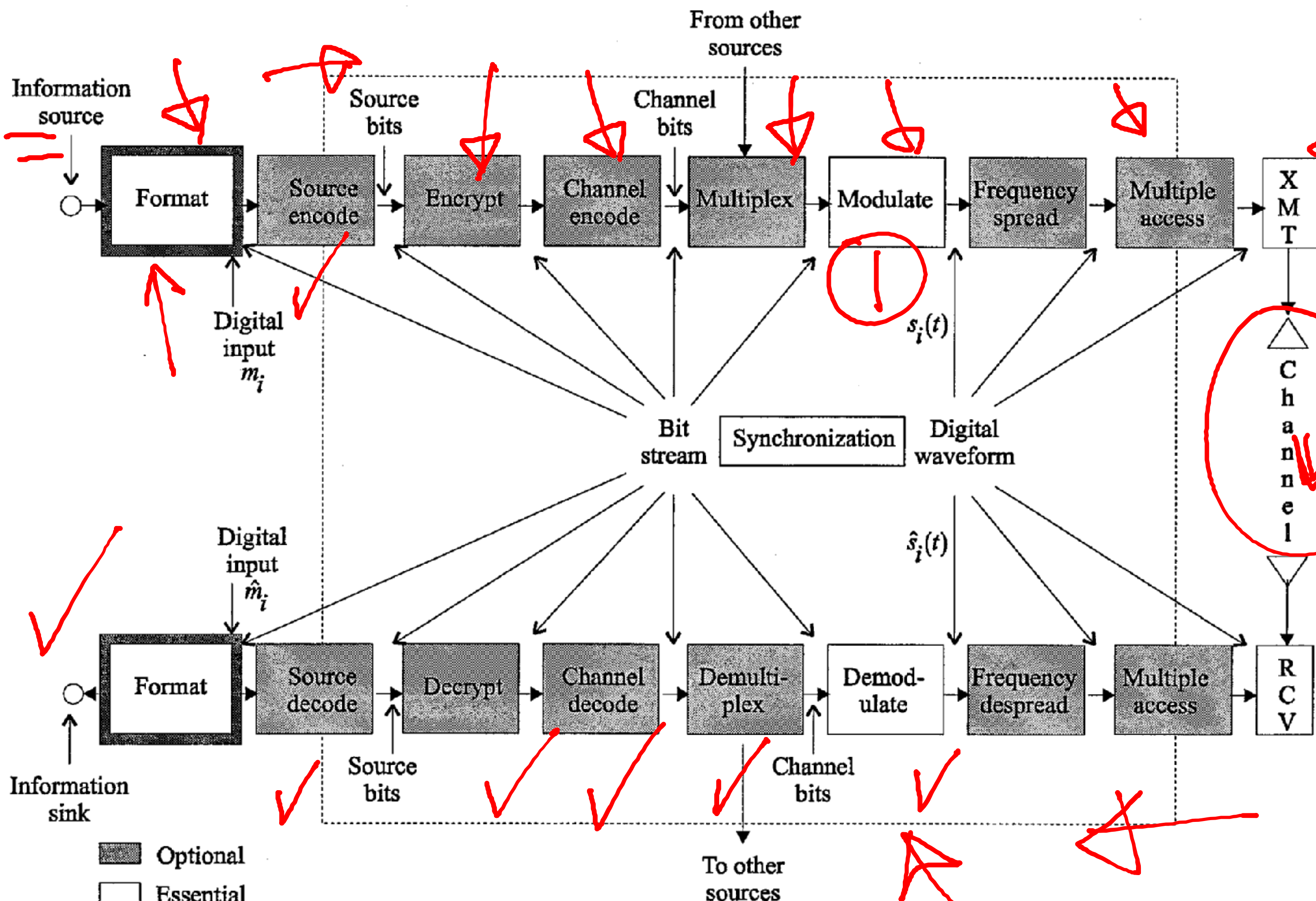
Communication Basics

An End-to-End Model

digital modulation



From Bits to Symbols to Pulses

bits → symbols

- Information is a sequence of bits 00100111010 … …

0:

1

-1

- For BPSK, two symbols and each carries one bit

One symbol

0 0 1 0 0 1 1 1 0 1 0 0 1 0 1 1 0 0 0 1 0 0 …

each

00

01

10

11

- For QPSK, four symbols and each carries two bits

One symbol

00 10 01 11 01 00 10 11 00 01 00 …

2

-2

- For 16-QAM, 16 symbols and each carries 4 bits

$2^4 = 16$

higher speed

One symbol

0010 0111 0100 1011 0001 00 …

- Each symbol is transmitted as a single pulse, e.g., a rectangular pulse

"0" 1 "1"

