Introduction to Linear Circuit Analysis and Modelling

From DC to RF

MATLAB and SPICE Examples

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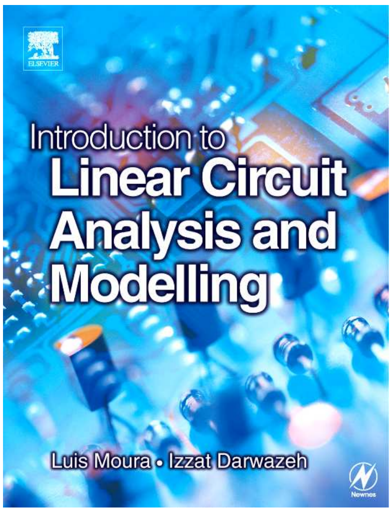

Introduction

MATLAB®[1] and OCTAVE[2] are numeric computation software packages which are used to solve engineering and scientific problems. SPICE is a general purpose circuit simulation program which originates from the University of California at Berkeley.

This manual contains numerous examples which make use of these software packages to study the key subjects discussed in each chapter of the book. Most of the examples are solved using both packages.

For the MATLAB solutions we show the analytical solution and its implementation as a MATLAB script (m-file). The MATLAB scripts allow the relevant numeric calculations. These scripts also valid for the OCTAVE software package.

For the SPICE solutions we provide the netlists of the circuits. This allows for the simulation of these circuits. All the netlists were written for the version 3f5.

We strongly recommend readers to get familiar with the subjects discussed in the book and the relevant analysis techniques prior to the study of the examples provided in this manual.

[1]MATLAB® is a registered trademark of the MathWorks, Inc. For MATLAB product information contact: The MathWorks, Inc., 3 Apple Drive, Natick, MA 01760-2098 USA.

[2]GNU OCTAVE is a Free Software. For more information write to the Free Software Foundation, 59 Temple Place - Suite 330, Boston, MA 02111-1307, USA.

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Chapter 1

Elementary electrical circuit analysis

1.1 Elementary circuits

Example 1.1 Determine the voltage at each node of the circuit of figure 1.1.

<ins>*Solution (using SPICE):*</ins>

```
* Circuit of figure 1.1

*----------------------netlist1----------------

V_1 1 0 dc 2

R_4 2 0 270
R_1 1 2 330
R_2 1 3 70
R_3 3 2 160
*---------------------------------------------

*---------------------------------------------

.dc V_1 0 2 2
.print dc v(1) v(2) v(3)
.end

*---------------------------------------------
```

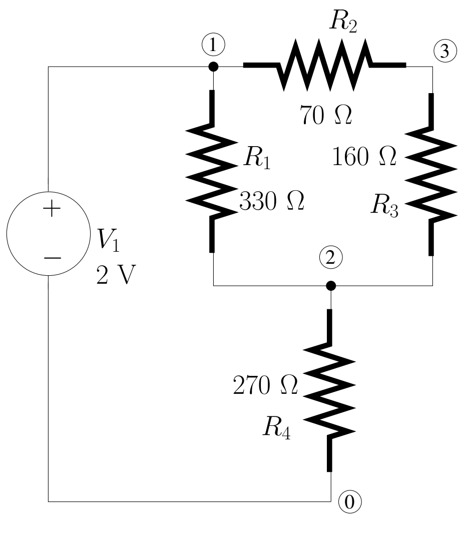


Figure 1.1: *DC circuit.*

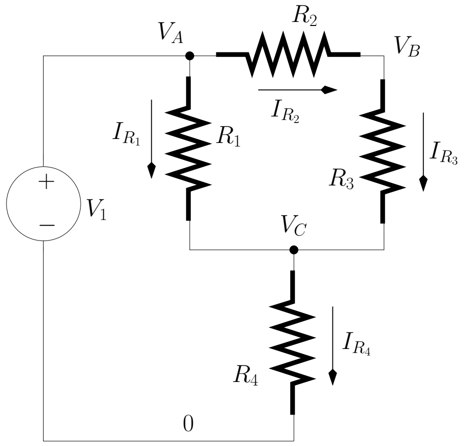


Figure 1.2: *DC circuit.*

Solution (using MATLAB/OCTAVE):

First, we write the following eqns (see also figure 1.2):

$$\begin{cases} I_{R_4} = I_{R_1} + I_{R_3} \\ I_{R_2} = I_{R_3} \\ V_A = V_1 \end{cases} \tag{1.1}$$

or

$$\begin{cases} \dfrac{V_C}{R_4} = \dfrac{V_1 - V_C}{R_1} + \dfrac{V_B - V_C}{R_3} \\ \\ \dfrac{V_1 - V_B}{R_2} = \dfrac{V_B - V_C}{R_3} \end{cases} \tag{1.2}$$

This set of eqns can be rewritten as follows:

$$\begin{cases} \dfrac{V_1}{R_1} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4}\right) V_C - \dfrac{V_B}{R_3} \\ \\ \dfrac{V_1}{R_2} = -\dfrac{V_C}{R_3} + \left(\dfrac{1}{R_3} + \dfrac{1}{R_2}\right) V_B \end{cases} \tag{1.3}$$

The last eqn can also be written in matrix form:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} \dfrac{V_1}{R_1} \\ \\ \dfrac{V_1}{R_2} \end{bmatrix} \tag{1.4}$$

$$[B] = \begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_3} + \dfrac{1}{R_4} & \dfrac{-1}{R_3} \\ \\ \dfrac{-1}{R_3} & \dfrac{1}{R_3} + \dfrac{1}{R_2} \end{bmatrix} \tag{1.5}$$

$$[C] = \begin{bmatrix} V_C \\ \\ V_B \end{bmatrix} \tag{1.6}$$

We can determine the unknown variables, V_B and V_C by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===========  mat_script1.m ===================
clear
V_1= 2;

R_1= 330;
R_2= 70;
R_3= 160;
R_4= 270;

B=[1/R_1+1/R_3+1/R_4 -1/R_3 ; ...
   -1/R_3 1/R_3+1/R_2]

A=[V_1/R_1; V_1/R_2]

C=inv(B)*A
%==============================================
```

`inv.m` is a built-in m-function which calculates the inverse of a matrix.

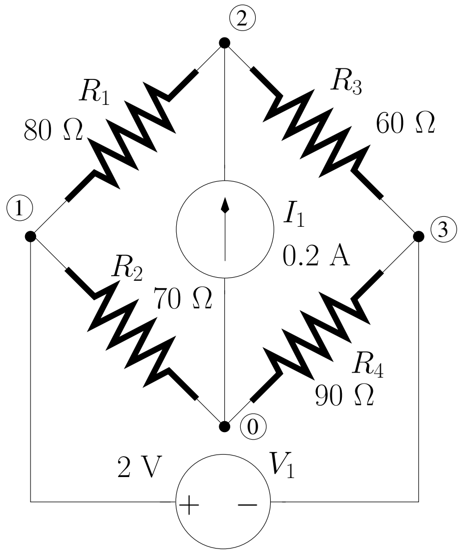


Figure 1.3: *DC circuit.*

Example 1.2 Determine the voltage at each node of the circuit of figure 1.3.

Solution (using SPICE):

```
* Circuit of figure 1.3

*-------------------netlist2-----------------

V_1 1 3 dc 2
I_1 0 2 dc 0.2

R_1 1 2 80
R_2 1 0 70
R_3 2 3 60
R_4 3 0 90
*-----------------------------------------

*-----------------------------------------

.dc V_1 0 2 2
.print dc v(1) v(2) v(3)
.end

*-----------------------------------------
```

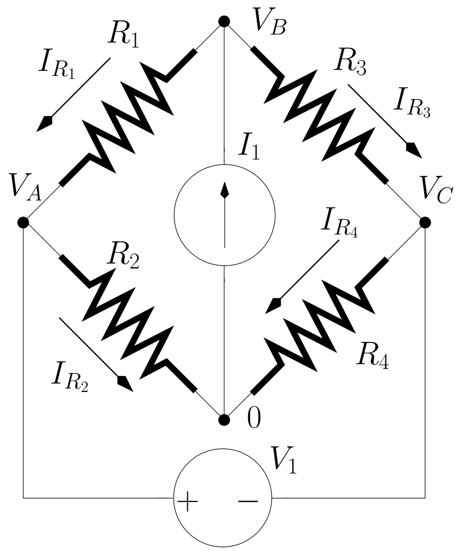


Figure 1.4: *DC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.4 we write the following set of eqns:

$$\begin{cases} I_1 = I_{R_1} + I_{R_3} \\ I_1 = I_{R_2} + I_{R_4} \\ V_1 = V_A - V_C \end{cases} \tag{1.7}$$

that is

$$\begin{cases} I_1 = \dfrac{V_B - V_A}{R_1} + \dfrac{V_B - V_C}{R_3} \\ \\ I_1 = \dfrac{V_A}{R_2} + \dfrac{V_C}{R_4} \\ \\ V_1 = V_A - V_C \end{cases} \tag{1.8}$$

The last eqn can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_1 \\ \\ I_1 \\ \\ V_1 \end{bmatrix} \tag{1.9}$$

$$[B] = \begin{bmatrix} \dfrac{-1}{R_1} & \dfrac{1}{R_3} + \dfrac{1}{R_1} & \dfrac{-1}{R_3} \\ \\ \dfrac{1}{R_2} & 0 & \dfrac{1}{R_4} \\ \\ 1 & 0 & -1 \end{bmatrix} \tag{1.10}$$

$$[C] = \begin{bmatrix} V_A \\ \\ V_B \\ \\ V_C \end{bmatrix} \tag{1.11}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===========  mat_script2.m ===================
clear
V_1= 2;
I_1= 0.2;
R_1= 80;
R_2= 70;
R_3= 60;
R_4= 90;

A=[I_1;I_1;V_1]
B=[-1/R_1 1/R_1+1/R_3 -1/R_3; ...
   1/R_2  0           1/R_4 ; ...
   1      0           -1        ]

C=inv(B)*A
%===============================================
```

Example 1.3 Determine the voltage at each node of the circuit of figure 1.5.

Solution (using SPICE):

```
* Circuit of figure 1.5

*-------------netlist3------------------------
V_1 2 1 dc 2
V_2 2 3 dc 3

R_1 1 0 90
R_2 2 0 680
R_3 4 0 180
R_4 3 4 220
*--------------------------------------------

*--------------------------------------------

.dc V_1 0 2 2
.print dc v(1) v(2) v(3) v(4)
.end

*--------------------------------------------
```

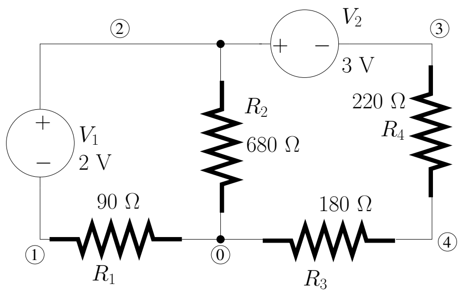


Figure 1.5: *DC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.6 we write the following set of eqns:

$$\begin{cases} I_{R_1} + I_{R_2} + I_{R_3} = 0 \\ I_{R_4} = I_{R_3} \\ V_B - V_A = V_1 \\ V_B - V_C = V_2 \end{cases} \tag{1.12}$$

This set of eqns can be written as:

$$\begin{cases} \dfrac{V_A}{R_1} + \dfrac{V_B}{R_2} + \dfrac{V_D}{R_3} = 0 \\[2ex] \dfrac{V_C - V_D}{R_4} = \dfrac{V_D}{R_3} \\[2ex] V_B - V_A = V_1 \\[2ex] V_B - V_C = V_2 \end{cases} \tag{1.13}$$

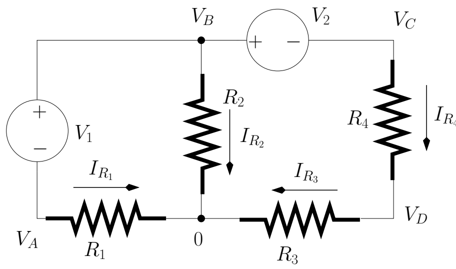


Figure 1.6: *DC circuit.*

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} 0 \\ 0 \\ V_1 \\ V_2 \end{bmatrix} \tag{1.14}$$

$$[B] = \begin{bmatrix} \frac{1}{R_1} & \frac{1}{R_3} & 0 & \frac{1}{R_3} \\[2ex] 0 & 0 & \frac{1}{R_4} & -\left(\frac{1}{R_3} + \frac{1}{R_4}\right) \\[2ex] -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \tag{1.15}$$

$$[C] = \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} \tag{1.16}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===========  mat_script3.m ===================
clear
V_1= 2
V_2= 3
R_1= 90
R_2= 680
R_3= 180
R_4= 220

A=[0;0;V_1;V_2];

B=[1/R_1 1/R_2 0     1/R_3        ;...
```

```
    0      0      1/R_4  -1/R_3-1/R_4; ...
    -1     1      0      0           ; ...
    0      1      -1     0           ]
C=inv(B)*A
%===============================================
```

1.2 Equivalent resistance

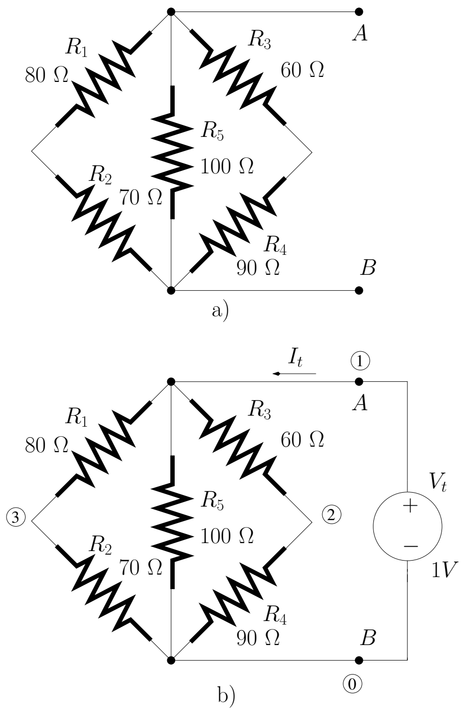


Figure 1.7: *a) Resistive circuit. b) Calculation of its equivalent resistance between points A and B.*

Example 1.4 Determine the equivalent resistance of the circuit of figure 1.7 a) between points A and B.

Solution (using SPICE):

```
* Circuit of figure 1.7 b)

*------------------netlist4--------------------
V_t 1 0 dc 1

R_1 1 3 80
R_2 3 0 70
R_3 1 2 60
R_4 2 0 90
R_5 1 0 100

*---------------------------------------------

*---------------------------------------------

.dc V_t 0 1 1
.print dc i(V_t)
.end

*---------------------------------------------
```

Note that a voltage source of 1 V is applied to the resistive circuit between points A and B. We can determine the equivalent resistance by first calculating the current provided by V_t, $I_t = -$i(V_t)[1]. Then, the equivalent resistance is calculated as indicated below:

$$R_{eq} = \frac{1}{I_t} \ (\Omega)$$

[1]Note that, in SPICE, the positive current is assumed to flow from the positive pole, through the source, to the negative pole.

Solution (using MATLAB/OCTAVE):

The equivalent resistance of the circuit of figure 1.7 a) can be determined after recognising that $R_1 + R_2$ is connected in parallel with R_5 and also with $R_3 + R_4$.

```
%===========  mat_script4.m ===================
clear

R_1= 80;
R_2= 70;
R_3= 60;
R_4= 90;
R_5= 100;

Req_x= parallel( R_1+R_2 , R_5)
Req= parallel( R_3+R_4 , Req_x)
%==============================================
```

In this script `parallel.m` is an m-function which calculates the equivalent resistance of a parallel combination of two resistances; R_1 and R_2.

```
%===========  parallel.m ===================
function R_eq=parallel(R1,R2)
%
%   function R_eq=parallel(R1,R2)
%
R_eq=(R1.*R2)./(R1+R2);

%==============================================
```

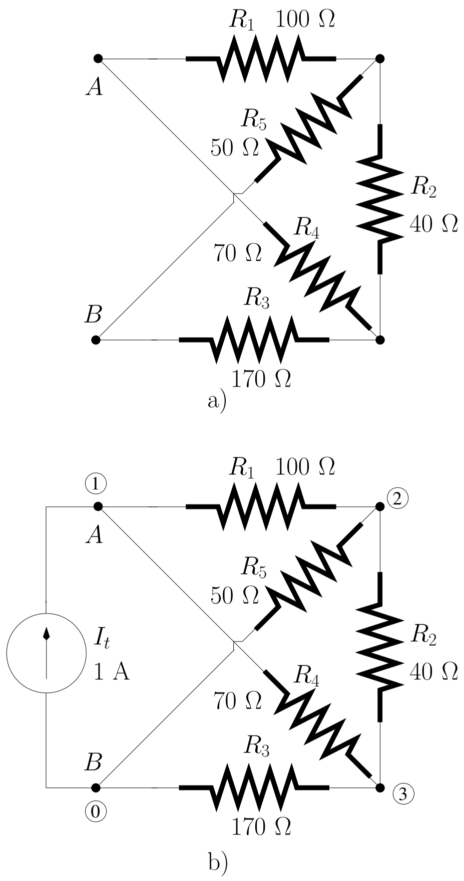


Figure 1.8: *a) Resistive circuit b) Calculation of the equivalent resistance between points A and B.*

Example 1.5 Determine the equivalent resistance of the circuit of figure 1.8 a) between points A and B.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 1.8 b)

*--------------------netlist5------------------
I_t 0 1 dc 1

R_1 1 2 100
R_2 2 3 40
R_3 3 0 170
R_4 1 3 70
R_5 2 0 50

*----------------------------------------------

*----------------------------------------------

.dc I_t 0 1 1
.print dc v(1)
.end

*----------------------------------------------
```

Note that a current source of 1 A is applied to the resistive circuit between points A and B. We can determine the equivalent resistance by calculating the voltage across the source, V_t. Then, the equivalent resistance is simply

$$R_{eq} \quad = \quad \frac{V_t}{1} \ (\Omega)$$

with V_t =`v(1)`.

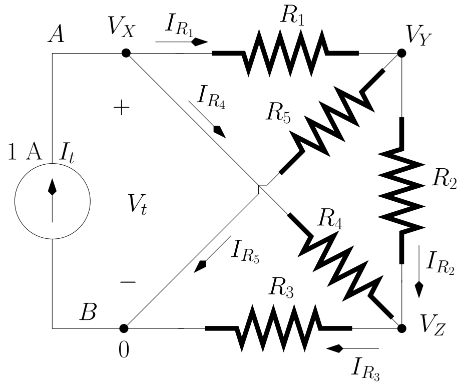


Figure 1.9: *Calculation of the equivalent resistance between nodes* A *and* B.

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.9 we can write

$$\begin{cases} I_t = I_{R_1} + I_{R_4} \\ I_t = I_{R_5} + I_{R_3} \\ I_{R_1} = I_{R_5} + I_{R_2} \\ V_X = V_t \end{cases} \tag{1.17}$$

that is,

$$\begin{cases} I_t = \dfrac{V_t - V_Y}{R_1} + \dfrac{V_t - V_Z}{R_4} \\[2ex] I_t = \dfrac{V_Y}{R_5} + \dfrac{V_Z}{R_3} \\[2ex] \dfrac{V_t - V_Y}{R_1} = \dfrac{V_Y}{R_5} + \dfrac{V_Y - V_Z}{R_2} \end{cases} \tag{1.18}$$

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_t \\ I_t \\ 0 \end{bmatrix} \tag{1.19}$$

$$[B] = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_4} & -\frac{1}{R_1} & -\frac{1}{R_4} \\[2ex] 0 & \frac{1}{R_5} & \frac{1}{R_3} \\[2ex] -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_2} \end{bmatrix} \tag{1.20}$$

$$[C] = \begin{bmatrix} V_t \\ V_Y \\ V_Z \end{bmatrix} \tag{1.21}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

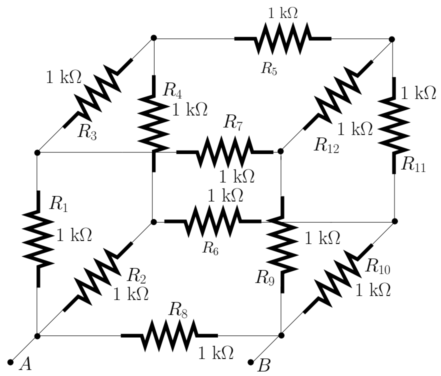
The equivalent resistance is:

$$R_{eq} = \frac{V_t}{1}\ \Omega$$

```
%===========  mat_script5.m ===================
clear

I_t= 1

R_1= 100
R_2= 40
R_3= 170
R_4= 70
R_5= 50

A=[I_t; I_t; 0]
B=[1/R_1+1/R_4 -1/R_1        -1/R_4; ...
```

```
   0            1/R_5         1/R_3 ; ...
   -1/R_1  1/R_1+1/R_2+1/R_5 -1/R_2    ]

C=inv(B)*A
Req=C(1)

%===============================================
```

Example 1.6 Determine the equivalent resistance of the circuit of figure 1.10 a) between points A and B.



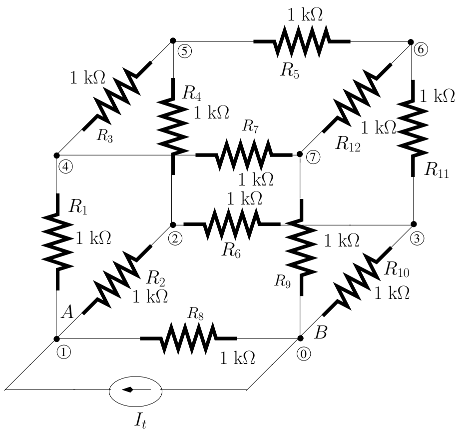


Figure 1.10: *a) Resistive circuit. b) Calculation of the equivalent resistance between points A and B.*

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 1.10 b)
*----------------------netlist5b---------------

I_t 0 1 dc 1

R_1  1 4 1k
R_2  1 2 1k
R_3  4 5 1k
R_4  2 5 1k
R_5  5 6 1k
R_6  2 3 1k
R_7  4 7 1k
R_8  1 0 1k
R_9  7 0 1k
R_10 3 0 1k
R_11 6 3 1k
R_12 6 7 1k

*---------------------------------------------

*---------------------------------------------
.dc I_t 0 1 1
.print dc v(1)
.end

*---------------------------------------------
```

The resistance between points A and B is equal to

$$R_{eq} = \frac{V_t}{1}\ \Omega$$

where V_t is the voltage across the current source, that is $V_t =$`v(1)`.

<u>*Solution (using MATLAB/OCTAVE):*</u>

For the circuit of 1.11 we can write

$$\begin{cases} I_t = I_a + I_b + I_c \\ I_t = I_b + I_k + I_l \\ I_a = I_e + I_d \\ I_d = I_f + I_m \\ I_m = I_j + I_h \\ I_k = I_g + I_h \\ I_l = I_e + I_j \end{cases} \tag{1.22}$$

Since all resistances are equal, we can write the previous set of eqns as follows:

$$\begin{cases} I_t = \dfrac{V_1 - V_4}{R} + \dfrac{V_1}{R} + \dfrac{V_1 - V_2}{R} \\ I_t = \dfrac{V_1}{R} + \dfrac{V_3}{R} + \dfrac{V_7}{R} \\ \dfrac{V_1 - V_4}{R} = \dfrac{V_4 - V_7}{R} + \dfrac{V_4 - V_5}{R} \\ \dfrac{V_4 - V_5}{R} = \dfrac{V_5 - V_2}{R} + \dfrac{V_5 - V_6}{R} \\ \dfrac{V_5 - V_6}{R} = \dfrac{V_6 - V_7}{R} + \dfrac{V_6 - V_3}{R} \\ \dfrac{V_3}{R} = \dfrac{V_2 - V_3}{R} + \dfrac{V_6 - V_3}{R} \\ \dfrac{V_7}{R} = \dfrac{V_4 - V_7}{R} + \dfrac{V_6 - V_7}{R} \end{cases} \tag{1.23}$$

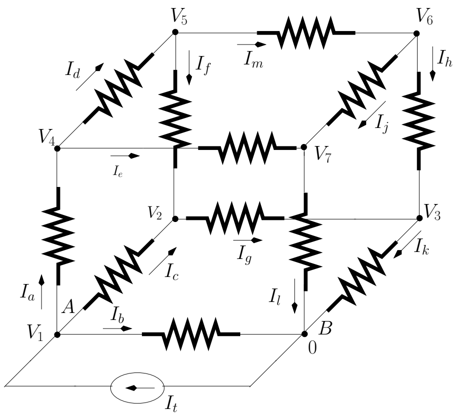


Figure 1.11: *Calculation of the equivalent resistance between points A and B.*

We can express these eqns in matrix form:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_t \\ I_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1.24}$$

$$[B] = \begin{bmatrix} \frac{3}{R} & -\frac{1}{R} & 0 & -\frac{1}{R} & 0 & 0 & 0 \\ \frac{1}{R} & 0 & \frac{1}{R} & 0 & 0 & 0 & \frac{1}{R} \\ -\frac{1}{R} & 0 & 0 & \frac{3}{R} & -\frac{1}{R} & 0 & -\frac{1}{R} \\ 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{3}{R} & 0 & -\frac{1}{R} \\ 0 & 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{3}{R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{3}{R} & 0 & 0 & -\frac{1}{R} & 0 \\ 0 & 0 & 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{3}{R} \end{bmatrix} \tag{1.25}$$

$$[C] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} \tag{1.26}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

The equivalent resistance is:

$$R_{eq} = \frac{V_1}{1} \; \Omega$$

```
%===========  mat_script5b.m ===================
clear

I_t= 1

R= 1000

A=[I_t; I_t; 0;  0;  0;  0;  0]

B=[3/R -1/R 0 -1/R  0  0  0   ;...
   1/R  0  1/R 0  0  0  1/R   ;...
  -1/R  0  0  3/R -1/R 0 -1/R ;...
   0 -1/R  0  -1/R 3/R 0 -1/R ;...
   0  0  -1/R 0 -1/R 3/R -1/R ;...
   0 -1/R  3/R 0 0 -1/R  0    ;...
   0  0  0 -1/R 0 -1/R 3/R     ]

C=inv(B)*A

Req=C(1)
%===============================================
```

1.3 Circuits containing controlled sources

Example 1.7 Determine the voltage at each node of the circuit of figure 1.12 a).

Solution (using SPICE):

```
* Circuit of figure 1.12 b)
*---------------------netlist6----------------

I_1 0 1 dc 0.25

R_2 1 2 45
R_3 3 0 70
R_1 1 10 90
Vsens 0 10 dc 0
F_Ai 3 2 Vsens 12
*---------------------------------------------

*---------------------------------------------

.dc I_1 0 0.25 0.25
.print dc v(1) v(2) v(3)
.end

*---------------------------------------------
```

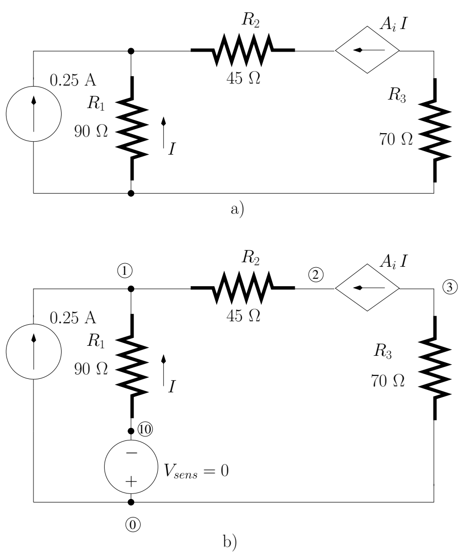


Figure 1.12: *a) DC circuit with current controlled-current source. b) Equivalent circuit.*

Note the inclusion of a voltage source $V_{sens} = 0$ which represents a short-circuit and does not influence the behaviour of the circuit. This source is necessary for the SPICE simulation since it allows us to identify the current I that controls the current controlled-current source.

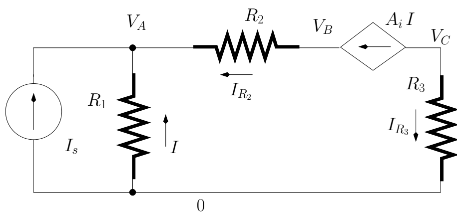


Figure 1.13: *DC circuit with current controlled-current source.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.13 we can write the following set of equations indicated below:

$$\begin{cases} I_s + I + I_{R_2} = 0 \\ I_{R_2} = A_i\, I \\ I = -\frac{V_A}{R_1} \\ I_{R_3} = -A_i\, I \end{cases} \tag{1.27}$$

that is

$$\begin{cases} I_s + I + \frac{V_B - V_A}{R_2} = 0 \\ \frac{V_B - V_A}{R_2} = A_i\, I \\ I = -\frac{V_A}{R_1} \\ \frac{V_C}{R_3} = -A_i\, I \end{cases} \tag{1.28}$$

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1.29}$$

$$[B] = \begin{bmatrix} \frac{1}{R_2} & -\frac{1}{R_2} & 0 & -1 \\ -\frac{1}{R_2} & \frac{1}{R_2} & 0 & -A_i \\ \frac{1}{R_1} & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{R_3} & A_i \end{bmatrix} \tag{1.30}$$

$$[C] = \begin{bmatrix} V_A \\ V_B \\ V_C \\ I \end{bmatrix} \tag{1.31}$$

We can determine the unknown variables by solving the following eqn using MATLAB or OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===========  mat_scrip6.m ===================
clear

I_s= 0.25;

R_2= 45;
R_3= 70;
R_1= 90;
```

```
A_i= 12;

A=[I_s;0;0;0]

B=[1/R_2 -1/R_2 0      -1  ;...
   -1/R_2 1/R_2 0      -A_i;...
   1/R_1  0     0      1   ;...
   0      0     1/R_3 A_i ];

C=inv(B)*A

%===============================================
```

Example 1.8 Determine the voltage at each node of the circuit of figure 1.14.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 1.14

*----------------------netlist7----------------

V_1 1 0 dc 5

R_1 1 2 68
R_2 2 0 10
R_3 3 0 100

G_Gm 0 3 2 0 0.5

*---------------------------------------------

*---------------------------------------------

.dc V_1 0 5 5
.print dc v(1) v(2) v(3)
.end

*---------------------------------------------
```

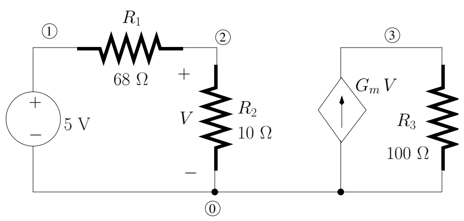


Figure 1.14: *DC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.15 we can write:

$$\begin{cases} V = V_B \\ \dfrac{V_C}{R_3} = G_m\, V \\ V = V_s \dfrac{R_2}{R_2 + R_1} \end{cases} \tag{1.32}$$

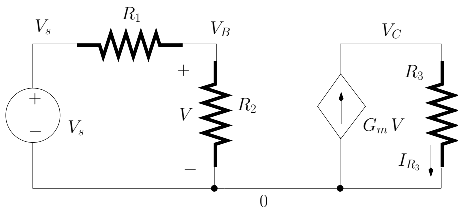


Figure 1.15: *DC circuit.*

that is,

$$\begin{aligned} V_B &= V_s \frac{R_2}{R_2 + R_1} \\ V_C &= R_3\, G_m\, V_s \frac{R_2}{R_2 + R_1} \end{aligned}$$

```
%===========  mat_script7.m ===================
clear

V_s= 5

R_1= 68
R_2= 10
R_3= 100
G_m= 0.5

V_B=V_s*R_2/(R_2+R_1)
V_C=R_3*G_m*V_s*R_2/(R_2+R_1)

%==============================================
```

Example 1.9 Determine the voltage at each node of the circuit of figure 1.16.

Solution (using SPICE):

```
* Circuit of figure 1.16
*------------------netlist8--------------------
V_1 1 0 dc 5

R_1 1 2 80
R_2 2 0 90
R_3 3 4 10
R_4 4 0 57

E_Av 2 3 2 0 10

*---------------------------------------------
*---------------------------------------------
.dc V_1 0 5 5
.print dc v(1) v(2) v(3) v(4)
.end
*---------------------------------------------
```

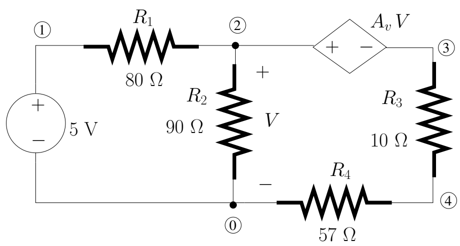


Figure 1.16: *DC circuit.*

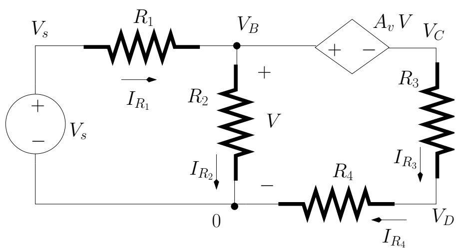


Figure 1.17: *DC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.17 we can write:

$$\begin{cases} I_{R_1} = I_{R_2} + I_{R_4} \\ V_B - V_C = A_v V_B \\ I_{R_1} = I_{R_2} + I_{R_3} \end{cases} \tag{1.33}$$

that is

$$\begin{cases} \dfrac{V_s - V_B}{R_1} = \dfrac{V_B}{R_2} + \dfrac{V_D}{R_4} \\ V_B - V_C = A_v V_B \\ \dfrac{V_s - V_B}{R_1} = \dfrac{V_B}{R_2} + \dfrac{V_C - V_D}{R_3} \end{cases} \tag{1.34}$$

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} \dfrac{V_s}{R_1} \\ 0 \\ \dfrac{V_s}{R_1} \end{bmatrix} \tag{1.35}$$

$$[B] = \begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & 0 & \dfrac{1}{R_4} \\ A_v - 1 & 1 & 0 \\ \dfrac{1}{R_1} + \dfrac{1}{R_2} & \dfrac{1}{R_3} & -\dfrac{1}{R_3} \end{bmatrix} \tag{1.36}$$

$$[C] = \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} \tag{1.37}$$

We can determine the unknown variables by solving the following eqn using MATLAB or OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===========  mat_script8.m ===================
clear

V_s= 5
R_1= 80
R_2= 90
R_3= 10
R_4= 57
A_v = 10

A=[V_s/R_1 ; 0 ; V_s/R_1];
B=[(1/R_1+1/R_2) 0 1/R_4  ;...
   (A_v-1)       1  0     ;...
   (1/R_1+1/R_2) 1/R_3 -1/R_3];

C=inv(B)*A
%===============================================
```

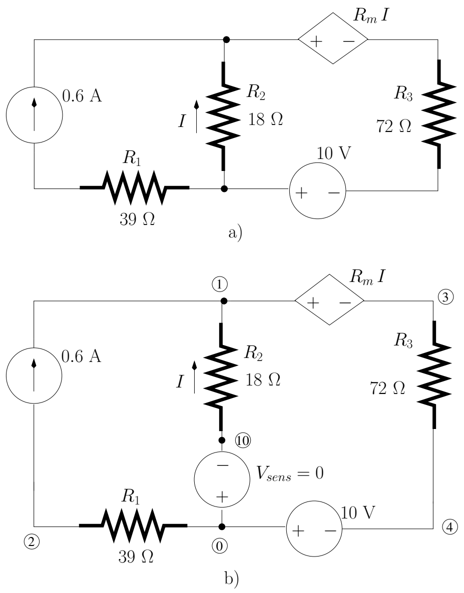


Figure 1.18: *a) DC circuit. b) Equivalent circuit*

Example 1.10 Determine the voltage at each node of the circuit of figure 1.18.

