Cardiovascular System Modeling

ECE1254 Course Project
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Motivation

- Heart disease and stroke are two of the three leading causes of death in Canada.
- Every 7 minutes in Canada, someone dies from heart disease or stroke.
- In 2008 cardiovascular disease accounted for 29% of all deaths in Canada (69,703 deaths – or more than 69,500).
- Heart disease and stroke costs the Canadian economy more than $20.9 billion every year in physician services, hospital costs, lost wages and decreased productivity

HEART & STROKE FOUNDATION

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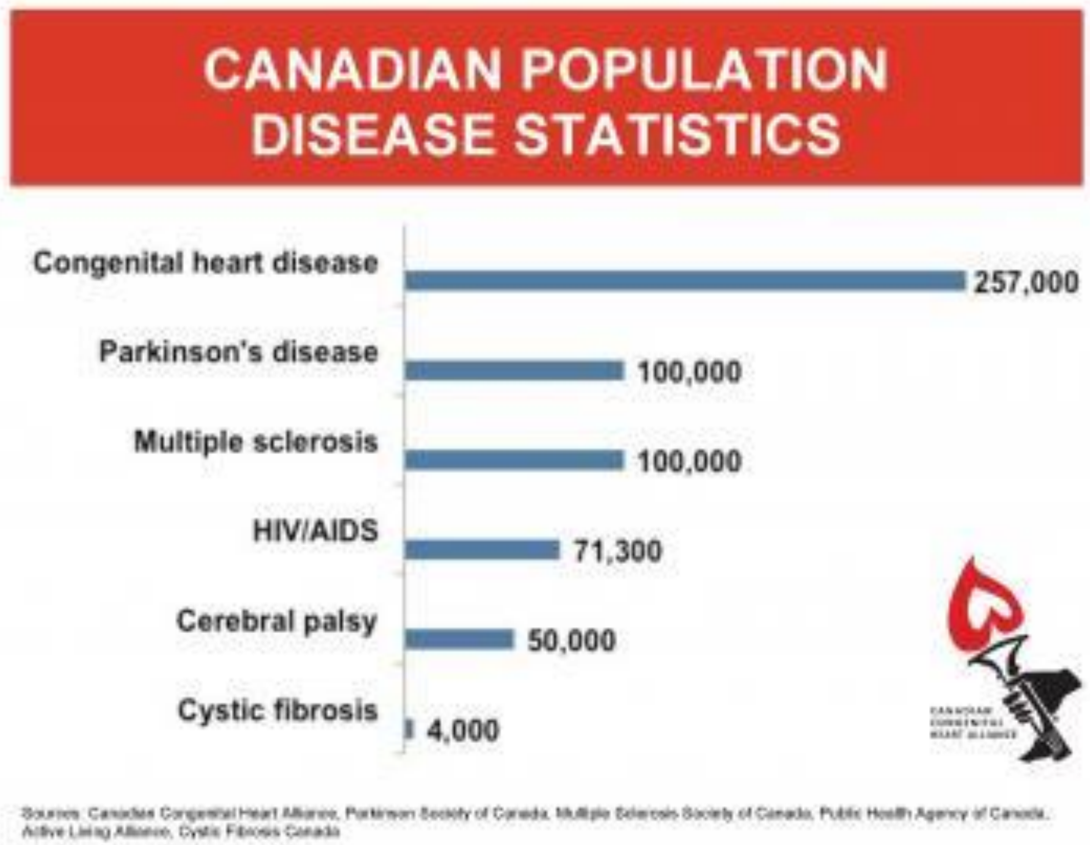
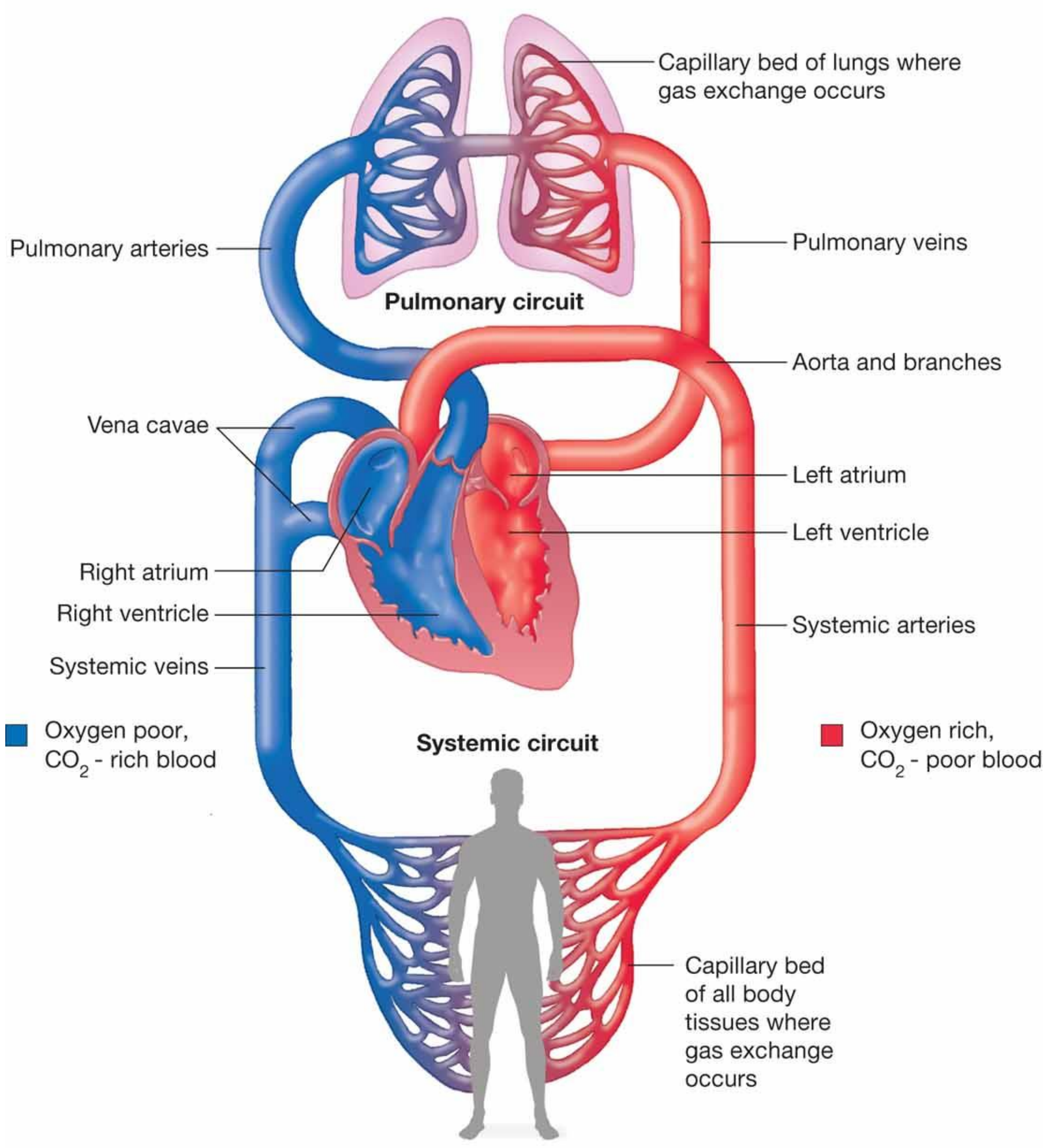
Motivation



