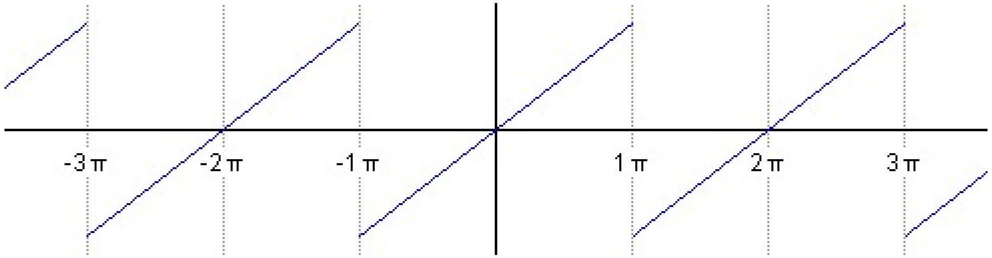
Wireless Communications Principles

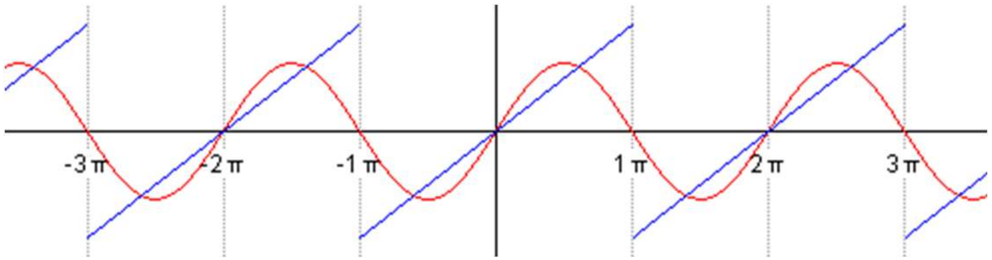
Signal representation basics

The Time and Frequency Domains

- Representation of the signal in the time domain:



- Representation of the signal in terms of a **Fourier series**:



- The **signal** can be approximated as a **sum of sinewaves** of appropriate frequencies, amplitudes and phases.

More reading: http://www.thefouriertransform.com

Time- and Frequency-Domain Representation of a Signal

- A continuous-time signal can be defined both in the **time-** or the **frequency-domain**.

- The **frequency-domain representation** $\boldsymbol{X(f)}$ is obtained from its **time-domain representation** $\boldsymbol{x(t)}$ via the Fourier transform:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} \, dt$$

- The time-domain representation $x(t)$ is obtained from the frequency domain representation $X(f)$ via the inverse Fourier transform:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} \, df$$

Time- and Frequency-Domain Representation of a System

- A linear time-invariant system can also be **represented both in** the **time-domain** or the **frequency-domain**.

- It is represented in the **time-domain** by the **impulse response *h*(*t*)**, i.e. the response of the system to an impulse.

- It is represented in the **frequency-domain** by the **frequency response *H*(*f*)**, i.e. the response of the signal to a complex exponential with frequency *f*.

- The impulse response and the frequency response are **Fourier transform pairs**, that is

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t}\, df \leftrightarrow H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t}\, dt$$

Fourier Transform Properties

Properties	Time-Domain	Frequency-Domain
Linearity	$c_1x_1(t) + c_2x_2(t)$	$c_1X_1(f) + c_2X_2(f)$
Delay	$x(t - t_0)$	$e^{-j2\pi f t_0}X(f)$
Modulation	$e^{j2\pi f_0 t}x(t)$	$X(f - f_0)$
Convolution	$\cancel{\int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)\,d\tau}$	$X_1(f)X_2(f)$
Multiplication	$x_1(t)x_2(t)$	$\cancel{\int_{-\infty}^{\infty} X_1(\theta)X_2(f-\theta)\,d\theta}$

Notes: $x_1(t) \leftrightarrow X_1(f)$ $\quad$ $x_2(t) \leftrightarrow X_2(f)$

Fourier Transform Pairs

Note: Π stands for rectangular function. Λ stands for triangular function.