Solution (using SPICE):

```
* Circuit of figure 1.18 b)
*--------------------netlist9------------------
I_1 2 1 dc 0.6
V_1 0 4 dc 10

R_1 2 0 39
R_3 3 4 72

R_2 1 10 18
Vsens 0 10 dc 0
H_Rm 1 3 Vsens 40

*----------------------------------------------
*----------------------------------------------
.dc V_1 0 10 10
.print dc v(1) v(2) v(3) v(4)
.end
*----------------------------------------------
```

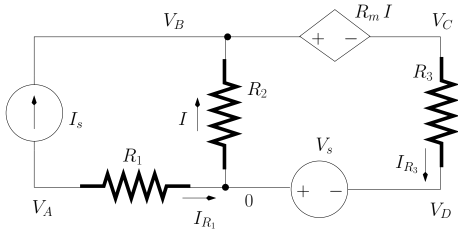


Figure 1.19: *DC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 1.19 we can write;

$$\begin{cases} V_B - V_C = R_m \dfrac{-V_B}{R_2} \\ V_D = -V_s \\ \dfrac{V_A}{R_1} + \dfrac{V_C - V_D}{R_3} = \dfrac{-V_B}{R_2} \\ \dfrac{V_A}{R_1} = -I_s \end{cases} \tag{1.38}$$

This eqn can be written is matrix form as follows

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} 0 \\ V_s \\ 0 \\ I_s \end{bmatrix} \tag{1.39}$$

$$[B] = \begin{bmatrix} 0 & \left(1 + \dfrac{R_M}{R_2}\right) & -1 & 0 \\ 0 & 0 & 0 & -1 \\ \dfrac{1}{R_1} & \dfrac{1}{R_2} & \dfrac{1}{R_3} & -\dfrac{1}{R_3} \\ -\dfrac{1}{R_1} & 0 & 0 & 0 \end{bmatrix} \tag{1.40}$$

$$[C] = \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} \tag{1.41}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

```
%===========  mat_script9.m ===================
clear

I_s= 0.6
V_s= 10
R_1= 39
R_3= 72
R_2= 18
R_m= 40

A=[0 ; V_s ; 0 ; I_s]
B=[0 (1+R_m/R_2) -1      0     ;...
   0 0            0     -1     ;...
   1/R_1 1/R_2    1/R_3 -1/R_3 ;...
  -1/R_1 0        0      0      ];
C=inv(B)*A
%==============================================
```

1.4 Electrical network theorems

1.4.1 Thévenin theorem

Example 1.11 Consider the circuit of figure 1.20. Determine the Thévenin equivalent circuit between nodes A and B.

Solution (using SPICE):

The following netlist allows us to obtain the Thévenin voltage which is `v(1)-v(7)`.

```
* Circuit of figure 1.20

*---------------------netlist10----------------

V_S1 4 3 dc 5
V_S2 2 1 dc 2
I_S  0 5 dc 1e-3

R_1 1 7 1k
R_2 6 7 1k
R_3 5 6 2k
R_4 4 5 4k
R_5 3 2 4k
R_6 6 0 1k
R_7 4 0 3k
R_8 2 0 4k
R_9 7 0 1k
*---------------------------------------------

*---------------------------------------------

.dc V_S1 0 5 5
.print dc v(1) v(7)
.end

*---------------------------------------------
```

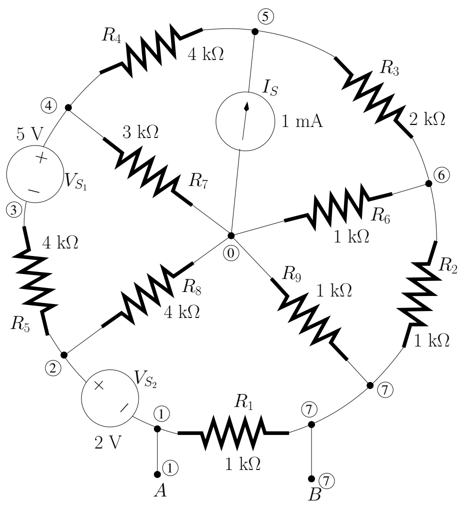


Figure 1.20: *Electrical network.*

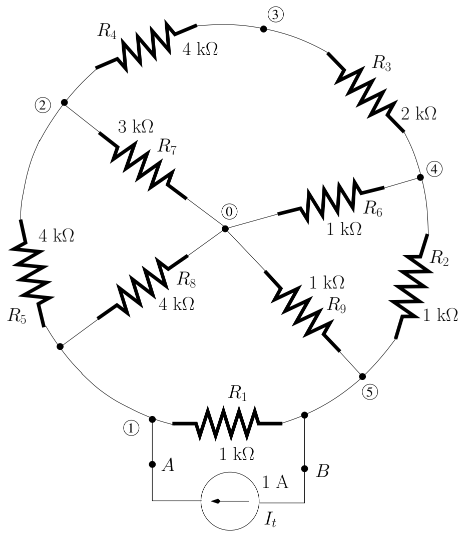


Figure 1.21: *Circuit for the calculation of* R_{Th}.

The following netlist allows us to obtain the Thévenin resistance which is

$$R_{Th} = \frac{\texttt{v(1)} - \texttt{v(5)}}{1} \quad \Omega$$

```
* Circuit of figure 1.21

*---------------------netlist11----------------

I_t  5 1 dc 1

R_1 1 5 1k
R_2 5 4 1k
R_3 3 4 2k
R_4 2 3 4k
R_5 2 1 4k
R_6 4 0 1k
R_7 2 0 3k
R_8 1 0 4k
R_9 5 0 1k
*---------------------------------------------

*---------------------------------------------

.dc I_t 0 1 1
.print dc v(1) v(5)
.end

*---------------------------------------------
```

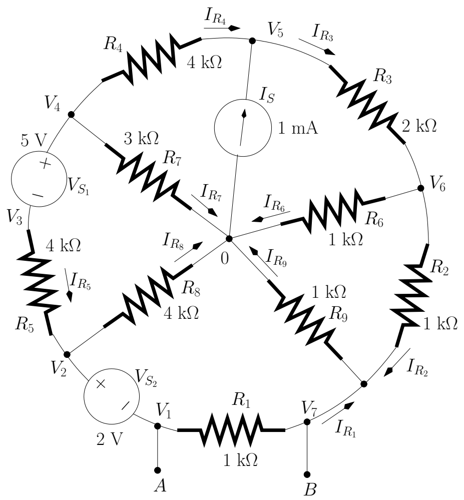


Figure 1.22: *Equivalent circuit for the calculation of the Thévenin voltage.*

<u>*Solution (using MATLAB/OCTAVE):*</u>

First we determine the Thévenin voltage, V_{Th}, of the circuit of figure 1.22. The Thévenin voltage is $V_1 - V_7$. For this circuit we can write:

$$\begin{cases} V_{S_1} = V_4 - V_3 \\ V_{S_2} = V_2 - V_1 \\ I_{R_9} = I_{R_1} + I_{R_2} \\ I_{R_3} = I_{R_6} + I_{R_2} \\ I_{R_3} = I_S + I_{R_4} \\ I_{R_5} + I_{R_7} + I_{R_4} = 0 \\ I_{R_5} = I_{R_1} + I_{R_8} \end{cases} \tag{1.42}$$

or

$$\begin{cases} V_{S_1} = V_4 - V_3 \\ V_{S_2} = V_2 - V_1 \\ \dfrac{V_7}{R_9} = \dfrac{V_1 - V_7}{R_1} + \dfrac{V_6 - V_7}{R_2} \\ \dfrac{V_5 - V_6}{R_3} = \dfrac{V_6}{R_6} + \dfrac{V_6 - V_7}{R_2} \\ \dfrac{V_5 - V_6}{R_3} = I_S + \dfrac{V_4 - V_5}{R_4} \\ \dfrac{V_3 - V_2}{R_5} + \dfrac{V_4}{R_7} + \dfrac{V_4 - V_5}{R_4} = 0 \\ \dfrac{V_3 - V_2}{R_5} = \dfrac{V_1 - V_7}{R_1} + \dfrac{V_2}{R_8} \end{cases} \tag{1.43}$$

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} V_{S_1} \\ V_{S_2} \\ 0 \\ 0 \\ I_S \\ 0 \\ 0 \end{bmatrix} \tag{1.44}$$

$$[B] = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & 0 & 0 & 0 & 0 & -\frac{1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_9} \\ 0 & 0 & 0 & 0 & -\frac{1}{R_3} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6} & -\frac{1}{R_2} \\ 0 & 0 & 0 & -\frac{1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_3} & 0 \\ 0 & -\frac{1}{R_5} & \frac{1}{R_5} & \frac{1}{R_4} + \frac{1}{R_7} & -\frac{1}{R_4} & 0 & 0 \\ \frac{1}{R_1} & \frac{1}{R_5} + \frac{1}{R_8} & -\frac{1}{R_5} & 0 & 0 & 0 & -\frac{1}{R_1} \end{bmatrix} \tag{1.45}$$

$$[C] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} \tag{1.46}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

```
%================  mat_script10.m  ============
clear

V_S1 = 5;
V_S2 = 2;
I_S  = 1e-3;

R_1 = 1e3;
R_2 = 1e3;
R_3 = 2e3;
R_4 = 4e3;
R_5 = 4e3;
R_6 = 1e3;
R_7 = 3e3;
R_8 = 4e3;
R_9 = 1e3;

A=[V_S1;V_S2;0;0;I_S;0;0];

B=[0 0 -1 1 0 0 0;
   -1 1 0 0 0 0 0 ;
   -1/R_1 0 0 0 0 -1/R_2 1/R_1+1/R_2+1/R_9;
   0 0 0 0 -1/R_3 1/R_2+1/R_3+1/R_6 -1/R_2;
   0 0 0 -1/R_4 1/R_3+1/R_4 -1/R_3 0;
   0 -1/R_5 1/R_5 1/R_4+1/R_7 -1/R_4 0 0;
   1/R_1 1/R_5+1/R_8 -1/R_5 0 0 0 -1/R_1 ];

C=inv(B)*A;

V_Th=C(1)-C(7)
%==============================================
```

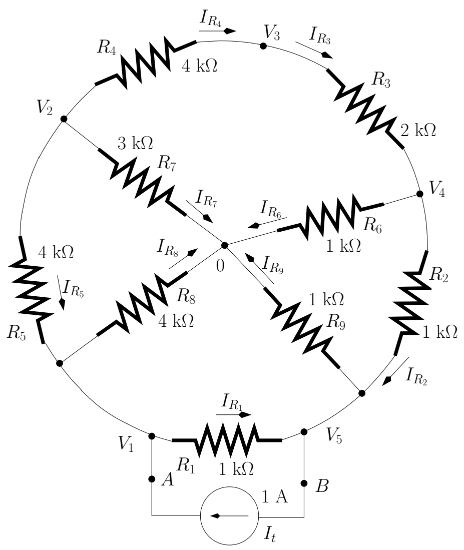


Figure 1.23: *Equivalent circuit for the calculation of the Thévenin resistance.*

Figure 1.23 shows the equivalent circuit for the calculation of the Thévenin resistance. We can write:

$$\begin{cases} I_t + I_{R_9} = I_{R_1} + I_{R_2} \\ I_t + I_{R_5} = I_{R_1} + I_{R_8} \\ I_{R_5} + I_{R_7} + I_{R_4} = 0 \\ I_{R_4} = I_{R_6} + I_{R_2} \end{cases} \tag{1.47}$$

Note that $I_{R_3} = I_{R_4}$ and that R_3 is connected in series with R_4. Hence we can write $R_{3,4} = R_3 + R_4$. Now, the set of eqns given by 1.47 can be written as

$$\begin{cases} I_t + \dfrac{V_5}{R_9} = \dfrac{V_1 - V_5}{R_1} + \dfrac{V_4 - V_5}{R_2} \\[2ex] I_t + \dfrac{V_2 - V_1}{R_5} = \dfrac{V_1 - V_5}{R_1} + \dfrac{V_1}{R_8} \\[2ex] \dfrac{V_2 - V_1}{R_5} + \dfrac{V_2}{R_7} + \dfrac{V_2 - V_4}{R_{3,4}} = 0 \\[2ex] \dfrac{V_2 - V_4}{R_{3,4}} = \dfrac{V_4}{R_6} + \dfrac{V_4 - V_5}{R_2} \end{cases} \tag{1.48}$$

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_t \\ I_t \\ 0 \\ 0 \end{bmatrix} \tag{1.49}$$

$$[B] = \begin{bmatrix} \dfrac{1}{R_1} & 0 & \dfrac{1}{R_2} & -\dfrac{1}{R_1} - \dfrac{1}{R_2} - \dfrac{1}{R_9} \\[2ex] \dfrac{1}{R_1} + \dfrac{1}{R_5} + \dfrac{1}{R_8} & -\dfrac{1}{R_5} & 0 & -\dfrac{1}{R_1} \\[2ex] -\dfrac{1}{R_5} & \dfrac{1}{R_{3,4}} + \dfrac{1}{R_5} + \dfrac{1}{R_7} & -\dfrac{1}{R_{3,4}} & 0 \\[2ex] 0 & -\dfrac{1}{R_{3,4}} & \dfrac{1}{R_{3,4}} + \dfrac{1}{R_2} + \dfrac{1}{R_6} & -\dfrac{1}{R_2} \end{bmatrix} \tag{1.50}$$

$$[C] = \begin{bmatrix} V_1 \\ V_2 \\ V_4 \\ V_5 \end{bmatrix} \tag{1.51}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

and the Thévenin resistance is:

$$R_{Th} \;\; = \;\; \frac{V_1 - V_5}{1} \;\; \Omega$$

```
%================  mat_script11.m  ===========
clear

I_t  = 1;
```

```
R_1 = 1e3;
R_2 = 1e3;
R_3 = 2e3;
R_4 = 4e3;
R_5 = 4e3;
R_6 = 1e3;
R_7 = 3e3;
R_8 = 4e3;
R_9 = 1e3;

R_34=R_3+R_4;


A=[I_t;I_t;0;0];

B=[1/R_1 0 1/R_2 -(1/R_1+1/R_2+1/R_9);
  1/R_1+1/R_5+1/R_8 -1/R_5 0 -1/R_1;
  -1/R_5 1/R_34+1/R_5+1/R_7 -1/R_34 0;
  0 -1/R_34 1/R_34+1/R_2+1/R_6 -1/R_2];

C=inv(B)*A;

R_Th=C(1)-C(4)

%===============================================
```

1.4.2 Norton theorem

Example 1.12 Consider the circuit of figure 1.20. Determine the Norton equivalent circuit between nodes A and B.

<u>*Solution (using SPICE):*</u>

The Norton resistance is equal to the Thévenin resistance which was calculated in the previous example. The following netlist allows us to obtain the Norton current.

```
* Circuit of figure 1.24

*----------------------netlist12----------------
V_S1 4 3 dc 5
V_S2 2 1 dc 2
I_S  0 5 dc 1e-3

R_2 6 1 1k
R_3 5 6 2k
R_4 4 5 4k
R_5 3 2 4k
R_6 6 0 1k
R_7 4 0 3k
R_8 2 0 4k
R_9 1 0 1k
*---------------------------------------------

*---------------------------------------------

.dc V_S1 0 5 5
.print dc i(V_S2)
.end

*---------------------------------------------
```

The Norton current, I_{Nt}, is equal to i(V_S2).

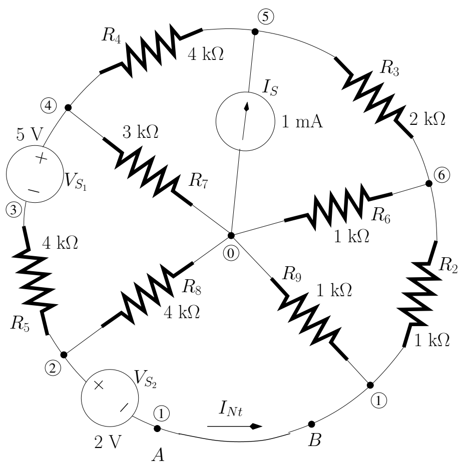


Figure 1.24: *Equivalent circuit for the calculation of the Norton current.*

<u>*Solution (using MATLAB/OCTAVE):*</u>

We determine the Norton current, I_{Nt}, of the circuit of figure 1.25. For this circuit we can write:

$$\begin{cases} V_{S_1} = V_4 - V_3 \\ V_{S_2} = V_2 - V_1 \\ I_{R_9} = I_{Nt} + I_{R_2} \\ I_{R_3} = I_{R_6} + I_{R_2} \\ I_{R_3} = I_S + I_{R_4} \\ I_{R_5} + I_{R_7} + I_{R_4} = 0 \\ I_{R_5} = I_{Nt} + I_{R_8} \end{cases} \tag{1.52}$$

or

$$\begin{cases} V_{S_1} = V_4 - V_3 \\ V_{S_2} = V_2 - V_1 \\ \dfrac{V_1}{R_9} = I_{Nt} + \dfrac{V_6 - V_1}{R_2} \\ \dfrac{V_5 - V_6}{R_3} = \dfrac{V_6}{R_6} + \dfrac{V_6 - V_1}{R_2} \\ \dfrac{V_5 - V_6}{R_3} = I_S + \dfrac{V_4 - V_5}{R_4} \\ \dfrac{V_3 - V_2}{R_5} + \dfrac{V_4}{R_7} + \dfrac{V_4 - V_5}{R_4} = 0 \\ \dfrac{V_3 - V_2}{R_5} = I_{Nt} + \dfrac{V_2}{R_8} \end{cases} \tag{1.53}$$

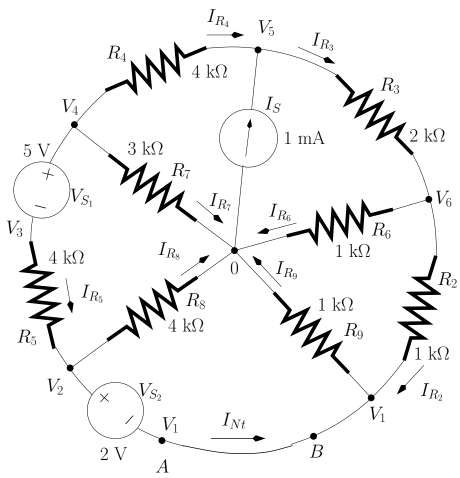


Figure 1.25: *Equivalent circuit for the calculation of the Norton current.*

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} V_{S_1} \\ V_{S_2} \\ 0 \\ 0 \\ I_S \\ 0 \\ 0 \end{bmatrix} \tag{1.54}$$

$$[B] = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_2} - \frac{1}{R_9} & 0 & 0 & 0 & 0 & \frac{1}{R_2} & 1 \\ -\frac{1}{R_2} & 0 & 0 & 0 & -\frac{1}{R_3} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_3} & 0 \\ 0 & -\frac{1}{R_5} & \frac{1}{R_5} & \frac{1}{R_4} + \frac{1}{R_7} & -\frac{1}{R_4} & 0 & 0 \\ 0 & \frac{1}{R_5} + \frac{1}{R_8} & -\frac{1}{R_5} & 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.55}$$

$$[C] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ I_{Nt} \end{bmatrix} \tag{1.56}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

```
%================  mat_script12.m  ============
clear

V_S1 = 5;
V_S2 = 2;
I_S  = 1e-3;

R_1 = 1e3;
R_2 = 1e3;
R_3 = 2e3;
R_4 = 4e3;
R_5 = 4e3;
R_6 = 1e3;
R_7 = 3e3;
R_8 = 4e3;
R_9 = 1e3;



A=[V_S1;V_S2;0;0;I_S;0;0];

B=[0 0 -1 1 0 0 0;
   -1 1 0 0 0 0 0 ;
   -1/R_2-1/R_9 0 0 0 0 1/R_2 1;
   -1/R_2 0 0 0 -1/R_3 1/R_2+1/R_3+1/R_6 0;
   0 0 0 -1/R_4 1/R_3+1/R_4 -1/R_3 0;
   0 -1/R_5 1/R_5 1/R_4+1/R_7 -1/R_4 0 0;
   0 1/R_5+1/R_8 -1/R_5 0 0 0 1 ];

C=inv(B)*A;

I_Nt=C(7)

%===============================================
```

1.4.3 Superposition theorem

Example 1.13 Consider the circuit of figure 1.26 a). Determine the contribution of each independent source to the voltage across R_4. $G_M = 2$ mS and $R_M = 500\ \Omega$.

<u>*Solution (using SPICE):*</u>

We start by calculating the contribution of V_S to the voltage across R_4. Note that the value of the current source is set to zero.

```
* Circuit of figure 1.26 b)

*----------------------netlist13a----------------
*-------------Contribution from V_S--------------

V_S 1 0 dc 2
I_S 10 3 dc 0

V_sens 2 10 dc 0

R_1 2 1 1k
R_2 3 4 5k
R_3 4 5 3.5k
R_4 5 0 1.5k
G_M 5 3 2 1 0.002
H_M 4 1 V_sens 500

*-----------------------------------------------

*-----------------------------------------------

.dc V_S 0 2 2
.print dc v(5)
.end
*-----------------------------------------------
```

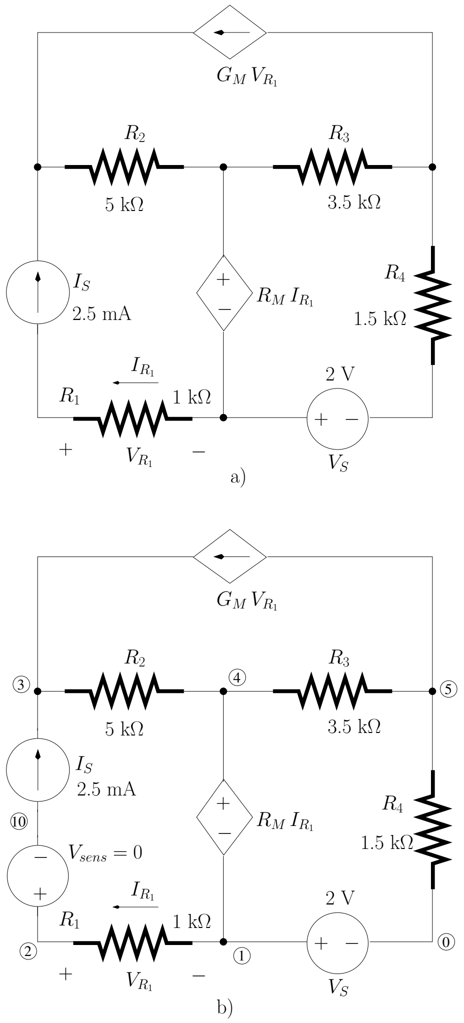


Figure 1.26: *a) DC circuit. b) Equivalent circuit.*

Now we calculate the contribution of I_S to the voltage across R_4. Note that the value of the voltage source is set to zero.

```
* Circuit of figure 1.26 b)

*-----------------------netlist13b----------------
*-------------Contribution from I_S--------------

V_S 1 0 dc 0
I_S 10 3 dc 2.5e-3

V_sens 2 10 dc 0

R_1 2 1 1k
R_2 3 4 5k
R_3 4 5 3.5k
R_4 5 0 1.5k
G_M 5 3 2 1 0.002
H_M 4 1 V_sens 500

*----------------------------------------------

*----------------------------------------------

.dc I_S 0 2.5e-3 2.5e-3
.print dc v(5)
.end
*----------------------------------------------
```

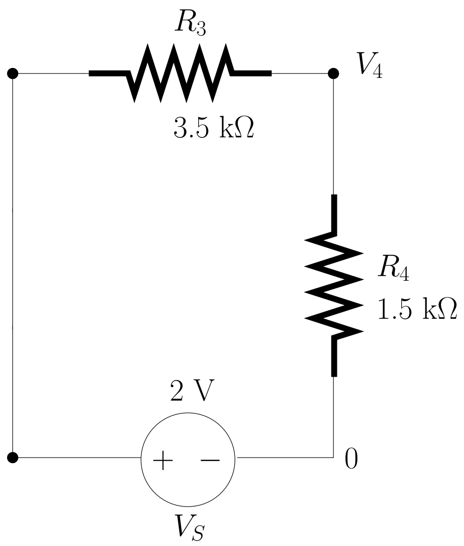


Figure 1.27: *Equivalent circuit to calculate the contribution of V_S to the voltage across R_4.*

Solution (using MATLAB/OCTAVE):

We start by calculating the contribution of V_S to the voltage across R_4. Figure 1.27 shows the equivalent circuit. Note that since the current source I_S has been replaced by an open-circuit there is no current flowing through R_1 ($I_{R_1} = 0$) and, therefore, there is no voltage across R_1 ($V_{R_1} = 0$). Hence, the current source controlled by V_{R_1} has been replaced by an open-circuit and the voltage source controlled by I_{R_1} has been replaced by a short-circuit.

The voltage across the resistance R_4 is

$$V_4 = V_S \frac{R_4}{R_4 + R_3}$$

```
%================  mat_script13a.m  ============
clear

V_S = 2;

R_1 = 1e3;
R_2 = 5e3;
R_3 = 3.5e3;
R_4 = 1.5e3;
G_M = 0.002;
R_M = 500;

V_4=V_S*R_4/(R_4+R_3)
%===============================================
```

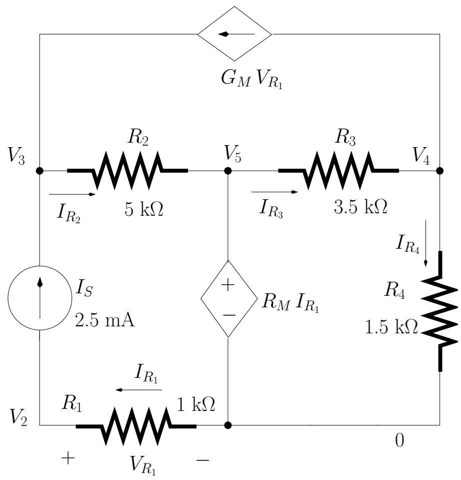


Figure 1.28: *Equivalent circuit to calculate the contribution of I_S to the voltage across R_4.*

Now we calculate the contribution of I_S to the voltage across R_4. Figure 1.28 shows the equivalent circuit. For this circuit we can write

$$\begin{cases} I_{R_2} = I_s + G_M V_{R_1} \\ I_{R_3} = I_{R_4} + G_M V_{R_1} \\ I_{R_1} = I_S \\ V_5 = R_M I_{R_1} \end{cases} \tag{1.57}$$

or

$$\begin{cases} \dfrac{V_3 - V_5}{R_2} = I_S + G_M V_2 \\[2ex] \dfrac{V_5 - V_4}{R_3} = \dfrac{V_4}{R_4} + G_M V_2 \\[2ex] -\dfrac{V_2}{R_1} = I_S \\[2ex] V_5 = R_M I_S \end{cases} \tag{1.58}$$

This last eqn can be written in a matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_S \\ 0 \\ I_S \\ I_S \end{bmatrix} \tag{1.59}$$

$$[B] = \begin{bmatrix} -G_M & \dfrac{1}{R_2} & 0 & -\dfrac{1}{R_2} \\[2ex] G_M & 0 & \dfrac{1}{R_3} + \dfrac{1}{R_4} & -\dfrac{1}{R_3} \\[2ex] -\dfrac{1}{R_1} & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & \dfrac{1}{R_M} \end{bmatrix} \tag{1.60}$$

$$[C] = \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} \tag{1.61}$$

We can determine the unknown variables by solving the following eqn:

$$[C] = [B]^{-1} \times [A]$$

```
%================  mat_script13.m  ===========
clear

I_S = 2.5e-3;

R_1 = 1e3;
R_2 = 5e3;
R_3 = 3.5e3;
R_4 = 1.5e3;
G_M = 0.002;
R_M = 500;

A=[I_S;0;I_S;I_S];
```

```
B=[-G_M 1/R_2 0 -1/R_2;
   G_M 0 1/R_3+1/R_4 -1/R_3;
   -1/R_1 0 0 0;
   0 0 0 1/R_M];

C=inv(B)*A;

V_4=C(3)
%===============================================
```

Chapter 2

Complex numbers: an introduction

Complex numbers can be entered in MATLAB or OCTAVE by using the letter i or j to express the imaginary number $j = \sqrt{-1}$. For example, the number $z = 3 + j\,4$ can be entered in MATLAB or OCTAVE as follows;

```
z = 3+4*j
```

Complex numbers can also be entered in these packages using the complex exponential (phasor) form. For example, $z = 3 + j\,4$ which is equal to $5\exp(j\,0.9273)$ can be entered as

```
z =5*exp(j*0.9273)
```

The elementary algebra of complex numbers using MATLAB or OCTAVE is straightforward. To sum $z_1 = 2 + j\,2$ to $z_2 = 3 - j$ we can enter

```
z1=2+j*2;
z2=3-j;

z3=z1+z2
```

This produces

```
z3 =

   5.0000 + 1.0000i
```

To subtract z_2 from z_1 we can enter

```
z4=z1-z2
```

which produces

```
z4 =

  -1.0000 + 3.0000i
```

To multiply z_1 by z_2 we can enter

```
z5=z1*z2
```

which produces

```
z5 =

   8.0000 + 4.0000i
```

The division z_1/z_2 can be effected as

```
z6=z1/z2
```

which produces

```
z6 =

   0.4000 + 0.8000i
```

All these operation can be carried out by MATLAB or OCTAVE if the complex numbers are expressed in the complex exponential form. For example if we want to add $z_a = \sqrt{2}\,\exp\left(-j\,\pi/3\right)$ to $z_b = 4.5\,\exp\left(j\,\pi/8\right)$ we can enter

```
za=sqrt(2)*exp(-j*pi/3);
zb=4.5*exp(j*pi/8);

zc=za+zb
```

which produces

```
zc =

   4.8646 + 0.4973i
```

In order to convert a complex number from its rectangular representation to the complex exponential representation we can use the functions `abs.m` and `angle.m` as follows:

```
zc_mag = abs(zc);
zc_ang = angle(zc);
```

To obtain the real part and the imaginary part of a complex number we can use the functions `real.m` and `imag.m` as follows:

```
zc_real = real(zc);
zc_imag = imag(zc);
```

To obtain the conjugate of a complex number we can use the function `conj.m` as follows:

```
zc_conj = conj(zc);
```

The Nth roots of a complex number can be calculated using the following script.

```
%=============================================
% Nth roots of a complex number Z

Z=input('Enter the complex number (a+j*b) ');

N=input('Enter the Nth root (N) ');

for k=1:N

 z=abs(Z)^(1/N)*exp(j*angle(Z)/N+j*2*pi*k/N)

end
%=============================================
```

Complex matrices can be entered in MATLAB or OCTAVE. For example, let us consider a complex matrix $[A]$ as indicated below:

$$A = \begin{bmatrix} 1+j & 2.4+j\,0.6 \\ \sqrt{2} & 3.1-j\,\pi \end{bmatrix} \tag{2.1}$$

This matrix can be entered in MATLAB or OCTAVE as follows:

```
A=[1+j  2.4+j*0.6 ; sqrt(2) 3.1-j*pi]
```

This produces

```
A =

   1.0000 + 1.0000i   2.4000 + 0.6000i
   1.4142             3.1000 - 3.1416i
```

