Butterworth filter $\boxed{|H(\omega)|^2 = \dfrac{1}{1+\omega^{2n}}}$ $\Rightarrow$ find $n$

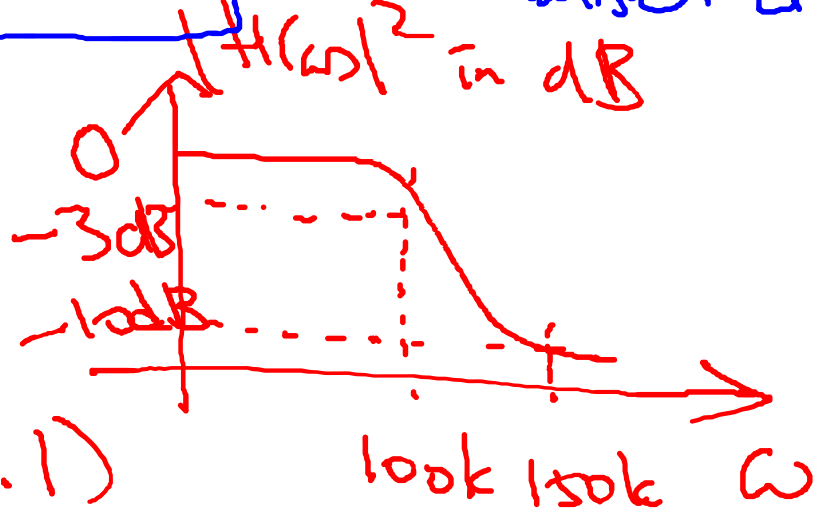
normalised LPF

(1) passband $0-\omega_c$ : $\omega_c = \boxed{100k}$ rad $s^{-1}$

(2) at $\omega_c$ : $-3$dB (or 0.5) ✓

(3) stopband at $\omega_r$ : $\omega_r = 150$ krad$s^{-1}$

(4) max gain at $\omega_r = -10$ dB (or 0.1)




$$|H(\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{100k}\right)^{2n}}$$

$$|H(150k)|^2 = \frac{1}{1+\left(\frac{150k}{100k}\right)^{2n}} \le 0.1$$

$\Rightarrow n \ge 2.7$

s/wc

$\Rightarrow$ choose $n=3$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$\Rightarrow H(s) = \frac{1}{\left(\frac{s}{100k}+1\right)\left(\left(\frac{s}{100k}\right)^2+\left(\frac{s}{100k}\right)+1\right)}$$

Chebyshev filter $|H(\omega)|^2 = \dfrac{1}{1+\epsilon^2 C_n(\omega)}$

$C_n(\omega) = \cos(n \cos^{-1}\omega)$

$x = \cos^{-1}\omega \Rightarrow C_n(\omega) = \cos nx$

$C_0(\omega) = \cos 0x = 1$

$C_1(\omega) = \cos x = \omega$

$C_2(\omega) = \cos 2x = 2\cos^2 x - 1 = 2\omega^2 - 1$

$C_3(\omega) = \cos 3x = 4\cos^3 x - 3\cos x = 4\omega^3 - 3\omega$

$C_{n+1}(\omega) = 2\omega C_n(\omega) - C_{n-1}(\omega)$

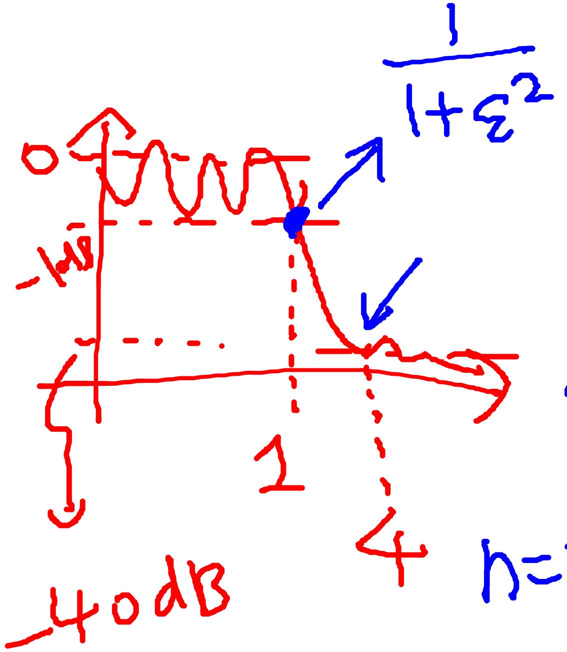
$\varepsilon = 0.5088$ and $n = 3$

Design a normalised Chebyshev LPF ($\omega_c = 1\ \text{rad}\ s^{-1}$)

$$|H(\omega)|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$

① max passband ripple (−1 dB)

② $\omega_r$ at 4 rad $s^{-1}$, max attenuation at least −40 dB




$$10\log\frac{1}{1+\varepsilon^2} = -1 \Rightarrow \varepsilon = \sqrt{10^{0.1}-1} = 0.5088 \checkmark$$

$n=1$: $10\log\dfrac{1}{1+(0.5088)^2\, C_1^2(4)} = 10\log\dfrac{1}{1+(0.5088)^2(4)^2} = -7\ \text{dB}$ ✗

$n=2 \Rightarrow 10\log\dfrac{1}{1+(0.5088)^2(2\times4^2-1)^2} = -24\ \text{dB}$ ✗

$n=3 \Rightarrow 10\log\dfrac{1}{1+(0.5088)^2(4\times4^3-3\times4)^2} = -42\ \text{dB} < -40\ \text{dB}$ ✓