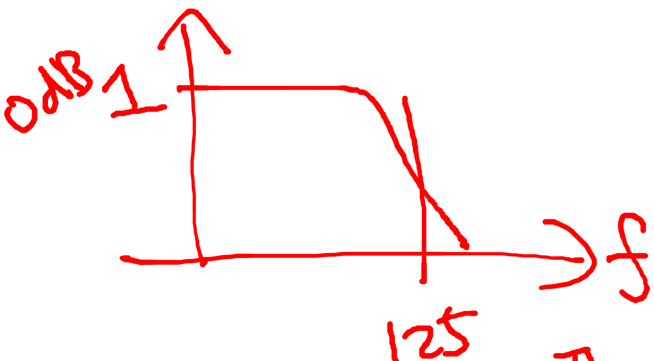
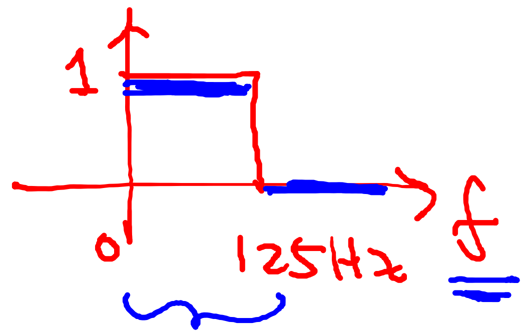
Example Linear Phase FIR Filter Design

- For the brick wall low pass response with a 1kHz sampling rate, a desired magnitude of unity in the pass band and a 125Hz -3dB cut off frequency, find the coefficient values appropriate to a 21-order FIR.





$N = 21$

$\Delta t = \frac{1}{1k}$

$$C_n = \frac{\Delta t}{\pi} \int_0^{\pi/\Delta t} H(\omega) \cos(n \omega \Delta t)\, d\omega$$

$$= \frac{(1/1k)}{\pi} \int_0^{2\pi(125)} H(\omega) \cos\left(\frac{n\omega}{1k}\right) d\omega$$

$$C_n = \frac{1}{1k\pi}\int_0^{250\pi} \cos\left(\frac{n\omega}{1k}\right) d\omega$$

$$\boxed{\int_0^{\theta} \cos(ax)\,dx = \frac{1}{a}\int_0^{\theta} d\sin ax = \frac{\sin a\theta}{a} = \theta\,\mathrm{sinc}(a\theta)}$$