Chapter 3

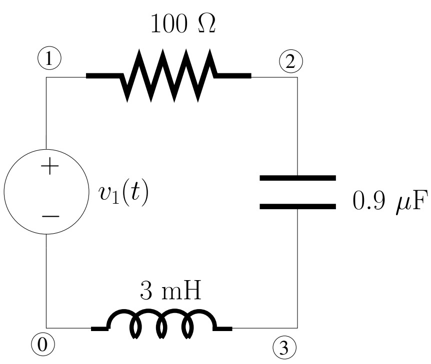
Frequency domain electrical signal and circuit analysis

3.1 AC circuits

Example 3.1 Determine the voltage at each node of the circuit of figure 3.1.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 3.1

*----------netlist1----------------------------

V_1 1 0 AC 10 45 *10 VOLTS, 45 DEGREES (PI/4)

R_1 1 2 100
L_1 3 0 3m
C_1 2 3 0.9u

*---------------------------------------------

*---------------------------------------------

.ac lin 2 4.7745k 4.775k *30 krad/s
.print ac v(1) v(2) v(3)
.end

*---------------------------------------------
```



$v_1(t) = 10\cos(30 \times 10^3 t + \pi/4)$ V

Figure 3.1: *AC circuit.*

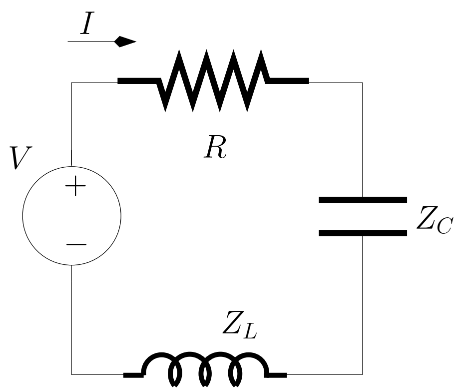


Figure 3.2: *AC circuit.*

Solution (using MATLAB/OCTAVE):

The impedances associated with the capacitor, Z_C and the inductor, Z_L, can be obtained as:

$$Z_C = \left.\frac{1}{j\,\omega\,C}\right|_{\omega=30 \text{ krad/s}}$$
$$Z_L = \left. j\,\omega\,L\right|_{\omega=30 \text{ krad/s}}$$

The (static) phasor associated with the voltage source is

$$V = 10\,\exp(j\,\pi/4)\ \text{V}$$

Since R is connected in series with C and with L we can determine the current I as follows:

$$I = \frac{V}{Z_L + Z_C + R}$$

The voltage across the resistance, V_R, can be obtained as:

$$V_R = R\,I$$

The voltage across the capacitance, V_C, can be obtained as:

$$V_C = Z_C\,I$$

The voltage across the inductance, V_L, can be obtained as:

$$V_L = Z_L\,I$$

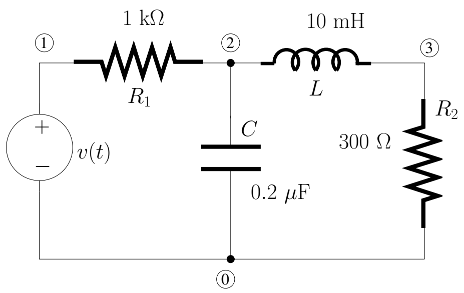
The numeric results can be obtained using MATLAB or OCTAVE

```
%===========  mat_script1.m ==================
clear
V= 10*exp(j*pi/4);

omega=30e3

R= 100;
L= 3e-3;
C= 0.9e-6;

Z_C= 1/(j*omega*C);
Z_L= j*omega*L;

I= V/(Z_L+Z_C+R)

V_R= I*R
V_L= I*Z_L
V_C= I*Z_C
%============================================
```

Example 3.2 Determine the voltage at each node of the circuit of figure 3.3.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 3.3

*------------netlist2-------------------------

V_1 1 0 AC 10 45

R_1 1 2 1k
L_1 2 3 10m
R_2 3 0 300
C_1 2 0 0.2u

*---------------------------------------------

*---------------------------------------------

.ac lin 2 4.7745k 4.775k
.print ac v(1) v(2) v(3)
.end

*---------------------------------------------
```



$v_1(t) = 10\cos(30 \times 10^3 t + \pi/4)$ V

Figure 3.3: *AC circuit.*

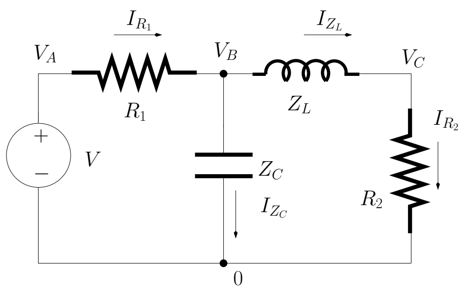


Figure 3.4: *AC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 3.4 we can write the following set of eqns:

$$\begin{cases} I_{R_1} = I_{Z_L} + I_{Z_C} \\ I_{Z_L} = I_{R_2} \\ V = V_A \end{cases} \tag{3.1}$$

This can also be written as

$$\begin{cases} \frac{V_A - V_B}{R_1} = \frac{V_B - V_C}{Z_L} + \frac{V_B}{Z_C} \\ \frac{V_B - V_C}{Z_L} = \frac{V_C}{R_2} \\ V = V_A \end{cases} \tag{3.2}$$

where Z_C and Z_L are given by:

$$\begin{aligned} Z_C &= \left.\frac{1}{j\,\omega\,C}\right|_{\omega=30\ \text{krad/s}} \\ Z_L &= \left. j\,\omega\,L\right|_{\omega=30\ \text{krad/s}} \end{aligned}$$

The eqn 3.2 can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.3}$$

$$[B] = \begin{bmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{Z_C} + \frac{1}{Z_L} & -\frac{1}{Z_L} \\ 0 & -\frac{1}{Z_L} & \frac{1}{R_2} + \frac{1}{Z_L} \\ 1 & 0 & 0 \end{bmatrix} \tag{3.4}$$

$$[C] = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} \tag{3.5}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===================  mat_script2.m ===========
clear
V= 10*exp(j*pi/4);
R_1= 1e3;
R_2= 300;
L= 10e-3;
C= 0.2e-6;
```

```
omega= 30e3;
Z_C= 1/(j*omega*C);
Z_L= j*omega*L;

B=[-1/R_1 (1/R_1+1/Z_L+1/Z_C) -1/Z_L;...
    0      -1/Z_L     1/R_2+1/Z_L; ...
    1       0         0    ];

A=[0 ; 0; V];

C=inv(B)*A
%===============================================
```

Example 3.3 Determine the voltage at each node of the circuit of figure 3.5.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 3.5

*-------------------netlist3-------------------

V_1 2 4 AC 10 45
I_1 0 2 AC 0.15 60

R_1 1 2 250
L_1 1 0 5m
R_2 3 2 200
C_1 3 4 0.7u
R_3 4 0 280
*---------------------------------------------

*---------------------------------------------

.ac lin 2 4.7745k 4.775k
.print ac v(1) v(2) v(3) v(4)
.end

*---------------------------------------------
```

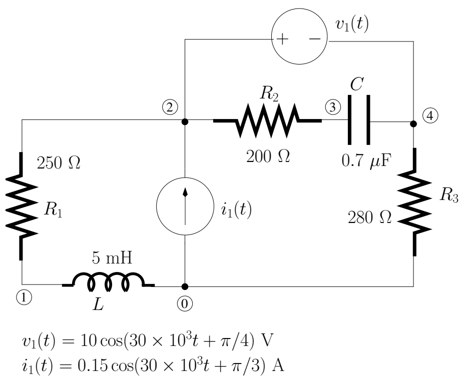


Figure 3.5: *AC circuit.*

<u>*Solution (using MATLAB/OCTAVE):*</u>

For the circuit of figure 3.6 we can write:

$$\begin{cases} I_{R_3} + I_{Z_L} = I_1 \\ I_{R_2} = I_{Z_C} \\ I_{R_1} = I_{Z_L} \\ V_A - V_C = V_1 \end{cases} \tag{3.6}$$

This can also be written as

$$\begin{cases} \dfrac{V_D}{Z_L} + \dfrac{V_C}{R_3} = I_1 \\ \dfrac{V_A - V_B}{R_2} = \dfrac{V_B - V_C}{Z_C} \\ \dfrac{V_A - V_D}{R_1} = \dfrac{V_D}{Z_L} \\ V_A - V_C = V_1 \end{cases} \tag{3.7}$$

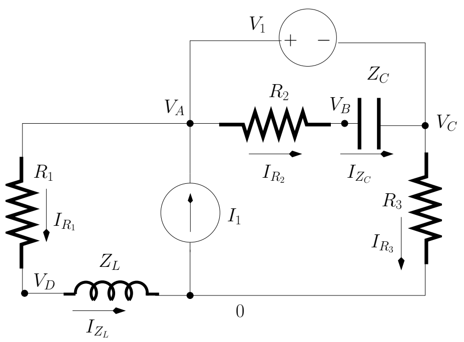


Figure 3.6: *AC circuit.*

where Z_C and Z_L are given by:

$$\begin{aligned} Z_C &= \left.\frac{1}{j\,\omega\,C}\right|_{\omega=30\ \text{krad/s}} \\ Z_L &= \left. j\,\omega\,L\right|_{\omega=30\ \text{krad/s}} \end{aligned}$$

The eqn 3.7 can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_1 \\ 0 \\ 0 \\ V_1 \end{bmatrix} \tag{3.8}$$

$$[B] = \begin{bmatrix} 0 & 0 & \frac{1}{R_3} & \frac{1}{Z_L} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{Z_C} & -\frac{1}{Z_C} & 0 \\ -\frac{1}{R_1} & 0 & 0 & \frac{1}{Z_L} + \frac{1}{R_1} \\ 1 & 0 & -1 & 0 \end{bmatrix} \tag{3.9}$$

$$[C] = \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} \tag{3.10}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%===================  mat_script3.m ===========
clear
V_1= 10*exp(j*pi/4);
I_1= 0.15*exp(j*pi/3);

R_1= 250
R_2= 200
R_3= 280
L= 5e-3
C= 0.7e-6

omega= 30e3;

Z_C= 1/(j*omega*C);
Z_L= j*omega*L;

B=[ 0 0 1/R_3 1/Z_L; ...
    -1/R_2 1/R_2+1/Z_C -1/Z_C 0; ...
    -1/R_1 0 0 1/Z_L+1/R_1;...
    1 0 -1 0];

A=[I_1; 0 ; 0; V_1];

C=inv(B)*A
%==============================================
```

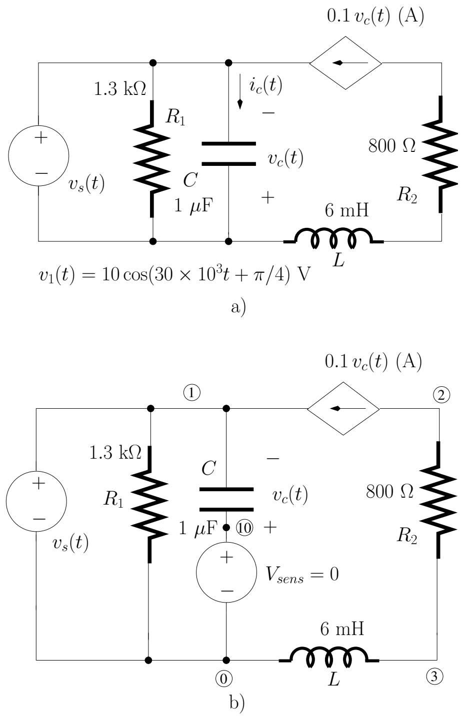


Figure 3.7: *AC circuit.*

Example 3.4 Determine the voltage at each node of the circuit of figure 3.7 a). Determine also the current through the capacitor.

Solution (using SPICE):

```
* Circuit of figure 3.7 b)

*--------------------netlist4-----------------
V_S 1 0 ac 10 45

R_1 1 0 1.3k
L_1 3 0 6m
R_2 3 2 800
C_1 1 10 1u
v_sens 10 0 dc 0 ac 0
G_1 2 1 10 1 0.1
*--------------------------------------------

*--------------------------------------------

.ac lin 2 4.7745k 4.775k
.print ac v(1) v(2) v(3) i(v_sens)
.end

*--------------------------------------------
```

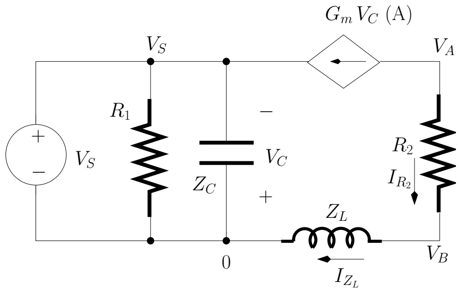


Figure 3.8: *AC circuit.*

Solution (using MATLAB/OCTAVE):

For the circuit of figure 3.8 we can write:

$$Z_C = \left.\frac{1}{j\,\omega\,C}\right|_{\omega=30 \text{ krad/s}}$$
$$Z_L = \left. j\,\omega\,L\right|_{\omega=30 \text{ krad/s}}$$

and

$$\begin{cases} V_C = -V_S \\ G_m\,V_C = -I_{R_2} \\ I_{R_2} = I_{Z_L} \end{cases} \tag{3.11}$$

This can also be written as

$$\begin{cases} G_m\,V_S = \dfrac{V_A - V_B}{R_2} \\ \dfrac{V_A - V_B}{R_2} = \dfrac{V_B}{Z_L} \end{cases} \tag{3.12}$$

The eqn 3.12 can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} G_m\,V_S \\ 0 \end{bmatrix} \tag{3.13}$$

$$[B] = \begin{bmatrix} \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{Z_L} \end{bmatrix} \tag{3.14}$$

$$[C] = \begin{bmatrix} V_A \\ V_B \end{bmatrix} \tag{3.15}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

The current through the capacitor is

$$I_{Z_C} = \frac{V_S}{Z_C}$$

```
%===================  mat_script4.m ==========
clear
V_S= 10*exp(j*pi/4);

R_1= 1.3e3;
R_2= 800;
C= 1e-6;
L= 6e-3;
G_m=0.1

omega= 30e3;
```

```
Z_C= 1/(j*omega*C);
Z_L= j*omega*L;

B=[1/R_2 -1/R_2; ...
   -1/R_2 1/R_2+1/Z_L];

A=[G_m*V_S; 0];

C=inv(B)*A

I_ZC=V_S/Z_C
%==============================================
```

Example 3.5 Consider the circuit of figure 3.9 a). Determine the average power dissipated by R_1. $v_s(t) = 6\cos(15 \times 10^3\, t)$ V, $i_s(t) = 3\cos(20 \times 10^3\, t + \pi/4)$ mA and $G_M = 2$ mS.

Solution (using SPICE):

Since the two sources have different frequencies we apply the superposition theorem to determine the contribution of each source to the average power dissipated in R_1.

```
* Circuit of figure 3.9 b)

*--------------------netlist4a1----------------
*-----------Contribution from v_s(t)-----------
V_s 1 4 ac 6
I_s 5 3 ac 0

R_1 1 2 180
L_1 4 0 10m
C_1 1 2 0.1u

R_2 5 0 200
C_2 2 0 0.3u
L_2 2 3 17m

G_M 3 2 2 0 2m
*---------------------------------------------

*---------------------------------------------

.ac lin 2 2.38725k 2.3873k
.print ac v(1) v(2)
.end
*---------------------------------------------
```

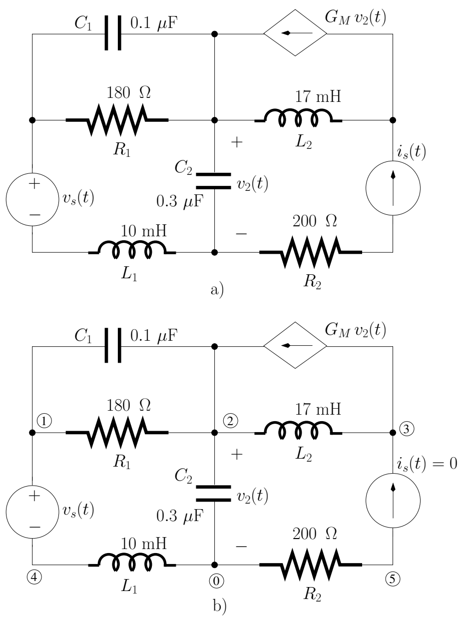


Figure 3.9: *a) AC circuit. b) Contribution from $v_s(t)$.*

The contribution of $v_s(t)$ to the average power dissipated in R_1 can be determined as follows:

$$P'_{R_1} = \frac{|\texttt{v(1)} - \texttt{v(2)}|^2}{2\,R_1}$$

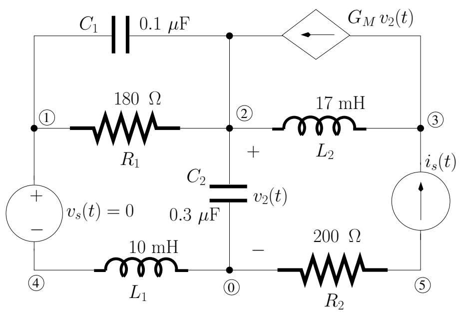


Figure 3.10: *Contribution from $i_s(t)$.*

```
* Circuit of figure 3.10

*--------------------netlist4a2----------------
*-----------Contribution from i_s(t)-----------

V_s 1 4 ac 0
I_s 5 3 ac 3e-3 45

R_1 1 2 180
L_1 4 0 10m
C_1 1 2 0.1u

R_2 5 0 200
C_2 2 0 0.3u
L_2 2 3 17m

G_M 3 2 2 0 2m
*----------------------------------------------

*----------------------------------------------

.ac lin 2 3.1830k 3.1831k
.print ac v(1) v(2)
.end
*----------------------------------------------
```

The contribution of $i_s(t)$ to the average power dissipated in R_1 can be determined as follows:

$$P''_{R_1} \quad = \quad \frac{|\texttt{v(1)} - \texttt{v(2)}|^2}{2\,R_1}$$

The total average power dissipated by R_1 is:

$$P_{R_1} \quad = \quad P'_{R_1} + P''_{R_1}$$

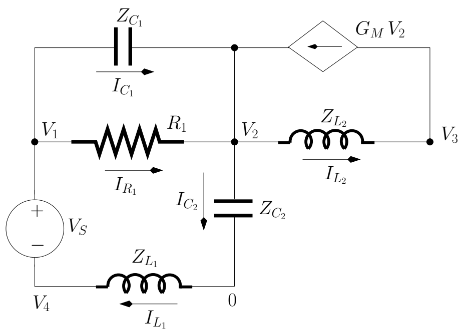


Figure 3.11: *Equivalent circuit for the calculation of the contribution of $v_s(t)$ to the average power dissipated in R_1.*

<u>*Solution (using MATLAB/OCTAVE):*</u>

We apply the superposition theorem to determine the contribution of each source to the average power dissipated in R_1. Figure 3.11 shows the equivalent circuit for the calculation of the contribution of $v_s(t)$ to the average power dissipated in R_1. For this circuit we can write:

$$\begin{cases} V_S = V_1 - V_4 \\ I_{C_2} = I_{L_1} \\ I_{R_1} + I_{C_1} + G_M\,V_2 = I_{C_2} + I_{L_2} \\ I_{L_2} = G_M\,V_2 \end{cases} \tag{3.16}$$

This can also be written as

$$\begin{cases} V_S = V_1 - V_4 \\[2ex] \dfrac{V_2}{Z_{C_2}} = -\dfrac{V_4}{Z_{L_1}} \\[2ex] \dfrac{V_1 - V_2}{R_1} + \dfrac{V_1 - V_2}{Z_{C_1}} + G_M\,V_2 = \dfrac{V_2}{Z_{C_2}} + \dfrac{V_2 - V_3}{Z_{L_2}} \\[2ex] \dfrac{V_2 - V_3}{Z_{L_2}} = G_M\,V_2 \end{cases} \tag{3.17}$$

where Z_{C_1}, Z_{C_2}, Z_{L_1} and Z_{L_2} are given by:

$$\begin{aligned} Z_{C_1} &= \left.\frac{1}{j\,\omega\,C_1}\right|_{\omega=15\ \text{krad/s}} \\ Z_{C_2} &= \left.\frac{1}{j\,\omega\,C_2}\right|_{\omega=15\ \text{krad/s}} \\ Z_{L_1} &= \left.j\,\omega\,L_1\right|_{\omega=15\ \text{krad/s}} \\ Z_{L_2} &= \left.j\,\omega\,L_2\right|_{\omega=15\ \text{krad/s}} \end{aligned}$$

The eqn 3.17 can be written in matrix form as follows:

$$[A_1] = [B_1] \times [C_1]$$

with

$$[A_1] = \begin{bmatrix} V_S \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.18}$$

$$[B_1] = \begin{bmatrix} 1 & 0 & 0 & -1 \\[1ex] 0 & \dfrac{1}{Z_{C_2}} & 0 & \dfrac{1}{Z_{L_1}} \\[2ex] -\dfrac{1}{R_1} - \dfrac{1}{Z_{C_1}} & \left(\dfrac{1}{R_1} + \dfrac{1}{Z_{C_1}} + \dfrac{1}{Z_{C_2}} + \dfrac{1}{Z_{L_2}} - G_M\right) & -\dfrac{1}{Z_{L_2}} & 0 \\[2ex] 0 & G_M - \dfrac{1}{Z_{L_2}} & \dfrac{1}{Z_{L_2}} & 0 \end{bmatrix} \tag{3.19}$$

$$[C_1] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \tag{3.20}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE (see `mat_script4a.m`):

$$[C_1] = [B_1]^{-1} \times [A_1]$$

The power dissipated in R_1 is

$$P'_{R_1} = \frac{|V_1 - V_2|^2}{2\,R_1}$$

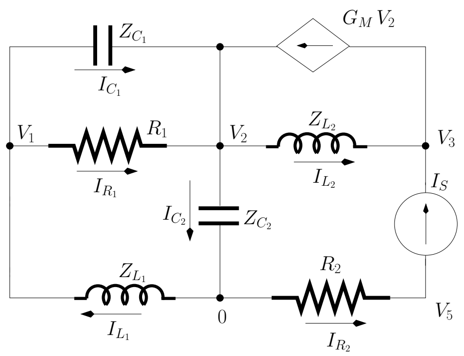


Figure 3.12: *Equivalent circuit for the calculation of the contribution of $i_s(t)$ to the average power dissipated in R_1.*

Figure 3.12 shows the equivalent circuit for the calculation of the contribution of $i_s(t)$ to the average power dissipated in R_1. For this circuit we can write:

$$\begin{cases} I_{C_2} = I_{L_1} + I_{R_2} \\ I_S = I_{R_2} \\ I_{R_1} + I_{C_1} + G_M\,V_2 = I_{C_2} + I_{L_2} \\ I_S = I_{L_2} = G_M\,V_2 \end{cases} \tag{3.21}$$

This can also be written as

$$\begin{cases} \dfrac{V_2}{Z_{C_2}} = -\dfrac{V_1}{Z_{L_1}} - \dfrac{V_5}{R_2} \\ I_S = -\dfrac{V_5}{R_2} \\ \dfrac{V_1 - V_2}{R_1} + \dfrac{V_1 - V_2}{Z_{C_1}} + G_M\,V_2 = \dfrac{V_2}{Z_{C_2}} + \dfrac{V_2 - V_3}{Z_{L_2}} \\ I_S + \dfrac{V_2 - V_3}{Z_{L_2}} = G_M\,V_2 \end{cases} \tag{3.22}$$

Now Z_{C_1}, Z_{C_2}, Z_{L_1} and Z_{L_2} are given by:

$$\begin{aligned} Z_{C_1} &= \left.\frac{1}{j\,\omega\,C_1}\right|_{\omega=20\ \text{krad/s}} \\ Z_{C_2} &= \left.\frac{1}{j\,\omega\,C_2}\right|_{\omega=20\ \text{krad/s}} \\ Z_{L_1} &= \left. j\,\omega\,L_1\right|_{\omega=20\ \text{krad/s}} \\ Z_{L_2} &= \left. j\,\omega\,L_2\right|_{\omega=20\ \text{krad/s}} \end{aligned}$$

The eqn 3.22 can be written in matrix form as follows:

$$[A_2] = [B_2] \times [C_2]$$

with

$$[A_2] = \begin{bmatrix} 0 \\ I_S \\ 0 \\ I_S \end{bmatrix} \tag{3.23}$$

$$[B_2] = \begin{bmatrix} \frac{1}{Z_{L_1}} & \frac{1}{Z_{C_2}} & 0 & \frac{1}{R_2} \\ 0 & 0 & 0 & -\frac{1}{R_2} \\ -\frac{1}{R_1} - \frac{1}{Z_{C_1}} & \left(\frac{1}{R_1} + \frac{1}{Z_{C_1}} + \frac{1}{Z_{C_2}} + \frac{1}{Z_{L_2}} - G_M\right) & -\frac{1}{Z_{L_2}} & 0 \\ 0 & G_M - \frac{1}{Z_{L_2}} & \frac{1}{Z_{L_2}} & 0 \end{bmatrix} \tag{3.24}$$

$$[C_2] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \end{bmatrix} \tag{3.25}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C_2] = [B_2]^{-1} \times [A_2]$$

The power dissipated in R_1 is

$$P''_{R_1} = \frac{|V_1 - V_2|^2}{2\,R_1}$$

The total average power dissipated by R_1 is $P'_{R_1} + P''_{R_1}$.

```
%==================  mat_script4a.m ===========
clear
V_S= 6;
I_S= 3e-3*exp(j*pi/4);
R_1= 180;
L_1= 10e-3;
C_1= 0.1e-6;
R_2= 200;
C_2= 0.3e-6;
L_2= 17e-3;

G_M=2e-3;

%++++++++++++++Contribution from V_S+++++++++++

omega= 15e3;

Z_C1= 1/(j*omega*C_1);
Z_L1= j*omega*L_1;
Z_C2= 1/(j*omega*C_2);
Z_L2= j*omega*L_2;


B1=[ 1 0 0 -1;
   0 1/Z_C2 0 1/Z_L1;
-1/R_1-1/Z_C1 1/R_1+1/Z_C1+1/Z_C2+1/Z_L2-G_M ...
                                       -1/Z_L2 0;
```

```
   0 G_M-1/Z_L2 1/Z_L2 0];

A1=[V_S; 0; 0; 0];

C1=inv(B1)*A1;
P_R1l= abs( C1(1)-C1(2) )^2/(2*R_1);

%+++++++++++++++Contribution from I_S+++++++++++

omega= 20e3;

Z_C1= 1/(j*omega*C_1);
Z_L1= j*omega*L_1;
Z_C2= 1/(j*omega*C_2);
Z_L2= j*omega*L_2;


B2=[1/Z_L1 1/Z_C2 0 1/R_2;
    0 0 0 -1/R_2
 -1/R_1-1/Z_C1 1/R_1+1/Z_C1+1/Z_C2+1/Z_L2-G_M ...
                                   -1/Z_L2 0;
   0 G_M-1/Z_L2 1/Z_L2 0];

A2=[0; I_S; 0 ; I_S];

C2=inv(B2)*A2;
P_R1ll= abs( C2(1)-C2(2) )^2/(2*R_1);

P_R1=P_R1l+P_R1ll
%================================================
```

3.2 Maximum power transfer

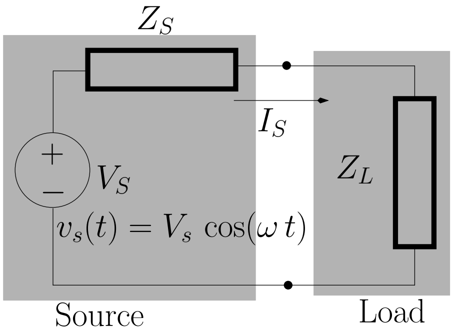


Figure 3.13: *AC circuit.*

Example 3.6 Study the variation of the average power transfer from a voltage source, with an output impedance $Z_S = R_S + j\, X_S$, to a load $Z_L = R_L + j\, X_L$.

Solution (using MATLAB/OCTAVE):

The average power dissipated by the load Z_L can be expressed as follows:

$$P_L \quad = \quad \frac{V_s^2}{2}\, \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

This equation can be rewritten as indicated below

$$P_L = \underbrace{\frac{V_s^2}{8\, R_S}}_{\substack{P_{L_{max}} \\ (R_L = R_S) \\ (X_L = -X_S)}} \times \frac{4\, \frac{R_L}{R_S}}{(1 + \frac{R_L}{R_S})^2 + \frac{(X_S + X_L)^2}{R_S}} \tag{3.26}$$

The function expressed by the last equation can be studied using the following m-script.

```
%===================  mat_script4b.m ===========
clear
clf

RL_over_RS=logspace(-2,2);

XLXS_over_RS=0:1:10;       % This variable repre-
                           % sents the quantity
                           % (X_S+X_L)^2/R_S

for k=1:length(XLXS_over_RS)

  PL_norm=4.*RL_over_RS./( (1+RL_over_RS).^2+...
                               XLXS_over_RS(k));

  semilogx(RL_over_RS,PL_norm)
  hold on
end
hold off
axis([1e-2 1e2 0 1.1])
title('Maximum power transfer')
ylabel('Average power normalised to P_{L_{max}}')
xlabel('R_L/R_S')
text(0.06,0.8,'(X_S+X_L)^2/R_S=0')
text(0.49,0.67,'1')
text(1,0.25,'10')
%===============================================
```

`logspace.m` is a built-in function which generates a logarithmically spaced vector. `loglog.m` is a built-in function to plot data with logarithmic scales. `semilogx.m` is a built-in function to plot data where a logarithmic (base 10) scale is used for the axis.

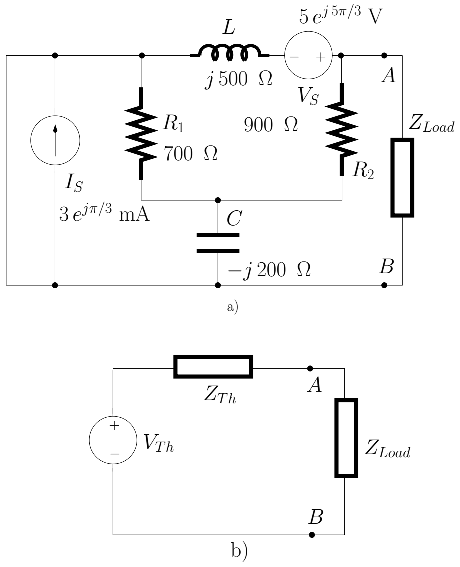


Figure 3.14: *a) AC circuit. b) Thévenin equivalent circuit.*

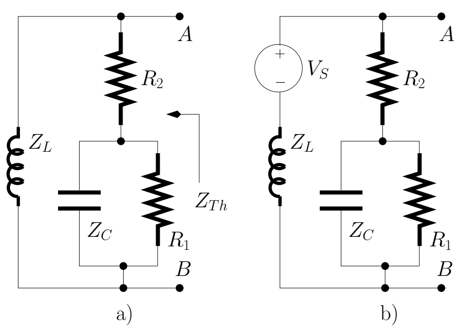


Figure 3.15: *a) Equivalent circuit to calculate Z_{Th}. b) Equivalent circuit to calculate V_{Th}.*

Example 3.7 Consider the circuit of figure 3.14 a). Determine the value of Z_{Load} for which there is maximum power transfer to this impedance. Determine the power dissipated in Z_{Load}.

Solution (using MATLAB/OCTAVE):

Figure 3.14 b) shows the Thévenin equivalent circuit. From this circuit it is clear that Z_{Load} must be equal to Z^*_{Th} so that maximum power transfer to this impedance is obtained (see also the previous example).

Figure 3.15 a) shows the equivalent circuit for the calculation of Z_{Th}. Note that V_S has been replaced by a short-circuit and I_S has been replaced by an open-circuit. From this figure it is clear that Z_{Th} can be obtained as follows:

$$Z_{Th} \;=\; Z_L \,||\, [R_2 + (R_1 \,||\, Z_C)]$$

with $Z_L = j\,500\ \Omega$ and $Z_C = -j\,200\ \Omega$.

Figure 3.15 b) shows the equivalent circuit for the calculation of V_{Th} which is the voltage between points A and B. From this circuit we can calculate V_{Th} as follows:

$$V_{Th} \;=\; V_S\,\frac{Z_A}{Z_A + Z_L}$$

with

$$Z_A \;=\; R_2 + (R_1 \,||\, Z_C)$$

The power power dissipated in $Z_{Load} = Z^*_{Th}$ is

$$P_{Load} = \frac{|V_{Th}|^2}{2\,R_{Th}}$$

where R_{Th} =Real $[Z_{Th}]$.