Digital Modulation

$\cos 2\pi f_c t$

$\sin(2\pi f_c t)$

- Maps symbols to waveforms for transmission over the air on a specific carrier frequency (speed and range consideration)

2 bits 2 symbol

symbols

- k bits to form a symbol, $M = \boxed{2^k}$ constellation points

2 bits 2 symbol

4 points

- Choice of modulation
  - Spectrum efficiency (bandwidth) ✓
  - Power efficiency (required signal power) ✓
  - Complexity ✓

3 bits/symbol

8 points

- Examples include QPSK, 8-PSK, 64-QAM … etc.
  - QPSK: two bits per symbol
  - 8-PSK: 3 bits per symbol
  - 64-QAM: 6 bits per symbol

$$s(t) = \mathrm{Re}\left\{\tilde{s}(t)e^{j2\pi f_c t}\right\}$$

General Expressions

$$\tilde{s}(t) = A(t)e^{j\phi(t)}$$

real

Imag

- A modulated signal:

Information

Amplitude

Frequency

Phase

Carrier

$\tilde{s}(t)$

signal constellation

$$s(t) = A(t)\cos(2\pi f_c t + \phi(t)) = \mathrm{Re}\left\{A(t)e^{j\phi(t)}e^{j2\pi f_c t}\right\}$$

$$= \underbrace{A(t)\cos\phi(t)\cos(2\pi f_c t)}_{\text{In Phase}} - \underbrace{A(t)\sin\phi(t)\sin(2\pi f_c t)}_{\text{Quadrature}}$$

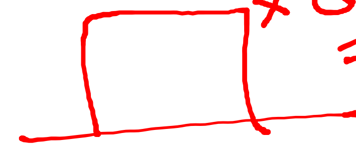
- With AWGN, the noise can be expressed similarly as

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$= \mathrm{Re}\left\{(n_I(t) + jn_Q(t))e^{j2\pi f_c t}\right\}$$

0110111010....

$\to d_1 \quad \to d_2 \quad \to d_3 \quad \ldots$ constellations



Linear Modulations

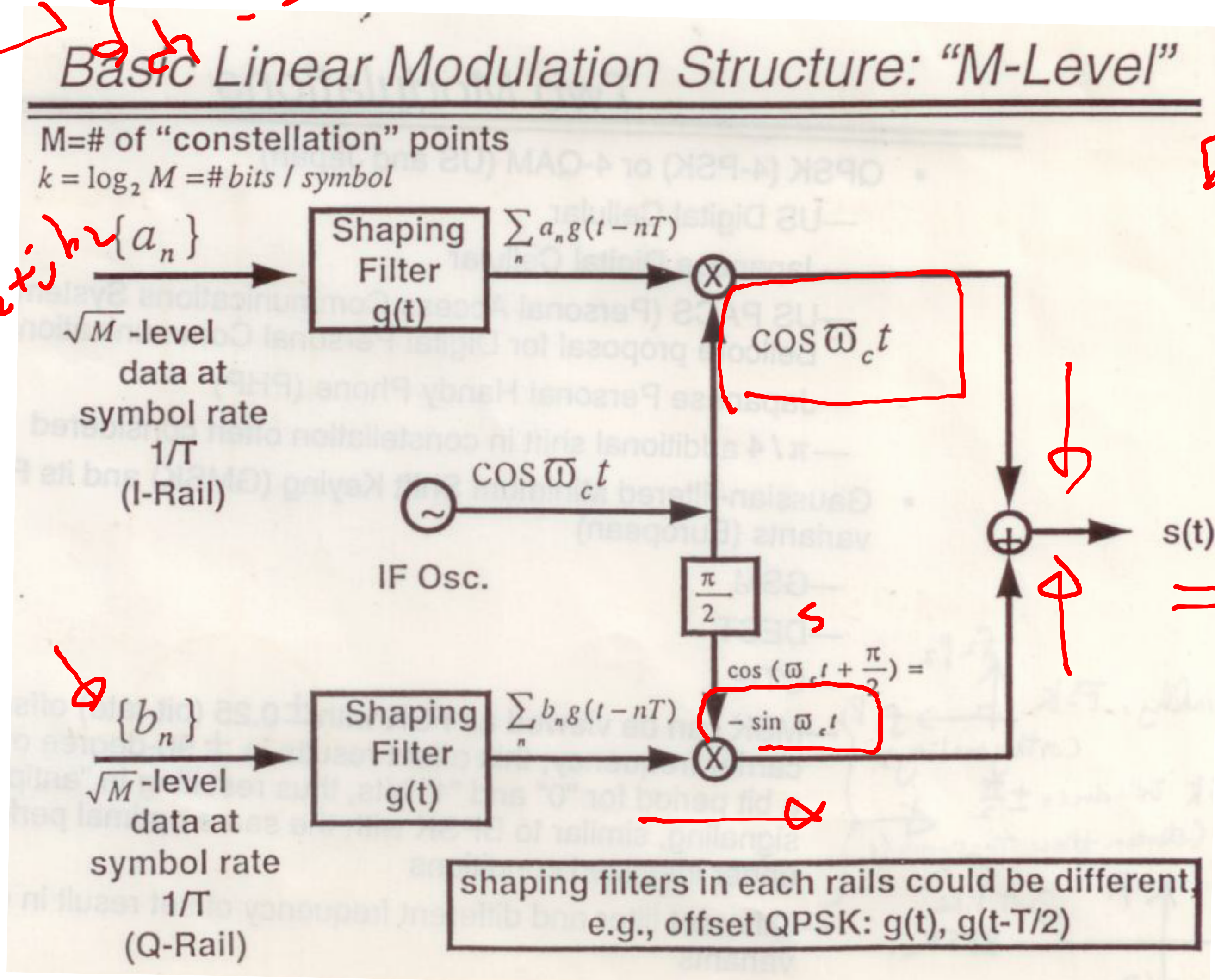
- Let u(t) be the baseband representation, i.e.,