CANADIAN POPULATION DISEASE STATISTICS
Congenital heart disease
257,000
Parkinson's disease
100,000
Multiple sclerosis
100,000
HIV/AIDS
71,300
Cerebral palsy
50,000
Cystic fibrosis
4,000

EVERY 7 MINUTES SOMEONE IN CANADA
DIES FROM HEART DISEASE OR STROKE
CANADA NEEDS A HEART HEALTH ACTION PLAN NOW

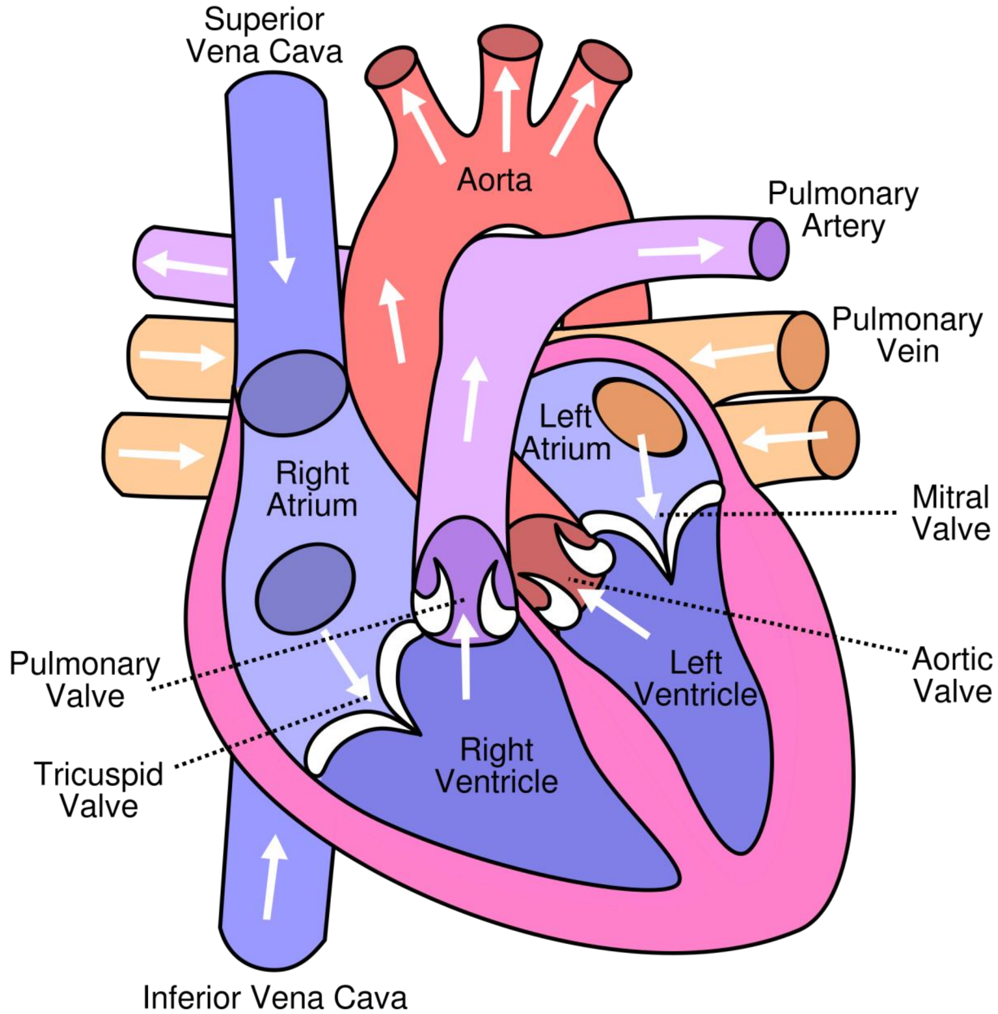
Anatomy of the Cardiovascular System



Two paths of circulation:

- Systemic
- Pulmonary

Anatomy of the Cardiovascular System



Four chambers:

- Left and right atria
- Left and right ventricles

Four heart valves:

- AV (atrioventricular) valves:

  I. *mitral valve – between the left atrium and the left ventricle*

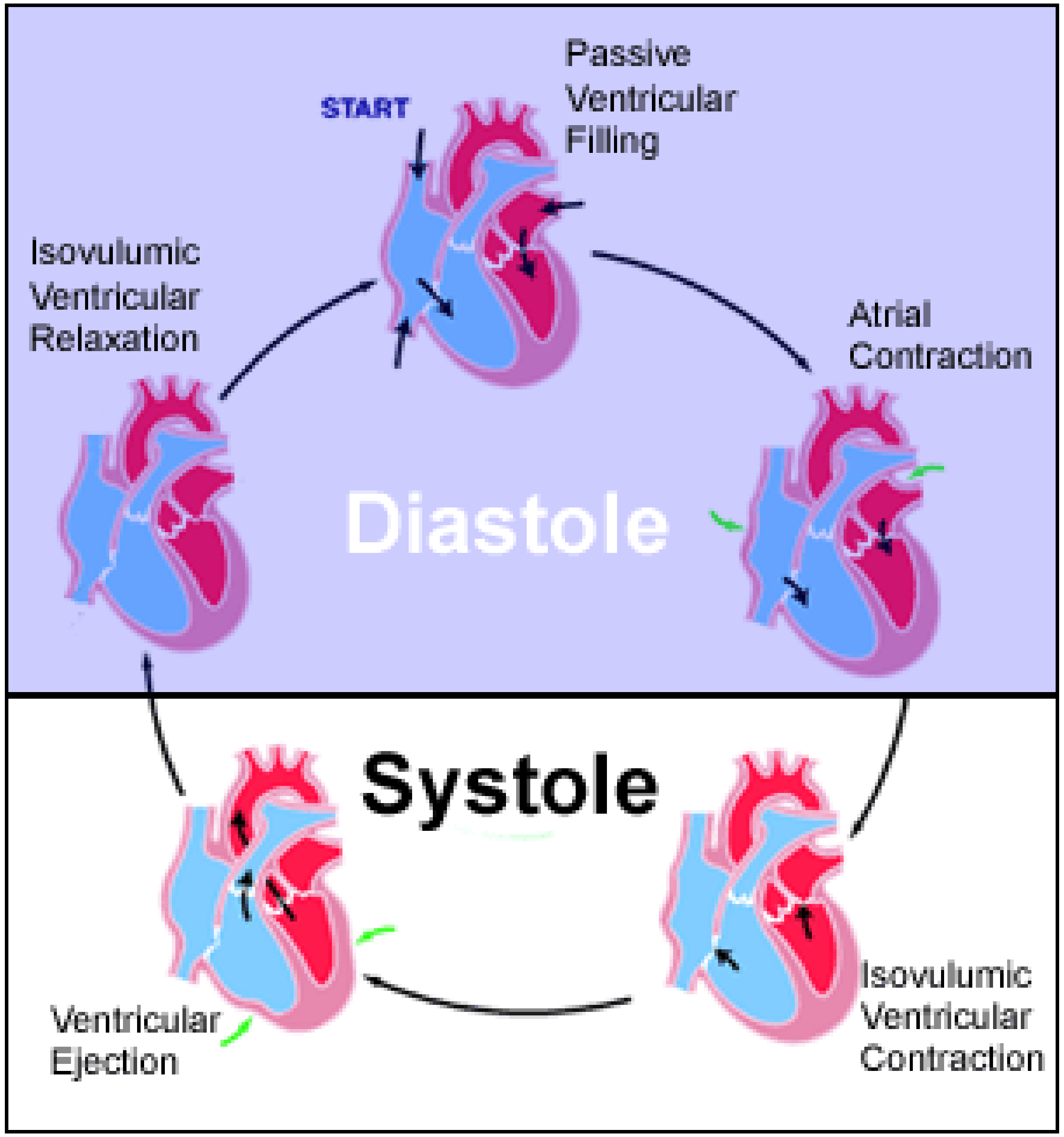
  II. *tricuspid valve – between the right atrium and the right ventricle*