```
%===================  mat_script4c.m ===========
clear

Z_L=j*500;
Z_C=-j*200;
R_1=700;
R_2=900;
V_S=3*exp(j*5*pi/3);
Z_A=R_2+parallel(R_1,Z_C);

Z_Th=parallel(Z_L,Z_A);

Z_LOAD=conj(Z_Th)

V_Th=V_S*Z_A/(Z_A+Z_L);

P_LOAD=abs(V_Th)^2/(2*real(Z_Th))
%===============================================
```

`parallel.m` is an m-function presented in section 1.2.

3.3 Transfer functions

Example 3.8 Plot the phase and the magnitude of the voltage transfer function V_O/V_1 of the circuit of figure 3.16 for frequencies ranging from 10 Hz to 100kHz.

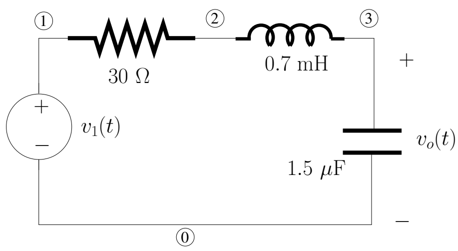


Figure 3.16: *AC circuit.*

Solution (using SPICE):

```
* Circuit of figure 3.16

*-------------------netlist5-------------------

V_1 1 0 AC 1

R_1 1 2 30
L_1 2 3 0.7m
C_1 3 0 1.5u

*---------------------------------------------

*---------------------------------------------

.ac DEC 10 10 100k
.print ac v(3)
.plot ac vm(3) vp(3)
.end

*---------------------------------------------
```

In order to obtain the voltage transfer function, the circuit must be driven by an AC voltage source. If the magnitude and the phase of this source are one volt and zero degrees, respectively, the output voltage represents the transfer function as follows:

$$V_O(f) = H(f) \times 1 \text{ (V)}$$

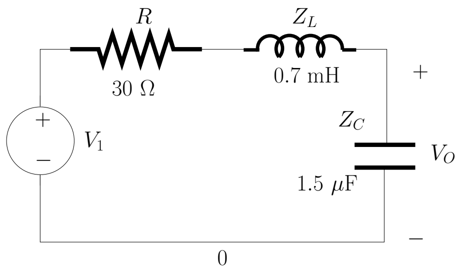


Figure 3.17: *AC circuit.*

Solution (using MATLAB/OCTAVE):

The transfer function V_O/V_1 can be determined as follows:

$$\begin{aligned} H(f) &= \frac{V_O}{V_1} \\ &= \frac{Z_C}{Z_C + Z_L + R} \end{aligned}$$

with

$$\begin{aligned} Z_C &= \frac{1}{j\,\omega\,C} \\ Z_L &= j\,\omega\,L \end{aligned}$$

```
%===================  mat_script5.m ===========
clear
R= 30;
L= 0.7e-3;
C= 1.5e-6;

f= logspace(1,5);
omega= 2*pi.*f;

Z_C= 1./(j.*omega.*C);
Z_L= j.*omega.*L;

H_f=Z_C./(Z_C+Z_L+R);

subplot(211)
loglog(f,abs(H_f))
title('Magnitude of the transfer function')
xlabel('Frequency (Hz)')
ylabel('Amplitude')
axis([10 1e5 1e-3 10])

subplot(212)
semilogx(f,angle(H_f))
title('Phase of the transfer function')
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
axis([10 1e5 -3.5 0.5])
%==============================================
```

3.4 Fourier series

Example 3.9 Determine the Fourier series of the voltage $v_c(t)$. Sketch $v_c(t)$. $v_s(t)$ is a periodic square wave with period $T = 1$ s and amplitudes ranging from -1 to 1 V.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 3.18

*---------------------netlist6-----------------

V_1 1 0 pulse(-1 1 0 0.01 0.01 0.48 1)

R_1 1 2 2k
C_1 2 0 100u

*---------------------------------------------

*---------------------------------------------

.tran 0.01 5 3
.print tran v(2)
.plot tran v(2)
.four 1 v(2)
.end

*---------------------------------------------
```

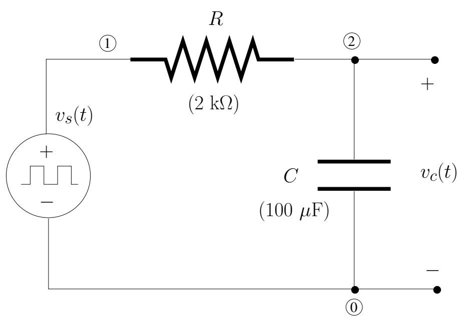


Figure 3.18: *RC circuit.*

The `.four` statement is the indication for SPICE to compute the Fourier series of `v(2)`. The output produced by this statement is $2 \times c_n$. These coefficients are given by

$$c_n = \frac{1}{T} \int_{t_o}^{t_o+T} \texttt{v(2)}(t)\ dt$$

Solution (using MATLAB/OCTAVE):

The Fourier coefficients of the input signal are given by:

$$V_{S_n} = \begin{cases} \frac{2\,A}{j\,\pi\,n} & \text{if} \quad |n| \text{ is odd} \\ \\ 0 & \text{if} \quad |n| \text{ is even} \end{cases} \tag{3.27}$$

The circuit transfer function is:

$$\begin{aligned} H\,(f) &= \frac{V_C}{V_S} \\ &= \frac{1}{1 + 2\,\pi\,f\,\tau} \end{aligned} \tag{3.28}$$

with $\tau = R\,C$. The Fourier of the output signal are given by:

$$V_{C_n} \quad = \quad V_{S_n} \times H\left(\frac{n}{T}\right) \tag{3.29}$$

that is

$$V_{C_n} = \begin{cases} \frac{2\,A}{j\,\pi\,n} \times \frac{1}{1+2\,\pi\,\frac{n}{T}\,\tau} & \text{if} \quad |n| \text{ is odd} \\ \\ 0 & \text{if} \quad |n| \text{ is even} \end{cases} \tag{3.30}$$

The output signal can be expressed as:

$$v_c(t) \quad = \quad V_{C_0} + \sum_{n=1}^{\infty} 2\,|V_{C_n}|\,\cos\left(2\,\pi\frac{n}{T}\,t + \angle(V_{C_n})\right)$$

```
%==================  mat_script6.m ===========

clear
clf

%++++++++ Circuit data ++++++
R= 2e3;
C= 100e-6;
tau=R*C;

% H_f=1./(1+j.*omega.*tau); Transfer
                           % function

%++++++++ Input signal data ++++++

T=1;                        % period

A=1;                        % Amplitude

%++++++++ Output signal ++++++++++

time=3:0.002:5;

N=31;                       % Number of
                            % harmonics

vc_t=zeros(size(time));     % Output signal,
                            % v_c(t)
```

```
V_cn= zeros(size([1:1:N]));% Fourier Coeff

for k=1:2:N

  omega_k=2*pi*k/T;
  V_cn(k)=2*A/(j*pi*k)/(1+j.*omega_k*tau);
  vc_t=vc_t + 2*abs(V_cn(k)).* ...
       cos(omega_k.*time+angle( V_cn(k)));

end

plot(time,vc_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_c(t)')

%===============================================
```

3.5 Convolution

Example 3.10 Determine the output voltage $v_c(t)$. Sketch $v_c(t)$.

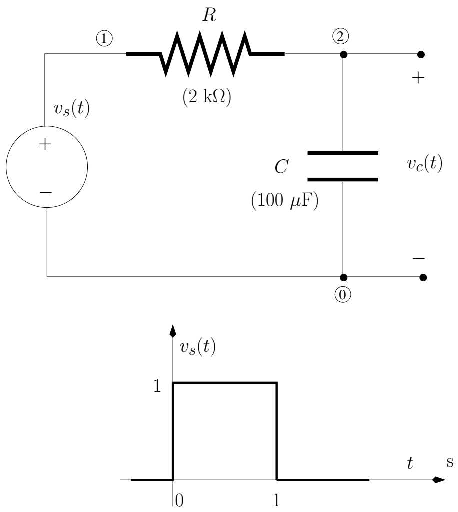


Figure 3.19: *RC circuit.*

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 3.19

*---------------------------netlist 7---------

V_1 1 0 pulse(0 1 0 0.001 0.001 0.99 4)

R_1 1 2 2k
C_1 2 0 100u

*----------------------------------------------

*----------------------------------------------

.tran 0.001 4 0
.print tran v(2)
.plot tran v(2)
.end

*----------------------------------------------
```

Solution (using MATLAB/OCTAVE):

The circuit impulse response is:

$$h(t) \quad = \quad \frac{1}{\tau}\, e^{-t/\tau}\, u(t)$$

The input voltage can be represented as:

$$v_s(t) \quad = \quad \text{rect}\,(t-0.5) \;\; \text{V}$$

The output voltage can be calculated as follows:

$$v_c(t) = h(t) * v_s(t)$$

This convolution operation can be calculated numerically by MATLAB or OCTAVE.

```
%===================  mat_script7.m ===========
clear
clf
%+++++++++ Input signal data ++++++

T=1;      %period
dt=T/500;
time=0:dt:2*T;

A=1;      %Amplitude
vs_t= A.*rect(time-T/2,T);

%+++++++++ Circuit data ++++++
R= 2e3;
C= 100e-6;
tau=R*C;

h_t=1./tau.*exp(-time./tau).*unitstep(time);

%+++++++++ Output signal ++++++++++

vc_t=conv(h_t,vs_t).*dt;


time2=0:dt:4*T;

subplot(211)
plot(time,vs_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_s(t)')
axis([-0.2 2 -0.2 1.2])

subplot(212)
plot(time2,vc_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_c(t)')
axis([-0.2 2 -0.2 1.2])

%===============================================
```

`rect.m` and `unitstep.m` are m-functions which implement the rectangular and the unit-step functions, respectively, as defined in appendix A. `conv.m` is a built-in m-function.

```
%===================     rect.m     ===========

function y=rect(t,tau)
%
% Rectangular function;  y=rect(t,tau)

y=zeros(size(t));

I1=find(t>-tau/2 & t<=tau/2);

if  (isempty(I1)~=1)
 y(I1)=ones(size(I1));
end

%===============================================
```

```
%===================   unitstep.m   ===========

function y=unitstep(x)
%
%   Unit-step function y=unitstep(x)
%

y=zeros(size(x));

I=find(x>0);

if (isempty(I)==0)
 y(I)=ones(size(I));
end

%===============================================
```

Chapter 4

Natural and forced responses circuit analysis

4.1 Natural Response

4.1.1 RC circuit

Example 4.1 Consider the circuit of figure 4.1. Determine the voltage across the capacitor for $t \geq 0$. At $t = 0$, the capacitor is initially charged showing 5 Volts across its terminals.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 4.1
*---------------------netlist1-----------------

R_1 0 1 2k
C_1 1 0 100u IC=5

*----------------------------------------------

*----------------------------------------------

.tran 0.025 2 0.001 UIC
.print tran v(1)
.plot tran v(1)
.end

*----------------------------------------------
```

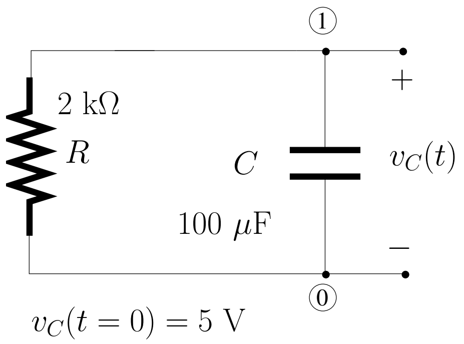


Figure 4.1: *RC circuit.*

Solution (using MATLAB/OCTAVE):

The voltage across the capacitor for $t \geq 0$ is:

$$v_C(t) \quad = \quad V_{co}\, e^{-t/\tau}\, u(t)$$

with $V_{co} = 5$ V and $\tau = RC = 0.2$ s.

```
%============  mat_script1.m =================
clear
clf
R= 2e3;
C= 100e-6;
tau=R*C;
Vco=5;

time=0:tau/100:5*tau;

vc_t=Vco.*exp(-time./tau).*unitstep(time);

plot(time,vc_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_c(t)')
axis([0 5*tau 0 6])
%============================================
```

4.1.2 RL circuit

Example 4.2 Consider the circuit of figure 4.2. Determine the voltage across the inductor for $t \geq 0$. At $t = 0$ the current through the inductor is 1 mA.

Solution (using SPICE):

```
* Circuit of figure 4.2

*---------------------netlist2-----------------

R_1 0 1 20
L_1 1 0 100m IC=1e-3

*---------------------------------------------

*---------------------------------------------

.tran 0.25m 30m 0.001m UIC
.print tran v(1)
.plot tran v(1)
.end

*---------------------------------------------
```

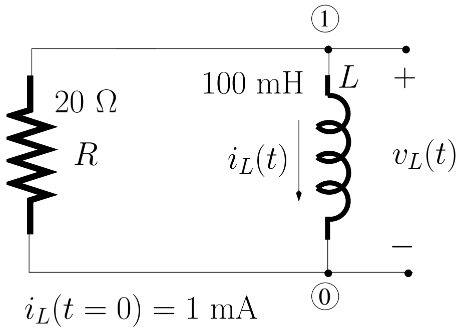


Figure 4.2: *RL circuit.*

Solution (using MATLAB/OCTAVE):

The voltage across the inductor for $t \geq 0$ is:

$$v_L(t) \quad = \quad -R\, I_{lo}\, e^{-t/\tau}\, u(t)$$

with $I_{lo} = 1$ mA and $\tau = L/R = 5$ ms.

```
%============  mat_script2.m =================
clear
clf
R= 20;
L= 100e-3;
tau=L/R;
Ilo=1e-3;

time=0:tau/100:5*tau;

vl_t=-R*Ilo.*exp(-time./tau).*unitstep(time);

plot(time,vl_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_L(t)')
axis([0 5*tau -21e-3 0])
%============================================
```

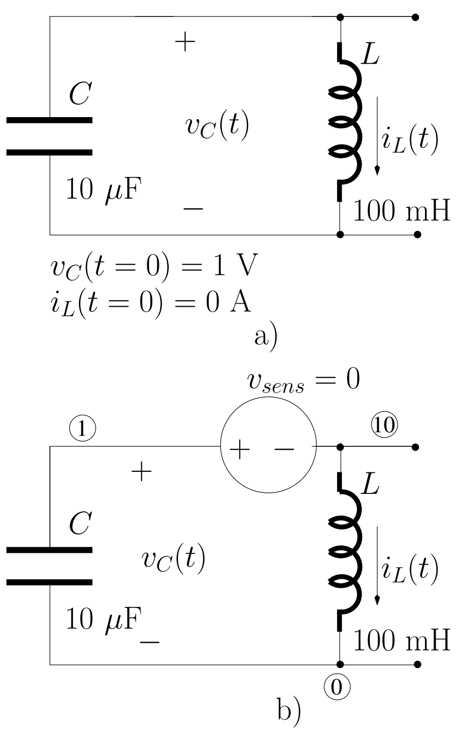


Figure 4.3: *LC circuit.*

4.1.3 LC circuit

Example 4.3 Consider the circuit of figure 4.3 a). Determine the voltage across the capacitor and the current through the inductor for $t \geq 0$.

Solution (using SPICE):

```
* Circuit of figure 4.3 b)
*-----------------netlist3---------------------

C_1 1 0 10u IC=1
V_sens 10 1 dc 0
L_1 10 0 100m IC=0

*---------------------------------------------

*---------------------------------------------

.tran 0.1m 15m 0.001m UIC
*
.print tran v(1) i(v_sens)
.plot tran v(1) i(v_sens)
.end
*---------------------------------------------
```

Solution (using MATLAB/OCTAVE):

The current through the inductor and the voltage across the capacitor, for $t \geq 0$, are given by:

$$\begin{aligned} i(t) &= -\sqrt{\frac{C}{L}}\, V_{co} \sin\left(\frac{1}{\sqrt{L\,C}}\, t\right) u(t) \\ v_{LC}(t) &= V_{co} \cos\left(\frac{1}{\sqrt{L\,C}}\, t\right) u(t) \end{aligned}$$

where $V_{co} = 1$ V.

```
%============  mat_script3.m =================
clear
clf
L= 100e-3;
C=10e-6;

omega_o=1/sqrt(L*C);
Vco=1;

time=0:1e-5:15e-3;

vc_t=Vco.*cos(omega_o.*time).*unitstep(time);
il_t=-sqrt(C/L)*Vco.*sin(omega_o.*time).*...
      unitstep(time);

subplot(211)
plot(time,vc_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_C(t)')

subplot(212)
plot(time,il_t)
xlabel('time (s)')
ylabel('Amplitude (A)')
title('i_L(t)')

%==============================================
```

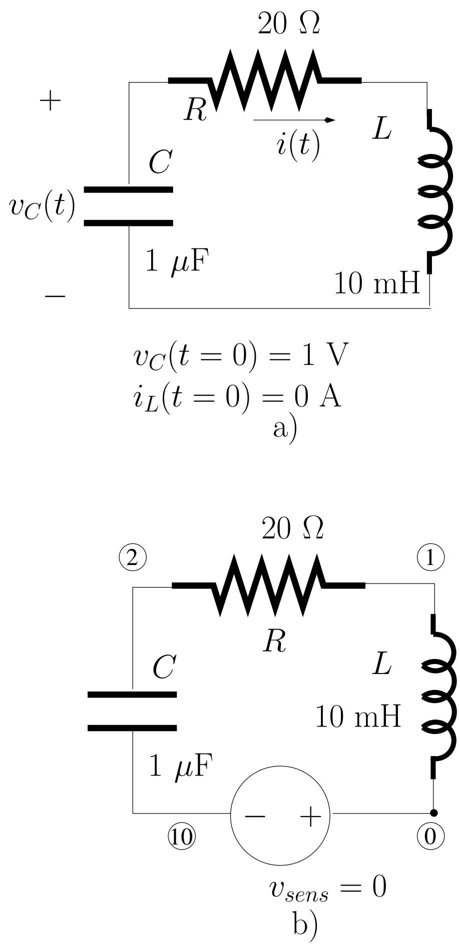


Figure 4.4: *RLC circuit.*

4.1.4 RLC circuit

Example 4.4 Consider the circuit of figure 4.4 a). Determine the current $i(t)$ for $t \geq 0$.

Solution (using SPICE):

```
* Circuit of figure 4.4 b)

*---------------------netlist4-----------------

L_1 1 0 100m IC=0
C_1 2 10 10u IC=1
R_1 1 2 20
v_sens 10 0 dc 0
*---------------------------------------------

*---------------------------------------------

.tran 0.1m 15m 0.001m UIC
.print tran i(v_sens)
.plot tran i(v_sens)
.end

*---------------------------------------------
```

Solution (using MATLAB/OCTAVE):

The current $i_L(t)$ can be expressed as follows ($t \geq 0$):

$$i(t) = C\,V_{co}\,\frac{\omega_n}{\sqrt{1-\eta^2}}\sin\left(\omega_n\,\sqrt{1-\eta^2}\,t\right)\,e^{-t\,\eta\,w_n}\,u(t)$$

with

$$\begin{aligned}\omega_n &= \frac{1}{\sqrt{L\,C}}\\ &= 1\ \text{krad/s}\\ \eta &= \frac{1}{2}\,R\,\sqrt{\frac{C}{L}}\\ &= 0.1\end{aligned}$$

```
%============  mat_script4.m =================
clear
clf
L= 100e-3;
C=10e-6;
R=20;

omega_n=1/sqrt(L*C);
eta=0.5*R*sqrt(C/L);
Vco=1;

time=0:1e-5:15e-3;

il_t=C.*Vco*omega_n/sqrt(1-eta^2).*...
     exp(-time.*omega_n.*eta).*...
     sin(sqrt(1-eta^2)*omega_n.*time).*...
      unitstep(time);

plot(time,il_t)
xlabel('time (s)')
ylabel('Amplitude (A)')
title('i_L(t)')
%==============================================
```

4.2 Response to the step function

4.2.1 RC circuit

Example 4.5 Determine the voltage across the capacitor of the circuit of figure 4.5. Assume zero initial conditions.

Solution (using SPICE):

```
* Circuit of figure 4.5
*----------------------netlist5---------------
V_1 1 0 pulse(0 1 0 0.01 0.01 2 3)
R_1 1 2 2k
C_1 2 0 100u
*---------------------------------------------
*---------------------------------------------
.tran 0.025 1.5 0
.print tran v(2)
.plot tran v(2)
.end
*---------------------------------------------
```

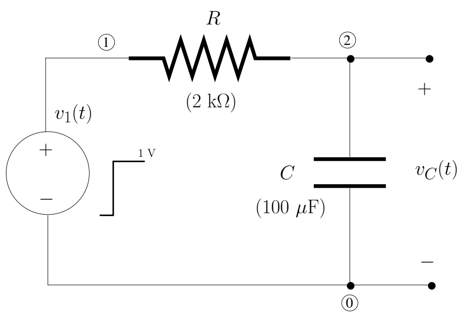


Figure 4.5: *RC circuit.*

Solution (using MATLAB/OCTAVE):

The voltage across the capacitor is given by:

$$v_C(t) \quad = \quad V_s \left(1 - e^{-t/\tau}\right) u(t)$$

with $V_s = 1$ V and $\tau = RC = 0.2$ s.

```
%============  mat_script5.m =================
clear
clf

R= 2e3;
C= 100e-6;
tau= R*C;
Vs= 1;

time= 0:tau/10:5*tau;

vc_t= Vs.*(1-exp(-time./tau)).*unitstep(time);

plot(time,vc_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_C(t)')
%============================================
```

4.2.2 RL circuit

Example 4.6 Determine the voltage across the inductor of the circuit of figure 4.6 for $t \geq 0$. Assume zero initial conditions.

<u>*Solution (using SPICE):*</u>

```
*-----------------netlist5b--------------------
V_1 1 0 pulse(0 1 0 1e-6 1e-6 2e-4 3e-4)

R_1 1 2 20
L_1 2 0 1e-3

*---------------------------------------------

*---------------------------------------------

.tran 0.2e-6 1.5e-4 0
.print tran v(2)
.plot tran v(2)
.end
*---------------------------------------------
```

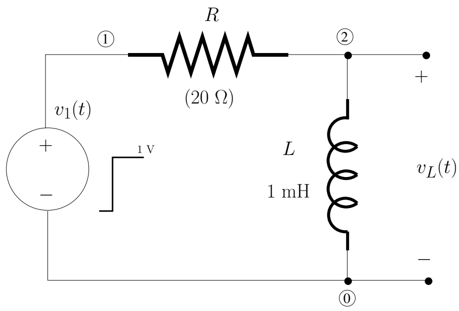


Figure 4.6: *RL circuit.*

Solution (using MATLAB/OCTAVE):

The voltage across the inductor is given by:

$$v_L(t) \quad = \quad V_s\, e^{-t/\tau}\, u(t)$$

with $V_s = 1$ V and $\tau = L/R$.

```
%============  mat_script5b.m ================
clear
clf

R= 20;
L= 1e-3;
tau= L/R;
Vs= 1;

time= 0:tau/100:3*tau;

vL_t= Vs.*exp(-time./tau).*unitstep(time);

plot(time,vL_t)
xlabel('time (s)')
ylabel('Amplitude (V)')
title('v_L(t)')
%==============================================
```

4.2.3 RLC circuit

Example 4.7 Determine the current through the inductor of the circuit of figure 4.7 a) for $t \geq 0$. Assume zero initial conditions.

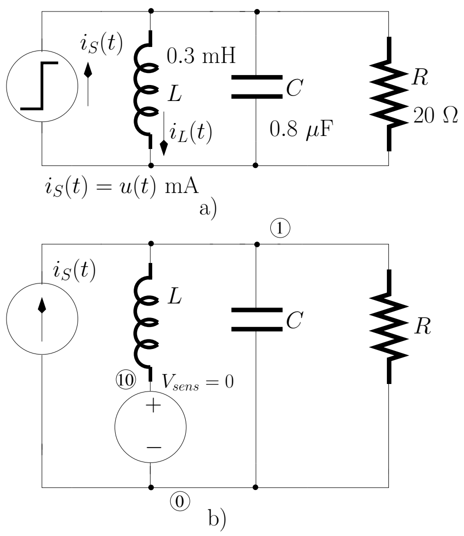


Figure 4.7: *a) RLC circuit. b) Equivalent circuit.*

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 4.7 b)

*-----------------netlist6--------------------

I_1 0 1 pulse(0 1e-3 0 0.01m 0.01m 2m 3m)

R_1 1 0 20
C_1 1 0 0.8u
L_1 1 10 0.3m
v_sens 10 0 dc 0
*--------------------------------------------

*--------------------------------------------

.tran 0.01m 0.25m 0
.print tran i(v_sens)
.plot tran i(v_sens)
.end

*--------------------------------------------
```

Solution (using MATLAB/OCTAVE):

The current through the inductor is given by:

$$i_L(t) \quad = \quad I_s \left[1 - \frac{e^{-t\,\eta\,\omega_n}}{\sqrt{1-\eta^2}}\, \sin\left(\sqrt{1-\eta^2}\,\omega_n\,t + \phi'\right)\right]\, u(t)$$

with

$$\phi' = \tan^{-1}\left(\frac{\sqrt{1-\eta^2}}{\eta}\right)$$

and

$$\begin{aligned} \omega_n &= \frac{1}{\sqrt{L\,C}} \\ &= 64.6 \;\; \text{krad/s} \\ \eta &= \frac{1}{2\,R}\sqrt{\frac{L}{C}} \\ &= 0.48 \end{aligned}$$

```
%============  mat_script6.m =================
clear
clf

R= 20;
C= 0.8e-6;
L=0.3e-3;

t= 0:1e-6:2.5e-4;
Is= 1e-3;
omega_n=1/sqrt(L*C);
eta=0.5/R*sqrt(L/C);
phi=atan(sqrt(1-eta^2)/eta);

il_t=(1-exp(-omega_n*eta.*t)/sqrt(1-eta^2)...
      .* sin(sqrt(1-eta^2)*omega_n.*t+phi))...
      .* Is .* unitstep(t);

plot(t,il_t)
xlabel('time (s)')
ylabel('Amplitude (A)')
title('i_L(t)')
%==============================================
```

Chapter 5

Electrical two-port network analysis

5.1 Z-parameters

Example 5.1 Determine the Z-parameters of the circuit of figure 5.1 a) for frequencies ranging from 10 Hz to 100kHz.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 5.1 b)
*-------------------netlis1---------------------

I_1 0 1 dc 0 ac 1

L_1 1 3 10m
C_1 3 0 3u
L_2 2 3 10m
R_1 3 0 10e9

I_2 0 2 dc 0 ac 0
*-----------------------------------------------

*-----------------------------------------------

.ac dec 10 10 100k
.plot ac vm(2) vp(2) vm(1) vp(1)
.end

*-----------------------------------------------
```

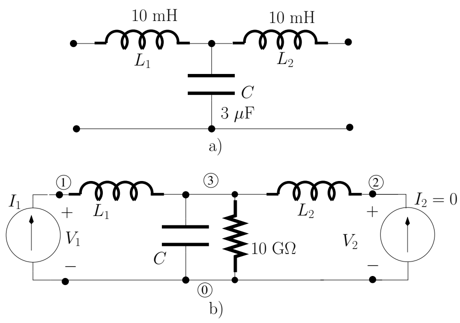


Figure 5.1: *a) AC circuit. b) Calculation of Z_{11} and of Z_{21}.*

The circuit is driven, from port one, by a current source providing 1 A. Now Z_{11} and Z_{21} can be determined as follows:

$$Z_{11} = \frac{V_1}{1} \ (\Omega)$$
$$Z_{21} = \frac{V_2}{1} \ (\Omega)$$

with $V_1 = $ `v(1)` and $V_2 = $ `v(2)`. Note the existence of a zero amp current source at port 2 which implements an open-circuit. There is also a very high resistance (10 GΩ) across the capacitor. The purpose of this resistance is to avoid SPICE convergence problems.

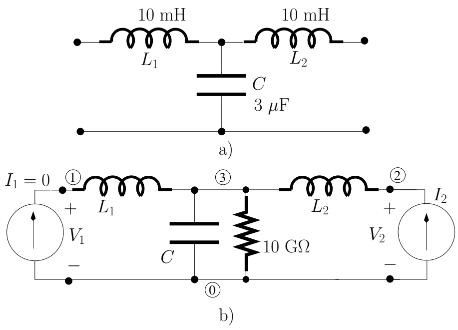


Figure 5.2: *a) AC circuit. b) Calculation of Z_{12} and Z_{22}.*

```
* Circuit of figure 5.2 b)

*-----------------------netlis2----------------
I_2 0 2 dc 0 ac 1

L_1 1 3 10m
C_1 3 0 3u
L_2 2 3 10m
R_1 3 0 10e9

I_1 0 1 dc 0 ac 0
*---------------------------------------------

*---------------------------------------------

.ac dec 10 10 100k
.plot ac vm(2) vp(2) vm(1) vp(1)
.end

*---------------------------------------------
```

Now, the circuit is driven, from port two, by a current source providing 1 A. Z_{12} and Z_{22} can be determined as follows:

$$Z_{12} = \frac{V_1}{1} \ (\Omega)$$
$$Z_{22} = \frac{V_2}{1} \ (\Omega)$$

with $V_1 =$ `v(1)` and $V_2 =$ `v(2)`.

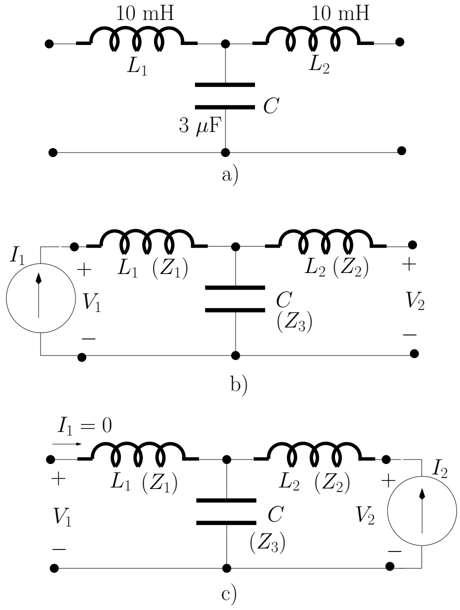


Figure 5.3: *a) AC circuit. b) Calculation of Z_{11} and Z_{21}. c) Calculation of Z_{12} and Z_{22}.*

Solution (using MATLAB/OCTAVE):

The Z-parameters of the circuit of figure 5.3 a) are:

$$\begin{aligned}
Z_{11} &= Z_1 + Z_3 \\
&= j\,\omega\,L_1 + \frac{1}{j\omega\,C} \\
&= \frac{1-\omega^2\,L_1\,C}{j\,\omega\,C} \\
Z_{21} &= Z_3 \\
&= \frac{1}{j\omega\,C} \\
Z_{22} &= Z_2 + Z_3 \\
&= \frac{1-\omega^2\,L_2\,C}{j\,\omega\,C} \\
Z_{12} &= Z_{21}
\end{aligned}$$

```
%============  mat_script1.m =================
clear
clf
C= 3e-6;
L_1=10e-3;
L_2=10e-3
f=logspace(1,5);
omega=2*pi.*f;

Z_11=(1-omega.^2.*L_1.*C)./(j*omega.*C);
Z_21=1./(j*omega.*C);
Z_22=(1-omega.^2.*L_2.*C)./(j*omega.*C);
Z_12=1./(j*omega.*C);

subplot(221)
semilogx(f,abs(Z_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{11}')

subplot(222)
semilogx(f,abs(Z_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{12}')

subplot(223)
semilogx(f,abs(Z_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{21}')

subplot(224)
semilogx(f,abs(Z_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{22}')
%============================================
```

5.2 Y-parameters

Example 5.2 Determine the Y-parameters of the circuit of figure 5.4 a) for frequencies ranging from 100 kHz to 100 MHz.

Solution (using SPICE):

```
* Circuit of figure 5.4 b)

*---------------------netlist3----------------
V_1 1 0 ac 1
V_2 2 0 ac 0

C_1 1 0 3n
L_1 1 10 1u
R_1 10 2 1e-9
C_2 2 0 1n
*--------------------------------------------

*--------------------------------------------

.ac dec 10 100k 100000k
.plot ac abs(i(V_2)) -phase(i(V_2))
.plot ac abs(i(V_1)) -phase(i(V_1))

.end
*--------------------------------------------
```

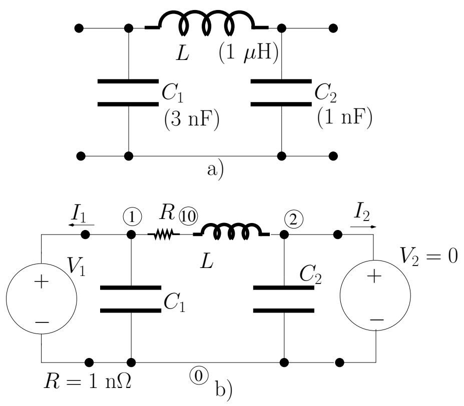


Figure 5.4: *a) AC circuit. b) Calculation of* Y_{11} *and* Y_{21}.

We include a very small resistance (1 nΩ) to avoid convergence problems. The circuit is driven, from port one, by a voltage source supplying 1 V. Y_{11} and Y_{21} can be determined as follows[1]:

$$
\begin{aligned}
Y_{11} &= -\frac{I_1}{1} \quad (\mathrm{S}) \\
Y_{21} &= -\frac{I_2}{1} \quad (\mathrm{S})
\end{aligned}
$$

with $I_1 = \texttt{i(V\_1)}$ and $I_2 = \texttt{i(V\_2)}$. Note that the short-circuit is implemented with a voltage source supplying zero Volts, $V_2 = 0$.

[1]Recall that, in SPICE, the positive current is assumed to flow from the positive pole, through the source, to the negative pole.

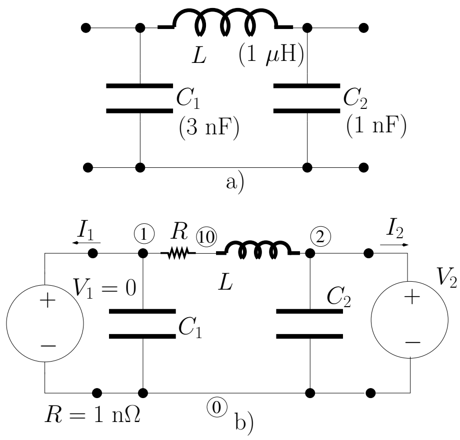


Figure 5.5: *a) AC circuit. b) Calculation of Y_{12} and Y_{22}.*

```
* Circuit of figure 5.5 b)

*--------------------netlist4-------------------
V_1 1 0 ac 0
V_2 2 0 ac 1

C_1 1 0 3n
L_1 1 10 1u
R_1 10 2 1e-9
C_2 2 0 1n

*-----------------------------------------------

*-----------------------------------------------

.ac dec 10 100k 100000k
.plot ac abs(i(V_2)) -phase(i(V_2))
.plot ac abs(i(V_1)) -phase(i(V_1))

.end
*-----------------------------------------------
```

The circuit is driven, from port two, by a voltage source supplying 1 V. Y_{12} and Y_{22} can be determined as follows:

$$Y_{12} = -\frac{I_1}{1} \text{ (S)}$$
$$Y_{22} = -\frac{I_2}{1} \text{ (S)}$$

with $I_1 =$ `i(V_1)` and $I_2 =$ `i(V_2)`.

