequencies of human speech?

Modulation solves this problem. You'll learn more about modulation later, but it's the shifting of a signal to a different set of frequencies – the recorded sound is shifted to a different range of frequencies when it's broadcast, so the signals have no overlapping frequencies, minimising interference.

It's therefore evident that frequency analysis is fundamental to communications engineering, where it's given a name: **Fourier analysis**.

The **Fourier series** is the basis of the **Fourier transform**, which are methods of analysing the frequency spectrum of signal. They are fundamental to communications engineering so you should get to know Fourier analysis intimately.

This reading reviews integration by parts and some discrete identities, and you'll be introduced to two new functions, the cardinal sine (sinc) function, and the Dirac delta function, all of which will be heavily used in Fourier analysis. You might like to refer to this reading during the lecture.

Fourier analysis might seem maths-heavy at first, but you'll eventually learn ways to avoid this!

Useful identities and methods for Fourier series

Integration between a function and an exponential

This type of integration occurs quite often, and you can find a table of integrals on Moodle. You'll need to know integration by parts, however, to fully understand this process.

Given a function $y(x)$ and constant a:

$$f(x) = \int_{x_1}^{x_2} y e^{ax} dx$$

$$= \left[y \frac{1}{a} e^{ax}\right]_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{dy}{dx} \frac{1}{a} e^{ax} dx$$

$$= \frac{1}{a}\left([ye^{ax}]_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{dy}{dx} e^{ax} dx \right)$$

This is the general solution, and evaluating it can vary greatly depending on the function $y(x)$.

If $y(x)$ is a polynomial you have to continue applying integration by parts until the derivative is zero.

If $y(x)$ is sine or cosine, then the integration by parts integral will eventually resemble the original integral $f(x)$, allowing you to substitute $f(x)$ into the equation. You can then rearrange to solve for the variable f. This is more of a special case that you should be aware of.

Always look to see if the quantity $[ye^{ax}]_{x_1}^{x_2}$ can be simplified by evaluating the limits. If $y(x)$ is zero at either of the boundaries, it makes the simplification much easier!

Since the Fourier series is discrete, sometimes you can simplify trigonometric expressions. For any integer n:

$$\cos n\pi = (-1)^n$$

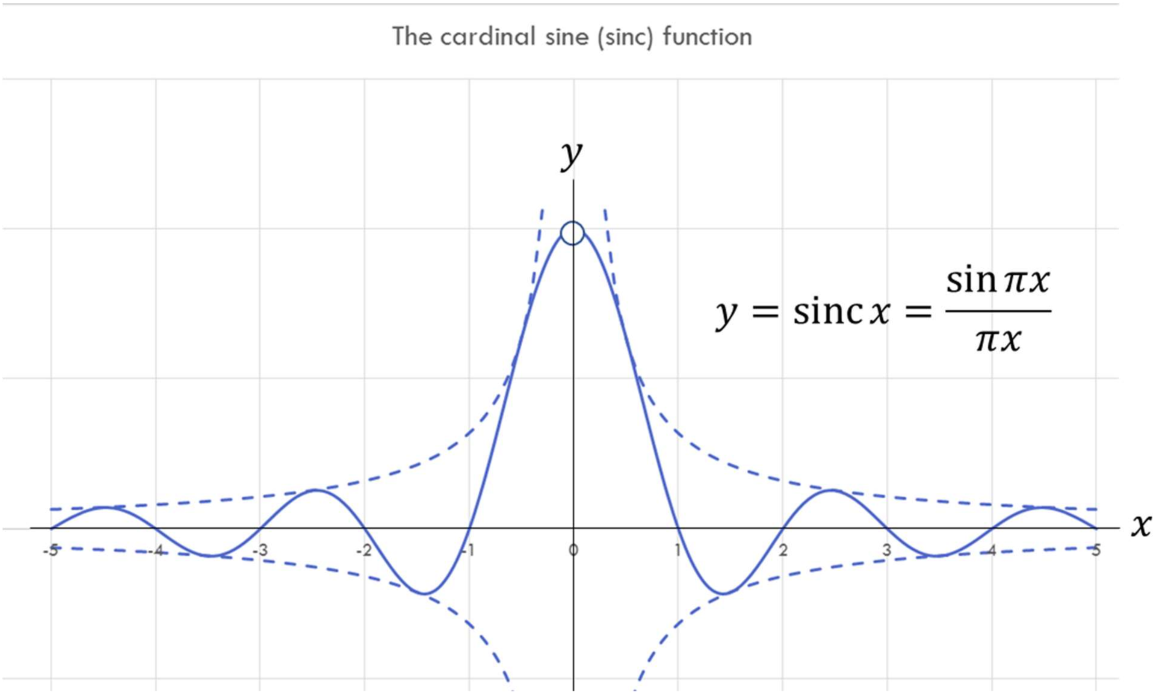
$$\sin n\pi = 0$$

$$\cos\left(n\pi + \frac{\pi}{2}\right) = 0$$

$$\sin\left(n\pi + \frac{\pi}{2}\right) = (-1)^n$$

The cardinal sine (sinc) function

This function pops up quite often in Fourier analysis and is often associated with ideal signals. In mathematics it's sometimes defined differently, without the π factors, but in electrical engineering it's defined as the following:



The sinc function is a multiplication of the sine function and the reciprocal function, giving it an interesting shape.

Its envelope (in dotted lines) is formed by $\pm\frac{1}{\pi x}$, which is helpful when sketching.

The central maximum is undefined, but taking the limit shows that it approaches 1. In programmes like Matlab, the central maximum is defined to be 1 to avoid errors with non-defined points.

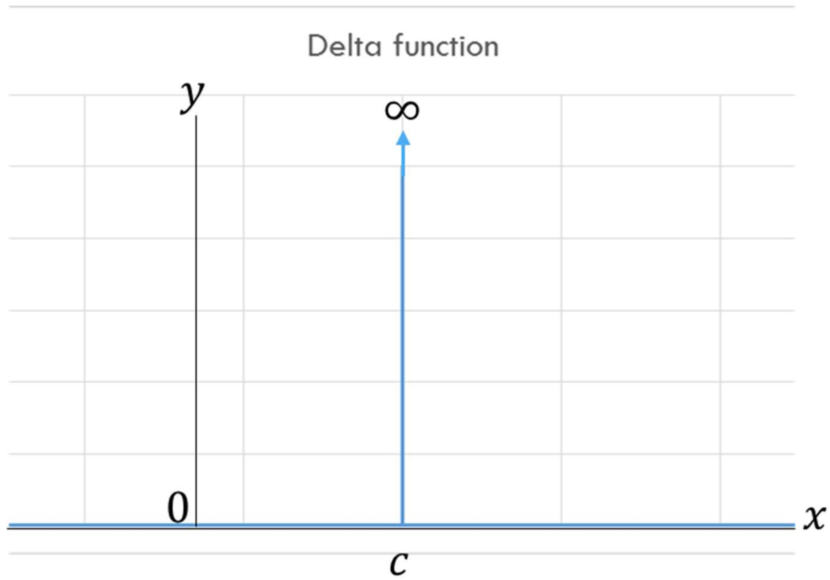
The roots come from the sine function. Whenever sinc's argument x is an integer, sinc will be zero.

The local extrema decrease in absolute value the further away they are from the central maximum.

Very far from the centre, sinc approaches zero and appears to be flat.

The Dirac delta function

This is not technically a function, but it's quite useful in engineering because of its properties. It's also known as an impulse.



The variable c is some constant.

There are some properties you should be aware of:

$$\int_{-\infty}^{\infty} \delta(x-c)dx = 1$$

How does this make sense? Don't worry about that, just know it!

For any function $y(x)$:

$$\int_{-\infty}^{\infty} y(x)\delta(x-c)dx = y(c)$$

This is to say that the infinite integral of the product of any function with the delta function is equal to the value of that function at the delta's infinity point. A bit hard to conceptualise, but important to know!

An interesting way of defining the delta function involves sinc. Let's say we multiplied the sinc function by infinity, then multiplied its argument by infinity. Its central maximum would approach infinity, and the rest of the function would be zero because the argument is infinity. Mathematically, for a variable n:

$$\delta(x-c) = \lim_{n\to\infty}(n \operatorname{sinc} n(x-c))$$