- Semilunar valves:

  I. *aortic valve – between left ventricle and aorta*

  II. *pulmonic valve – between right ventricle and pulmonary artery*

Five Stages of Cardiac Cycle



Systole:

- Isovolumetric ventricular contraction
- Ventricular ejection

Diastole:

- Isovolumetric ventricular relaxation
- Passive ventricular filling
- Atrial contraction

Analogy between Cardiovascular and Electrical Parameters

Cardiovascular	Unit	Electrical	Unit
blood volume (V)	mL	electric charge (Q)	C
flow rate (F)	mL/s	current (I)	A
pressure (P)	mmHg	potential (V)	V

Cardiovascular	Relation	Unit	Electrical	Relation	Unit
vessel resistance	$R_c = \Delta P / F$	mmHg • s / mL	resistance	$R_e = \Delta V / I$	Ω
vessel compliance	$C_c \frac{dP}{dt} = F$	mL / mmHg	capacitance	$C_e \frac{dV}{dt} = I$	F
blood inertia	$L_c \frac{dF}{dt} = P$	mmHg • mL / s²	inductance	$L_e \frac{dF}{dt} = V$	H
heart valve	$F = \begin{cases} 0 & if \Delta P < 0 \\ \frac{P}{R_c} & if\ \Delta P \geq 0 \end{cases}$	mL / s	diode	$I = \begin{cases} 0 & if \Delta V < 0 \\ \frac{V}{R_e} & if\ \Delta V \geq 0 \end{cases}$	A

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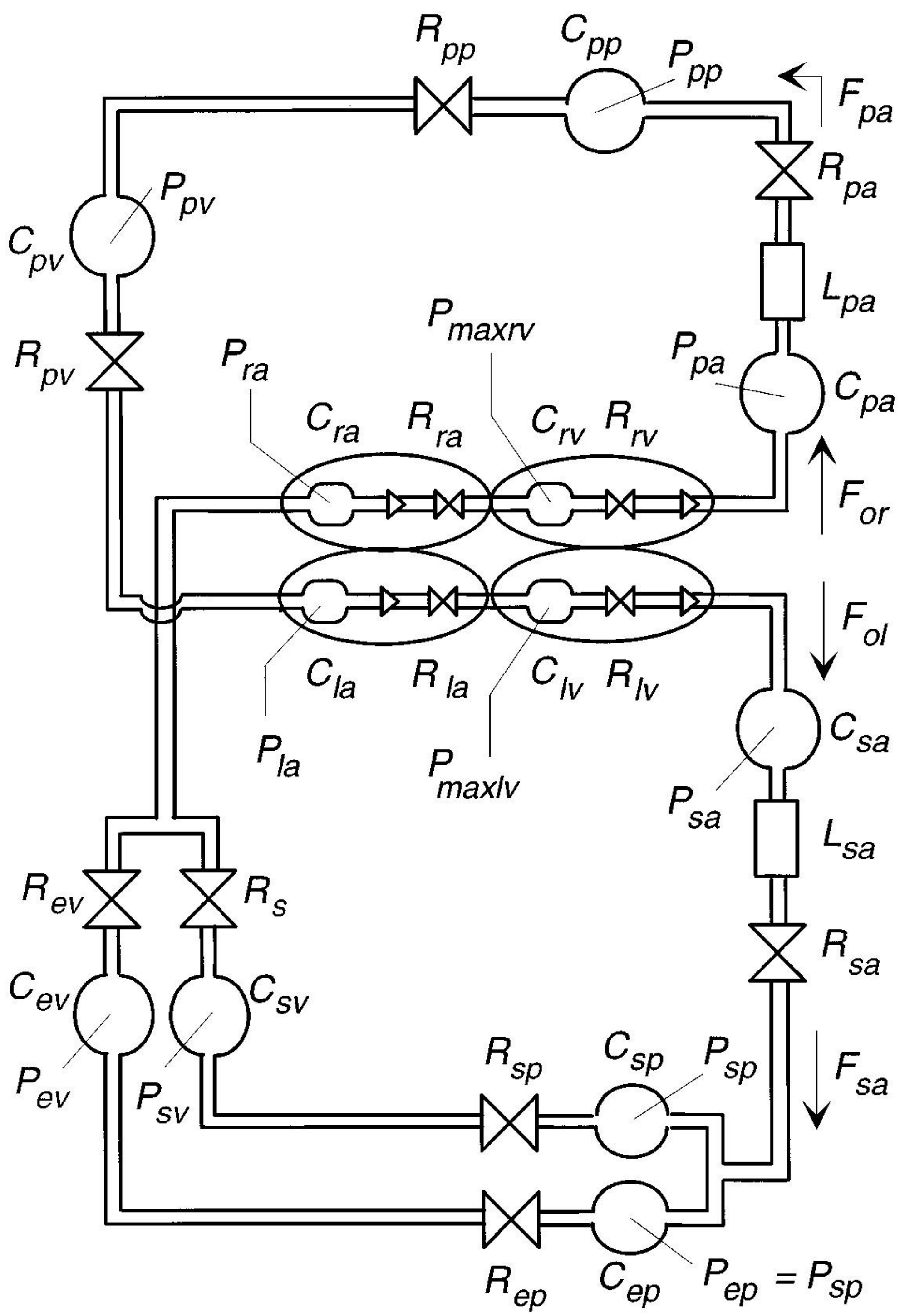
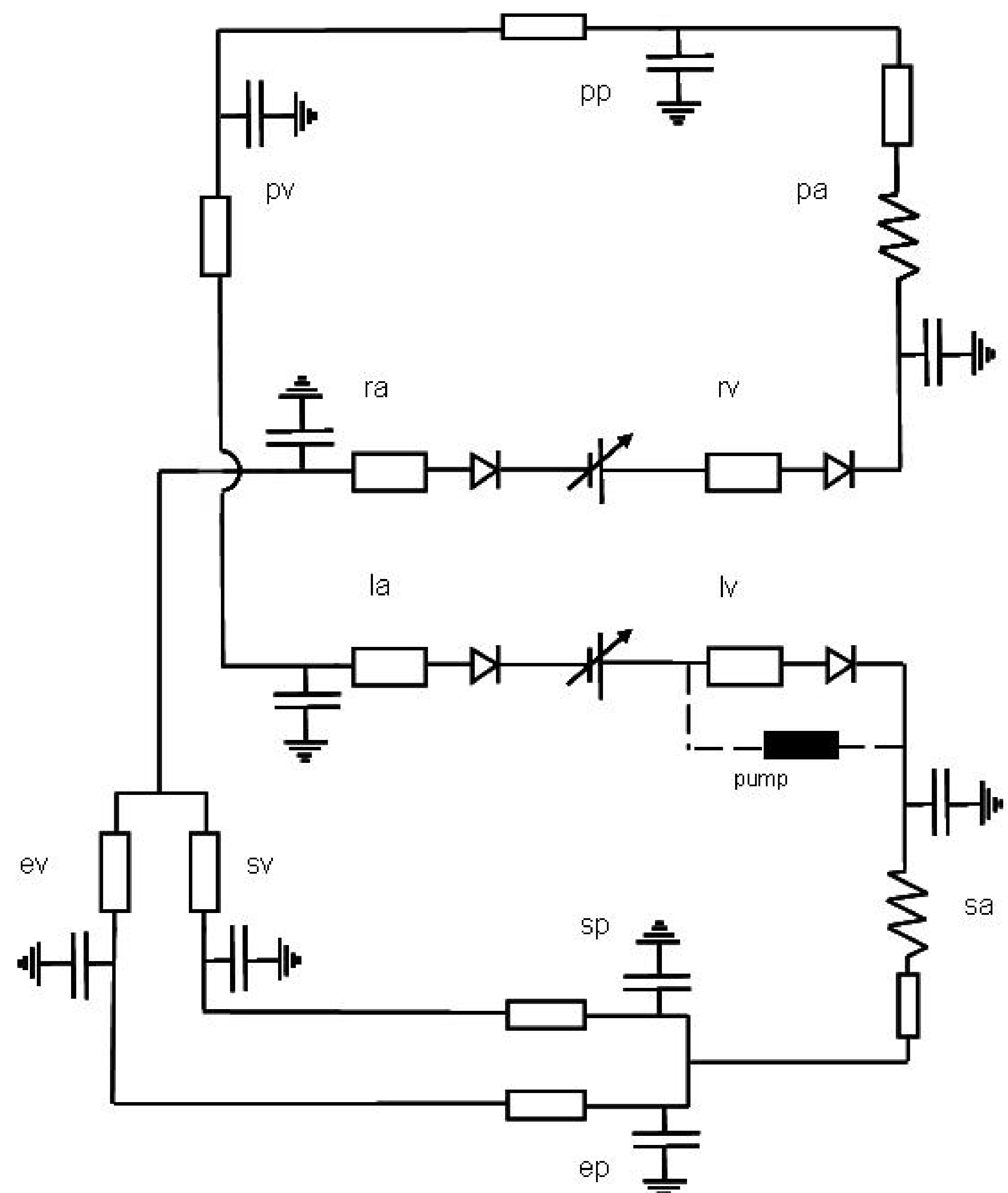
System Modeling