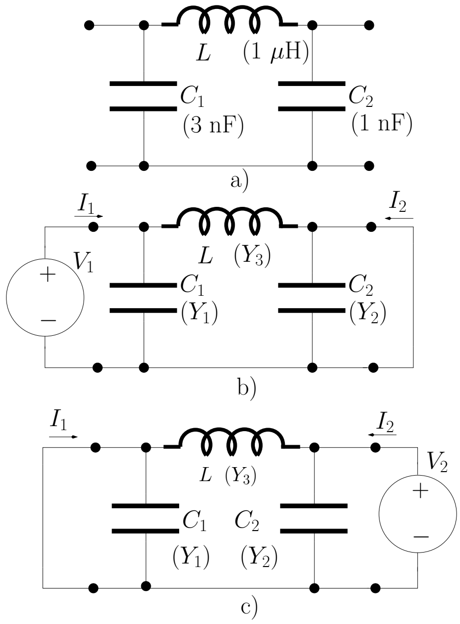


Figure 5.6: *a) AC circuit. b) Calculation of Y_{11} and Y_{21}. c) Calculation of Y_{12} and Y_{22}.*

Solution (using MATLAB/OCTAVE):

The Y-parameters of the circuit of figure 5.6 a) are:

$$
\begin{aligned}
Y_{11} &= \frac{1-\omega^2\,L\,C_1}{j\,\omega\,L}\\
Y_{21} &= -\frac{1}{j\omega\,L}\\
Y_{22} &= \frac{1-\omega^2\,L\,C_2}{j\,\omega\,L}\\
Y_{12} &= Y_{21}
\end{aligned}
$$

```
%============  mat_script2.m =================
clear
clf

C_1= 3e-9;
L=1e-6;
C_2=1e-9

f=logspace(5,8);
omega=2*pi.*f;

Y_11=(1-omega.^2.*L.*C_1)./(j*omega.*L);
Y_21=-1./(j*omega.*L);
Y_22=(1-omega.^2.*L.*C_2)./(j*omega.*L);
Y_12=-1./(j*omega.*L);

subplot(221)
semilogx(f,abs(Y_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{11}')

subplot(222)
semilogx(f,abs(Y_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{12}')

subplot(223)
semilogx(f,abs(Y_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{21}')

subplot(224)
semilogx(f,abs(Y_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{22}')
%============================================
```

5.3 Chain parameters

Example 5.3 Consider the circuit of figure 5.7 a) which represents an electrical model for a bipolar transistor. Determine its chain parameters for frequencies ranging from 10 kHz to 100 MHz. $R_x = 5\ \Omega$, $R_\pi = 2.5$ kΩ, $R_o = 100$ kΩ, $g_m = 40$ mS, $C_\pi = 10$ pF and $C_\mu = 1$ pF.

Solution (using SPICE):

```
* Circuit of figure 5.7 b)

*--------------------------netlist5------------
V_1 1 0 ac 1

R_x  1 3 5
R_pi 3 0 2.5k
C_pi 3 0 10p
C_mu 3 2 1p
Ro   2 0 100k
G_Gm 2 0 3 0 40m
*--------------------------------------------

*--------------------------------------------

.ac dec 10 1e4 1e9
.plot ac abs(V(2)) phase(V(2))

.end
*--------------------------------------------
```

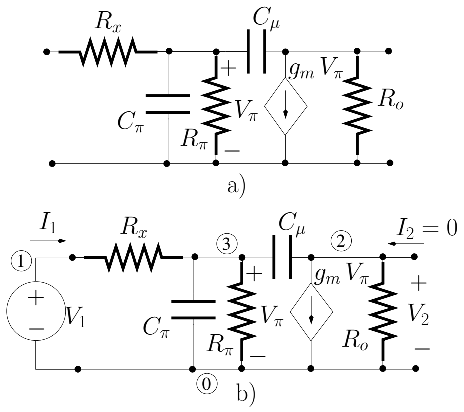


Figure 5.7: *a) Electrical model for a transistor. b) Calculation of A_{11}.*

A_{11} can be determined as follows:

$$A_{11} = \frac{1}{V_2}$$

with $V_2 =$ `v(2)`.

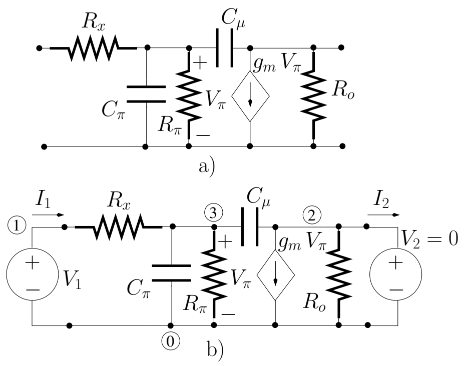


Figure 5.8: *a) Electrical model for a transistor. b) Calculation of A_{12}.*

```
* Circuit of figure 5.8 b)

*---------------netlist6-----------------------

V_1 1 0 ac 1
V_2 2 0 ac 0

R_x  1 3 5
R_pi 3 0 2.5k
C_pi 3 0 10p
C_mu 3 2 1p
Ro   2 0 100k
G_Gm 2 0 3 0 40m
*---------------------------------------------

*---------------------------------------------

.ac dec 10 1e4 1e9
.plot ac abs(i(V_2)) phase(i(V_2))

.end
*---------------------------------------------
```

A_{12} can be determined as follows:

$$A_{12} = \frac{1}{I_2} \ (\Omega)$$

with $I_2 = $ `i(V_2)`

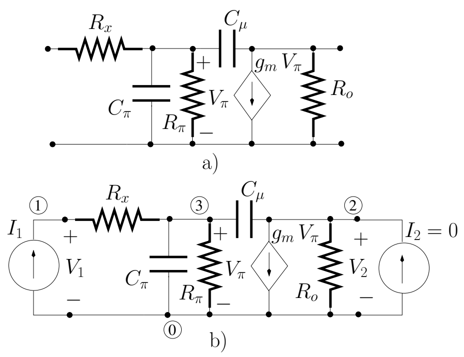


Figure 5.9: *a) Electrical model for a transistor. b) Calculation of A_{21}.*

```
* Circuit of figure 5.9 b)

*-----------------netlist7--------------------
I_1 0 1 ac 1
I_2 0 2 ac 0

R_x  1 3 5
R_pi 3 0 2.5k
C_pi 3 0 10p
C_mu 3 2 1p
Ro   2 0 100k
G_Gm 2 0 3 0 40m
*---------------------------------------------

*---------------------------------------------

.ac dec 10 1e4 1e9
.plot ac abs(V(2)) phase(V(2))

.end

*---------------------------------------------
```

A_{21} can be determined as follows:

$$A_{21} = \frac{1}{V_2} \text{ (S)}$$

with $V_2 =$ `v(2)`.

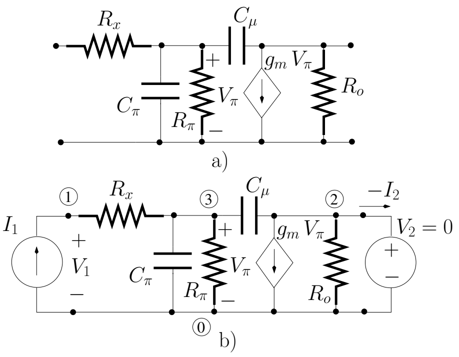


Figure 5.10: *a) Electrical model for a transistor. b) Calculation of A_{22}.*

```
* Circuit of figure 5.10 b)

*----------------netlist8---------------------
I_1 0 1 ac 1
V_2 2 0 ac 0

R_x  1 3 5
R_pi 3 0 2.5k
C_pi 3 0 10p
C_mu 3 2 1p
Ro   2 0 100k
G_Gm 2 0 3 0 40m
*---------------------------------------------

*---------------------------------------------

.ac dec 10 1e4 1e9
.plot ac abs(i(V_2)) phase(i(V_2))

.end

*---------------------------------------------
```

A_{22} can be determined as follows:

$$A_{22} = \frac{1}{I_2}$$

with $I_2 =$ `i(V_2)`.

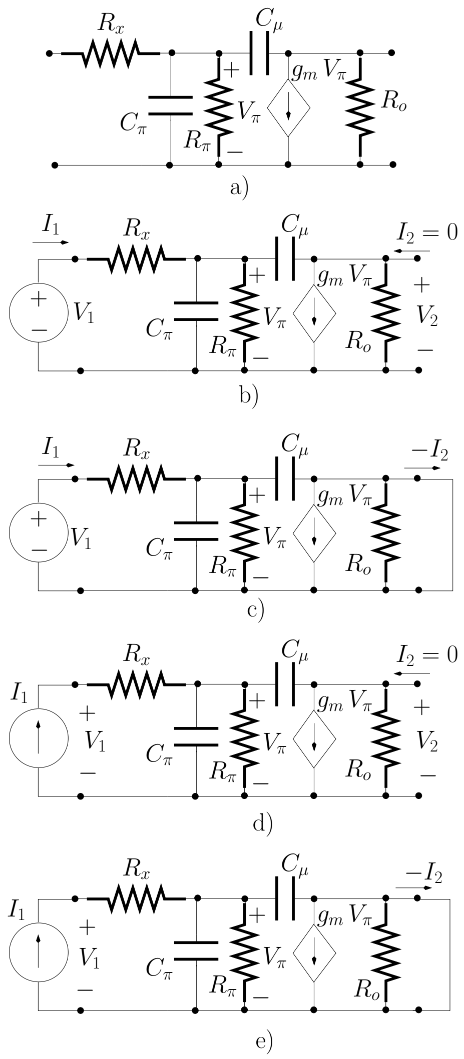


Figure 5.11: *a) Electrical model for a transistor. b) Calculation of A_{11}. c) Calculation of A_{12}. d) Calculation of A_{21}. e) Calculation of A_{22}.*

Solution (using MATLAB/OCTAVE):

Figure 5.11 b) shows the equivalent circuit for the calculation of A_{11}. For this circuit we can write:

$$\begin{cases} \dfrac{V_1 - V_\pi}{R_x} = \dfrac{V_\pi}{R_\pi} + V_\pi\, j\,\omega\, C_\pi + (V_\pi - V_2)\, j\,\omega\, C_\mu \\ (V_\pi - V_2)\, j\,\omega\, C_\mu = g_m\, V_\pi + \dfrac{V_2}{R_o} \end{cases} \tag{5.1}$$

This set of eqns can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} \dfrac{V_1}{R_x} \\ 0 \end{bmatrix} \tag{5.2}$$

$$[B] = \begin{bmatrix} \dfrac{1}{R_x} + \dfrac{1}{R_\pi} + j\,\omega\,(C_\mu + C_\pi) & -j\,\omega\, C_\mu \\ g_m - j\,\omega\, C_\mu & \dfrac{1}{R_o} + j\,\omega\, C_\mu \end{bmatrix} \tag{5.3}$$

$$[C] = \begin{bmatrix} V_\pi \\ V_2 \end{bmatrix} \tag{5.4}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%============  mat_script3.m ================
clear
clf

V_1 = 1;

Rx  = 5
Rpi = 2.5e3
Cpi = 10e-12
Cmu = 1e-12
Ro  = 100e3
gm  = 40e-3

freq=logspace(4,9);
V_2=zeros(size(freq));

A=[V_1/Rx; 0];

for k=1:length(freq)

 omega=2*pi*freq(k);

 B=[1/Rx+1/Rpi+j*omega*(Cpi+Cmu) -j*omega*Cmu;...
     gm-j*omega*Cmu       1/Ro+j*omega*Cmu];
```

```
 C=inv(B)*A;

 V_2(k)=C(2);

end
A_11=V_1./V_2;

subplot(211)
semilogx(freq,abs(A_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude ')
title('Magnitude of A_{11}')

subplot(212)
semilogx(freq,angle(A_11))
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
title('Phase of A_{11}')
%==============================================
```

Figure 5.11 c) shows the equivalent circuit for the calculation of A_{12}. For this circuit we can write:

$$\begin{cases} \dfrac{V_1 - V_\pi}{R_x} = \dfrac{V_\pi}{R_\pi} + V_\pi\, j\,\omega\, C_\pi + V_\pi\, j\,\omega\, C_\mu \\ \\ I_2 = g_m\, V_\pi - V_\pi\, j\,\omega\, C_\mu \end{cases} \tag{5.5}$$

Solving this set of eqns in order to obtain $V_1/(-I_2)$ we get A_{12}:

$$A_{12} \quad = \quad \frac{R_\pi + R_x + j\,\omega\,(C_\mu + C_\pi)\,R_\pi\,R_x}{R_\pi\,(j\,\omega\,C_\mu - g_m)}$$

```
%============  mat_script4.m =================

clear
clf

Rx  = 5
Rpi = 2.5e3
Cpi = 10e-12
Cmu = 1e-12
Ro  = 100e3
gm  = 40e-3

freq=logspace(4,9);

omega=2.*pi.*freq;

A_12=(Rx+Rpi+j.*omega.*(Cpi+Cmu).*Rpi.*Rx)...
     ./((j*omega*Cmu-gm).*Rpi);

subplot(211)
semilogx(freq,abs(A_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude ')
title('Magnitude of A_{12}')

subplot(212)
semilogx(freq,angle(A_12))
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
title('Phase of A_{12}')
%============================================
```

Figure 5.11 d) shows the equivalent circuit for the calculation of A_{21}. For this circuit we can write:

$$\begin{cases} I_1 = \dfrac{V_\pi}{R_\pi} + V_\pi\, j\,\omega\, C_\pi + (V_\pi - V_2)\, j\,\omega\, C_\mu \\ \\ (V_\pi - V_2)\, j\,\omega\, C_\mu = g_m\, V_\pi + \dfrac{V_2}{R_o} \end{cases} \tag{5.6}$$

This set of eqns can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} I_1 \\ \\ 0 \end{bmatrix} \tag{5.7}$$

$$[B] = \begin{bmatrix} \dfrac{1}{R_\pi} + j\,\omega\,(C_\mu + C_\pi) & -j\,\omega\, C_\mu \\ \\ g_m - j\,\omega\, C_\mu & \dfrac{1}{R_o} + j\,\omega\, C_\mu \end{bmatrix} \tag{5.8}$$

$$[C] = \begin{bmatrix} V_\pi \\ \\ V_2 \end{bmatrix} \tag{5.9}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

```
%============  mat_script5.m =================
clear
clf

I_1 = 1;

Rx  = 5
Rpi = 2.5e3
Cpi = 10e-12
Cmu = 1e-12
Ro  = 100e3
gm  = 40e-3

freq=logspace(4,9);
V_2=zeros(size(freq));

A=[I_1; 0];

for k=1:length(freq)

 omega=2*pi*freq(k);

 B=[1/Rpi+j*omega*(Cpi+Cmu) -j*omega*Cmu;...
     gm-j*omega*Cmu       1/Ro+j*omega*Cmu];

 C=inv(B)*A;

 V_2(k)=C(2);
```

```
end
A_21=I_1./V_2;

subplot(211)
semilogx(freq,abs(A_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude ')
title('Magnitude of A_{21}')

subplot(212)
semilogx(freq,angle(A_21))
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
title('Phase of A_{21}')
%==============================================
```

Figure 5.11 e) shows the equivalent circuit for the calculation of A_{22}. For this circuit we can write:

$$\begin{cases} I_1 = \frac{V_\pi}{R_\pi} + V_\pi\, j\,\omega\, C_\pi + V_\pi\, j\,\omega\, C_\mu \\ \\ V_\pi\, j\,\omega\, C_\mu = g_m\, V_\pi - I_2 \end{cases} \tag{5.10}$$

Solving to obtain $I_1/(-I_2)$ we can express A_{22} as follows:

$$A_{22} \quad = \quad \frac{1 + j\,\omega\,(C_\mu + C_\pi)\,R_\pi}{R_\pi\,(j\,\omega\,C_\mu - g_m)}$$

```
%==================  mat_script6.m ===========

clear
clf

Rx  = 5
Rpi = 2.5e3
Cpi = 10e-12
Cmu = 1e-12
Ro  = 100e3
gm  = 40e-3

freq=logspace(4,9);

omega=2.*pi.*freq;

A_22=(1+j.*omega.*(Cpi+Cmu).*Rpi)...
     ./((j*omega*Cmu-gm).*Rpi);

subplot(211)
semilogx(freq,abs(A_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude ')
title('Magnitude of A_{22}')

subplot(212)
semilogx(freq,angle(A_22))
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
title('Phase of A_{22}')
%===============================================
```

5.4 Series connection

Example 5.4 Determine the Z-parameters of the circuit of figure 5.12 a) considering that this circuit results from the series connection of two *two-port* circuits as shown in figure 5.12 b). Consider frequencies ranging from 10 Hz to 100kHz.

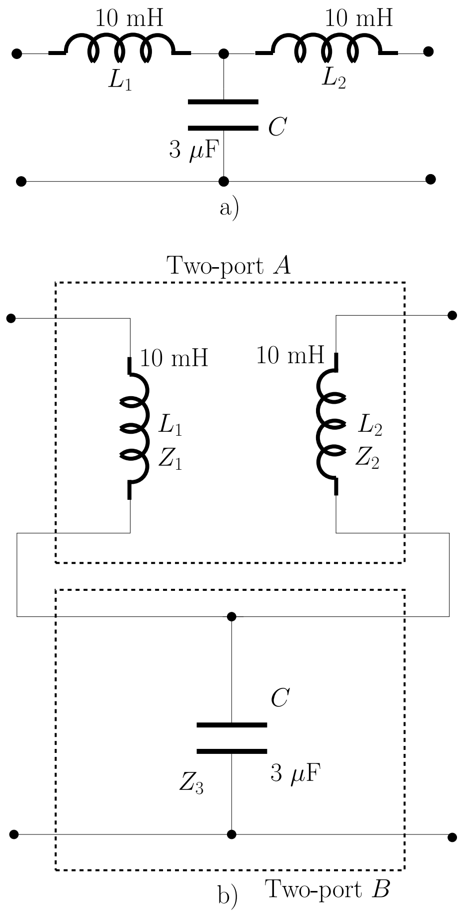


Figure 5.12: *a) Two-port circuit. b) Equivalent circuit.*

Solution (using MATLAB/OCTAVE):

The two-port A can be characterised by the following Z-parameters:

$$[Z_A] = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \tag{5.11}$$

that is

$$[Z_A] = \begin{bmatrix} j\,\omega\,L_1 & 0 \\ 0 & j\,\omega\,L_2 \end{bmatrix} \tag{5.12}$$

The two-port B can be characterised by the following Z-parameters:

$$[Z_B] = \begin{bmatrix} Z_3 & Z_3 \\ Z_3 & Z_3 \end{bmatrix} \tag{5.13}$$

that is

$$[Z_B] = \begin{bmatrix} (j\,\omega\,C)^{-1} & (j\,\omega\,C)^{-1} \\ (j\,\omega\,C)^{-1} & (j\,\omega\,C)^{-1} \end{bmatrix} \tag{5.14}$$

The two-port of figure 5.12 a) can be characterised by Z-parameters given by:

$$[Z_{eq}] \quad = \quad [Z_A] + [Z_B]$$

```
%============  mat_script7.m ================

clear
clf

C= 3e-6;
L_1=10e-3;
L_2=10e-3;

f=logspace(1,5);
omega=2*pi.*f;
Z_11=zeros(size(f));
Z_21=zeros(size(f));
Z_22=zeros(size(f));
Z_12=zeros(size(f));

for k=1:length(f)

   Z_A=[j*omega(k)*L_1 0; ...
        0 j*omega(k)*L_2];

   Z_B=[1/(j*omega(k)*C) 1/(j*omega(k)*C); ...
```

```
          1/(j*omega(k)*C) 1/(j*omega(k)*C)];

   Z_eq=Z_A+Z_B;

   Z_11(k)=Z_eq(1,1);
   Z_21(k)=Z_eq(2,1);
   Z_22(k)=Z_eq(2,2);
   Z_12(k)=Z_eq(1,2);

end

subplot(221)
semilogx(f,abs(Z_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{11}')

subplot(222)
semilogx(f,abs(Z_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{12}')


subplot(223)
semilogx(f,abs(Z_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{21}')

subplot(224)
semilogx(f,abs(Z_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of Z_{22}')
%=============================================
```

5.5 Parallel connection

Example 5.5 Determine the Y-parameters of the circuit of figure 5.13 a) considering that this circuit results from the parallel connection of two *two-port* circuits as shown in figure 5.13 b). Consider frequencies ranging from 100 kHz to 100 MHz.

Solution (using MATLAB/OCTAVE):

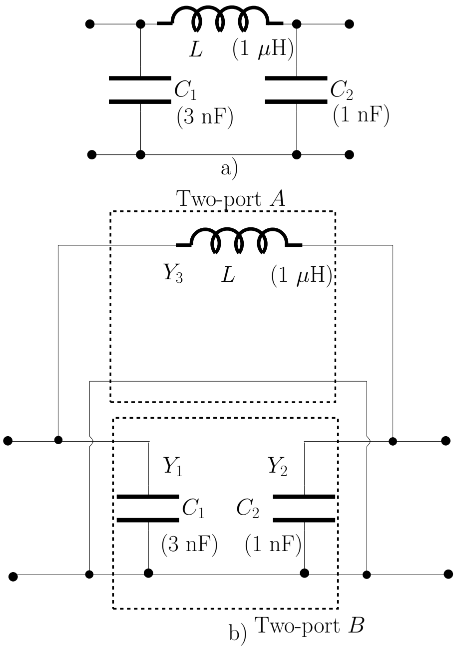


Figure 5.13: *a) Two-port circuit. b) Equivalent circuit.*

The two-port A can be characterised by the following Y-parameters:

$$[Y_A] = \begin{bmatrix} Y_3 & -Y_3 \\ -Y_3 & Y_3 \end{bmatrix} \tag{5.15}$$

that is

$$[Y_A] = \begin{bmatrix} (j\,\omega\,L)^{-1} & -(j\,\omega\,L)^{-1} \\ -(j\,\omega\,L)^{-1} & (j\,\omega\,L)^{-1} \end{bmatrix} \tag{5.16}$$

The two-port B can be characterised by the following Y-parameters:

$$[Y_B] = \begin{bmatrix} Y_1 & 0 \\ 0 & Y_2 \end{bmatrix} \tag{5.17}$$

that is

$$[Y_B] = \begin{bmatrix} j\,\omega\,C_1 & 0 \\ 0 & j\,\omega\,C_2 \end{bmatrix} \tag{5.18}$$

The two-port of figure 5.13 a) can be characterised by Y-parameters given by:

$$[Y_{eq}] \;=\; [Y_A] + [Y_B]$$

```
%============  mat_script8.m =================

clear
clf

C_1= 3e-9;
L=1e-6;
C_2=1e-9;

f=logspace(5,8);
omega=2*pi.*f;

Y_11=zeros(size(f));
Y_21=zeros(size(f));
Y_22=zeros(size(f));
Y_12=zeros(size(f));

for k=1:length(f)

   Y_A=[1/(j*omega(k)*L) -1/(j*omega(k)*L); ...
        -1/(j*omega(k)*L) 1/(j*omega(k)*L)];
```

```
   Y_B=[j*omega(k)*C_1 0; ...
        0 j*omega(k)*C_2];

   Y_eq=Y_A+Y_B;

   Y_11(k)=Y_eq(1,1);
   Y_21(k)=Y_eq(2,1);
   Y_22(k)=Y_eq(2,2);
   Y_12(k)=Y_eq(1,2);

end

subplot(221)
semilogx(f,abs(Y_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{11}')

subplot(222)
semilogx(f,abs(Y_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{12}')

subplot(223)
semilogx(f,abs(Y_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{21}')

subplot(224)
semilogx(f,abs(Y_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{22}')

%==============================================
```

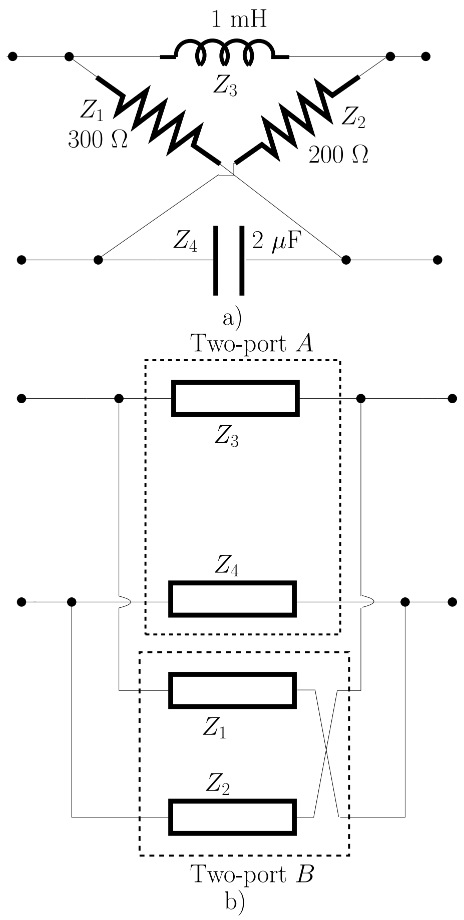


Figure 5.14: *a) Two-port circuit. b) Equivalent circuit.*

Example 5.6 Determine the Y-parameters of the circuit of figure 5.14 a) considering that this circuit results from the parallel connection of two *two-port* circuits as shown in figure 5.14 b). Consider frequencies ranging from 100 Hz to 100 kHz.

Solution (using MATLAB/OCTAVE):

The two-port A can be characterised by the following Y-parameters:

$$[Y_A] = \begin{bmatrix} \dfrac{1}{Z_3 + Z_4} & \dfrac{-1}{Z_3 + Z_4} \\ \dfrac{-1}{Z_3 + Z_4} & \dfrac{1}{Z_3 + Z_4} \end{bmatrix} \tag{5.19}$$

with $Z_3 = j\,\omega\,L$ and $Z_4 = (j\,\omega\,C)^{-1}$.

The two-port B can be characterised by the following Y-parameters:

$$[Y_B] = \begin{bmatrix} \dfrac{1}{Z_1 + Z_2} & \dfrac{1}{Z_1 + Z_2} \\ \dfrac{1}{Z_1 + Z_2} & \dfrac{1}{Z_1 + Z_2} \end{bmatrix} \tag{5.20}$$

with $Z_1 = 300\ \Omega$ and $Z_2 = 200\ \Omega$. The two-port of figure 5.14 a) can be characterised by Y-parameters given by:

$$[Y_{eq}] = [Y_A] + [Y_B]$$

```
%===============  mat_script8b.m =============

clear
clf

C=2e-6;
L=1e-3;

f=logspace(2,5);
omega=2*pi.*f;

Z1=300;
Z2=200;

Y_11=zeros(size(f));
Y_21=zeros(size(f));
Y_22=zeros(size(f));
Y_12=zeros(size(f));

for k=1:length(f)

   Z3=j.*omega(k).*L;
   Z4=1./(j.*omega(k).*C);

   Y_A=[1./(Z3+Z4) -1./(Z3+Z4); ...
       -1./(Z3+Z4)   1./(Z3+Z4)];

   Y_B=[1./(Z1+Z2)   1./(Z1+Z2); ...
        1./(Z1+Z2)   1./(Z1+Z2)];
```

```
   Y_eq=Y_A+Y_B;

   Y_11(k)=Y_eq(1,1);
   Y_21(k)=Y_eq(2,1);
   Y_22(k)=Y_eq(2,2);
   Y_12(k)=Y_eq(1,2);

end

subplot(221)
semilogx(f,abs(Y_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{11}')

subplot(222)
semilogx(f,abs(Y_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{12}')

subplot(223)
semilogx(f,abs(Y_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{21}')

subplot(224)
semilogx(f,abs(Y_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of Y_{22}')
%==============================================
```

5.6 Chain connection

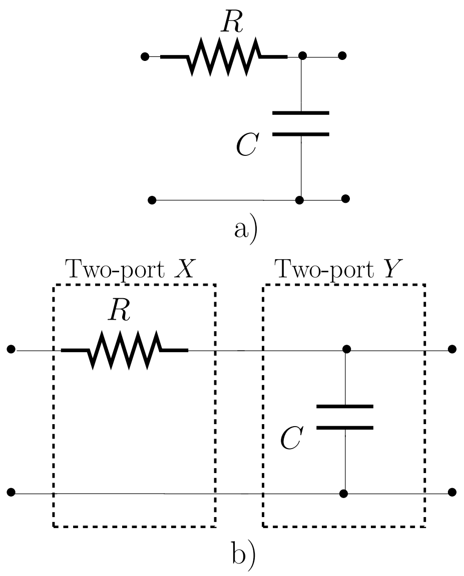


Figure 5.15: *a) Two-port circuit. b) Equivalent circuit.*

Example 5.7 Determine the chain-parameters of the circuit of figure 5.15 a) considering that this circuit results from the chain connection of two *two-port* circuits as shown in figure 5.15 b). Consider frequencies ranging from 100 kHz to 100 MHz.

Solution (using MATLAB/OCTAVE):

The two-port X can be characterised by the following chain parameters:

$$[A_X] = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \tag{5.21}$$

The two-port Y can be characterised by the following chain parameters:

$$[A_Y] = \begin{bmatrix} 1 & 0 \\ j\,\omega\,C & 1 \end{bmatrix} \tag{5.22}$$

The two-port of figure 5.12 a) can be characterised by chain parameters given by:

$$[A_{eq}] \;=\; [A_X] \times [A_Y]$$

```
%================  mat_script9.m =============
clear
clf

C= 3e-9;
R= 100;

f=logspace(5,8);
omega=2*pi.*f;

A_11=zeros(size(f));
A_21=zeros(size(f));
A_22=zeros(size(f));
A_12=zeros(size(f));

for k=1:length(f)

   A_X=[1 R; ...
        0 1];

   A_Y=[1 0; ...
        j*omega(k)*C 1];

   A_eq=A_X*A_Y;

   A_11(k)=A_eq(1,1);
   A_21(k)=A_eq(2,1);
   A_22(k)=A_eq(2,2);
   A_12(k)=A_eq(1,2);

end
```

```
subplot(221)
semilogx(f,abs(A_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('Magnitude of A_{11}')

subplot(222)
semilogx(f,abs(A_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude (Ohm)')
title('Magnitude of A_{12}')

subplot(223)
semilogx(f,abs(A_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude (S)')
title('Magnitude of A_{21}')

subplot(224)
semilogx(f,abs(A_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude ')
title('Magnitude of A_{22}')
%==============================================
```

5.7 Conversion between parameters

We use the tables appearing in the book in appendix C for conversions between the various parameters.

5.7.1 Chain to admittance

Example 5.8 Write a script-file to convert chain parameters into admittance parameters.

<u>Solution:</u>

```
%==============================================
function [y11,y12,y21,y22]=a2y(a11,a12,a21,a22)
%
%function [y11,y12,y21,y22]=a2y(a11,a12,a21,a22)
% CONVERSION OF CHAIN TO Y PARAMETERS
% aij are vectors--chain parameters versus the
% frequency
% yij are vectors--admittance parameters versus
% the frequency

      DET=a11.*a22-a21.*a12;

     y11=a22./a12;
     y12=-DET./a12;
     y21=-1./a12;
     y22=a11./a12;
%==============================================
```

5.7.2 Impedance to admittance

Example 5.9 Write a script-file to convert impedance parameters into admittance parameters.

Solution (using MATLAB/OCTAVE):

```
%==============================================
function [y11,y12,y21,y22]=z2y(z11,z12,z21,z22)
%
%function [y11,y12,y21,y22]=z2y(z11,z12,z21,z22)
% CONVERSION OF Z PARAMETERS TO Y PARAMETERS
% zij are vectors--impedance parameters versus
% the frequency)
% yij are vectors--admittance parameters versus
% the frequency

      DET=z11.*z22-z21.*z12;

     y11=z22./DET;
     y12=-z12./DET;
     y21=-z21./DET;
     y22=z11./DET;
%==============================================
```

5.7.3 Impedance to chain

Example 5.10 Write a script-file to convert impedance parameters into chain parameters.

Solution:

```
%==============================================
function [a11,a12,a21,a22]=z2a(z11,z12,z21,z22)
%
%function [a11,a12,a21,a22]=z2a(z11,z12,z21,z22)
% CONVERSION OF Z PARAMETERS TO CHAIN PARAMETERS
% zij are vectors--impedance parameters versus
% the frequency
% aij are vectors--chain parameters versus the
% frequency

      DET=z11.*z22-z21.*z12;

     a11=z11./z21;
     a12=DET./z21;
     a21=1./z21;
     a22=z22./z21;

%==============================================
```

5.7.4 Admittance to chain

Example 5.11 Write a script-file to convert admittance parameters into chain parameters.

<u>Solution:</u>

```
%==============================================

function [a11,a12,a21,a22]=y2a(y11,y12,y21,y22)

