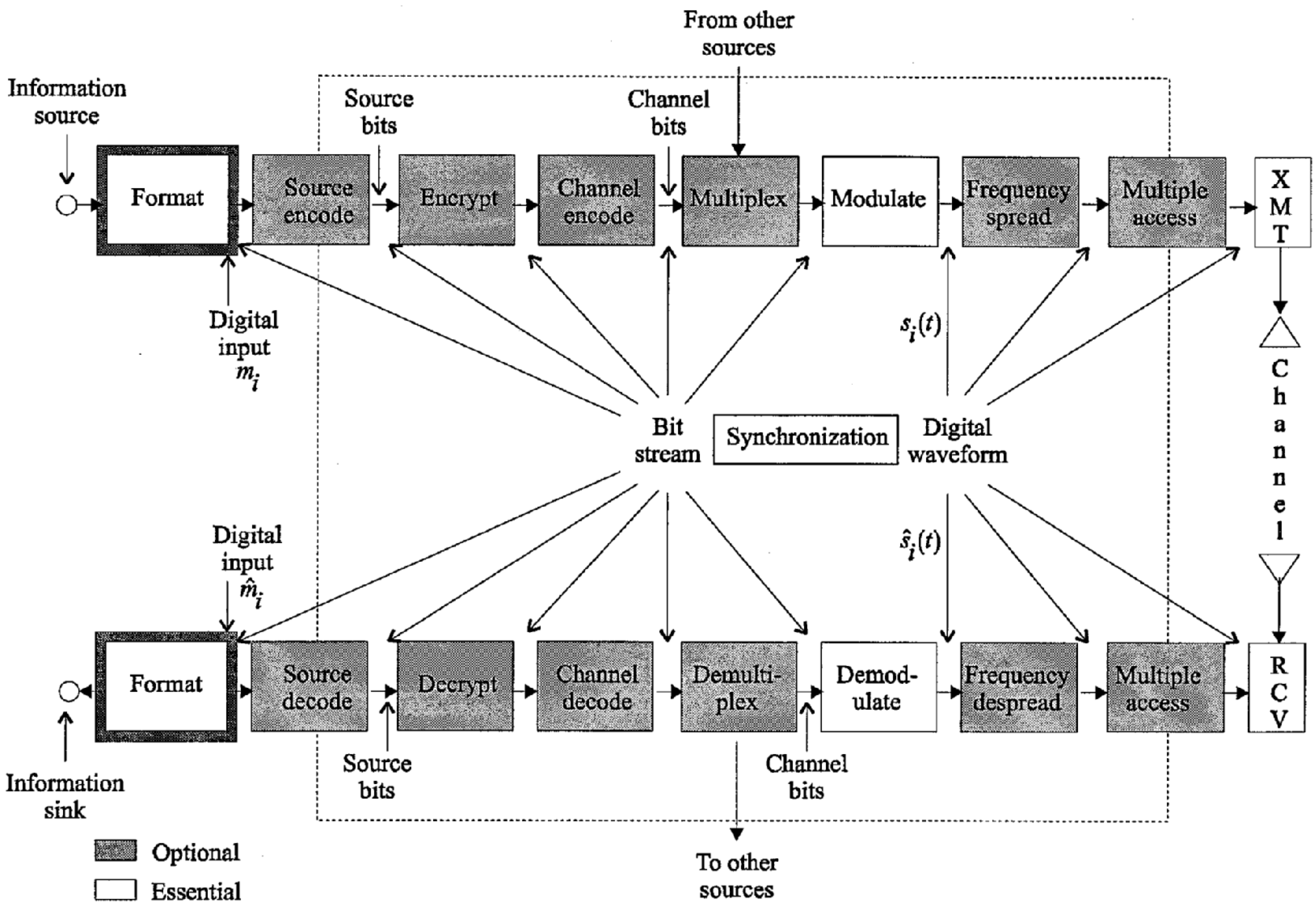
Pre-LAB 2: OFDM Simulation

Baseband Digital Data Transmission



From Bits to Symbols to Pulses

- Information is a sequence of bits 00100111010 … …

- For BPSK, two symbols and each carries one bit

  One symbol

  0 0 1 0 0 1 1 1 0 1 0 0 1 0 1 1 0 0 0 1 0 0 …

- For QPSK, four symbols and each carries two bits

  One symbol

  00 10 01 11 01 00 10 11 00 01 00 …

- For 16-QAM, 16 symbols and each carries 4 bits

  One symbol

  0010 0111 0100 1011 0001 00 …

- Each symbol is transmitted as a single pulse, e.g., a rectangular pulse

From SER to BER Calculations

- When higher-order modulations are used, in simulations, we don't need to simulate the bit sequence but the symbol sequence
- In that case, our simulations will obtain the **Symbol Error Rate (SER)**
- Converting SER into BER is easy!
- For example, if QPSK is used and we obtain the SER, then