State Equations Governing the Ventricular Blood Flow

$$\frac{dV_{lv}}{dt} = F_{i,l} - F_{o,l} \qquad (6)$$

$$F_{i,l} = \begin{cases} 0 & if\, P_{la} < P_{lv} \\ \frac{P_{la} - P_{lv}}{R_{la}} & if\ P_{la} \geq P_{lv} \end{cases}$$

$$F_{o,l} = \begin{cases} 0 & if\, P_{max,lv} < P_{sa} \\ \frac{P_{max,lv} - P_{sa}}{R_{lv}} & if\ P_{max,lv} \geq P_{sa} \end{cases}$$

$$R_{lv} = k_{r,lv}\, P_{max,lv} \qquad (8.a)$$

$$P_{lv} = P_{sa} \qquad (8.b)$$

$$P_{max,lv}(t) = E_n(t) E_{max,lv}(V_{lv} - V_{u,lv}) + [1 - E_n(t)] P_{0,lv}(exp(k_{E,lv} V_{lv}) - 1) \qquad (8.c)$$

$$\frac{dV_{rv}}{dt} = F_{i,r} - F_{o,r} \qquad (7)$$

$$F_{i,r} = \begin{cases} 0 & if\, P_{ra} < P_{rv} \\ \frac{P_{ra} - P_{rv}}{R_{ra}} & if\ P_{ra} \geq P_{rv} \end{cases}$$

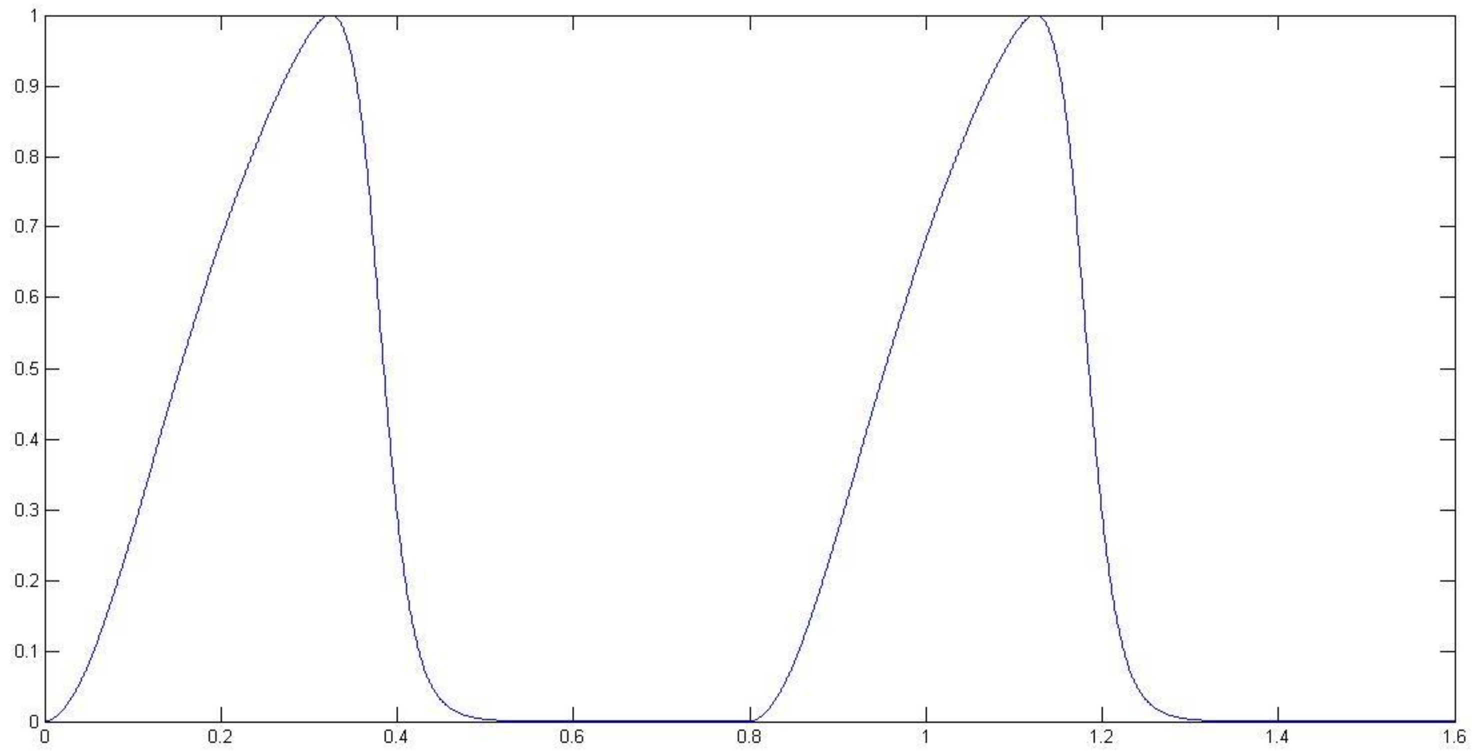
$$F_{o,r} = \begin{cases} 0 & if\, P_{max,rv} < P_{pa} \\ \frac{P_{max,rv} - P_{pa}}{R_{rv}} & if\ P_{max,rv} \geq P_{pa} \end{cases}$$

$$R_{rv} = k_{r,rv}\, P_{max,rv} \qquad (9.a)$$

$$P_{rv} = P_{pa} \qquad (9.b)$$

$$P_{max,rv}(t) = E_n(t) E_{max,rv}(V_{rv} - V_{u,rv}) + [1 - E_n(t)] P_{0,rv}(exp(k_{E,rv} V_{rv}) - 1) \qquad (9.c)$$