$$u(t) = \boxed{\sum_n d_n g(t - nT)} \text{ where } d_n = a_n + jb_n$$

$$s(t) = \underbrace{\left(\sum_n a_n g(t-nT)\right)}_{I(t),\ \text{In-phase}} \cos(2\pi f_c t) - \underbrace{\left(\sum_n b_n g(t-nT)\right)}_{Q(t),\ \text{Quadrature}} \sin(2\pi f_c t)$$

  - T is the symbol period
  - d_n is in general a sequence of complex numbers representing the information sequences
  - The mapping of an information bit sequence into d_n is determined by the particular modulation scheme
  - g(t) is the signalling waveform or pulse shape

Basic Linear Modulation Structure: “M-Level”



Signal Constellations

4 bits per symbol

- M-QAM (Square)

$$a_n = \pm 1, \pm 3, \ldots, \pm\left(\sqrt{M} - 1\right)$$

$$b_n = \pm 1, \pm 3, \ldots, \pm\left(\sqrt{M} - 1\right)$$

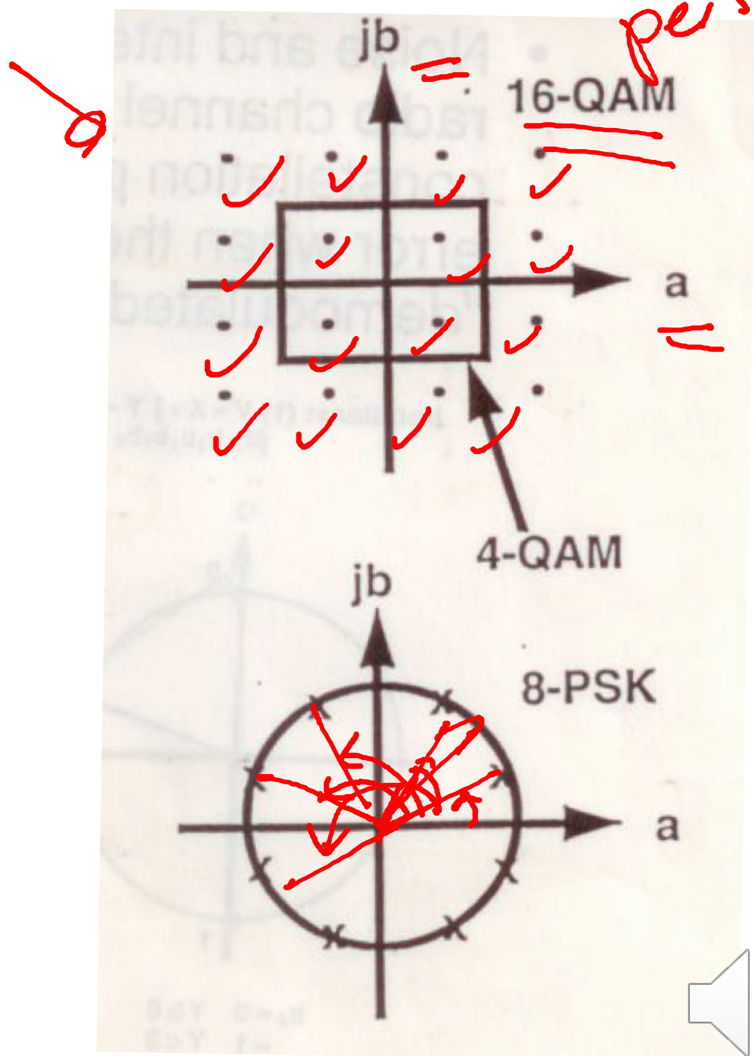
a_n, b_n are independent

- M-PSK (Circular)

$$a_n = \cos\phi_n$$

$$b_n = \sin\phi_n$$

each ϕ_n is one of M uniformly spaced phase values



Noise

$y(t) = s(t) + n(t)$

- Thermal noise
  - Caused by the random thermally excited vibrations of charge carriers in a conductor; 'Brownian motion of electricity'
  - AWGN – Additive White Gaussian Noise; additive, frequency independent, statistically described by Gaussian random variables
  - **Noise Power Formula** (the Boltzmann's model)

$$\boxed{N_t = kTB}$$

Noise power

Noise bandwidth

Boltzmann's constant

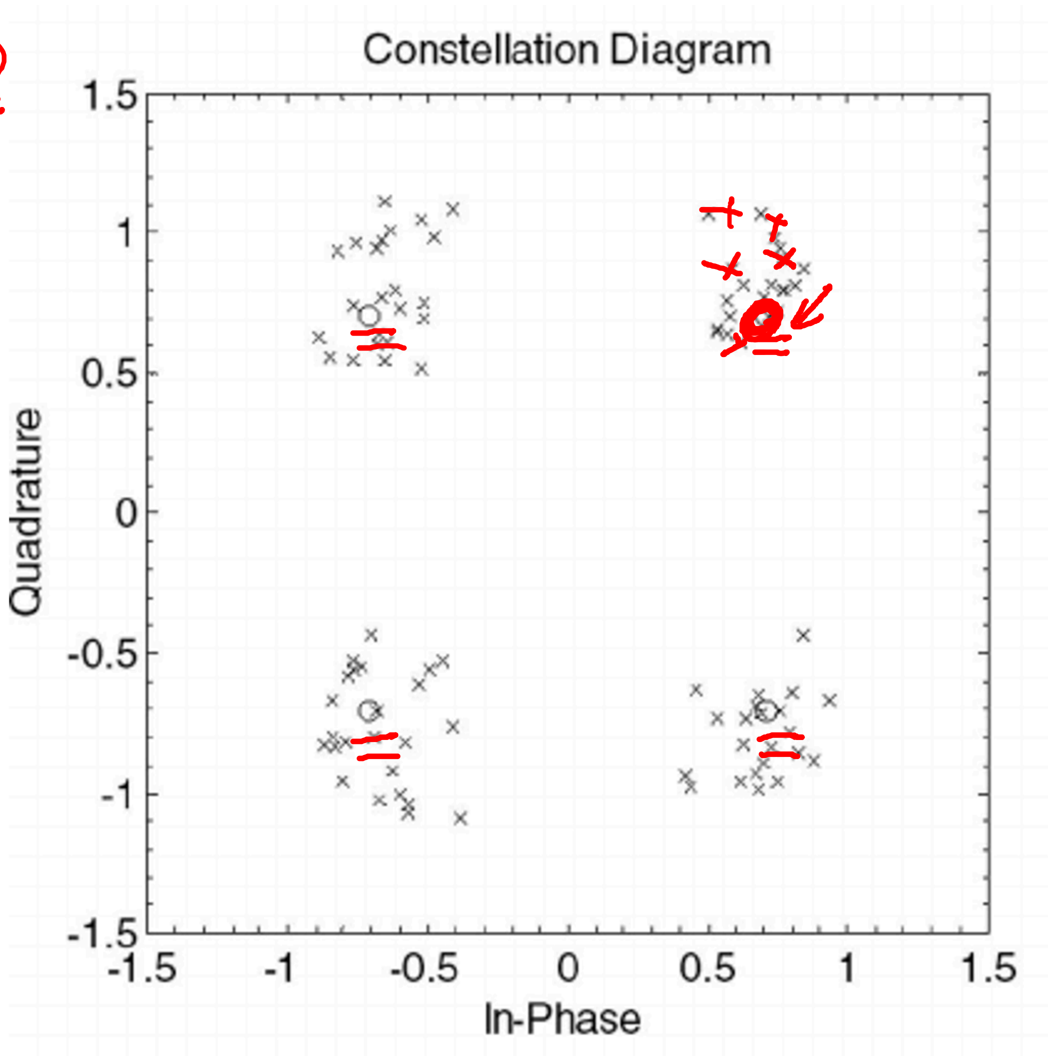
$1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$