$$\mathrm{SER} = 1 - (1 - \mathrm{BER})^2$$
$$\Rightarrow \mathrm{BER} \approx \frac{\mathrm{SER}}{2}$$

Channel Models for Wireless

- A general model is difficult to get and measurement is necessary

- We use the "PATH LOSS+LT FADE+ST FADE" model for analysis

- ❑ Use **path loss** model for dependency on d

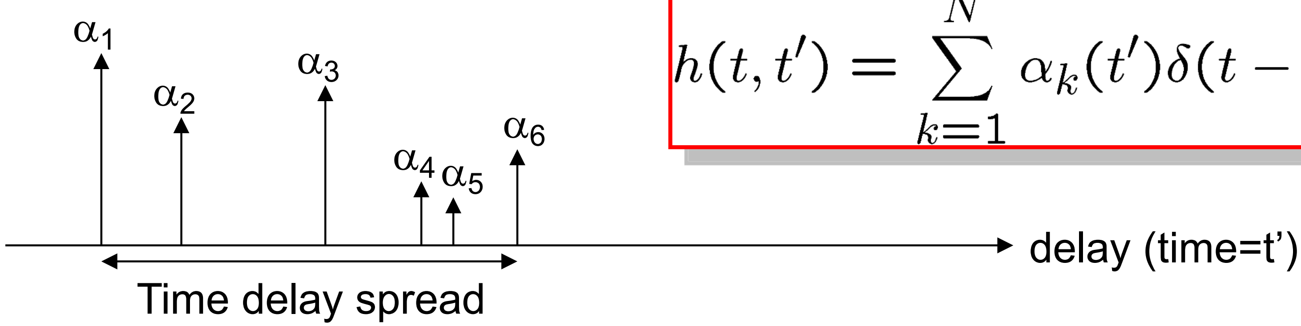
$$P_r \propto \frac{P_t}{d^{\varepsilon}}$$

- ❑ LT **Shadow** Fading=loss due to the characteristics of the environment

- ❑ ST **Rayleigh** Fading=the interference effect due to difference in paths' lengths $\rightarrow$ This is the most terrible problem making the link unstable!

Multipath Fading

- Results from
  - Interference between multiple path with different path lengths (in λ)
  - Movement of the mobile of environment makes this effect time varying
- A snapshot of the channel response may be

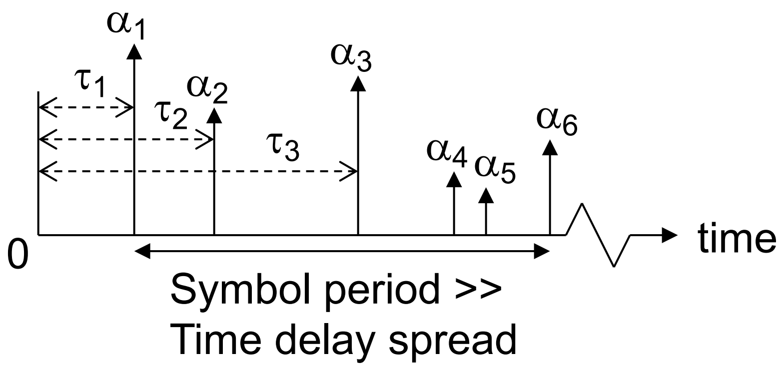


$$\boxed{h(t,t') = \sum_{k=1}^{N} \alpha_k(t')\delta(t-\tau_k(t'))}$$

  - α_n captures the reflections, attenuation, phase shift, etc for a particular path
  - Many paths arriving almost in all time within the time delay spread

Rayleigh Flat Fading

- If the symbol period is much greater than the time delay spread,



$$\boxed{\begin{aligned} h(t) &= \sum_k \alpha_k \delta(t - \tau_k) \\ H(f) &= \sum_k \alpha_k e^{-j2\pi f \tau_k} \approx \sum_k \alpha_k \end{aligned}}$$