Ventricular Elastance Function



Heart Rate = 75 beats/s $\quad T_{cycle} = 0.8s$

$$t_n = \frac{t}{0.2+0.1555\, T_{cycle}}$$

$$E_n(t_n) = 1.553174 \left[\frac{(t_n/0.7)^{1.9}}{1+(t_n/0.7)^{1.9}}\right] \left[\frac{1}{1+(t_n/1.173474)^{21.9}}\right]$$

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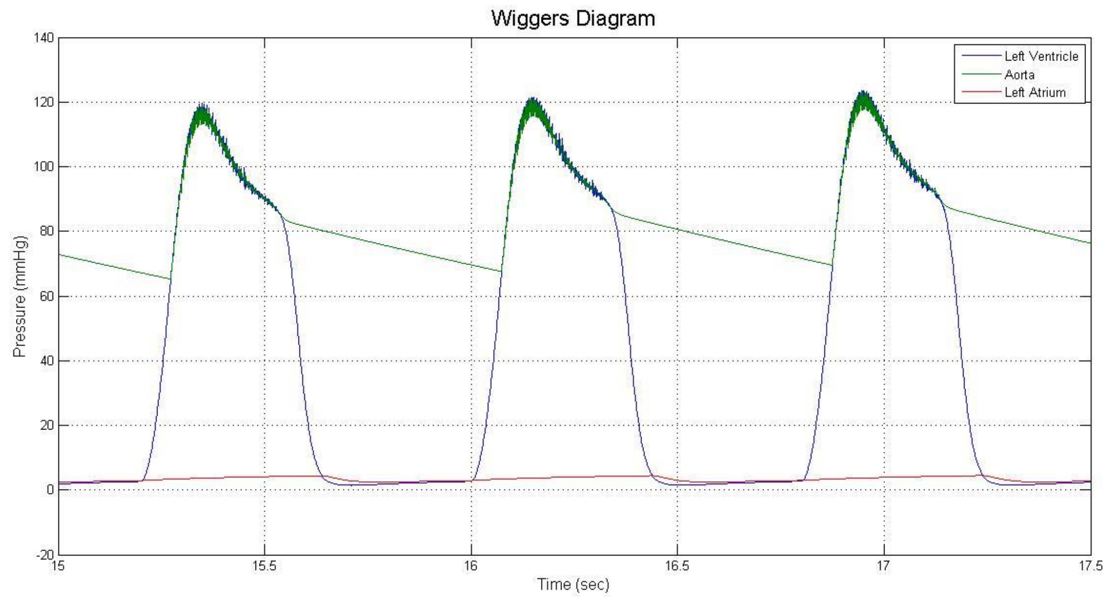
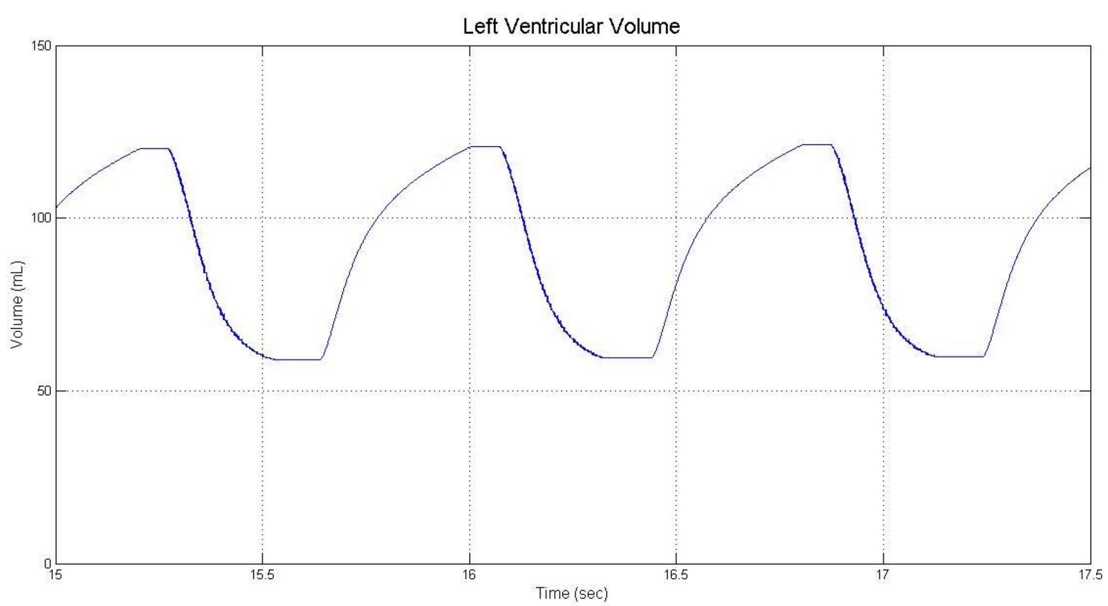
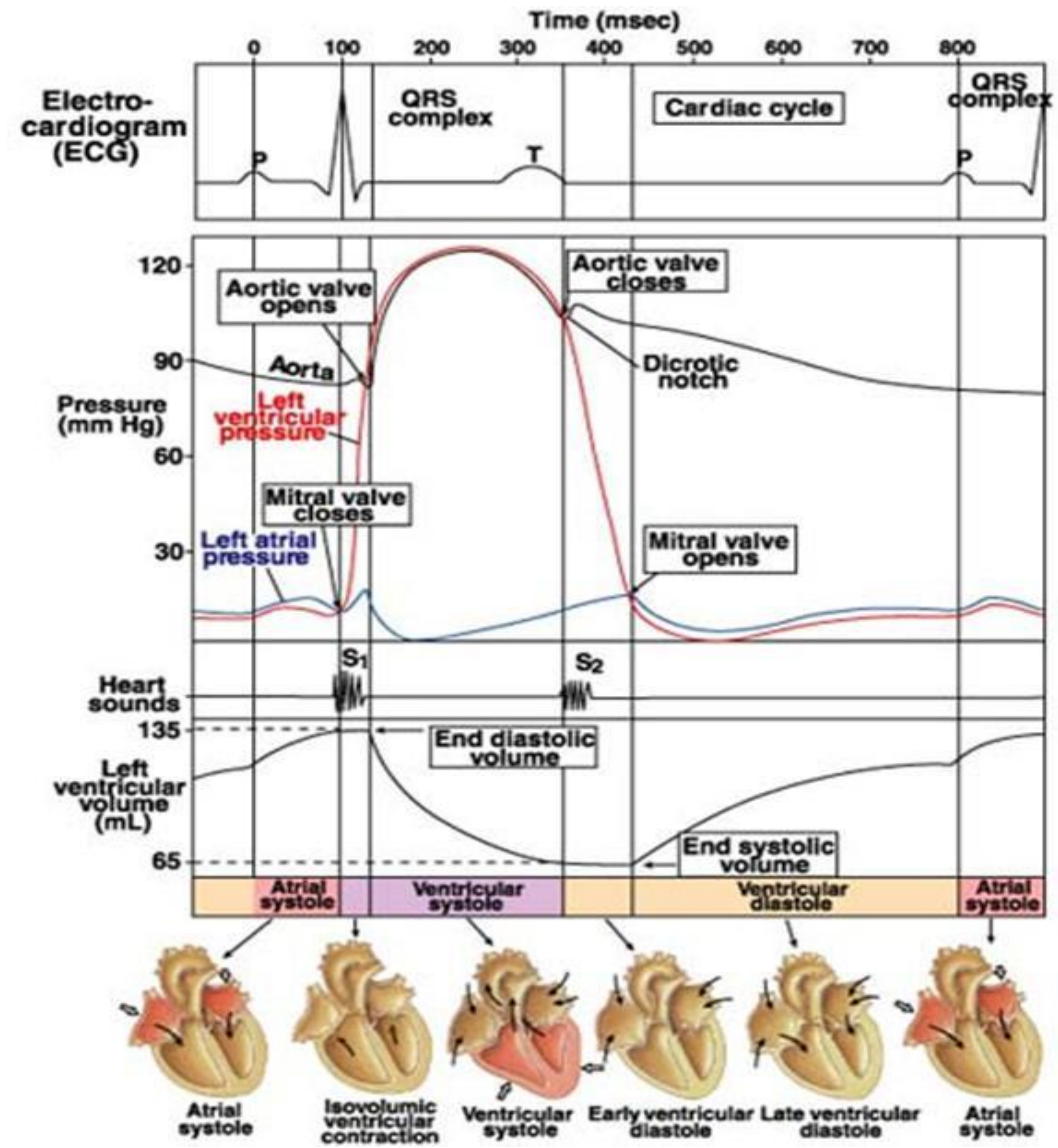
Simulation Results – Wiggers Diagram