$x(t)$	$X(f)$
$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$
$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$
$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$
$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
$\exp(-\alpha\|t\|),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$

Euler

steemit.com

Signals in Time Domain

- For a signal, *s*(*t*), in time domain. If *s*(*t*) is the voltage across the unit resistance

✓ The Signal Energy: $E_s = \int_{-\infty}^{\infty} |s(t)|^2 \, dt$

with unit (joule)

✓ The Signal Power: $P_s = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 \, dt$

with unit (watt)

✓ $s(t)$ is an energy-type signal if and only if

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 \, dt < \infty.$$

✓ If E_s is infinite, $s(t)$ is a power-type signal

Energy and Energy Spectral Density

- Energy of a signal *s*(*t*) with spectrum *S*(*f*):

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2\, dt = \int_{-\infty}^{\infty} s(t)s^*(t)dt = \int_{-\infty}^{\infty} s(t)\left[\int_{-\infty}^{\infty} S^*(f)e^{-j2\pi ft}df\right]dt$$

$$= \int_{-\infty}^{\infty} S^*(f)\left[\int_{-\infty}^{\infty} s(t)e^{-j2\pi ft}dt\right]df = \int_{-\infty}^{\infty} S^*(f)S(f)df = \int_{-\infty}^{\infty} |S(f)|^2\, df$$

- Energy spectral density is defined as : $U_s(f) = |S(f)|^2$

The energy spectral density tells us where the energy is distributed in frequency domain. Then, the energy of a signal can be calculated according to its energy spectral density, as

$$E_s = \int_{-\infty}^{\infty} U_s(f)df$$

Parseval's Theorem: $E_s = \int_{-\infty}^{\infty} |s(t)|^2\, dt = \int_{-\infty}^{\infty} |S(f)|^2\, df$

A signal's energy can be calculated based on its waveform function or its energy spectral density (ESD)

Power and Power Spectrum

- Power of power-type signal *s*(*t*):

$$s_T(t) = \begin{cases} s(t) & -T/2 < t < T/2 \\ 0 & otherwise \end{cases}$$

$$P_s = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2\, dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |s_T(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |S_T(f)|^2\, df = \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{T} |S_T(f)|^2\, df$$

- Power spectral density or simply power spectrum is defined as:

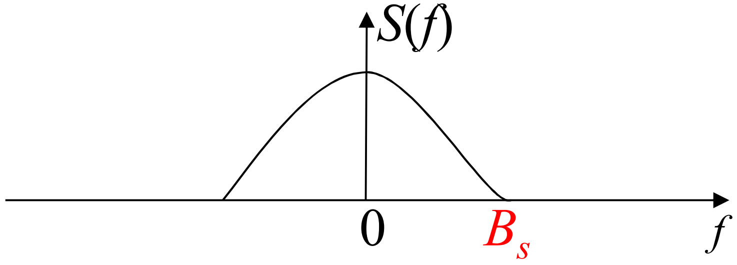
$$G_s(f) = \lim_{T\to\infty} \frac{1}{T} |S_T(f)|^2$$

- The power spectrum also tells where the energy or power of the signal is distributed over frequency.
- If the power spectrum of a signal is given, the power of the signal can be calculated as

$$\boxed{P_s = \int_{-\infty}^{\infty} G_s(f)df}$$

Signal Bandwidth

- Bandwidth of signal $s(t)$: the amount of **positive** frequency spectrum that signal $s(t)$ occupies.



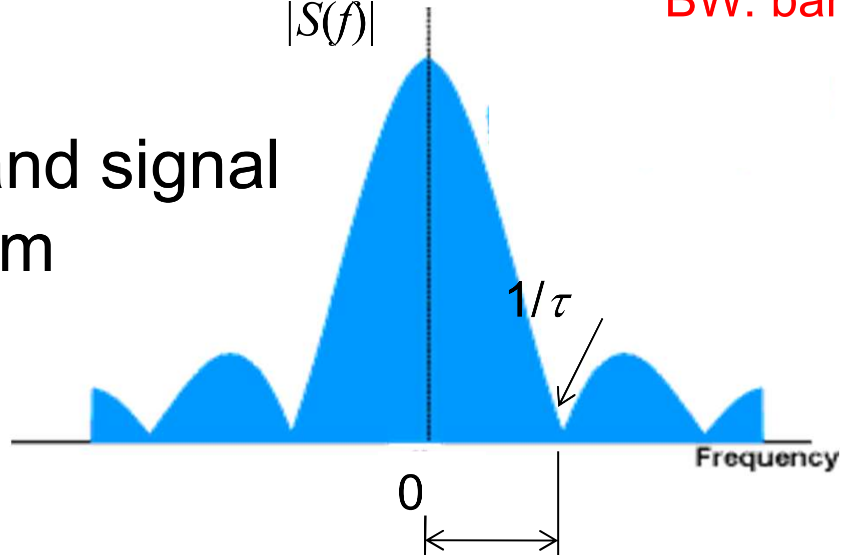
- Signal bandwidth provides a measure of the extent of **significant spectral content** of the signal for **positive frequencies**.