%function [a11,a12,a21,a22]=y2a(y11,y12,y21,y22)
% CONVERSION OF Y PARAMETERS TO CHAIN PARAMETERS
% aij are vectors--chain parameters versus
% the frequency
% yij are vectors--admittance parameters versus
% the frequency

      DET=y11.*y22-y21.*y12;

     a11=-y22./y21;
     a12=-1./y21;
     a21=-DET./y21;
     a22=-y11./y21;
%==============================================
```

5.7.5 Chain to impedance

Example 5.12 Write a script-file to convert chain parameters into impedance parameters.

<u>Solution:</u>

```
%==============================================
function [z11,z12,z21,z22]=a2z(a11,a12,a21,a22)
%
%function [z11,z12,z21,z22]=a2z(a11,a12,a21,a22)
% CONVERSION OF CHAIN PARAMETERS TO z PARAMETERS
% aij are vectors--chain parameters versus
% the frequency
% zij are vectors--impedance parameters versus
% the frequency

      DET=a11.*a22-a21.*a12;

     z11=a11./a21;
     z12=DET./a21;
     z21=1./a21;
     z22=a22./a21;
%==============================================
```

5.7.6 Admittance to impedance

Example 5.13 Write a script-file to convert admittance parameters into impedance parameters.

Solution:

```
%==============================================
function [z11,z12,z21,z22]=y2z(y11,y12,y21,y22)

%function [z11,z12,z21,z22]=y2z(y11,y12,y21,y22)
% CONVERSION OF Y PARAMETERS TO Z PARAMETERS
% zij are vectors--impedance parameters versus
% the frequency
% yij are vectors--admittance parameters versus
% the frequency

      DET=y11.*y22-y21.*y12

     z11=y22./DET;
     z12=-y12./DET;
     z21=-y21./DET;
     z22=y11./DET;
%==============================================
```

5.8 Computer-aided electrical analysis

Example 5.14 Write a script-file to compute the equivalent chain matrix of the cascade of two *two-port* circuits.

Solution:

```
%==========================================================================
function [a11,a12,a21,a22]=CHAIN(x11,x12,x21,x22, x , w11,w12,w21,w22, w)

% [a11,a12,a21,a22]=chain(x11,x12,x21,x22, x , w11,w12,w21,w22, w)
% CALCULATES THE EQUIVALENT CHAIN MATRIX OF THE CASCADE OF TWO TWO-PORTS
% 'x' INDICATES THE REPRESENTATION FOR THE FIRST MATRIX :z,y,a
% 'w' INDICATES THE REPRESENTATION FOR THE SECOND MATRIX :z,y,a

if (x == 'a')

  ax11=x11;
  ax12=x12;
  ax21=x21;
  ax22=x22;

else

  eval([' [ax11,ax12,ax21,ax22]=' x '2a(x11,x12,x21,x22);']);

end

if (w == 'a')

  aw11=w11;
  aw12=w12;
  aw21=w21;
  aw22=w22;

else

  eval([' [aw11,aw12,aw21,aw22]=' w '2a(w11,w12,w21,w22);']);

end

a11=ax11.*aw11  + ax12.*aw21;
a12=ax11.*aw12  + ax12.*aw22;
a21=ax21.*aw11  + ax22.*aw21;
a22=ax21.*aw12  + ax22.*aw22;
%==========================================================================
```

Example 5.15 Write a script-file to compute the equivalent admittance matrix of the parallel connection of two *two-port* circuits.

<u>*Solution:*</u>

```
%==========================================================================
function [y11,y12,y21,y22]=PARALLEL(x11,x12,x21,x22,x,w11,w12,w21,w22,w)

% [y11,y12,y21,y22]=PARALLEL(x11,x12,x21,x22, 'x', w11,w12,w21,w22, 'w')
% CALCULATES THE EQUIVALENT [Y] MATRIX OF THE PARALLEL CONNECTION OF
% TWO TWO-PORT CIRCUITS
% 'x' INDICATES THE REPRESENTATION FOR THE FIRST MATRIX :z,y,a
% 'w' INDICATES THE REPRESENTATION FOR THE SECOND MATRIX :z,y,a

if (x == 'y')

 ya11=x11;
 ya12=x12;
 ya21=x21;
 ya22=x22;

else

  eval([' [ya11,ya12,ya21,ya22]=' x '2y(x11,x12,x21,x22);']);

end


if (w == 'y')

 yb11=w11;
 yb12=w12;
 yb21=w21;
 yb22=w22;

else

  eval([' [yb11,yb12,yb21,yb22]=' w '2y(w11,w12,w21,w22);']);

end

y11= ya11 + yb11 ;
y12= ya12 + yb12 ;
y21= ya21 + yb21 ;
y22= ya22 + yb22 ;
%==========================================================================
```

Example 5.16 Write a script-file to compute the equivalent impedance matrix of the series connection of two *two-port* circuits.

<u>*Solution:*</u>

```
%============================================================================
function [z11,z12,z21,z22]=SERIES(x11,x12,x21,x22, x , w11,w12,w21,w22, w)

% [z11,z12,z21,z22]=SERIES(x11,x12,x21,x22, 'x', w11,w12,w21,w22, 'w')
% CALCULATES THE EQUIVALENT [Z] MATRIX OF THE SERIES CONNECTION OF TWO
% TWO-PORT CIRCUITS
% 'x' INDICATES WHICH REPRESENTATION FOR THE FIRST MATRIX :z,y,a
% 'w' INDICATES WHICH REPRESENTATION FOR THE SECOND MATRIX :z,y,a

if (x == 'z')

 za11=x11;
 za12=x12;
 za21=x21;
 za22=x22;

else

  eval([' [za11,za12,za21,za22]=' x '2z(x11,x12,x21,x22);']);

end


if (w == 'z')

 zb11=w11;
 zb12=w12;
 zb21=w21;
 zb22=w22;

else

  eval([' [zb11,zb12,zb21,zb22]=' w '2z(w11,w12,w21,w22);']);

end

z11= za11 + zb11 ;
z12= za12 + zb12 ;
z21= za21 + zb21 ;
z22= za22 + zb22 ;
%============================================================================
```

Example 5.17 Consider the circuit of figure 5.16 which represents an electrical model of an amplifier. Determine the voltage gain V_o/V_s for frequencies ranging from 1 kHz to 10 GHz. $C_i = 1$ μF, $C_{gs} = 10$ pF, $C_{gd} = 1$ pF, $R_S = 100$ Ω, $G_m = 20$ mS, $R_L = 10$ kΩ, $R_o = 70$ kΩ.

Solution (using MATLAB/OCTAVE):

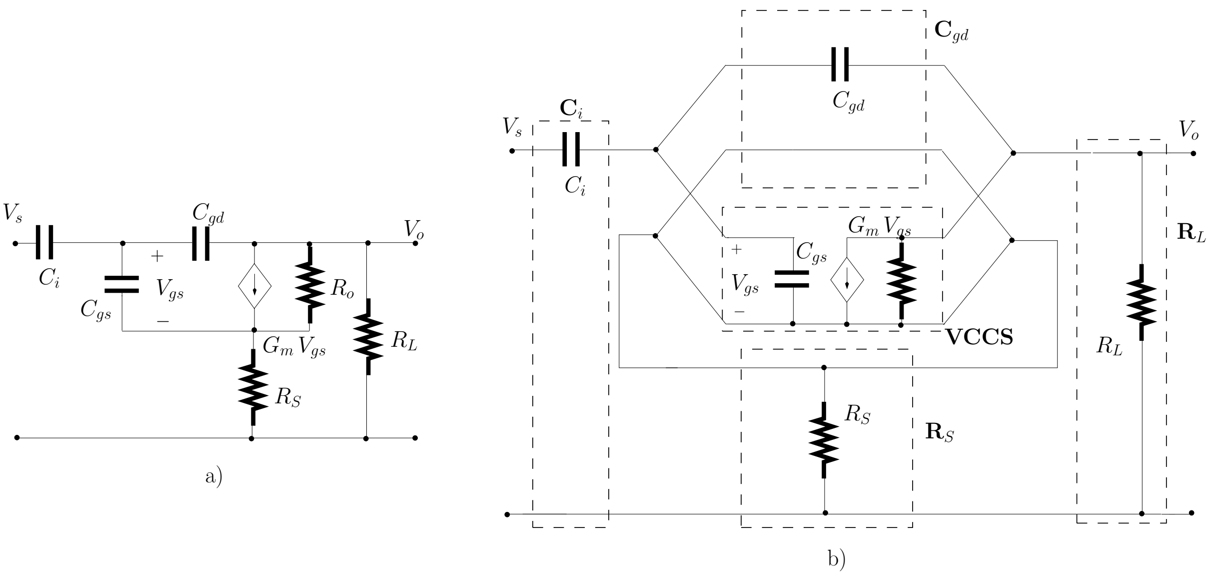


Figure 5.16: *a) Electrical model for an amplifier. b) Equivalent circuit.*

```
%===============================mat_script10.m============================
clear
clf

C_i=1e-6;
Cgs=10e-12;
Cgd=1e-12;
Gm =20e-3;
Ro =70e3
RL =10e3
RS =100;

f=logspace(3,10);
omega=2*pi.*f;

%+++++++++++++++++++++++++++++++VCCS (y-parameters)++++++++++++++++++++++

VCCS_11=j.*omega.*Cgs;
VCCS_12=zeros(size(f));
VCCS_21=Gm.*ones(size(f));
VCCS_22=1/Ro.*ones(size(f));

%++++++++++++++++++++++++++++++++ CGD (y-parameters)+++++++++++++++++++++

CGD_11= j.*omega.*Cgd;
CGD_12=-j.*omega.*Cgd;
```

```
CGD_21=-j.*omega.*Cgd;
CGD_22= j.*omega.*Cgd;

%+++++++++++++++++++++++++++++++++ RS (z-parameters)+++++++++++++++++++++

RS_11=RS.*ones(size(f));
RS_12=RS.*ones(size(f));
RS_21=RS.*ones(size(f));
RS_22=RS.*ones(size(f));

%++++++++++++++++++++++++++++++ RL (z-parameters)++++++++++++++++++++++++

RL_11=RL.*ones(size(f));
RL_12=RL.*ones(size(f));
RL_21=RL.*ones(size(f));
RL_22=RL.*ones(size(f));

%+++++++++++++++++++++++++++ Ci (y-parameters)+++++++++++++++++++++++++++

Ci_11= j.*omega.*C_i;
Ci_12=-j.*omega.*C_i;
Ci_21=-j.*omega.*C_i;
Ci_22= j.*omega.*C_i;

%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
%++++++++++++++++++++++++++++++++++++++Xa = VCCS and CGD ++++++++++++++++

[Xa_11,Xa_12,Xa_21,Xa_22]=PARALLEL(VCCS_11,VCCS_12,VCCS_21,VCCS_22,'y'...
                                   ,CGD_11,CGD_12,CGD_21,CGD_22,'y');

%++++++++++++++++++++++Xb = VCCS, CGD and RS ++++++++++++++++++++++++++++

[Xb_11,Xb_12,Xb_21,Xb_22]=SERIES(Xa_11,Xa_12,Xa_21,Xa_22,'y',...
                                 RS_11,RS_12,RS_21,RS_22,'z');

%+++++++++++++++++++++Xc = VCCS, CGD, RS and RL +++++++++++++++++++++++++

[Xc_11,Xc_12,Xc_21,Xc_22]=CHAIN(Xb_11,Xb_12,Xb_21,Xb_22,'z',...
                                RL_11,RL_12,RL_21,RL_22,'z');

%++++++++++++++++++++++++Xd = VCCS, CGD, RS, RL and Ci+++++++++++++++++++

[Xd_11,Xd_12,Xd_21,Xd_22]=CHAIN(Xc_11,Xc_12,Xc_21,Xc_22,'a',...
                                Ci_11,Ci_12,Ci_21,Ci_22,'y');

subplot(211)
semilogx(f,abs(1./Xd_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('Magnitude of the voltage gain')

subplot(212)
semilogx(f,angle(1./Xd_11))
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
title('Phase of the voltage gain')
%===========================================================================
```

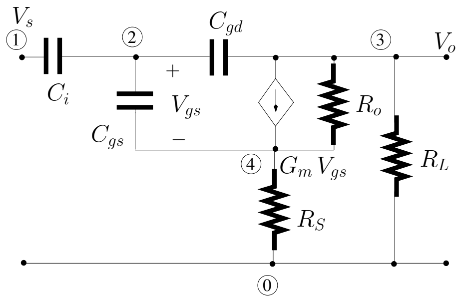


Figure 5.17: *Electrical model for an amplifier.*

Solution (using SPICE):

```
* Circuit of figure 5.17

*-----------------netlist9--------------------
V_s 1 0 ac 1

R_S  4 0 100
R_x  4 2 1e9
R_o  3 4 70k
C_gs 2 4 10p
C_i  1 2 1u
C_mu 3 2 1p
R_L   3 0 10k
G_Gm 3 4 2 4 20m
*---------------------------------------------

*---------------------------------------------

.ac dec 10 1e3 1e10
.plot ac abs(V(3)) phase(V(3))

.end

*---------------------------------------------
```

Chapter 6

Basic electronic amplifier building blocks

6.1 Operational Amplifiers

Example 6.1 Consider the Non-inverting amplifier of figure 6.1 a). Determine the voltage gain v_o/v_s and shows that it is approximately equal to $1 + R_1/R_2$. Show that the input terminals of the op-amp are nearly at the same voltage potential (virtual short-circuit). Assume a non-ideal op-amp with input resistance of 50 MΩ, output resistance of 40 Ω and open-loop voltage gain of 10^5.

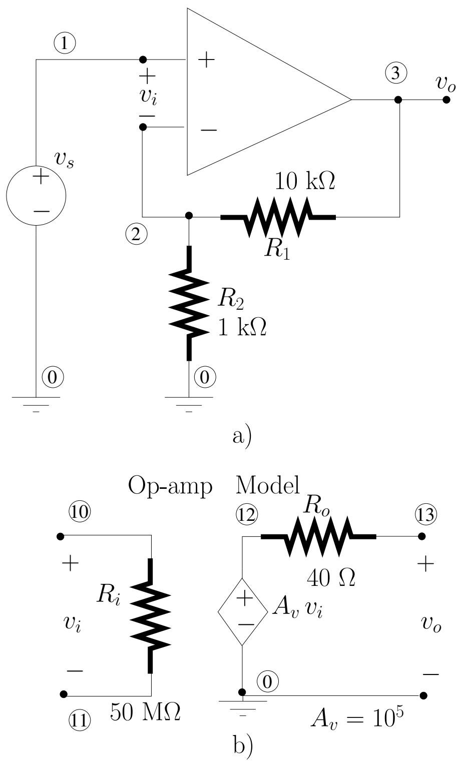


Figure 6.1: *a) Non-inverting amplifier. b) Op-Amp electrical model.*

Solution (using SPICE):

```
* Circuit of figure 6.1
*----------------netlist1----------------------
v_s 1 0 dc 1

R_2 2 0 1k
R_1 2 3 10k

x1 1 2 3 opamp
*---------------------------------------------
.subckt opamp 10 11 13  * See fig. 6.1 b)
R_i 10 11 5e7
R_o 12 13 40
E_Av 12 0 10 11 1e5
.ends
*---------------------------------------------

.dc v_s 0 1 1
.print dc v(2) v(3)

.end
*---------------------------------------------
```

Since the input voltage $v_s = 1$ V, the voltage gain is equal to:

$$A_v = \frac{\texttt{v(3)}}{1}$$

Solution (using MATLAB/OCTAVE):

For the circuit of figure 6.2 we can write:

$$\begin{cases} v_s = v_i + v_f \\ \dfrac{v_i}{R_i} = \dfrac{v_f}{R_2} + \dfrac{v_f - v_o}{R_1} \\ \dfrac{A_v\, v_i - v_o}{R_o} = \dfrac{v_o - v_f}{R_1} \end{cases} \tag{6.1}$$

The last eqn can be written in matrix form as follows:

$$[A] = [B] \times [C]$$

with

$$[A] = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix} \tag{6.2}$$

$$[B] = \begin{bmatrix} 1 & 1 & 0 \\ -\dfrac{1}{R_i} & \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_1} \\ \dfrac{A_v}{R_o} & \dfrac{1}{R_1} & -\dfrac{1}{R_1} - \dfrac{1}{R_o} \end{bmatrix} \tag{6.3}$$

$$[C] = \begin{bmatrix} v_i \\ v_f \\ v_o \end{bmatrix} \tag{6.4}$$

We can determine the unknown variables by solving the following eqn using MATLAB/OCTAVE:

$$[C] = [B]^{-1} \times [A]$$

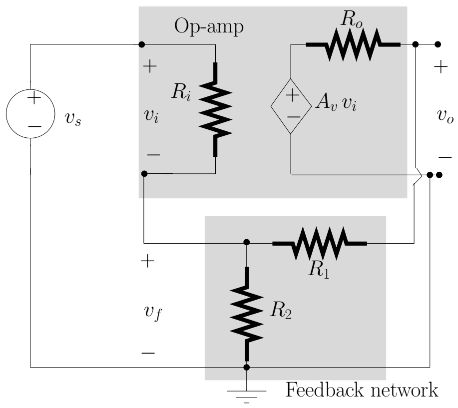


Figure 6.2: *Non-inverting amplifier equivalent model.*

```
%============  mat_script1.m =================
clear
v_s= 1;

R_1= 10e3;
R_2= 1e3;
A_v= 1e5;
R_i= 1e7;
R_o= 40;

B=[ 1 1 0; ...
    -1/R_i 1/R_1+1/R_2 -1/R_1;
    A_v/R_o 1/R_1  -1/R_o-1/R_1 ];

A=[v_s; 0; 0];

C=inv(B)*A;
```

```
v_i=C(1)
v_o=C(3)
A_v=v_o/v_s
%=============================================
```

6.2 Active devices

Example 6.2 Plot the DC characteristics of a bipolar junction transistor; $I_C = f(V_{CE}, V_{BE})$. Consider $I_S = 30 \times 10^{-15}$ A, $\beta_F = 220$, and $\beta_R = 1$. See also the circuit of figure 6.3.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 6.3

*-----------------netlist2-------------------
V_CE 2 0 dc 10
V_BE  1 0 dc 0.67

Q1 2 1 0 QGEN
*--------------------------------------------
.model QGEN NPN IS=30e-15 BF=220 BR=1
*--------------------------------------------

.dc V_CE 0.02 0.5 0.01 V_BE 0.6 0.7 0.02
.plot dc -i(V_CE)
.print dc -i(V_CE)
.end
*--------------------------------------------
```

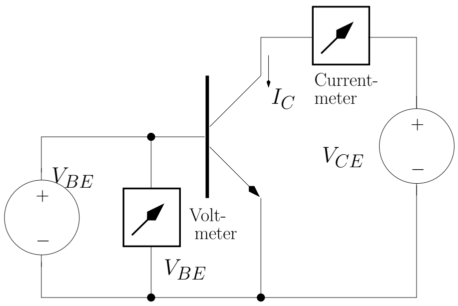


Figure 6.3: *Measurement of the DC characteristics of a BJT, $I_C = f(V_{CE}, V_{BE})$.*

<u>*Solution (using MATLAB/OCTAVE):*</u>

The Ebers-Moll model for the bipolar transistor can be written as:

$$I_E = \underbrace{I_{S_E}\left(e^{V_{BE}/V_T}-1\right)}_{\text{Diode effect}} - \underbrace{\alpha_R\, I_{S_C}\left(e^{V_{BC}/V_T}-1\right)}_{\text{Reverse transistor effect}} \quad (6.5)$$

$$I_C = \underbrace{\alpha_F\, I_{S_E}\left(e^{V_{BE}/V_T}-1\right)}_{\text{Forward transistor effect}} - \underbrace{I_{S_C}\left(e^{V_{BC}/V_T}-1\right)}_{\text{Diode effect}} \quad (6.6)$$

$$I_B = I_E - I_C \quad (6.7)$$

with

$$\alpha_F\, I_{S_E} = \alpha_R\, I_{S_C}$$

$$\alpha_F = \frac{\beta_F}{\beta_F+1}$$

$$\alpha_R = \frac{\beta_R}{\beta_R+1}$$

```
%============  mat_script2.m =================
clear

Beta_F=220;
Beta_R=1;
I_S=30e-15;


alpha_R=Beta_R/(Beta_R+1);
alpha_F=Beta_F/(Beta_F+1);

I_SE=(Beta_F+1)/Beta_F*I_S;
I_SC=(Beta_R+1)/Beta_R*I_S;

VT=27e-3;

V_CE=0.02:0.01:0.5;

V_BE_Vector=[ 0.6:0.02:0.75];

for k=1:length(V_BE_Vector)

  V_BE=V_BE_Vector(k);
  V_BC=V_BE-V_CE;
  I_E=I_SE.*(exp(V_BE./VT)-1)...
     -alpha_R.*I_SC.*(exp(V_BC./VT)-1);
  I_C=alpha_F.* I_SE.*(exp(V_BE./VT)-1)...
     -I_SC.*(exp(V_BC./VT)-1);
  I_B=I_E-I_C;

  plot(V_CE,I_C)
  hold on
  end

hold off
%==============================================
```

Example 6.3 Plot the DC characteristics of an insulated gate FET; $I_D = f(V_{DS}, V_{GS})$. Consider $K_n = 0.2$ mA/V^2 $W/L = 20$, $V_{Th} = 1$. Refer to the circuit of figure 6.4.

<u>*Solution (using SPICE):*</u>

```
* Circuit of figure 6.4

*----------------netlist3---------------------
V_DS 2 0 dc 10
V_GS 1 0 dc 1.3

Mn 2 1 0 0 modn L=10u W=200u
*---------------------------------------------
.model modn nmos level=1 VTO=1 KP=0.2e-3
*---------------------------------------------

.dc V_DS 0 10 0.01 V_GS 1 2 0.2
.plot dc -i(V_DS)
.print dc -i(V_DS)
.end
*---------------------------------------------
```

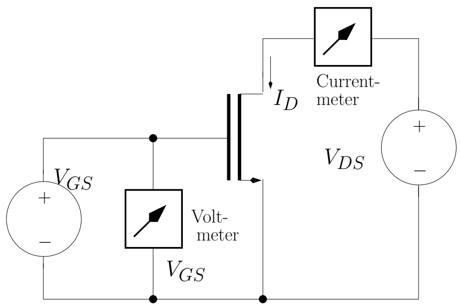


Figure 6.4: *Measurement of the DC characteristics of a IGFET,* $I_D = f(V_{DS}, V_{GS})$.

Solution (using MATLAB/OCTAVE):

The DC model that we use to characterise n-channel FETs is defined by the following set of equations:

$$I_{DS} = \begin{cases} k_n \frac{W}{L}\left[(V_{GS}-V_{Th})\,V_{DS} - \frac{1}{2}V_{DS}^2\right] & , V_{DS} \le V_{GS}-V_{Th} \text{ and } V_{GS} > V_{Th} \text{ (Triode region)} \\ \frac{1}{2}k_n \frac{W}{L}\left[V_{GS}-V_{Th}\right]^2 & , V_{DS} > V_{GS}-V_{Th} \text{ and } V_{GS} > V_{Th} \text{ (Saturation region)} \\ 0 & , V_{GS} \le V_{Th} \text{ (Cut-off region)} \end{cases} \tag{6.8}$$

```
%============  mat_script3.m =================
clear

Kn=0.2e-3;
W=200e-6
L=10e-6;

knWoL=Kn.*W/L;

VTh=1;

VDS=0:0.01:10;
IDS=zeros(size(VDS));
VGSk=1:0.2:2;

for k=1:length(VGSk)
  vgs=VGSk(k);

  vds_th=vgs-VTh;

  I_triode=find(VDS <= vds_th);
  I_sat=find(VDS >= vds_th);
  if isempty(I_triode)==0
   IDS(I_triode)=knWoL.*(vds_th.* ...
    VDS(I_triode)-VDS(I_triode).^2./2 );
  end
  if isempty(I_sat)==0
   IDS(I_sat)=0.5.*knWoL.*(vds_th).^2;
  end

  plot(VDS,IDS)
  hold on
end

hold off
%==============================================
```

Example 6.4 Determine f_T of the BJT of figure 6.5. Assume $I_S = 30 \times 10^{-15}$ A, $\beta_F = 220$, $\beta_R = 1$, $C_\mu = 2$ pF and $C_\pi = 10$ pF.

<ins>*Solution (using SPICE):*</ins>

```
* Circuit of figure 6.5

*-------------------netlist4-------------------
V_CC 2 0 dc 5
i_b  0 1 dc 0.05m ac 0.01m

Q1 2 1 0 QGEN
*---------------------------------------------
.model QGEN NPN IS=30e-15 BF=220
  + BR=1 CJC=2p CJE=10p
*---------------------------------------------

.ac DEC 10 1e2 1e10
.plot ac abs(i(V_CC))
.print ac abs(i(V_CC))
.end
*---------------------------------------------
```

f_t is the frequency at which the current gain becomes equal to unity corresponding to $|i_c| = |i_b| = 10\ \mu$A.

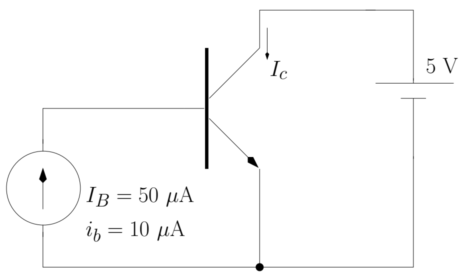


Figure 6.5: *Measurement of f_T.*

Solution (using MATLAB/OCTAVE):

The short-circuit current gain is given by:

$$\begin{aligned}\frac{i_c}{i_b} &= \frac{r_\pi\,(g_m - j\,\omega\,C_\mu)}{1 + j\,\omega\,(C_\mu + C_\pi)\,r_\pi} \\ &\simeq \frac{\beta_F}{1 + j\,\omega\,(C_\mu + C_\pi)\,r_\pi}\end{aligned}$$

The unity-gain bandwidth, f_T, can be calculated as follows:

$$f_T \simeq \frac{g_m}{2\,\pi\,(C_\mu + C_\pi)}$$

```
%============  mat_script4.m =================
clear

Beta=220;
Cpi=10e-12;
Cmu=2e-12;
IB=0.05e-3;

IC=IB*Beta;
VT=27e-3;

gm=IC/VT;
rpi=Beta/gm;

freq=logspace(2,10)
omega=2*pi.*freq;

H=rpi.*(gm-j.*omega.*Cmu)...
  ./(1+j.*omega.*(Cpi+Cmu)*rpi);

ft=gm/(2*pi*(Cmu+Cpi))

loglog(freq, abs(H))

%==============================================
```

6.3 Common-emitter amplifier

Example 6.5 Plot the voltage gain v_o/v_s as a function of frequency and find the bandwidth of the common-emitter amplifier shown in figure 6.6. Assume $I_S = 30 \times 10^{-15}$ A, $\beta_F = 220$, $\beta_R = 1$, $C_\mu = 2$ pF and $C_\pi = 10$ pF.

Solution (using SPICE):

```
* Circuit of figure 6.6

*-------------------netlist5-----------------
V_CC 7 0 dc 10
v_s  1 0 dc 0 ac 1

R_s 1 2 100
R_1 7 3 9k
R_2 3 0 1k
R_E 4 0 300
R_C 7 5 5k
R_L 6 0 15k

C_B 2 3 5u
C_E 4 0 10u
C_L 5 6 1u

Q1 5 3 4 QGEN
*-----------------------------------------------
.model QGEN NPN IS=30e-15 BF=220
+BR=1 CJC=2p CJE=10p
*-----------------------------------------------

.ac DEC 10 1e2 1e8
.plot ac abs(v(6)) phase(v(6))
.print ac abs(v(6)) phase(v(6))
.end
*-----------------------------------------------
```

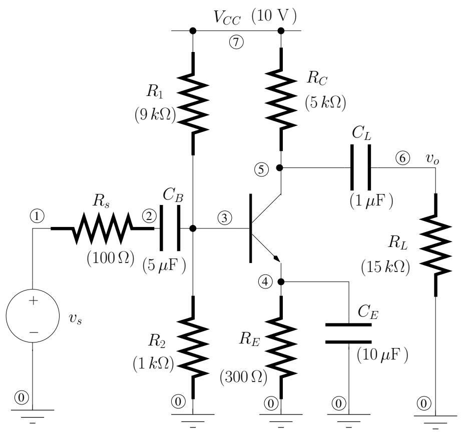


Figure 6.6: *Common-emitter amplifier.*

6.4 Common-base amplifier

Example 6.6 Plot the voltage gain v_o/v_s versus frequency and find the bandwidth of the common-base amplifier shown in figure 6.7. Assume $I_S = 30 \times 10^{-15}$ A, $\beta_F = 220$, $\beta_R = 1$, $C_\mu = 2$ pF and $C_\pi = 10$ pF.

Solution (using SPICE):

```
* Circuit of figure 6.7

*--------------------netlist5b-----------------
V_CC 7 0 dc 10
v_s  1 0 dc 0 ac 1

R_s 1 2 10
R_1 7 4 9k
R_2 4 0 1k
R_E 3 0 300
R_C 7 5 5k
R_L 6 0 15k

C_B 4 0 5u
C_E 3 2 10u
C_L 5 6 1u

Q1 5 4 3 QGEN
*---------------------------------------------
.model QGEN NPN IS=30e-15 BF=220
+BR=1 CJC=2p CJE=10p
*---------------------------------------------


.ac DEC 10 1e2 1e9
.plot ac abs(v(6)) phase(v(6))
.print ac abs(v(6)) phase(v(6))
.end
*---------------------------------------------
```

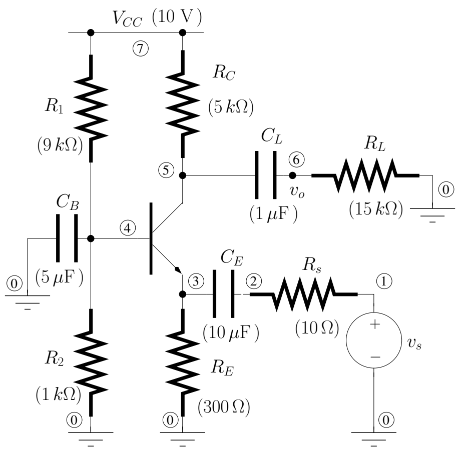


Figure 6.7: *Common-base amplifier.*

6.5 Common-collector amplifier

Example 6.7 Plot the voltage gain v_o/v_s versus frequency and find the bandwidth of the common-collector amplifier shown in figure 6.8. Assume $I_S = 30\times10^{-15}$ A, $\beta_F = 220$, $\beta_R = 1$, $C_\mu = 2$ pF and $C_\pi = 10$ pF.

Solution (using SPICE):

```
* Circuit of figure 6.8

*-------------------netlist5c-----------------
V_CC 7 0 dc 10
v_s  1 0 dc 0 ac 1

R_s 1 2 100
R_1 7 3 5k
R_2 3 0 7k
R_E 4 0 2.5k

C_B 2 3 5u

Q1 7 3 4 QGEN
*--------------------------------------------
.model QGEN NPN IS=30e-15 BF=220
+BR=1 CJC=2p CJE=10p
*--------------------------------------------

.ac DEC 10 1e1 1e10
.plot ac abs(v(4)) phase(v(4))
.print ac abs(v(4)) phase(v(4))
.end
*--------------------------------------------
```

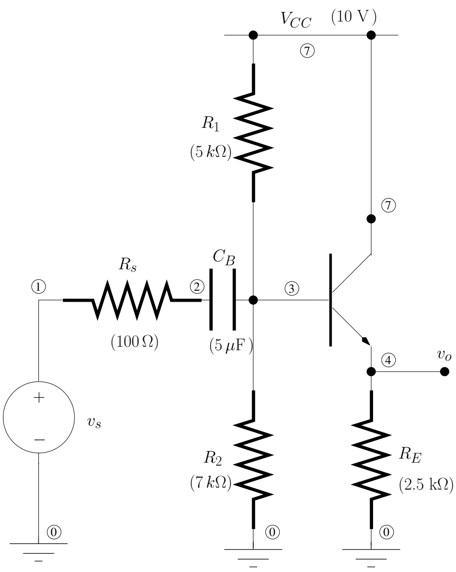


Figure 6.8: *Common-collector amplifier.*

6.6 Differential pair amplifier

Example 6.8 Plot the voltage gain v_o/v_s as a function of frequency of the differential pair amplifier shown in figure 6.9. Assume $K_n = 0.2$ mA/V^2, $W/L = 10$ and $V_{Th} = 1$.