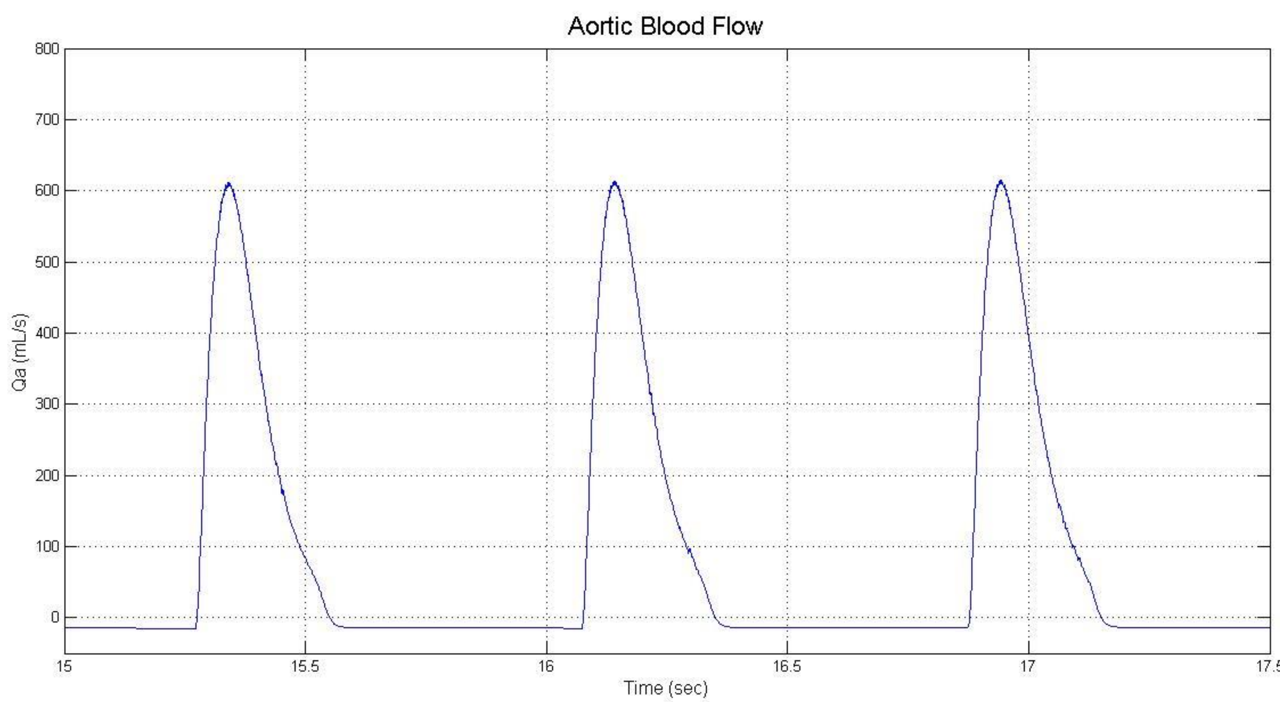




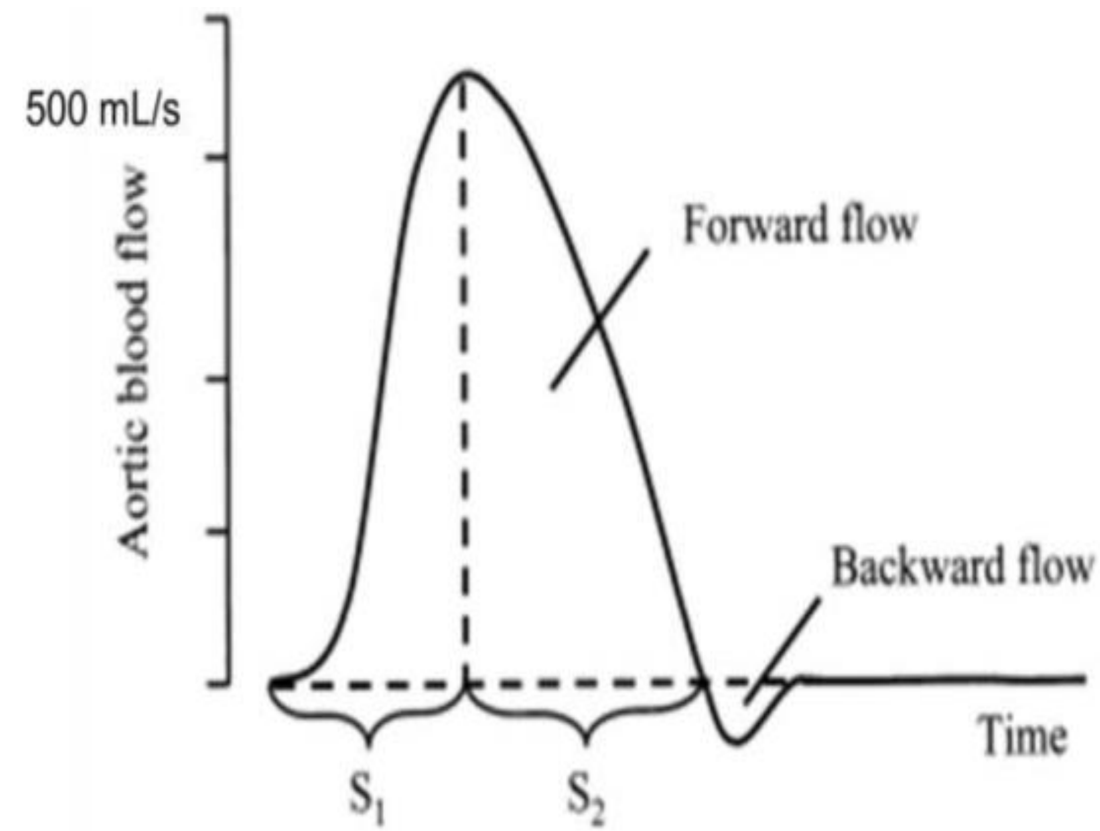


Comparison of simulated pressure and volume curves vs corresponding physiological data for the left heart