Baseband (Low-Pass) Signal Bandwidth

When the spectrum of a signal is symmetric with a **main lobe** bounded by nulls (frequencies at which the spectrum is zero), we may use the main lobe as a basis for defining the signal bandwidth.

BW: bandwidth

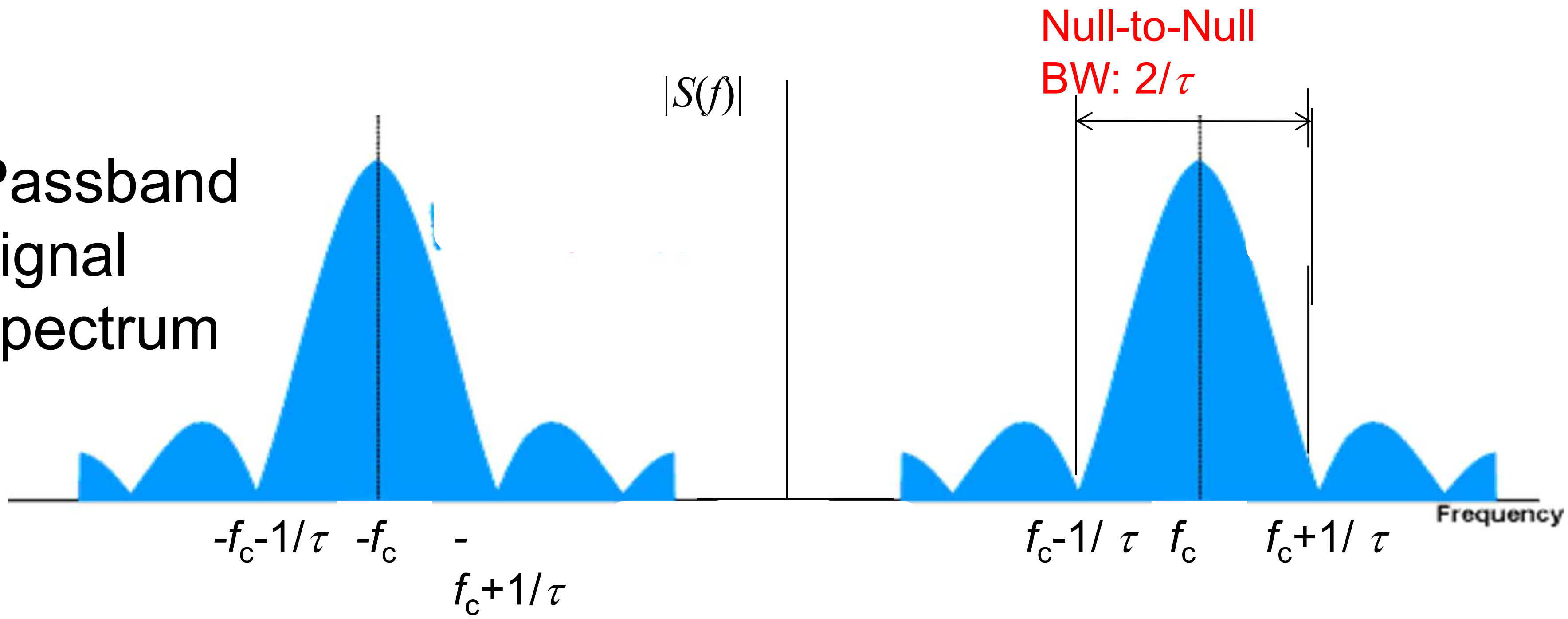
Baseband signal spectrum



Main Lobe
BW: $1/\tau$

Passband (Modulated) Signal Bandwidth