$$C_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

$n=0, n=1, 2, 3, \ldots, 10$

$C_{10} = 0.25$

$C_{11} = 0.2551$

$C_{12} = 0.1592$

$C_{13} = 0.075$

$C_{14} = 0$

$C_{15} = -0.045$

$C_{16} = -0.0531$

$C_{17} = -0.0322$

$C_{18} = 0$

$C_{19} = 0.025$

$C_{20} = 0.0318$

$C_9 = C_1 = 0.2551$ ✓

$C_8 = 0.1592$

$C_7 = 0.075$

$C_6 = 0$

$C_5 = -0.045$

$C_4 = -0.0531$

$C_3 = -0.0322$

$C_2 = 0$

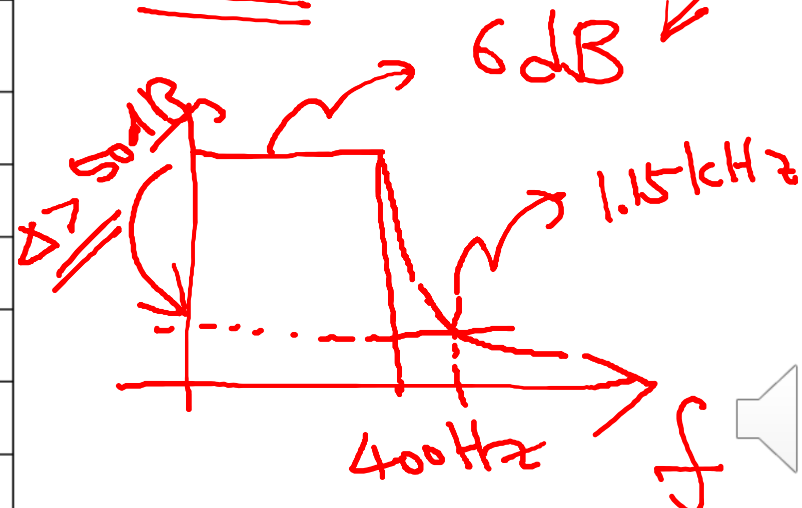
$C_1 = 0.025$

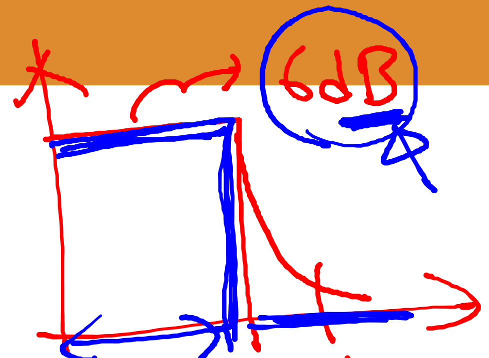
$C_0 = 0.0318$

Example Window Filter Calculation $f_s = 5kHz$

- Design a low pass FIR filter for a system with a 5KHz sample rate. Using an ideal brick wall freq response with a pass band gain of 6dB and a cut-off frequency of 400Hz.
- Calculate the required number of filter coefficients and weighted coefficient values such that the filter achieves a stop band attenuation of at least 50dB at all frequencies above 1.15KHz.

Window	Transition band (Hz)	Stopband rejection (dB)
Rectangular	$\frac{1}{N\Delta t}$	21
Hanning	$\frac{3.1}{N\Delta t}$	44
Hamming	$\boxed{\frac{3.3}{N\Delta t}}$	53
Kaiser, β =6	$\frac{4}{N\Delta t}$	63
Blackman	$\frac{5.5}{N\Delta t}$	74
Kaiser, β =9	$\frac{5.7}{N\Delta t}$	90





400

1.15k

Hamming window

$$\frac{3.3}{N\Delta t} = 1.15k - 400 = 750$$

$$\Rightarrow \frac{3.3}{N\left(\frac{1}{5k}\right)} = 750 \Rightarrow N = 22$$

$$\Delta t = \frac{1}{5k}$$

$N = 23$ ✗ Type I

$M = 11$ ✓ FIR

$$C_n = \frac{\left(\frac{1}{\Delta t}\right)}{\pi}\int_0^{\frac{\pi}{\Delta t}} H(\omega)\cos(n\omega\Delta t)\,d\omega \quad 0 \text{ to } 2\pi(400)$$

($\frac{1}{\Delta t} = 5k$, $\frac{\pi}{\Delta t} \to 800\pi$, $H(\omega) \to 2$, $\Delta t = \frac{1}{5k}$)

$$20\log_{10}|H(\omega)| = 6$$

$$H(\omega) = 2$$

$$C_n = \frac{2}{5k\pi}\int_0^{800\pi}\cos\left(\frac{n\omega}{5k}\right)d\omega$$

$n = 0, 1, 2, 3, \ldots$

$C_n = 0.32$ sinc(0.[illegible] $n\pi$)

$C_0 = 0.32$

$$\mathrm{sinc}(x) = \frac{\sin x}{x}$$

$x = 0$ $\mathrm{sinc}(0) = ? \Rightarrow$

$$C_n = 0.32\,\mathrm{Sinc}(0.16 n\pi) \quad (M = 11)$$

$n = 0, 1, 2, \ldots, 11$

$$W_n = 0.54 + 0.46 \cos\frac{n\pi}{M}$$

$$= 0.54 + 0.46 \cos\frac{n\pi}{11}$$

$n = 0, 1, 2, \ldots, 11$

$$\boxed{a_n = C_n \times W_n}$$

When choose other window filter, a(n) should be calculated

$C_0 = 0.32$

$C_1 = 0.2026$

$C_2 = -0.0017$

$C_3 = -0.0675$

$C_4 = 0.0017$

$C_5 = 0.0405$

$C_6 = -0.0017$

$C_7 = -0.0289$

$C_8 = 0.0017$

$C_9 = 0.0225$

$C_{10} = -0.0017$

$C_{11} = -0.0183$

$a_0 = 0.32$

$a_1 = 0.1989$

$a_2 = -0.0016$

$a_3 = -0.0568$

$a_4 = 0.0012$

$a_5 = 0.0245$

$a_6 = -0.0008$

$a_7 = -0.0101$

$a_8 = 0.0004$

$a_9 = 0.0034$

$a_{10} = -0.0002$

$a_{11} = -0.0005$

$a_{10} = 0.1989$

$a_{9} = -0.0016$

$a_{8} = -0.0568$

$a_{7} = 0.0012$

$a_{6} = 0.0245$

$a_{5} = -0.0008$

$a_{4} = -0.0101$

$a_{3} = 0.0004$

$a_{2} = 0.0034$

$a_{1} = -0.0002$

$a_{0} = -0.0015$

$a_{11} = 0.32$

$a_{12} = 0.1989$

$a_{13} = -0.0016$

$a_{14} = -0.0568$

$a_{15} = 0.0012$

$a_{16} = 0.0245$

$a_{17} = -0.0008$

$a_{18} = -0.0101$

$a_{19} = 0.0004$

$a_{20} = 0.0034$

$a_{21} = -0.0002$

$a_{22} = -0.0015$

orde

$\boxed{N=23}$

$\boxed{M=11}$

DONE!!