- By Central Limit Theorem

$$\begin{aligned} \alpha = \sum_k \alpha_k &= \mathtt{Re}\left\{\sum_k \alpha_k\right\} + j\mathtt{Im}\left\{\sum_k \alpha_k\right\} \\ &= x + jy \end{aligned}$$

Independent Gaussian random variables

Rayleigh Flat Fading

- Therefore, if $x + jy = re^{j\theta}$

$$f_{R,\Theta}(r,\theta) = f_\Theta(\theta) f_R(r) = \frac{1}{2\pi}\frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}}$$

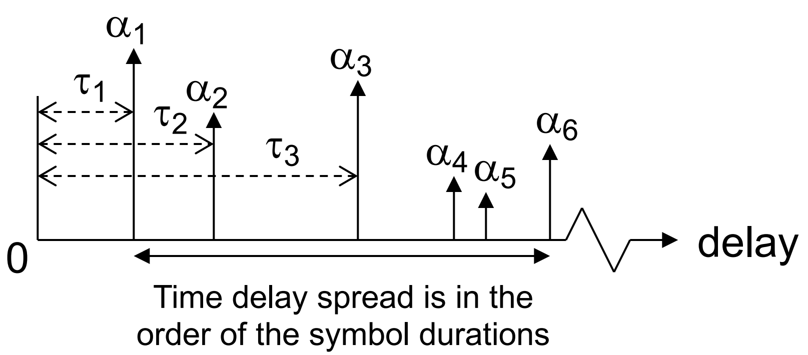
- **Phase** is uniform and **Magnitude** is Rayleigh

$$\boxed{\begin{aligned} f_R(r) &= \begin{cases} \frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}} & \text{if } r \geq 0 \\ 0 & \text{if } r < 0 \end{cases} \\ f_\Theta(\theta) &= \begin{cases} \frac{1}{2\pi} & \text{if } 0 \leq \theta \leq 2\pi \\ 0 & \text{otherwise} \end{cases} \end{aligned}}$$

Rayleigh Frequency-Selective Fading

- When delay spread is more significant, the multiple paths cause **inter-symbol interference (ISI)**, i.e., the delay copies of a symbol are jamming the other transmitted symbols

- Usually, it is modelled as the multi-ray model



  - All $\{|\alpha_k|\}$ are independent and Rayleigh distributed
  - All inter-arrival times $\{\tau_{k+1} - \tau_k\}$ are exponentially distributed
  - Number of rays (or paths) is Poisson distributed
  - $E[|\alpha_k|^2]$ usually follows an exponential power profile

Signal Model for ISI Channels

- Let us consider the transmitted data signal

Data symbols at time n
e.g., {+1,–1} for BPSK

$$\boxed{s(t) = \sum_{n=0}^{N-1} s_n f(t - nT)}$$

Symbol duration

Pulse shaping function, e.g., sinc functions

- And we have the ISI channel

$$\boxed{h(t) = \sum_{\ell=0}^{L-1} \alpha_\ell \delta(t - \tau_\ell)}$$

- The received signal can therefore be expressed as

$$\begin{aligned} y(t) &= s(t) * h(t) + n(t) \\ &= \int_{-\infty}^{\infty} s(x) h(t-x) dx + n(t) \\ &= \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} s_n f(x - nT) \sum_{\ell=0}^{L-1} \alpha_\ell \delta(t - x - \tau_\ell) dx + n(t) \end{aligned}$$

Signal Model for ISI Channels

- We are only interested in samples at time t=mT, so we have

$$y_m = \sum_{n=0}^{N-1} s_n \left[\sum_{\ell=0}^{L-1} \alpha_\ell f((m-n)T - \tau_\ell) \right] + n_m = \sum_{n=0}^{N-1} s_n g_{m,n} + n_m$$

$$\Rightarrow \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ \vdots \\ \\ \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} g_0 & 0 & 0 & \cdots & & & \cdots & 0 \\ g_1 & g_0 & 0 & 0 & \cdots & & & 0 \\ g_2 & g_1 & g_0 & 0 & \ddots & \ddots & & 0 \\ g_3 & g_2 & g_1 & g_0 & 0 & \ddots & & 0 \\ 0 & g_3 & g_2 & g_1 & g_0 & 0 & & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & \\ 0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 & g_0 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ \vdots \\ \vdots \\ \\ \\ s_{N-1} \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ \vdots \\ \vdots \\ \\ \\ n_{N-1} \end{bmatrix}$$