Solution (using SPICE):

```
* Circuit of figure 6.9

*----------------netlist6----------------------
V_DD 5 0 dc 5
V_SS 6 0 dc -5

V_s 1 0 dc 0 ac 1

Mn1 3 1 2 6 modn L=10u W=100u
Mn2 4 0 2 6 modn L=10u W=100u

Mn3 2 7 6 6 modn L=10u W=100u
Mn4 7 7 6 6 modn L=10u W=100u

R_D1 5 3 6k
R_D2 5 4 6k
R_S  7 0 3k

*---------------------------------------------
.model modn nmos level=1 VTO=1 KP=0.2e-3
*---------------------------------------------

.ac DEC 10 1e2 1e8
.plot ac abs(v(4)) phase(v(4))
.print ac abs(v(4)) phase(v(4))

.end
*---------------------------------------------
```

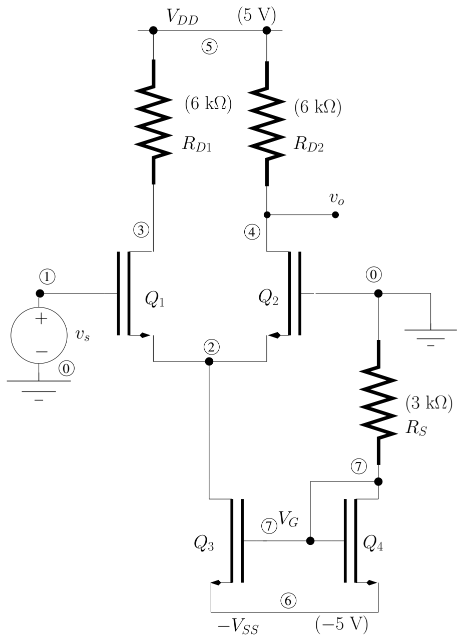


Figure 6.9: *Differential pair amplifier.*

Chapter 7

RF circuit analysis techniques

7.1 Transmission lines

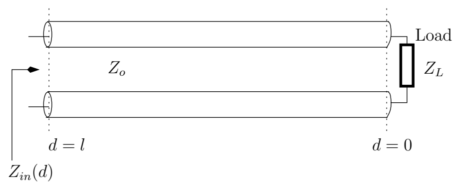


Figure 7.1: *Transmission line.*

Example 7.1 Consider the transmission line of figure 7.1. Determine the line input impedance, $Z_{in}(d)$, for $0 < \beta\, d < 0$ and for the following situations:

1. $Z_L/Z_o = 0.1$
2. $Z_L/Z_o = 1$
3. $Z_L/Z_o = 10$

Solution (using MATLAB/OCTAVE):

$$\begin{aligned}\frac{Z_{in}(d)}{Z_o} &= \frac{Z_L + j\, Z_o\, \tan(\beta\, l)}{Z_o + j\, Z_L\, \tan(\beta\, l)} \\ &= \frac{\frac{Z_L}{Z_o} + j\ \tan(\beta\, l)}{1 + j\,\frac{Z_L}{Z_o}\, \tan(\beta\, l)}\end{aligned}$$

```
%============  mat_script1.m =================
clear
clf
beta_d=0:0.02:2*pi;

ZL_over_Zo=...
    input(' Input the ratio  Z_L/Z_o ');

Zin_Norm=(ZL_over_Zo+i.*tan(beta_d))...
        ./(1+ZL_over_Zo.*i.*tan(beta_d));

subplot(211)
plot(beta_d./(pi),abs(Zin_Norm))
xlabel('beta d (normalised to pi)')
ylabel('|Z_{in}|/Z_o')

subplot(212)
plot(beta_d./(pi),angle(Zin_Norm)./(pi))
xlabel('beta d (normalised to pi)')
ylabel('Phase(Z_{in}) (normalised to pi)')
axis([0 2 -0.5 0.5])
%==============================================
```

Example 7.2 Consider the transmission line of figure 7.1 with an inductance per metre of 550 nH and a capacitance per metre of 100 pF. This line has a length $l = 13$ m and is terminated by a load $Z_L = 25\ \Omega$.Determine the line input impedance, $Z_{in}(d = l)$, for $\omega = 27$ krad/s.

Solution (using MATLAB/OCTAVE):

$$Z_{in}(d) \quad = \quad Z_o \, \frac{Z_L + j\, Z_o\, \tan(\beta\, l)}{Z_o + j\, Z_L\, \tan(\beta\, l)}$$

```
%============  mat_script2.m =================
clear

C=input(' Capacitance per meter (F)... ');
L=input(' Inductance per meter (H)... ');
l=input(' Length of the TX line (m)...');
ZL=input(' Load impedance (Ohm)...');
omega=input(' Frequency (rad/s)...');

Zo=sqrt(L/C);

beta=omega*sqrt(L*C);

Zin=Zo*(ZL+j*Zo*tan(beta*l))...
    /(Zo+j*ZL*tan(beta*l))

%==============================================
```

Example 7.3 V_s is a DC voltage source with a resistive output impedance, $Z_s = R_s = 10\ \Omega$, applied to an open-circuit transmission line with characteristic impedance $Z_o = 50\ \Omega$. Assuming that the source is switched-on at $t = 0$, show that the current provided by the source tends to zero as $t \to \infty$. The delay of the line is 3 μs.

Solution (using SPICE):

```
*------------------netlist2b-------------------

v_s 1 0 pulse(0 1 0 0.01u 0.01u 35u 40u)

R_s 1 2 10

T_line 2 0 3 0 ZO=50 TD=3u

R_L 3 0 1e12
*----------------------------------------------

*----------------------------------------------

.tran 0.01u 30u 0
.plot tran v(3) -i(v_s)

.end
*----------------------------------------------
```

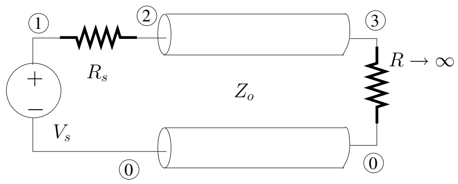


Figure 7.2: *Open-circuit transmission line driven by a DC voltage source.*

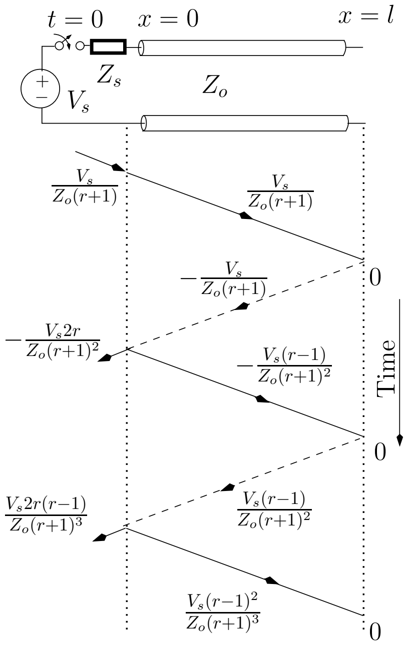


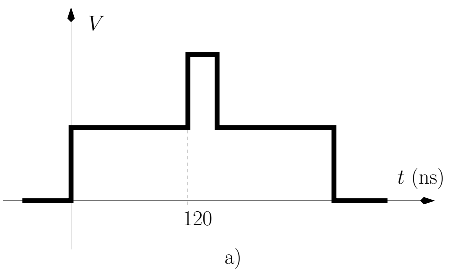
Figure 7.3: *Transient analysis for the current.*

Solution (using MATLAB/OCTAVE):

Figure 7.3 illustrates the transient analysis for the current provided by the source V_s where $r = R_s/Z_o$. The current provided by the source as t increases, I_s, can be obtained as follows:

$$\begin{aligned} I_s &= \frac{V_s}{Z_o\,(1+r)} - \frac{V_s\,2\,r}{Z_o\,(1+r)^2}\sum_{k=0}^{\infty}\left(\frac{1-r}{r+1}\right)^k \\ &= \frac{V_s}{Z_o}\left(\frac{1}{1+r} - \frac{2\,r}{(1+r)^2}\times\frac{1+r}{2\,r}\right) \\ &= 0 \end{aligned}$$

```
%============  mat_script2b.m =================
clear
clf

Zo=input('characteristic impedance (Ohm)...');
Rs=input('Source resistance (Ohm)...');

r=Zo/Rs;

N=10;

I_s=1/(1+r);

plot([0 0],[0 I_s])

hold on

for n=1:N

   k=n-1;
   plot([2*k 2*(k+1)],[I_s I_s])
   I_sn=I_s-2*r/((1+r)^2)* ((1-r)/(r+1))^k;
   plot([2*(k+1) 2*(k+1)],[I_s I_sn])
   I_s=I_sn;

end
hold off
xlabel('Time (normalised to the delay time)')
ylabel('Current (normalised to V_s/Z_o)')
title('Current provided by V_s')
%==============================================
```



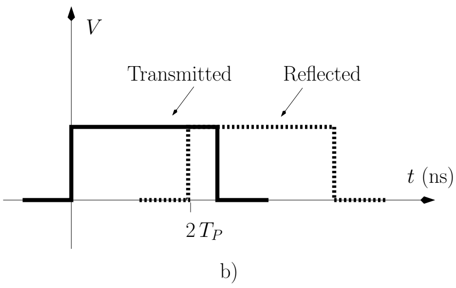


Figure 7.4: *Waveforms monitored at the input of a faulty cable.*

Example 7.4 We want to determine the location of a failure in a coaxial cable. In order to identify this location we send a 150 ns pulse through the cable. Figure 7.4 a) shows the waveform monitored at the input of the cable. Determine the location of the fault knowing that the cable has an inductance per metre of 250 nH and a capacitance per metre of 100 pF.

Solution (using MATLAB/OCTAVE):

Figure 7.4 b) shows the waveform monitored at the input of the cable as the sum of the 150 ns pulse with its delayed replica. From this figure it is clear that the fault is an open-circuit ($\Gamma = 1$). The distance where this fault occurs is obtained from the following eqn:

$$x = \frac{1}{2} T_P \frac{1}{\sqrt{LC}}$$

```
%============  mat_script2c.m ================
clear

L=250e-9;
C=100e-12;
total_delay=120e-9;

Zo=sqrt(L/C);
speed_prop=1/sqrt(L*C);

length=speed_prop*total_delay/2

%==========================================
```

Example 7.5 Simulate the faulty cable discussed in the previous example using SPICE and observe the test pulse monitored at the input of the cable.

Solution (using SPICE):

```
*-----------------netlist2c--------------------

v_s 1 0 pulse(0 1 0 0.01n 0.01n 150n 400n)
R_s 1 2 50

T_line 2 0 3 0 ZO=50 TD=60n

R_L 3 0 1e12
*-----------------------------------------------

*-----------------------------------------------

.tran 0.01n 310n 0
.plot tran v(2)

.end
```

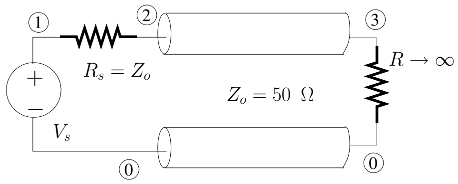


Figure 7.5: *Simulating a faulty cable (open-circuit).*

7.2 S-parameters

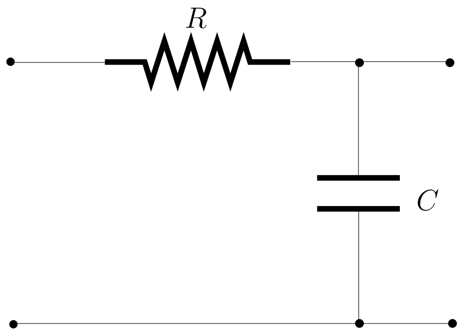


Figure 7.6: *RC Circuit.*

Example 7.6 Determine the S-parameters of an RC low-pass filter, $R = 70\ \Omega$, $C = 1$ nF. Use the frequency range 100 kHz$< f <$100 MHz and assume a reference impedance $Z_o = 50\ \Omega$.

Solution (using MATLAB/OCTAVE):

The S-parameters are given by the following expressions:

$$\begin{aligned}
S_{11} &= \frac{R + j\omega C Z_o(R - Z_o)}{R + 2Z_o + 2j\omega C Z_o(R + Z_o)} \\
S_{21} &= \frac{2\,Z_o}{2Z_o + R + j\omega C Z_o(R + Z_o)} \\
S_{22} &= \frac{R - j\omega C Z_o(Z_o + R)}{R + 2Z_o + j\omega C Z_o(Z_o + R)} \\
S_{12} &= \frac{2\,Z_o}{2Z_o + R + j\omega C Z_o(R + Z_o)}
\end{aligned}$$

```
%============  mat_script3.m =================

clear
clf

C=1e-9;
R=70;

freq=logspace(5,8);
omega=2.*pi.*freq;

Zo=50;

S_11=(R+j.*omega.*C.*Zo.*(R-Zo))...
      ./(R+2*Zo+2.*j.*omega.*C.*Zo.*(R+Zo));
S_21=(2*Zo)...
      ./(R+2*Zo+2.*j.*omega.*C.*Zo.*(R+Zo));
S_22=(R-j.*omega.*C.*Zo.*(R+Zo))...
      ./(R+2*Zo+2.*j.*omega.*C.*Zo.*(R+Zo));
S_12=(2*Zo)...
      ./(R+2*Zo+2.*j.*omega.*C.*Zo.*(R+Zo));

subplot(211)
semilogx(freq, abs(S_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('S_{11}')

subplot(212)
semilogx(freq, angle(S_11))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

A=input('Press enter for next plot...');

subplot(211)
semilogx(freq, abs(S_21))
xlabel('Frequency (Hz)')
```

```
ylabel('Amplitude')
title('S_{21}')

subplot(212)
semilogx(freq, angle(S_21))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

A=input('Press enter for next plot...');

subplot(211)
semilogx(freq, abs(S_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('S_{22}')

subplot(212)
semilogx(freq, angle(S_22))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

A=input('Press enter for next plot...');

subplot(211)
semilogx(freq, abs(S_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('S_{12}')

subplot(212)
semilogx(freq, angle(S_12))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

%=============================================
```

Example 7.7 Determine the S-parameters of the FET small-signal high-frequency model. Assume $k_n\, W/L = 40$ mA/V^2, $C_{gs} = 3$ pF, $C_{gd} = 1.5$ pF, $V_A = 60$ V and $I_D = 10$ mA. $Z_o = 50\ \Omega$. Plot the S-parameters for a frequency range 1 MHz–10 GHz.

<u>*Solution (using SPICE):*</u>

<u>Calculation of S_{11} and of S_{21}.</u>

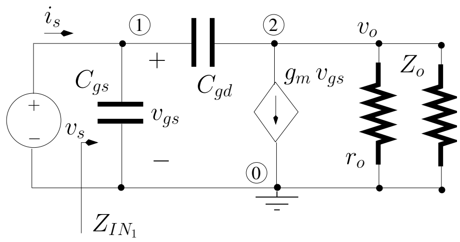


Figure 7.7: *Calculation of S_{11} and of S_{21}.*

g_m and r_o can be obtained as follows:

$$\begin{aligned} g_m &= \sqrt{k_n \frac{W}{L}\, 2\, I_D} \\ &= 0.0283\ \text{S} \\ r_o &= \frac{V_A}{I_D} \\ &= 6\ \text{k}\Omega \end{aligned}$$

```
* Circuit of figure 7.7
*--------------------netlist1------------------

v_s 1 0 ac 1

C_gd 1 2 1.5p
C_gs 1 0 3p
R_ro 2 0 6k
R_Zo 2 0 50
G_gm 2 0 1 0 28.3m

*---------------------------------------------
*---------------------------------------------

.ac DEC 10 1e6 1e10
.print ac abs(v(2)) phase(v(2))
.print ac abs(i(v_s)) -phase(i(v_s))

.end
*---------------------------------------------
```

$$\begin{aligned} S_{11} &= \frac{Z_{IN_1} - Z_o}{Z_{IN_1} + Z_o} \\ S_{21} &= \frac{2\, A_v}{1 + \frac{Z_o}{Z_{IN_1}}} \end{aligned}$$

Z_{IN_1} and A_v can be obtained as follows:

$$\begin{aligned} Z_{IN_1} &= -\frac{1}{\texttt{i(v\_s)}}\ \ (\Omega) \\ A_v &= \frac{\texttt{v(2)}}{1} \end{aligned}$$

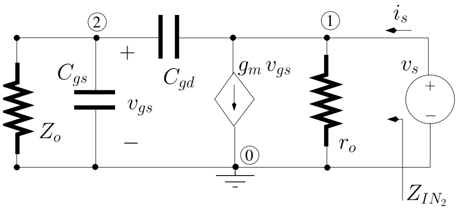


Figure 7.8: *Calculation of S_{12} and of S_{22}.*

Calculation of S_{12} and of S_{22}.

```
* Circuit of figure 7.8

*----------------------netlist2----------------

V_s 1 0 ac 1

C_gd 1 2 1.5p
C_gs 2 0 3p
R_ro 1 0 6k
R_Zo 2 0 50
G_gm 1 0 2 0 28.3m

*----------------------------------------------
*----------------------------------------------

.ac DEC 10 1e6 1e10
.print ac abs(v(2)) phase(v(2))
.print ac abs(i(V_s)) -phase(i(V_s))

.end
*----------------------------------------------
```

$$S_{22} = \frac{Z_{IN_2} - Z_o}{Z_{IN_2} + Z_o}$$

$$S_{12} = \frac{2\,A_{v_2}}{1 + \frac{Z_o}{Z_{IN_2}}}$$

Z_{IN_2} and A_{v_2} can be obtained as follows:

$$Z_{IN_2} = -\frac{1}{\texttt{i(v\_s)}} \ (\Omega)$$

$$A_{v_2} = \frac{\texttt{v(2)}}{1}$$

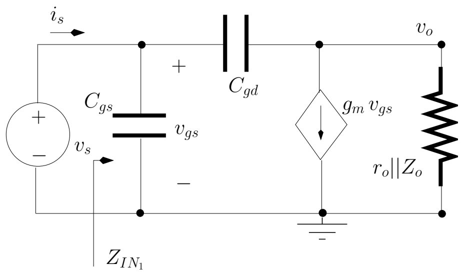


Figure 7.9: *Calculation of S_{11} and of S_{21}.*

Solution (using MATLAB/OCTAVE):

Calculation of S_{11} and of S_{21}.

$$S_{11} \quad = \quad \frac{Z_{IN_1} - Z_o}{Z_{IN_1} + Z_o}$$

$$S_{21} \quad = \quad \frac{2\, A_v}{1 + \frac{Z_o}{Z_{IN_1}}}$$

where Z_{IN_1} and $A_v = v_o/v_s$can be calculated from the circuit of figure 7.9.

$$Z_{IN_1} =$$

$$\frac{r_o + Z_o + j\,\omega\, C_{gd}\, r_o\, Z_o}{j\,\omega\, [(C_{gd} + C_{gs})\,(r_o + Z_o) + C_{gd}\, g_m\, r_o\, Z_o] - \omega^2\, C_{gd}\, C_{gs}\, r_o\, Z_o}$$

$$A_{v_1} \quad = \quad \frac{r_o\, Z_o(j\,\omega\, C_{gd} - g_m)}{j\,\omega\, C_{gd}\, r_o\, Z_o + r_o + Z_o}$$

```
%============  mat_script4.m =================
clear
clf

KnWoverL=40e-3;
Cgs=3e-12;
Cgd=1.5e-12;
VA=60;
ID=10e-3;
gm=sqrt(KnWoverL*2*ID);
ro=VA/ID;

freq=logspace(6,10);
omega=2.*pi.*freq;

Zo=50;


Z_IN1=(ro+Zo+j.*omega.*Cgd.*ro.*Zo)...
     ./(j.*omega.*((Cgs+Cgd)*(ro+Zo)+...
     Cgd*gm*ro*Zo)-omega.^2.*Cgs.*Cgd.*ro.*Zo);

A_v=(ro.*Zo.*(j.*omega.*Cgd-gm))...
        ./(j.*omega.*Cgd.*ro.*Zo+ro+Zo);

S_11=(Z_IN1-Zo)./(Z_IN1+Zo);
S_21=2.*A_v./(1+Zo./Z_IN1);
subplot(211)
semilogx(freq, abs(S_11))
xlabel('Frequency (Hz)')
```

```
ylabel('Amplitude')
title('S_{11}')

subplot(212)
semilogx(freq, angle(S_11))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

A=input('press enter to display next plot');

subplot(211)
semilogx(freq, abs(S_21))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('S_{21}')

subplot(212)
semilogx(freq, angle(S_21))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

%=============================================
```

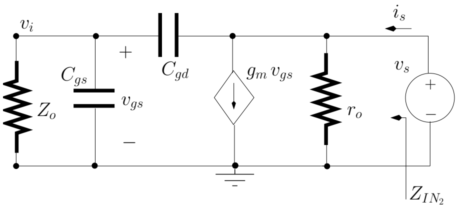


Figure 7.10: *Calculation of S_{12} and of S_{22}.*

Calculation of S_{12} and of S_{22}.

$$S_{22} = \frac{Z_{IN_2} - Z_o}{Z_{IN_2} + Z_o}$$

$$S_{12} = \frac{2\,A_{v_2}}{1 + \frac{Z_o}{Z_{IN_2}}}$$

where Z_{IN_2} and $A_{v_2} = v_i/v_s$ can be calculated from the circuit of figure 7.10.

$$Z_{IN_2} =$$

$$\frac{r_o[j\,\omega\,(C_{gd} + C_{gs})\,Z_o + 1]}{1 + j\,\omega\,(C_{gd} + C_{gs})\,Z_o + j\,\omega\,C_{gd}\,r_o\,(g_m\,Z_o + 1) - \omega^2\,C_{gd}\,C_{gs}\,r_o\,Z_o}$$

$$A_{v_2} = \frac{j\,\omega\,C_{gd}\,Z_o}{1 + j\,\omega\,(C_{gd} + C_{gs})\,Z_o}$$

```
%============  mat_script5.m =================
clear
clf

KnWoverL=40e-3;
Cgs=3e-12;
Cgd=1.5e-12;
VA=60;
ID=10e-3;
gm=sqrt(KnWoverL*2*ID);
ro=VA/ID;

freq=logspace(6,10);
omega=2.*pi.*freq;

Zo=50;

Z_IN2=ro.*(1+j.*omega.*(Cgs+Cgd).*Zo )...
       ./(1+j.*omega.*(Cgs+Cgd).*Zo+...
      j.*omega.*Cgd.*ro.*(gm*Zo+1)...
      -omega.^2.*Cgs.*Cgd.*ro.*Zo);

A_v2=(Zo.*j.*omega.*Cgd)...
       ./(1+j.*omega.*(Cgs+Cgd)*Zo);

S_22=(Z_IN2-Zo)./(Z_IN2+Zo);
S_12=2.*A_v2./(1+Zo./Z_IN2);
subplot(211)
semilogx(freq, abs(S_22))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('S_{22}')

subplot(212)
```

```
semilogx(freq, angle(S_22))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

A=input('press enter to display next plot');

subplot(211)
semilogx(freq, abs(S_12))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('S_{12}')

subplot(212)
semilogx(freq, angle(S_12))
xlabel('Frequency (Hz)')
ylabel('Phase (rad)')

%=============================================
```

7.3 Smith chart

Example 7.8 Write a script-file to plot the Smith chart.

Solution:

The Smith chart can be drawn by plotting the two families of circles described by the following eqns:

$$\begin{aligned}\left(U - \frac{r}{r+1}\right)^2 + V^2 &= \frac{1}{(r+1)^2}\\ (U-1)^2\left(V - \frac{1}{x}\right)^2 &= \frac{1}{x^2}\end{aligned}$$

where U and V represent the two orthogonal axis of the reflection coefficient plane and r and x represents the normalised resistance and reactance values.

```
%============ smith_chart.m  =================
clf
clear

x=[-100:0.01: 100];
r=[0  0.5 1 2 4 8 16 32 64];
for k=1:length(r)
   U=(r(k).^2-1+x.^2)./((r(k)+1).^2+x.^2);
   Uu=(r(k).^2-1)./((r(k)+1).^2);
   V=2.*x./((r(k)+1).^2+x.^2);
   plot(U,V)
    if (r(k)<=8)
    xxx=num2str(r(k));
    text(Uu+0.005,0.025,xxx)
     end
hold on
end

r=[0:0.01: 64];

x=[-20 -8 -6 -5 -4 -3 -2 -1 ...
   -0.5 0 0.5 1 2 3 4 5 6 8 20];

for k=1:length(x)
   U=(r.^2-1+x(k).^2)./((r+1).^2+x(k).^2);
   V=2.*x(k)./((r+1).^2+x(k).^2);
   Uu=(-1+x(k).^2)./((1).^2+x(k).^2);
   Vv=2.*x(k)./((1).^2+x(k).^2);

   plot(U,V)

hold on
    if (x(k)>0 & x(k)~=0.5)
    xxx=num2str(abs(x(k)));
    text(Uu+0.01,Vv+0.04,xxx)
     end
    if (x(k)<0 & x(k)~=-0.5)
    xxx=num2str(abs(x(k)));
    text(Uu+0.01,Vv-0.04,xxx)
```

```
    end
    if ( x(k)==0.5)
    xxx=num2str(abs(x(k)));
    text(Uu-0.05,Vv+0.07,xxx)
     end
    if ( x(k)==-0.5)
    xxx=num2str(abs(x(k)));
    text(Uu-0.05,Vv-0.07,xxx)
     end

end
%====
x=[-1:0.01: 1];
r=[0:0.1:1];
for k=1:length(r)
   U=(r(k).^2-1+x.^2)./((r(k)+1).^2+x.^2);
   V=2.*x./((r(k)+1).^2+x.^2);
   plot(U,V)
hold on
end

r=[0:0.01:1];

x=[-1:0.1:1 ];

for k=1:length(x)
   U=(r.^2-1+x(k).^2)./((r+1).^2+x(k).^2);
   V=2.*x(k)./((r+1).^2+x(k).^2);
   plot(U,V)
hold on
end
%====
x=[-4:0.01: 4];
r=[1:0.2:4];
for k=1:length(r)
   U=(r(k).^2-1+x.^2)./((r(k)+1).^2+x.^2);
   V=2.*x./((r(k)+1).^2+x.^2);
   plot(U,V)
hold on
end

r=[0:0.01:4];

x=[-4:0.2:-1 1:0.2:4];

for k=1:length(x)
   U=(r.^2-1+x(k).^2)./((r+1).^2+x(k).^2);
   V=2.*x(k)./((r+1).^2+x(k).^2);
   plot(U,V)
hold on
end
%====
x=[-8:0.01: 8];
r=[4:1:8];
for k=1:length(r)
   U=(r(k).^2-1+x.^2)./((r(k)+1).^2+x.^2);
   V=2.*x./((r(k)+1).^2+x.^2);
```

```
   plot(U,V)
hold on
end

r=[0:0.01:8];

x=[-8:1:-4 4:1:8];

for k=1:length(x)
   U=(r.^2-1+x(k).^2)./((r+1).^2+x(k).^2);
   V=2.*x(k)./((r+1).^2+x(k).^2);
   plot(U,V)
hold on
end

hold off

axis off
%==============================================
```

Example 7.9 Represent the following impedances on the Smith chart. $Z_o = 50\ \Omega$.

1. $Z_1 = 100 + j\,75\ \ \Omega$
2. $Z_2 = 80 - j\,25\ \ \Omega$
3. $Z_3 = 10 + j\,45\ \ \Omega$

Solution (using MATLAB/OCTAVE):

```
%============  mat_script5.m =================
clear
clf

smith_chart
hold on

Zo=50;

Z1=100+j*75;
Z2=80-j*25;
Z3=10+j*45;

z1=Z1/Zo;
z2=Z2/Zo;
z3=Z3/Zo;

[u1,v1]=zy2smith(real([z1 z2 z3]),...
          imag([z1 z2 z3]));

plot(u1,v1,'ro',u1,v1,'r*')
hold off
%==============================================
```

`smith_chart.m` is the m-script derived in the previous example and `zy2smith.m` is an m-function which maps normalised impedances (or normalised admittances) into the reflection coefficient plane (U, V).

```
%============ zy2smith.m  ====================
function [U,V]=zy2smith(r,x)
%
%function [U,V]=zy2smith(r,x)
%

   U=(r.^2-1+x.^2)./((r+1).^2+x.^2);
   V=2.*x./((r+1).^2+x.^2);

%==============================================
```

Example 7.10 Represent the following admittances on the Smith chart. $Z_o = 50\ \Omega$.

1. $Y_1 = 3.7 - j\,8.3$ mS
2. $Y_2 = 5.9 + j\,14.2$ mS
3. $Y_3 = 2 + j\,7.1$ mS

Solution (using MATLAB/OCTAVE):

```
%============  mat_script7.m =================
clear
clf

smith_chart
hold on

Yo=1/50;

Y1=(3.7-j*8.3).*1e-3;
Y2=(5.9+j*14.2).*1e-3;
Y3=(2+j*7.1).*1e-3;

y1=Y1/Yo;
y2=Y2/Yo;
y3=Y3/Yo;

[u1,v1]=zy2smith(real([y1 y2 y3]),...
          imag([y1 y2 y3]));

plot(u1,v1,'ro',u1,v1,'r*')
hold off
%============================================
```

Chapter 8

Noise in electronic circuits

8.1 Equivalent noise bandwidth

Example 8.1 Determine the equivalent noise bandwidth of the following transfer functions. Consider $\eta < 1$.

1.

$$H_1(\omega) \quad = \quad \frac{\omega_n^2}{-\omega^2 + 2\,j\,\eta\,\omega_n\,\omega + \omega_n^2}$$

2.

$$H_2(\omega) \quad = \quad \frac{2\,j\,\eta\,\omega_n\,\omega}{-\omega^2 + 2\,j\,\eta\,\omega_n\,\omega + \omega_n^2}$$

3.

$$H_3(\omega) \quad = \quad \frac{\omega_n^2 + 2\,j\,\eta\,\omega_n\,\omega}{-\omega^2 + 2\,j\,\eta\,\omega_n\,\omega + \omega_n^2}$$

Solution (using MATLAB/OCTAVE):

Normalising the frequency such that $\omega' = \omega/\omega_n$ the transfer functions mentioned above can be represented as follows:

$$\begin{aligned}
H_1(\omega') &= \frac{1}{-\omega'^2 + 2\,j\,\eta\,\omega' + 1} \\
H_2(\omega') &= \frac{2\,j\,\eta\,\omega'}{-\omega'^2 + 2\,j\,\eta\,\omega' + 1} \\
H_3(\omega') &= \frac{2\,j\,\eta\,\omega' + 1}{-\omega'^2 + 2\,j\,\eta\,\omega' + 1}
\end{aligned}$$

Now, the equivalent noise bandwidths, B_N, normalised to ω_n can be determined according to:

$$\begin{aligned}
\frac{B_N}{\omega_n} &= \int_{-\infty}^{\infty} |H_k(\omega')|^2\,d\,\omega'\ , \quad k = 1,2,3 \\
&\simeq 2\int_{0}^{10} |H_k(\omega')|^2\,d\,\omega'\ , \quad k = 1,2,3
\end{aligned}$$

```
%============  mat_script1a.m ================

clear
clf
```

```
eta=0.15:0.01:0.95;

d_omega=0.01;
omega=2.*pi.*[0:d_omega:10];

for k=1:length(eta)
  H_1=1./(1+2.*j.*eta(k).*omega-omega.^2);
  H_2=2.*j.*eta(k).*omega./(1+2.*j.*eta(k).*...
                              omega-omega.^2);
  Bn1(k)=2*sum(abs(H_1).^2).*d_omega;
  Bn2(k)=2*sum(abs(H_2).^2).*d_omega;
  Bn3(k)=Bn1(k)+Bn2(k);
end

plot(eta,Bn1,eta,Bn2,':',eta,Bn3,'--')
%==============================================
```

8.2 Conversion between noise representations

8.2.1 Chain to admittance

Example 8.2 Write a script-file to convert the chain noise representation into the admittance noise representation.

Solution:

```
%=============================a2y_noisy.m=================================
function [y11,y12,y21,y22,Cy11,Cy12,Cy21,Cy22]=a2y_noisy(a11,a12,a21,a22,...
%                                                       Ca11,Ca12,Ca21,Ca22)
%
% function [y11,y12,y21,y22,Cy11,Cy12,Cy21,Cy22]=a2y_noisy(a11,a12,a21,
%                                                   a22,Ca11,Ca12,Ca21,Ca22)
% CONVERSION OF A PARAMETERS TO Y PARAMETERS (noisy representations)
% a_ij are vectors - electrical chain parameters versus the frequency
% Ca_ij are vectors - correlation chain parameters versus the frequency
% y_ij are vectors - admittance parameters versus the frequency
% Cy_ij are vectors - correlation admittance parameters versus the frequency

     [y11,y12,y21,y22]=a2y(a11,a12,a21,a22);

     T11=-y11;
     T12=ones(size(y12));
     T21=-y21;
     T22=zeros(size(y12));

     Tc11=conj(T11);
     Tc12=conj(T21);
     Tc21=conj(T12);
     Tc22=conj(T22);

     Cy11= T11.*Ca11.*Tc11+T11.*Ca12.*Tc21+T12.*Ca21.*Tc11+T12.*Ca22.*Tc21;
     Cy12= T11.*Ca11.*Tc12+T11.*Ca12.*Tc22+T12.*Ca21.*Tc12+T12.*Ca22.*Tc22;
     Cy21= T21.*Ca11.*Tc11+T21.*Ca12.*Tc21+T22.*Ca21.*Tc11+T22.*Ca22.*Tc21;
     Cy22= T21.*Ca11.*Tc12+T21.*Ca12.*Tc22+T22.*Ca21.*Tc12+T22.*Ca22.*Tc22;

%=========================================================================
```

8.2.2 Chain to impedance

Example 8.3 Write a script-file to convert the chain noise representation into the impedance noise representation.