Simulation Results – Aortic Blood Flow Rate



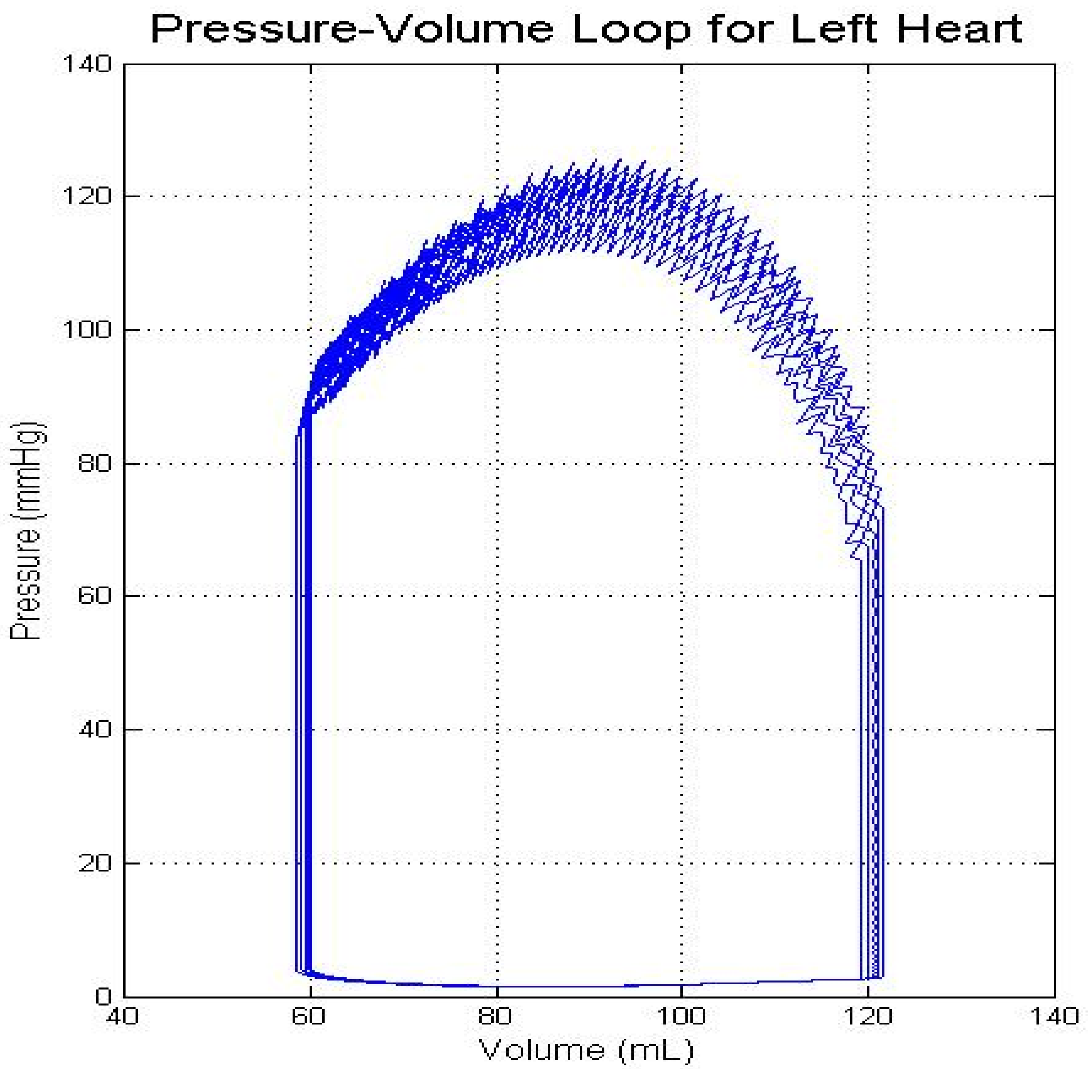
(a)



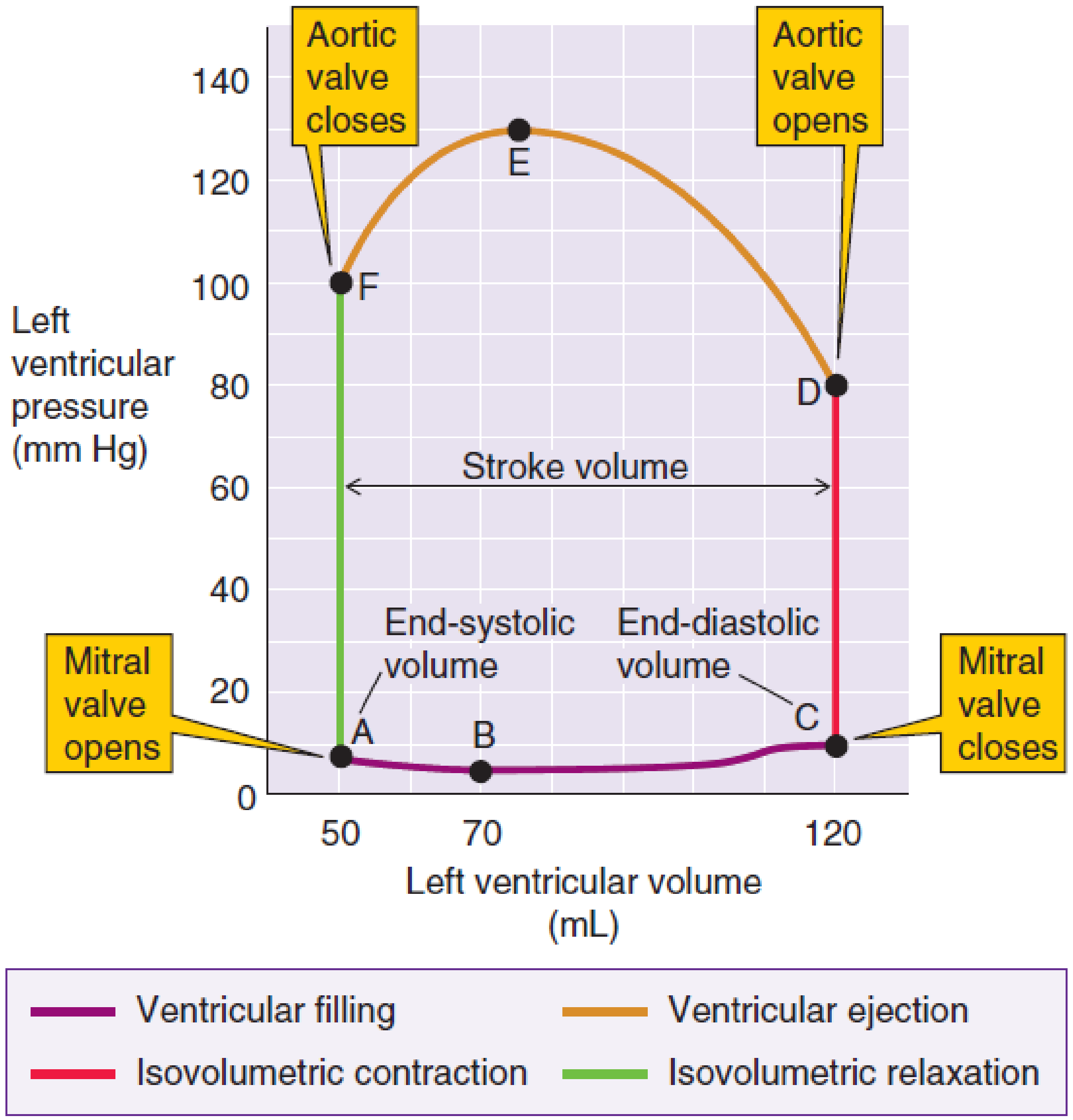
(b)