A Toeplitz channel matrix

From Toeplitz to Circulant ISI Channels

- ISI is a serious problem causing error in detection
- The channel matrix can be made circulant by adding a cyclic prefix at the transmitter side and removing it at the receiver side

Rx-side cyclic prefix

Tx-side cyclic prefix

$$
\begin{bmatrix} y_{-3} \\ y_{-2} \\ y_{-1} \\ y_0 \\ y_1 \\ y_2 \\ \vdots \\ \vdots \\ \\ \\ y_{N-1} \end{bmatrix}
=
\begin{bmatrix}
g_0 & 0 & 0 & \cdots & & & \cdots & 0 \\
g_1 & g_0 & 0 & 0 & \cdots & & & 0 \\
g_2 & g_1 & g_0 & 0 & \ddots & \ddots & & 0 \\
g_0 & 0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 \\
g_1 & g_0 & 0 & 0 & \cdots & 0 & g_3 & g_2 \\
g_2 & g_1 & g_0 & 0 & \ddots & \ddots & 0 & g_3 \\
g_3 & g_2 & g_1 & g_0 & 0 & \ddots & & 0 \\
0 & g_3 & g_2 & g_1 & g_0 & 0 & & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\
 & & \ddots & \ddots & \ddots & \ddots & \ddots & \\
0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 & g_0
\end{bmatrix}
\begin{bmatrix} s_{N-3} \\ s_{N-2} \\ s_{N-1} \\ s_0 \\ s_1 \\ s_2 \\ \vdots \\ \vdots \\ \\ \\ s_{N-1} \end{bmatrix}
+
\begin{bmatrix} n_{-3} \\ n_{-2} \\ n_{-1} \\ n_0 \\ n_1 \\ n_2 \\ \vdots \\ \vdots \\ \\ \\ n_{N-1} \end{bmatrix}
$$

A circulant channel matrix

OFDM: Converting ISI into Parallel Channels

- After removing the cyclic prefix at the receiver side, we have

$$
\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ \vdots \\ \\ \\ y_{N-1} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
g_0 & 0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 \\
g_1 & g_0 & 0 & 0 & \cdots & 0 & g_3 & g_2 \\
g_2 & g_1 & g_0 & 0 & \ddots & \ddots & 0 & g_3 \\
g_3 & g_2 & g_1 & g_0 & 0 & \ddots & & 0 \\
0 & g_3 & g_2 & g_1 & g_0 & 0 & & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\
 & & \ddots & \ddots & \ddots & \ddots & \ddots & \\
0 & 0 & \cdots & 0 & g_3 & g_2 & g_1 & g_0
\end{bmatrix}}_{\mathbf{G}}
\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ \vdots \\ \vdots \\ \\ \\ s_{N-1} \end{bmatrix}
+
\begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ \vdots \\ \vdots \\ \\ \\ n_{N-1} \end{bmatrix}
$$

- G can be diagonalised by DFT matrices so that

$$\mathbf{F}\mathbf{G}\mathbf{F}^{-1} \text{ is } \textit{diagonal} \text{ where } \mathbf{F} = \left[\frac{1}{\sqrt{N}} e^{-\frac{j2\pi}{N}(k-1)(l-1)}\right]_{k,l}$$

OFDM: Converting ISI into Parallel Channels

- Therefore, the transmission process of OFDM is

$$\boxed{\mathbf{s} \overset{\text{IDFT}}{\longrightarrow} \mathbf{x} = \mathbf{F}^{-1}\mathbf{s} \overset{\text{add cyclic prefix}}{\longrightarrow} \mathbf{x}'}$$

- At the receiver side, remove cyclic prefix and then DFT

$$\boxed{\mathbf{y}' \overset{\text{remove cyclic prefix}}{\longrightarrow} \mathbf{y} \overset{\text{DFT}}{\longrightarrow} \tilde{\mathbf{s}} = \mathbf{F}\mathbf{y}}$$

- The result of this is that

$$\tilde{\mathbf{s}} = \begin{bmatrix} H_0 & 0 & \cdots & 0 \\ 0 & H_1 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & H_{N-1} \end{bmatrix} \mathbf{s} + \boldsymbol{\eta}$$

- Or

$$\boxed{\tilde{s}_n = H_n s_n + \eta_n \quad \text{with no ISI}}$$