Nonlinear Systems

Motivation for Simulating Nonlinear Systems

- In principle the performance of linear systems can be tackled by analytical means but the study of nonlinear systems by such means is by and large intractable

- Hence, simulation is generally the appropriate tool for most nonlinear com systems because simulation of such systems is no more difficult than for linear systems given the model

- **Note that** the transform methods for linear systems cannot strictly be applied since superposition does not hold! The nonlinear system, therefore, generally has to be simulated in the *time domain*

Modelling Considerations

- For nonlinear system models, a number of model-type descriptors are:

    1. ***Memoryless models***
    2. ***Models with memory***
    3. Baseband models
    4. Bandpass models
    5. Block (input/output) models
    6. Analytical models
    7. Nonlinear differential equation models

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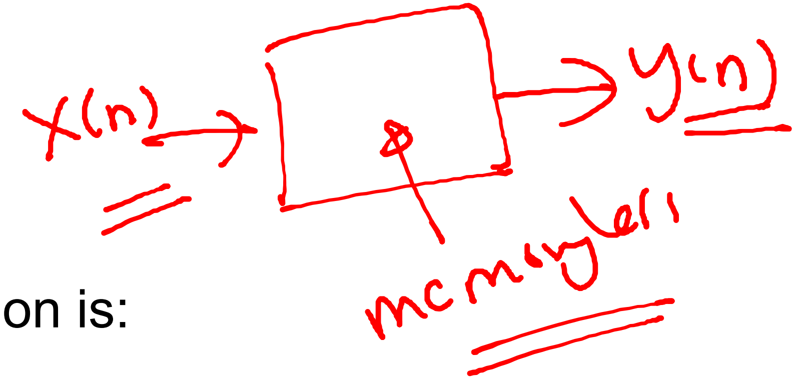
Modelling Considerations



- The *most significant* categorical distinction is:
  - Zero-memory nonlinearity (ZMNL)
  - Nonlinearity with memory (NLWM)

- The term “memoryless” implies that the output of a device is a function of the input signal at the present instance only!

- Memoryless models are an idealisation: **No** physical device is truly (input) frequency independent. Rather, as the BW of an input signal increases, we can expect filtering effects to become manifest

Memoryless Nonlinearities

- **Memoryless Baseband Nonlinearities** – This model is characterised by a simple functional relationship of the form

$$y(t) = F\left[x(t)\right]$$

- Our definition of a baseband nonlinearity means that x(t) is a baseband signal, meaning that its power (or energy) is spectrally concentrated around zero frequency and this will also be true of y(t)

- Certain nonlinearity can be given in analytical form, e.g., a diode

$$\boxed{I} = I_s\left(e^{\lambda V} - 1\right)$$

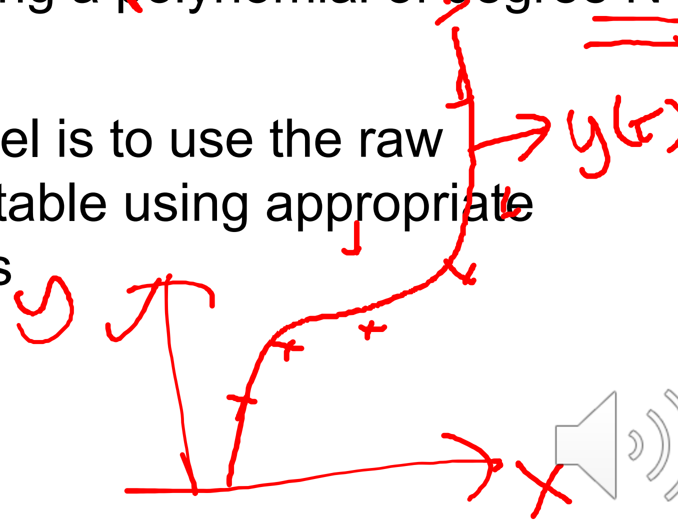
- The exponential can be expanded into a power series which could be truncated after a moderate number of terms with acceptable error

Memoryless Nonlinearities

- This suggests that F might also be representable by a power series, or by an orthogonal function expansion, or by a polynomial in x. Thus,

$$\boxed{y(t) = F\left[x(t)\right] \approx \sum_{n=0}^{N} a_n x^n(t)}$$

- The coefficients a_n may be obtained by fitting a polynomial of degree N
- Often, the most efficient and accurate model is to use the raw experimental data themselves in a lookup table using appropriate interpolation between tabulated data points



LTI

$f_s \geq 2W$

Memoryless Nonlinearities

$X(f) * X(f) \leftarrow \approx B$ $\quad f_s$?

- **Estimating the Sampling Rate for Nonlinear Systems** – Given a nonlinear model, the sampling rate in simulation must be increased

$X(f) \overset{n-1}{*} X(f)$

- Suppose that the input x(t) is bandlimited to ±B/2 and assume that the polynomial approximation on the previous slide holds. Then, we have

$x(t) \rightarrow X(f)$

$x^2(t) \Rightarrow X(f) * X(f)$

$$Y(f) = a_0\delta(f) + \sum_{n=1}^{N} a_n \left[X(f) \overset{n-1}{*} X(f) \right]$$

(n-1)-fold convolution

$X(f) * X(f) * X(f)$

- Hence, the term $x^n(t)$ has BW, nB (i.e., ±nB/2)

$y(t) = \sum_{n=0} a_n x^n(t)$

$\mathcal{F}\{x^n(t)\} = X(f) * X(f) * \cdots * X(f)$

$Y(f) = \sum_{n=0} a_n \mathcal{F}\{x^n(t)\}$

2 B no aliasing

Memoryless Nonlinearities

$2NB$ conservative estimate

- As we know, the sampling rate for any bandlimited system has to be at least twice the BW in order not to introduce aliasing distortion
- To avoid aliasing error for a nonlinear system, it would seem that we must have $\boxed{f_s > 2NB.}$ However,
  - For many nonlinear systems the coefficients a_n decrease with n
  - The magnitude of the spectrum of $x^n(t)$ is not uniformly distributed over nB. Indeed, as n increases, we generally expect the spectrum of $x^n(t)$ to be increasingly concentrated around zero frequency
- Hence if the sampling rate is less than 2NB, only the relatively low valued tails of the spectrum are aliased $\rightarrow$ error is small!