Absolute temperature

4-QAM or QPSK

Constellations with Noise

$y(t) = S(t) + n(t)$



$y(t) = s(t) + n(t)$

Demodulation

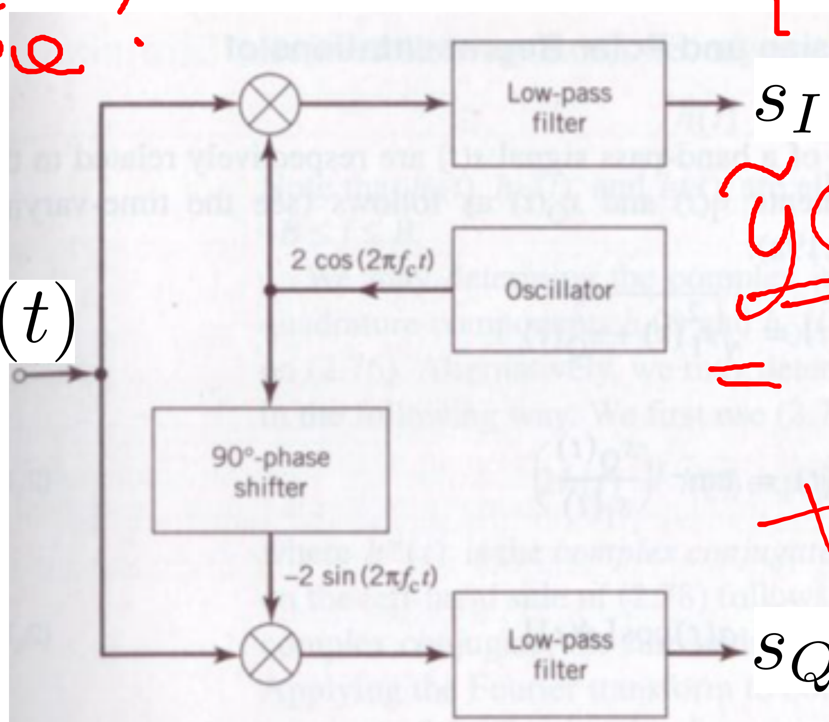
estimate what $s(t)$ is?

- Coherent detection requires recovery of the phase of the carrier

$$\boxed{s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) = \mathrm{Re}\left\{(s_I(t) + j s_Q(t))e^{j2\pi f_c t}\right\}}$$

$y \to s_I, s_Q$?

$s(t) + n(t)$



$s_I(t) + n_I(t)$

$s_Q(t) + n_Q(t)$

s_I, s_Q

$\tilde{y}(t) = (s_I + j s_Q) + (n_I + j n_Q)$

SER & BER

$y = (1+j) + n$ symbol

From SER to BER Calculations

① $1 - \text{SER} = (1 - \text{BER})^2$

- **Bit-error-rate (BER)** is the probability that a bit is decoded in error
- **Symbol-error-rate (SER)** measures the probability that a symbol is detected in error

2 bits/symbol

BER small

- Converting SER into BER is easy!
- For example, if QPSK is used and we obtain the SER, then

$= 1 - (1 - 2\text{BER})$

$(1 - \text{BER})^2 \approx 1 - 2\text{BER}$

16-QAM

256-QAM

② $$\text{SER} = 1 - (1 - \text{BER})^2$$

$$\Rightarrow \text{BER} \approx \frac{\text{SER}}{2}$$

$\text{SER} \approx 2\text{BER}$