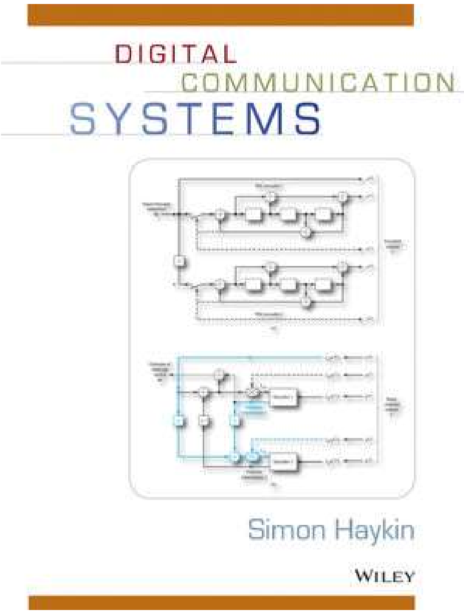
Passband signal spectrum

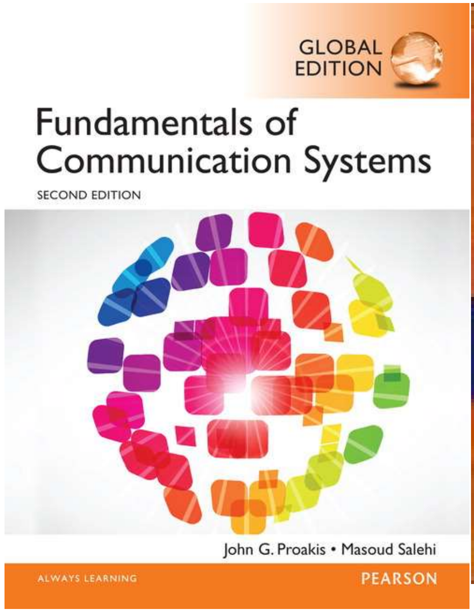


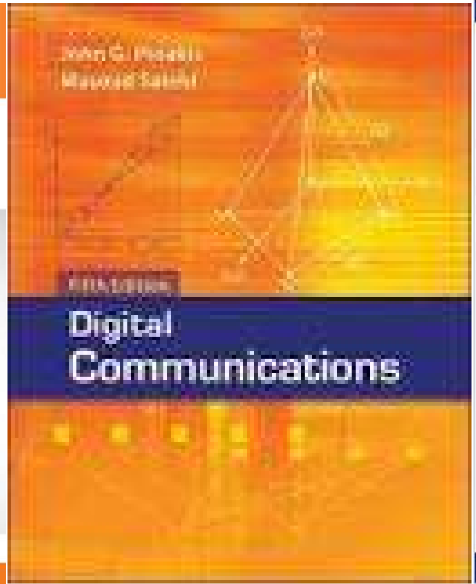
f_c--carrier frequency for modulation

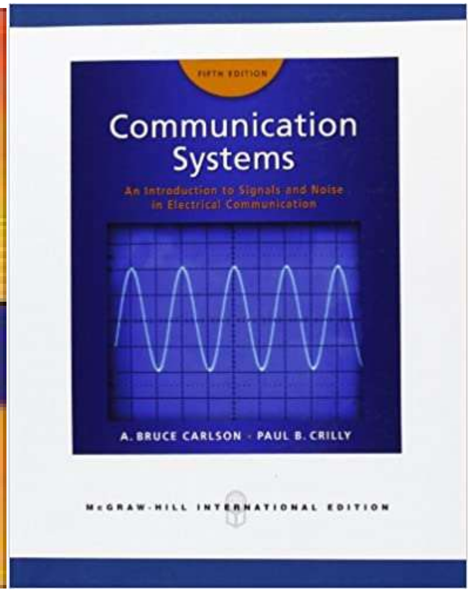
Read

- Chapter 2 in one of these…









… or a similar chapter on signals and systems basics in communications book – you can skim through if you feel confident about the material