Comparison between (a) simulated aortic flow rate curve vs (b) corresponding physiological data for the left heart

Simulation Results – Pressure-Volume Diagram



(a)

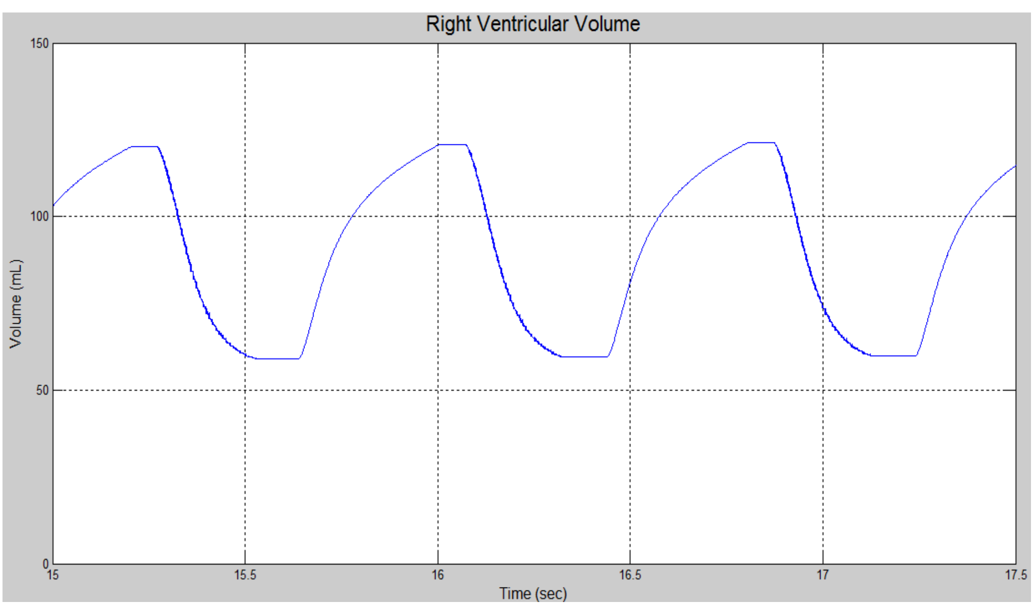


(b)

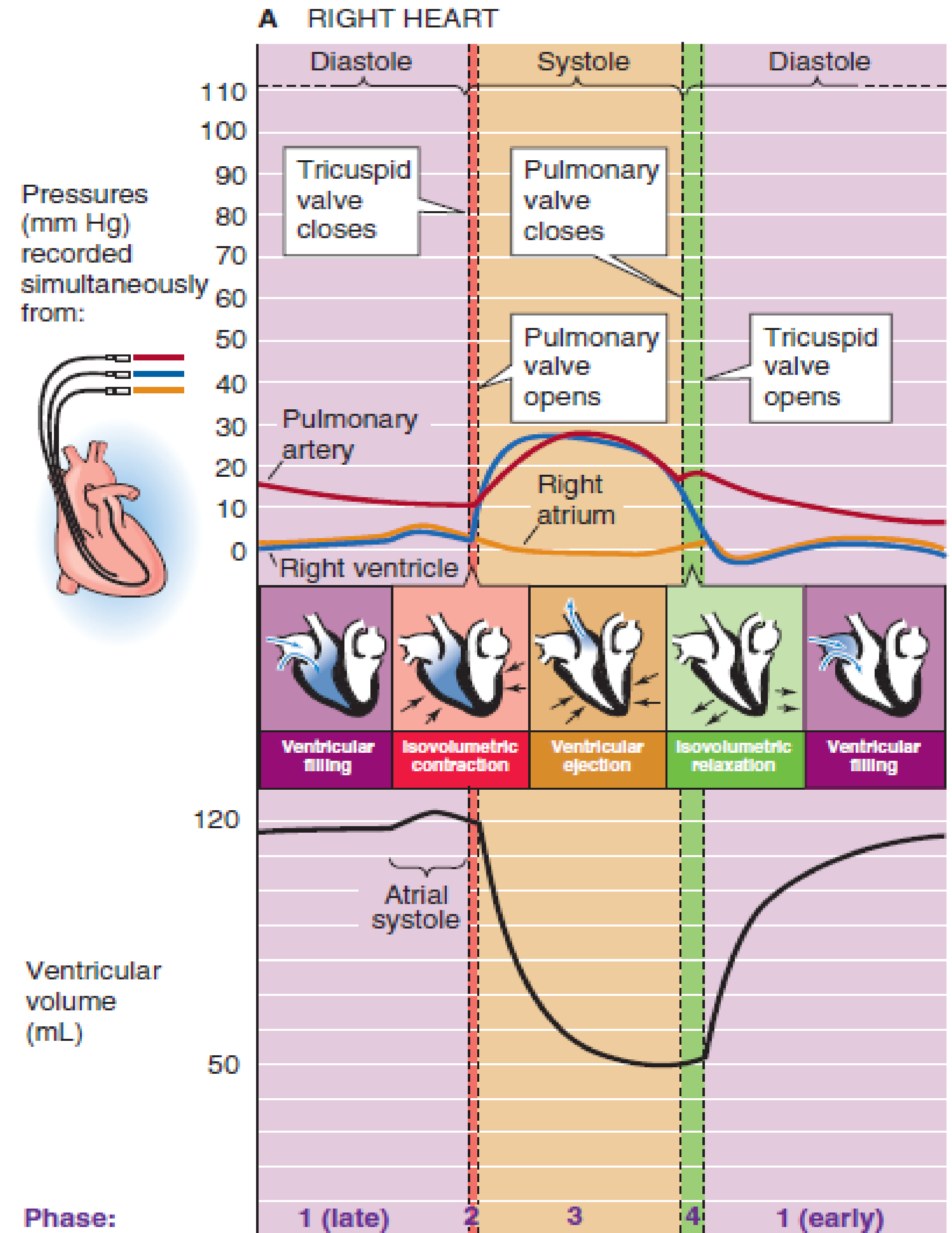
Comparison between (a) simulated pressure-volume loop vs (b) corresponding physiological data for the left heart

Pressure and Volume Plots for the Right Heart

(a)



(b)



(c)

Comparison between (a), (b) simulated pressure and volume curves vs (c) corresponding physiological data for the rightt heart

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Discussion of Results

- The model only assumes elastic ventricles, but atria to be inelastic

- Four heart valves can be modeled with more real diodes instead of the ideal diodes.

- More segments such as head and limbs can be included in the model to achieve higher degree of accuracy and capture more details of hemodynamics

Thanks!

Questions?

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