Solution:

```
%=============================a2z_noisy.m=================================
function [z11,z12,z21,z22,Cz11,Cz12,Cz21,Cz22]=a2z_noisy(a11,a12,a21,a22,...
%                                                        Ca11,Ca12,Ca21,Ca22)
%
% function [z11,z12,z21,z22,Cz11,Cz12,Cz21,Cz22]=a2z_noisy(a11,a12,a21,a22,
                                                          Ca11,Ca12,Ca21,Ca22)
% CONVERSION OF A PARAMETERS TO Z PARAMETERS (noisy representations)
% a_ij are vectors - electrical chain parameters versus the frequency
% Ca_ij are vectors - correlation chain parameters versus the frequency
% z_ij are vectors - impedance parameters versus the frequency
% Cz_ij are vectors - impedance correlation parameters versus the frequency

     [z11,z12,z21,z22]=a2z(a11,a12,a21,a22);

     T11=ones(size(z11));
     T12=-z11;
     T21=zeros(size(y12));
     T22=-z21;

     Tc11=conj(T11);
     Tc12=conj(T21);
     Tc21=conj(T12);
     Tc22=conj(T22);

     Cz11= T11.*Ca11.*Tc11+T11.*Ca12.*Tc21+T12.*Ca21.*Tc11+T12.*Ca22.*Tc21;
     Cz12= T11.*Ca11.*Tc12+T11.*Ca12.*Tc22+T12.*Ca21.*Tc12+T12.*Ca22.*Tc22;
     Cz21= T21.*Ca11.*Tc11+T21.*Ca12.*Tc21+T22.*Ca21.*Tc11+T22.*Ca22.*Tc21;
     Cz22= T21.*Ca11.*Tc12+T21.*Ca12.*Tc22+T22.*Ca21.*Tc12+T22.*Ca22.*Tc22;

%==========================================================================
```

8.2.3 Impedance to chain

Example 8.4 Write a script-file to convert the impedance noise representation into the chain noise representation.

Solution:

```
%============================z2a_noisy.m=================================
function [a11,a12,a21,a22,Ca11,Ca12,Ca21,Ca22]=z2a_noisy(z11,z12,z21,z22,...
%                                                        Cz11,Cz12,Cz21,Cz22)
%
% function [a11,a12,a21,a22,Ca11,Ca12,Ca21,Ca22]=z2a_noisy(y11,z12,z21,
                                                   z22,Cz11,Cz12,Cz21,Cz22)
% CONVERSION OF A PARAMETERS TO Z PARAMETERS (noisy representations)
% a_ij are vectors - electrical chain parameters versus the frequency
% Ca_ij are vectors - chain correlation parameters versus the frequency
% y_ij are vectors - admittance parameters versus the frequency
% Cy_ij are vectors - admittance correlation parameters versus the frequency

     [a11,a12,a21,a22]=z2a(z11,z12,z21,z22);

     T11=ones(size(z11));
     T12=-a11;
     T21=zeros(size(z11));
     T22=-a21;

     Tc11=conj(T11);
     Tc12=conj(T21);
     Tc21=conj(T12);
     Tc22=conj(T22);

     Ca11= T11.*Cz11.*Tc11+T11.*Cz12.*Tc21+T12.*Cz21.*Tc11+T12.*Cz22.*Tc21;
     Ca12= T11.*Cz11.*Tc12+T11.*Cz12.*Tc22+T12.*Cz21.*Tc12+T12.*Cz22.*Tc22;
     Ca21= T21.*Cz11.*Tc11+T21.*Cz12.*Tc21+T22.*Cz21.*Tc11+T22.*Cz22.*Tc21;
     Ca22= T21.*Cz11.*Tc12+T21.*Cz12.*Tc22+T22.*Cz21.*Tc12+T22.*Cz22.*Tc22;
%=========================================================================
```

8.2.4 Impedance to admittance

Example 8.5 Write a script-file to convert the impedance noise representation into the admittance noise representation.

<u>*Solution:*</u>

```
%============================z2y_noisy.m=================================
function [y11,y12,y21,y22,Cy11,Cy12,Cy21,Cy22]=z2y_noisy(z11,z12,z21,z22,...
                                                       Cz11,Cz12,Cz21,Cz22)
%
% function [y11,y12,y21,y22,Cy11,Cy12,Cy21,Cy22]=z2y_noisy(y11,z12,z21,z22,
%                                                       Cz11,Cz12,Cz21,Cz22)
% CONVERSION OF A PARAMETERS TO Z PARAMETERS (noisy representations)
% z_ij are vectors - electrical impedance parameters versus the frequency
% Cz_ij are vectors - impedance correlation parameters versus the frequency
% y_ij are vectors - admittance parameters versus the frequency
% Cy_ij are vectors - admittance correlation parameters versus the frequency

     [y11,y12,y21,y22]=z2y(z11,z12,z21,z22);

     T11=y11;
     T12=y12;
     T21=y21;
     T22=y22;

     Tc11=conj(T11);
     Tc12=conj(T21);
     Tc21=conj(T12);
     Tc22=conj(T22);

     Cy11= T11.*Cz11.*Tc11+T11.*Cz12.*Tc21+T12.*Cz21.*Tc11+T12.*Cz22.*Tc21;
     Cy12= T11.*Cz11.*Tc12+T11.*Cz12.*Tc22+T12.*Cz21.*Tc12+T12.*Cz22.*Tc22;
     Cy21= T21.*Cz11.*Tc11+T21.*Cz12.*Tc21+T22.*Cz21.*Tc11+T22.*Cz22.*Tc21;
     Cy22= T21.*Cz11.*Tc12+T21.*Cz12.*Tc22+T22.*Cz21.*Tc12+T22.*Cz22.*Tc22;


%=========================================================================
```

8.2.5 Admittance to chain

Example 8.6 Write a script-file to convert the admittance noise representation into the chain noise representation.

Solution:

```
%=============================y2a_noisy.m=================================
function [a11,a12,a21,a22,Ca11,Ca12,Ca21,Ca22]=y2a_noisy(y11,y12,y21,y22,...
                                                        Cy11,Cy12,Cy21,Cy22)
%
% function [a11,a12,a21,a22,Ca11,Ca12,Ca21,Ca22]=y2a_noisy(y11,y12,y21,y22,
%                                                        Cy11,Cy12,Cy21,Cy22)
% CONVERSION OF A PARAMETERS TO Z PARAMETERS (noisy representations)
% a_ij are vectors - electrical chain parameters versus the frequency
% Ca_ij are vectors - correlation chain parameters versus the frequency
% y_ij are vectors - admittance parameters versus the frequency
% Cy_ij are vectors - admittance correlation parameters versus the frequency

     [a11,a12,a21,a22]=y2a(y11,y12,y21,y22);

     T11=zeros(size(a11));
     T12=a12;
     T21=ones(size(a21));
     T22=a22;

     Tc11=conj(T11);
     Tc12=conj(T21);
     Tc21=conj(T12);
     Tc22=conj(T22);

     Ca11= T11.*Cy11.*Tc11+T11.*Cy12.*Tc21+T12.*Cy21.*Tc11+T12.*Cy22.*Tc21;
     Ca12= T11.*Cy11.*Tc12+T11.*Cy12.*Tc22+T12.*Cy21.*Tc12+T12.*Cy22.*Tc22;
     Ca21= T21.*Cy11.*Tc11+T21.*Cy12.*Tc21+T22.*Cy21.*Tc11+T22.*Cy22.*Tc21;
     Ca22= T21.*Cy11.*Tc12+T21.*Cy12.*Tc22+T22.*Cy21.*Tc12+T22.*Cy22.*Tc22;

%==========================================================================
```

8.2.6 Admittance to impedance

Example 8.7 Write a script-file to convert the admittance noise representation into the impedance noise representation.

<u>*Solution:*</u>

```
%============================y2z_noisy.m=================================
function [z11,z12,z21,z22,Cz11,Cz12,Cz21,Cz22]=y2z_noisy(y11,y12,y21,y22,...
                                                       Cy11,Cy12,Cy21,Cy22)
%
% function [z11,z12,z21,z22,Cz11,Cz12,Cz21,Cz22]=y2z_noisy(y11,y12,y21,y22,
%                                                       Cy11,Cy12,Cy21,Cy22)
% CONVERSION OF A PARAMETERS TO Z PARAMETERS (noisy representations)
% z_ij are vectors - electrical impedance parameters versus the frequency
% Cz_ij are vectors - impedance correlation parameters versus the frequency
% y_ij are vectors - admittance parameters versus the frequency
% Cy_ij are vectors - admittance correlation parameters versus the frequency

     [z11,z12,z21,z22]=y2z(y11,y12,y21,y22);

     T11=z11;
     T12=z12;
     T21=z21;
     T22=z22;

     Tc11=conj(T11);
     Tc12=conj(T21);
     Tc21=conj(T12);
     Tc22=conj(T22);

     Cz11= T11.*Cy11.*Tc11+T11.*Cy12.*Tc21+T12.*Cy21.*Tc11+T12.*Cy22.*Tc21;
     Cz12= T11.*Cy11.*Tc12+T11.*Cy12.*Tc22+T12.*Cy21.*Tc12+T12.*Cy22.*Tc22;
     Cz21= T21.*Cy11.*Tc11+T21.*Cy12.*Tc21+T22.*Cy21.*Tc11+T22.*Cy22.*Tc21;
     Cz22= T21.*Cy11.*Tc12+T21.*Cy12.*Tc22+T22.*Cy21.*Tc12+T22.*Cy22.*Tc22;

%=======================================================================
```

8.3 Computer-aided noise analysis

8.3.1 Chain connection

Example 8.8 Write a script-file to compute the chain representation of the chain of two noisy two-port circuits.

Solution:

```
%=========================CHAIN_noisy.m==================================
function [a11,a12,a21,a22,Ca11,Ca12,Ca21,Ca22]=CHAIN_noisy(x11,x12,x21,...
        x22,Cx11,Cx12,Cx21,Cx22, x ,w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22,w)
%
% [a11,a12,a21,a22,Ca11,Ca12,Ca21,Ca22]=CHAIN_noisy(x11,x12,x21,x22,
%         Cx11,Cx12,Cx21,Cx22, 'x' ,w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22,'w')
%
% CALCULATES THE EQUIVALENT CHAIN MATRIX OF THE CASCADE OF TWO NOISY
% TWO-PORTS
% 'x' INDICATES WHICH REPRESENTATION FOR THE FIRST MATRIX :z,y,a
% 'w' INDICATES WHICH REPRESENTATION FOR THE SECOND MATRIX :z,y,a

if (x == 'a')

 ax11=x11;
 ax12=x12;
 ax21=x21;
 ax22=x22;

 Cax11=Cx11;
 Cax12=Cx12;
 Cax21=Cx21;
 Cax22=Cx22;

else

  eval([' [ax11,ax12,ax21,ax22,Cax11,Cax12,Cax21,Cax22]=' x '2a_noisy(x11,...
                                      x12,x21,x22,Cx11,Cx12,Cx21,Cx22);']);

end

if (w == 'a')

 aw11=w11;
 aw12=w12;
 aw21=w21;
 aw22=w22;

 Caw11=Cw11;
 Caw12=Cw12;
 Caw21=Cw21;
 Caw22=Cw22;

else

  eval([' [aw11,aw12,aw21,aw22,Caw11,Caw12,Caw21,Caw22]=' w '2a_noisy(w11,...
                                      w12,w21,w22,Cw11,Cw12,Cw21,Cw22);']);
```

```
end

a11=ax11.*aw11  + ax12.*aw21;
a12=ax11.*aw12  + ax12.*aw22;
a21=ax21.*aw11  + ax22.*aw21;
a22=ax21.*aw12  + ax22.*aw22;

 axc11=conj(ax11);
 axc12=conj(ax21);
 axc21=conj(ax12);
 axc22=conj(ax22);

Ca11= ax11.*Caw11.*axc11+ax11.*Caw12.*axc21+ax12.*Caw21.*axc11+...
                                          ax12.*Caw22.*axc21+Cax11;
Ca12= ax11.*Caw11.*axc12+ax11.*Caw12.*axc22+ax12.*Caw21.*axc12+...
                                         ax12.*Caw22.*axc22+Cax12;
Ca21= ax21.*Caw11.*axc11+ax21.*Caw12.*axc21+ax22.*Caw21.*axc11+...
                                          ax22.*Caw22.*axc21+Cax21;
Ca22= ax21.*Caw11.*axc12+ax21.*Caw12.*axc22+ax22.*Caw21.*axc12+...
                                          ax22.*Caw22.*axc22+Cax22;

%===========================================================================
```

8.3.2 Parallel connection

Example 8.9 Write a script-file to compute the admittance representation of the parallel connection of two noisy two-port circuits.

Solution:

```
%===============================PARALLEL_noisy================================
function [y11,y12,y21,y22,Cy11,Cy12,Cy21,Cy22]=PARALLEL_noisy(x11,x12,...
    x21,x22,Cx11,Cx12,Cx21,Cx22, x ,w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22,w)
%
% [y11,y12,y21,y22,Cy11,Cy12,Cy21,Cy22]=PARALLEL_noisy(x11,x12,x21,x22,
%        Cx11,Cx12,Cx21,Cx22, 'x' ,w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22,'w')
%
% CALCULATES THE EQUIVALENT ADMITTANCE MATRIX OF THE PARALLEL CONNECTION
% OF TWO NOISY TWO-PORTS
% 'x' INDICATES WHICH REPRESENTATION FOR THE FIRST MATRIX :z,y,a
% 'w' INDICATES WHICH REPRESENTATION FOR THE SECOND MATRIX :z,y,a

if (x == 'y')

 yx11=x11;
 yx12=x12;
 yx21=x21;
 yx22=x22;

 Cyx11=Cx11;
 Cyx12=Cx12;
 Cyx21=Cx21;
 Cyx22=Cx22;


else

  eval([' [yx11,yx12,yx21,yx22,Cyx11,Cyx12,Cyx21,Cyx22]=' x '2y_noisy(...
                                x11,x12,x21,x22,Cx11,Cx12,Cx21,Cx22);']);

end

if (w == 'y')

 yw11=w11;
 yw12=w12;
 yw21=w21;
 yw22=w22;

 Cyw11=Cw11;
 Cyw12=Cw12;
 Cyw21=Cw21;
 Cyw22=Cw22;

else

  eval([' [yw11,yw12,yw21,yw22,Cyw11,Cyw12,Cyw21,Cyw22]=' w '2y_noisy(...
                                w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22);']);
```

```
end

y11=yx11+yw11 ;
y12=yx12+yw12 ;
y21=yx21+yw21 ;
y22=yx22+yw22 ;

Cy11=Cyx11+Cyw11 ;
Cy12=Cyx12+Cyw12 ;
Cy21=Cyx21+Cyw21 ;
Cy22=Cyx22+Cyw22 ;
%===========================================================================
```

8.3.3 Series connection

Example 8.10 Write a script-file to compute the impedance representation of the series connection of two noisy two-port circuits.

Solution:

```
%=============================SERIES_noisy.m==============================
function [z11,z12,z21,z22,Cz11,Cz12,Cz21,Cz22]=SERIES_noisy(x11,x12,x21,...
         x22,Cx11,Cx12,Cx21,Cx22, x ,w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22,w)
%
% [z11,z12,z21,z22,Cz11,Cz12,Cz21,Cz22]=SERIES_noisy(x11,x12,x21,x22,
%        Cx11,Cx12,Cx21,Cx22, 'x' ,w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22,'w')
%
% CALCULATES THE EQUIVALENT IMPEDANCE MATRIX OF THE SERIES CONNECTION
%  OF TWO NOISY TWO-PORTS
% 'x' INDICATES WHICH REPRESENTATION FOR THE FIRST MATRIX :z,y,a
% 'w' INDICATES WHICH REPRESENTATION FOR THE SECOND MATRIX :z,y,a

if (x == 'z')

 zx11=x11;
 zx12=x12;
 zx21=x21;
 zx22=x22;

 Czx11=Cx11;
 Czx12=Cx12;
 Czx21=Cx21;
 Czx22=Cx22;


else

  eval([' [zx11,zx12,zx21,zx22,Czx11,Czx12,Czx21,Czx22]=' x '2z_noisy(...
                              x11,x12,x21,x22,Cx11,Cx12,Cx21,Cx22);']);

end

if (w == 'z')

 zw11=w11;
 zw12=w12;
 zw21=w21;
 zw22=w22;

 Czw11=Cw11;
 Czw12=Cw12;
 Czw21=Cw21;
 Czw22=Cw22;

else

  eval([' [zw11,zw12,zw21,zw22,Czw11,Czw12,Czw21,Czw22]=' w '2z_noisy(...
                              w11,w12,w21,w22,Cw11,Cw12,Cw21,Cw22);']);
```

```
end

z11=zx11+zw11 ;
z12=zx12+zw12 ;
z21=zx21+zw21 ;
z22=zx22+zw22 ;

Cz11=Czx11+Czw11 ;
Cz12=Czx12+Czw12 ;
Cz21=Czx21+Czw21 ;
Cz22=Czx22+Czw22 ;

%==========================================================================
```

8.3.4 Common-emitter amplifier

Example 8.11 Determine the equivalent input noise voltage of the common-emitter amplifier shown in figure 6.6 for frequencies ranging from 100 Hz to 10 MHz. Assume $I_S = 33 \times 10^{-16}$ A, $\beta_F = 250$, $\beta_R = 1$, $C_\mu = 2$ pF and $C_\pi = 10$ pF.

Solution (using SPICE):

```
* Circuit of figure 8.1

*------------------netlist1------------------

V_CC 7 0 dc 10
v_s  1 0 dc 0 ac 1

R_s 1 2 100
R_1 7 3 9k
R_2 3 0 1k
R_E 4 0 300
R_C 7 5 5k
R_L 6 0 15k

C_B 2 3 5u
C_E 4 0 10u
C_L 5 6 1u

Q1 5 3 4 QGEN
*--------------------------------------------
.model QGEN NPN IS=33e-16 BF=250
+ CJC=2e-12 CJE=10e-12
*--------------------------------------------

*.ac DEC 10 1e2 1e7
.noise v(6) v_s DEC 10 1e2 1e7 1
*.plot noise inoise
.plot noise inoise_spectrum

.end
*--------------------------------------------
```

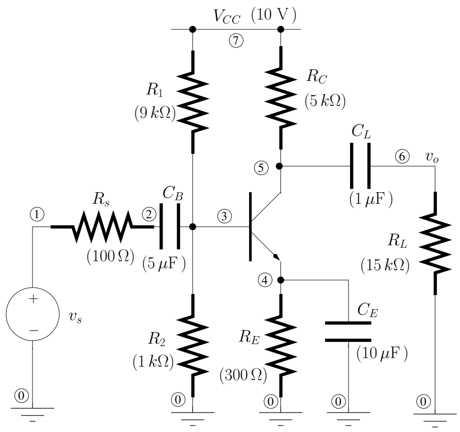


Figure 8.1: *Common-emitter amplifier.*

Note that the noise spectra produced by SPICE are unilateral. The bilateral spectral density of the equivalent input noise voltage is

$$\sqrt{\frac{\texttt{inoise\_spectrum}}{2}}$$

Solution (using MATLAB/OCTAVE):

Figure 8.2 shows the small-signal equivalent circuit for the common-emitter including the various noise sources. The noise sources which

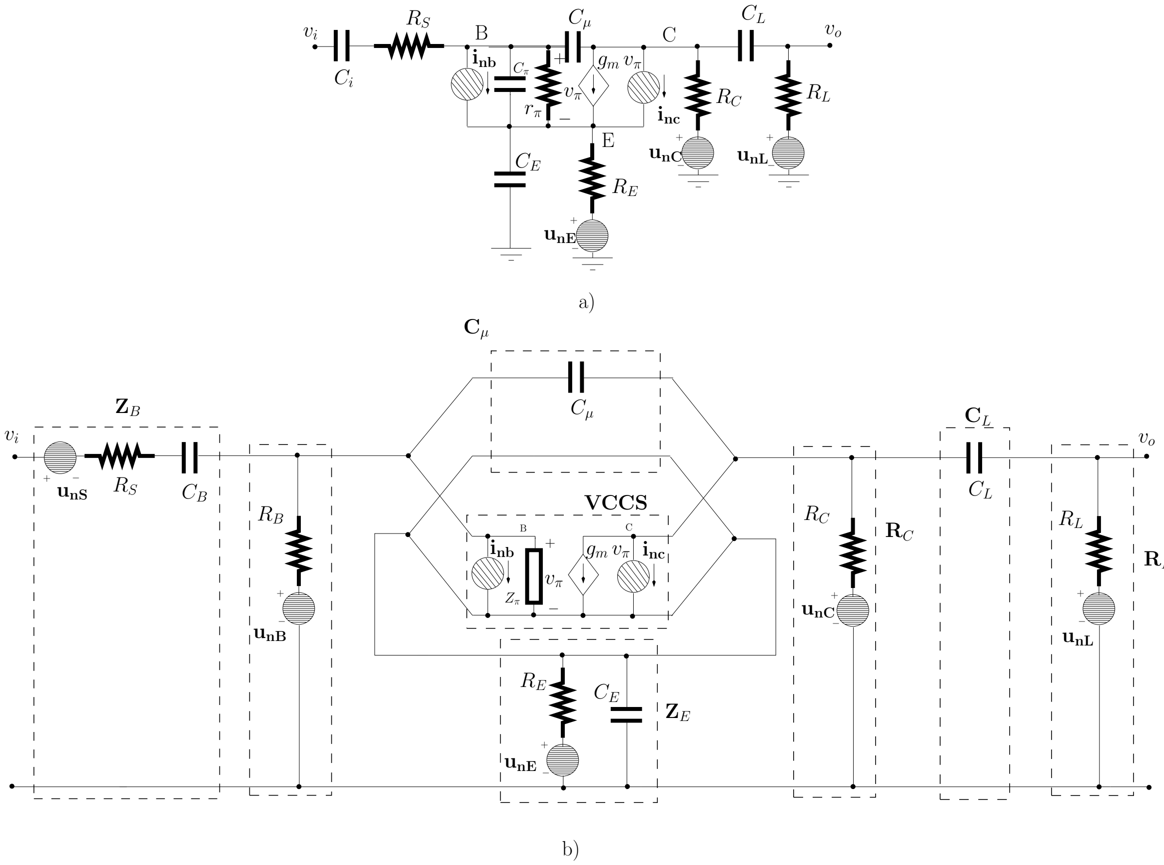


Figure 8.2: *a) Small-signal model for the common-emitter amplifier. b) Equivalent circuit.*

model the noise generated by the transistor are characterised by the following spectral densities:

$$\mathbf{i_{nb}} = q\,I_B$$
$$\mathbf{i_{nc}} = q\,I_C$$

The $1/f$ noise is neglected.

```
%========================  mat_script1.m ================================
clear
clf

CB=5e-6;
CE=10e-6;
CL=1e-6
Cpi=10e-12;
Cmu=2e-12;
RL =15e3;
RE=300;
RC=5e3
Rs=100;
R1=1e3;
R2=9e3;
```

```
IC=1e-3;
hFE=250;

q=1.6e-19;
Kb=1.38e-23;
Temp=300;
VT=Kb*Temp/q;
RB=R1*R2/(R1+R2);
f=logspace(2,7);
omega=2*pi.*f;

Gm =IC/VT;
IB=IC/hFE;
r_pi=hFE/Gm;

%+++++++++++++++VCCS (y-representation)++++++++++++++++++++++++++++++++

VCCS_11=j.*omega.*Cpi+1/r_pi;
VCCS_12=zeros(size(f));
VCCS_21=Gm.*ones(size(f));
VCCS_22=zeros(size(f));

C_VCCS_11=q*IB.*ones(size(f));
C_VCCS_12=zeros(size(f));
C_VCCS_21=zeros(size(f));
C_VCCS_22=q*IC.*ones(size(f));

%+++++++++++++++ Cmu (y-representation)+++++++++++++++++++++++++++++++++

Cmu_11= j.*omega.*Cmu;
Cmu_12=-j.*omega.*Cmu;
Cmu_21=-j.*omega.*Cmu;
Cmu_22= j.*omega.*Cmu;

C_Cmu_11= zeros(size(f));
C_Cmu_12= zeros(size(f));
C_Cmu_21= zeros(size(f));
C_Cmu_22= zeros(size(f));

%+++++++++++++++ Rs and CB (y-representation)+++++++++++++++++++++++++++

RSCB_11=j.*omega.*CB./(1+j.*omega.*CB.*Rs);
RSCB_12=-j.*omega.*CB./(1+j.*omega.*CB.*Rs);
RSCB_21=-j.*omega.*CB./(1+j.*omega.*CB.*Rs);
RSCB_22=j.*omega.*CB./(1+j.*omega.*CB.*Rs);

C_RSCB_11=2.*Kb.*Temp.*real(RSCB_11);
C_RSCB_12=2.*Kb.*Temp.*real(RSCB_12);
C_RSCB_21=2.*Kb.*Temp.*real(RSCB_21);
C_RSCB_22=2.*Kb.*Temp.*real(RSCB_22);

%+++++++++++++++ RB (z-representation)++++++++++++++++++++++++++++++++++

RB_11=RB.*ones(size(f));
RB_12=RB.*ones(size(f));
RB_21=RB.*ones(size(f));
```

```
RB_22=RB.*ones(size(f));

C_RB_11=2.*Kb.*Temp.*real(RB_11);
C_RB_12=2.*Kb.*Temp.*real(RB_12);
C_RB_21=2.*Kb.*Temp.*real(RB_21);
C_RB_22=2.*Kb.*Temp.*real(RB_22);

%+++++++++++++++ RL (z-representation)+++++++++++++++++++++++++++++++++

RL_11=RL.*ones(size(f));
RL_12=RL.*ones(size(f));
RL_21=RL.*ones(size(f));
RL_22=RL.*ones(size(f));

C_RL_11=2.*Kb.*Temp.*real(RL_11);
C_RL_12=2.*Kb.*Temp.*real(RL_12);
C_RL_21=2.*Kb.*Temp.*real(RL_21);
C_RL_22=2.*Kb.*Temp.*real(RL_22);

%+++++++++++++++ CL (y-representation)+++++++++++++++++++++++++++++++++

CL_11= j.*omega.*CL;
CL_12=-j.*omega.*CL;
CL_21=-j.*omega.*CL;
CL_22= j.*omega.*CL;

C_CL_11= zeros(size(f));
C_CL_12= zeros(size(f));
C_CL_21= zeros(size(f));
C_CL_22= zeros(size(f));
%+++++++++++++++ RC (z-representation)+++++++++++++++++++++++++++++++++

RC_11=RC.*ones(size(f));
RC_12=RC.*ones(size(f));
RC_21=RC.*ones(size(f));
RC_22=RC.*ones(size(f));

C_RC_11=2.*Kb.*Temp.*real(RC_11);
C_RC_12=2.*Kb.*Temp.*real(RC_12);
C_RC_21=2.*Kb.*Temp.*real(RC_21);
C_RC_22=2.*Kb.*Temp.*real(RC_22);

%+++++++++++++++ RE and CE (z-representation)+++++++++++++++++++++++++++

RECE_11=RE./(1+j.*omega.*CE.*RE);
RECE_12=RE./(1+j.*omega.*CE.*RE);
RECE_21=RE./(1+j.*omega.*CE.*RE);
RECE_22=RE./(1+j.*omega.*CE.*RE);

C_RECE_11=2.*Kb.*Temp.*real(RECE_11);
C_RECE_12=2.*Kb.*Temp.*real(RECE_12);
C_RECE_21=2.*Kb.*Temp.*real(RECE_21);
C_RECE_22=2.*Kb.*Temp.*real(RECE_22);

%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

%++++++++++++++Xa = VCCS and Cmu +++++++++++++++++++++++++++++++++++++++++
```

```
[Xa_11,Xa_12,Xa_21,Xa_22,C_Xa_11,C_Xa_12,C_Xa_21,C_Xa_22]=PARALLEL_noisy(...
VCCS_11,VCCS_12,VCCS_21,VCCS_22,C_VCCS_11,C_VCCS_12,C_VCCS_21,C_VCCS_22,...
'y', Cmu_11,Cmu_12,Cmu_21,Cmu_22,C_Cmu_11,C_Cmu_12,C_Cmu_21,C_Cmu_22,'y');

%++++++++++++++Xb = VCCS, Cmu, RE and CE  ++++++++++++++++++++++++++++++++

[Xb_11,Xb_12,Xb_21,Xb_22, C_Xb_11,C_Xb_12,C_Xb_21,C_Xb_22]=SERIES_noisy(...
Xa_11,Xa_12,Xa_21,Xa_22,C_Xa_11,C_Xa_12,C_Xa_21,C_Xa_22,'y', RECE_11,...
RECE_12,RECE_21,RECE_22, C_RECE_11,C_RECE_12,C_RECE_21,C_RECE_22 ,'z');

%++++++++++++++Xc =  VCCS, Cmu, RE, CE, and RC++++++++++++++++++++++++++++

[Xc_11,Xc_12,Xc_21,Xc_22,C_Xc_11,C_Xc_12,C_Xc_21,C_Xc_22]=CHAIN_noisy(...
Xb_11,Xb_12,Xb_21,Xb_22, C_Xb_11,C_Xb_12,C_Xb_21,C_Xb_22,'z',RC_11,...
RC_12,RC_21,RC_22,C_RC_11,C_RC_12,C_RC_21,C_RC_22,'z');

%++++++++++++++Xd = VCCS, Cmu, RE, CE,  RC and CL ++++++++++++++++++++++++

[Xd_11,Xd_12,Xd_21,Xd_22,C_Xd_11,C_Xd_12,C_Xd_21,C_Xd_22]=CHAIN_noisy(...
Xc_11,Xc_12,Xc_21,Xc_22,C_Xc_11,C_Xc_12,C_Xc_21,C_Xc_22,'a',CL_11,...
CL_12,CL_21,CL_22,C_CL_11,C_CL_12,C_CL_21,C_CL_22,'y');

%++++++++++++++Xe= VCCS, Cmu, RE, CE,  RC , CL and R_L++++++++++++++++++++

[Xe_11,Xe_12,Xe_21,Xe_22,C_Xe_11,C_Xe_12,C_Xe_21,C_Xe_22]=CHAIN_noisy(...
Xd_11,Xd_12,Xd_21,Xd_22,C_Xd_11,C_Xd_12,C_Xd_21,C_Xd_22,'a', RL_11,...
RL_12,RL_21,RL_22,C_RL_11,C_RL_12,C_RL_21,C_RL_22,'z');


%++++++++++++++Xf= VCCS, Cmu, RE, CE,  RC , CL , R_L and RB+++++++++++++++

[Xf_11,Xf_12,Xf_21,Xf_22,C_Xf_11,C_Xf_12,C_Xf_21,C_Xf_22]=CHAIN_noisy(...
RB_11,RB_12,RB_21,RB_22,C_RB_11,C_RB_12,C_RB_21,C_RB_22,'z',Xe_11,Xe_12,...
Xe_21,Xe_22,C_Xe_11,C_Xe_12,C_Xe_21,C_Xe_22,'a');

%++++++++++++++Xg= VCCS, Cmu, RE, CE,  RC , CL , R_L , RB, Rs and CB ++++++

[Xg_11,Xg_12,Xg_21,Xg_22,C_Xg_11,C_Xg_12,C_Xg_21,C_Xg_22]=CHAIN_noisy(...
RSCB_11,RSCB_12,RSCB_21,RSCB_22,C_RSCB_11,C_RSCB_12,C_RSCB_21,...
C_RSCB_22,'y',Xf_11,Xf_12,Xf_21,Xf_22,C_Xf_11,C_Xf_12,C_Xf_21,C_Xf_22,'a');

subplot(211)
semilogx(f,abs(1./Xg_11))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('Magnitude of the voltage gain')

subplot(212)
semilogx(f,angle(1./Xg_11))
xlabel('Frequency (Hz)')
ylabel('Angle (rad)')
title('Phase of the voltage gain')

input('Next plot...');

subplot(111)
```

```
semilogx(f,sqrt(abs(C_Xg_11)))
xlabel('Frequency (Hz)')
ylabel('Volt/sqrt(Hz)')
title('RMS Input noise voltage')
%==========================================================================
```

8.3.5 Differential pair amplifier

Example 8.12 Determine the output noise voltage of the differential pair amplifier shown in figure 8.3. Assume $K_n = 0.2$ mA/V^2 $W/L = 10$, $V_{Th} = 1$.

Solution (using SPICE):

```
* Circuit of figure 8.3

*---------------------netlist2-----------------

V_DD 5 0 dc 5
V_SS 6 0 dc -5

V_s 1 0 dc 0 ac 1

Mn1 3 1 2 6 modn L=10u W=100u
Mn2 4 0 2 6 modn L=10u W=100u

Mn3 2 7 6 6 modn L=10u W=100u
Mn4 7 7 6 6 modn L=10u W=100u

R_D1 5 3 6k
R_D2 5 4 6k
R_S  7 0 3k

*---------------------------------------------
.model modn nmos level=1 VTO=1 KP=0.2e-3
*---------------------------------------------

*.ac DEC 10 1e2 1e8
.noise v(4) V_s DEC 10 1e2 1e8 1
*.plot noise onoise
.plot noise onoise_spectrum

.end
*---------------------------------------------
```

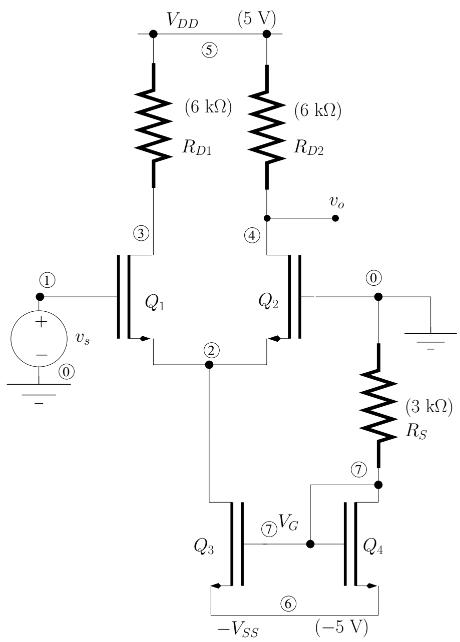


Figure 8.3: *Differential pair amplifier.*

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