Nonlinearities with Memory (NLWM)

- The memoryless models represent fairly accurately a variety of devices driven by narrowband inputs

- The suitability of a memoryless model for narrowband signals stems from the fact that many RF amplifiers have indeed a wide BW and over any relatively small portion of the band the transfer characteristic does look nearly frequency independent

$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$

- However, when we send "wideband" signals by which we mean that the BW of the signal is comparable to the inherent BW of the device, then we are bound to encounter some frequency-dependent behaviour

❑ **Analytical models** such as the Volterra series are, under some circumstances, able to represent exactly the functioning of a NLWM

Volterra Series Modelling

$y_n(t) = x \ast h_n$ (n-fold)

- The Volterra series approach to modelling is appealing due to its generality and its relatively intuitive decomposition of the response of a nonlinear system into that due to an equivalent linear filter plus additional terms produced by nonlinear behaviour

- A Volterra series is described by

$$\boxed{y(t) = \sum_{n=0}^{\infty} y_n(t)} = y_0(t) + y_1(t) + y_2(t) + \cdots$$

where the term of order n is given by an n-fold convolution

$$\boxed{y_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \underbrace{h_n(\tau_1, \ldots, \tau_n)}\, x(t-\tau_1) \cdots x(t-\tau_n)\, d\tau_1 \cdots d\tau_n}$$

Volterra Series Modelling

- Volterra series expansion provides an input-output relationship for nonlinear time-invariant continuous systems
- The functions h_0, $h_1(t)$, $h_2(t_1,t_2)$, … are called the ***Volterra kernels***
- The 0-th order term accounts for the response to a dc input
- The 1st order kernel is the IR of a linear system $h_n \otimes x$
- The higher-order kernels are higher-order IRs for nonlinear behaviours
- The computation necessary to produce an output sample from the n-th kernel box is that for n=1, raised to the n-th power, hence prohibitive

Volterra Series Modelling

- Block diagram interpretation of the Volterra series representation