Literature for conference paper

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Contents

CHAPTER 1

Chance-constrained stochastic programming under variable reliability levels with an application to humanitarian relief network design

Table 1: Summary of Model of [1]

PART 1: Theoretical Parts
Objective
min $\leftarrow$ the risk tolerance
Reformulation (in Value-at-risk): "equivalence relation"
$\mathbb{P}(T_k\mathbf{x} \geq \xi_k) \geq 1 - \epsilon_k \Leftrightarrow T_k\mathbf{x} \geq \mathbf{F}_{\xi_k}^{(-1)}(1 - \epsilon_k)$
Some theories used in the paper (Lemma 1)
Variable reduction
Valid inequality
PART 2: Modeling with application to humanitarian relief network design
Objective min $\leftarrow$ expected total delivery amount-weighted accessibilty score
and expected cost of tolerance (s.t. 11 constraints)

CHAPTER 2

A Chance-Constrained Programming (CCP) Approach to Solve the Energy Management Problem in Microgrids Considering Uncertainties of Renewable Energy Resources

Table 1: Summary of Model of [2]

Parameter and Variables
energy price
revenue function
cost function
Total demanded energy
Solar irradiance
Risk to reward
The energy level of the battery storage
Probability density function
Binary variables for charging of storage devices
Binary variables for on or off status of generators
Objective
min $\leftarrow$ the profit for the microgrid
Reformulation
Sample Average Approximation (SAA)

CHAPTER 3

Analytical Reformulation of Chance-Constrained Optimal Power Flow with Uncertain Load Control

Table 1: Summary of Model of [4]

Objective

min ← energy and reserve cost

s.t. 6 deterministic (Secondary frequency control + Power flow constraints)

and 19 prob constraints (re-dispatch constraints)

Uncertainty: Gaussian

Reformulation

Secondary frequency control constraints

- one-variable constraints → linear constraints
- two-variable constraints → 2-norms!!

Re-dispatch

- re-define some variables
- observe secondary frequency control: whether it is active or inactive.

Power flow constraints

Most difficult as all uncertainties lie in these constraints

Algorithm

Cutting plane (replace some complicated variables with slack variables)

CHAPTER 4

On deterministic reformulations of distributionally robust joint chance constrained optimization problems

Table 1: Summary of Model of [5]

DRCCP
$\min c^T x$,
s.t. $\inf_{\mathbb{P}\in\mathcal{P}} \mathbb{P}[\xi : F(x,\xi) \geq 0] \geq 1-\epsilon$
Method
• Define the feasible region $Z_D = \{x \in \mathbb{R}^n : \inf_{\mathbb{P}\in\mathcal{P}} \mathbb{P}[\xi : F(x,\xi) \geq 0] \geq 1-\epsilon\}$
• Propose a deterministic conservative approximation of Z_D.

CHAPTER 5

Chance-constrained optimization under limited distributional information: A review of reformulations based on sampling and distributional robustness

This is a summary of [3].

The chance-constrained problem CCP In a probaiblity space $(\Omega, \mathcal{F}, \mathbb{P}^0)$

$$\min_x \quad c^T x \tag{1a}$$
$$\text{s.t.} \quad \mathbb{P}^0(x \in \mathcal{P}(\omega)) \geq 1 - \epsilon \tag{1b}$$
$$x \in \mathcal{X}, \tag{1c}$$

where ω is a random variable vector with a distribution $mathbbP^0$, $\mathcal{X} \subset \mathbb{R}^n$ is compact set defined by the determinitic constraints on the decision variables x and risk level $\epsilon \leq 0.05$.

Properties of CCPs In this survey, the study is discussing on the linear chance constrained problems, i.e. polyhedral $\mathcal{P}(\omega)$. More precisely, let

$$\mathcal{P}(\omega) := \{x : T(\omega)x \geq r(\omega)\}, \tag{2}$$

where $T(\omega)$ is an $m \times n$ matrix of random constraint coefficients and $r(\omega) \in \mathbb{R}^m$ is a vector of random right-hand sides.

- $m = 1$: CCP is called *individual* CCP.
- $m > 1$: CCP is called *joint* CCP.

If for all $\omega \in \Omega, T(\omega) = T$, then the CCP has right-hand side uncertainty. In contrast, the *technology matrix* $T(\omega)$ is random, the CCP has left-hand side uncertainty. One of the reformulations is choosing the smaple space Ω to be finite, which means that the CCP is under finite discrete distribution.

CCPs under finite discrete distribution Given the probability space $(\Omega, 2^\Omega, \mathbb{P})$ The Sample Average Approach (SAA) of (1) is

$$\min_x \quad c^T x \tag{3a}$$
$$\text{s.t.} \quad \frac{1}{N} \sum_{i \in [N]} \mathbb{1}(x \notin (\omega_i)) \leq \epsilon, \tag{3b}$$
$$x \in \mathcal{X}, \tag{3c}$$

where $\mathbb{1}(\cdot)$ is the indicator function.

♣ This leads to MIP via the introduction of binary variables and big-M constraints.

1. **RHS uncertainty** The problem (3) becomes

$$\min_{x,t,z} \quad c^T x \tag{4a}$$
$$\text{s.t.} x \in \mathcal{X}, Tx = \bar{r} + t \tag{4b}$$
$$t_j \geq r_{i,j}(1 - z_i), \forall i \in [N], \forall j \in [m] \tag{4c}$$
$$\frac{1}{N} \sum_{i \in [N]} z_i \leq \epsilon \tag{4d}$$
$$t \in \mathbb{R}_+^m \tag{4e}$$
$$z \in \{0, 1\}^N, \tag{4f}$$

where $\bar{r} \in \mathbb{R}^m$ is chosen to satisfy $r(\omega_i) \geq \bar{r}, \forall i$ and $r_i = (r_{i,1}, \ldots, r_{i,m})^T$ denotes $r(\omega_i) - \bar{r}$. [...]

2. **LHS uncertainty** The problem (3) becomes

$$\min_{x,z} \quad c^T x \tag{5a}$$
$$\text{s.t.} x \in \mathcal{X} \tag{5b}$$
$$T(\omega_i)x \geq r(\omega_i) - M(\omega_i)z_i, \forall \in [N] \tag{5c}$$
$$\frac{1}{N} \sum_{i \in [N]} z_i \leq \epsilon \tag{5d}$$
$$z \in \{0, 1\}^N, \tag{5e}$$

where $M(\omega_i), i \in [N]$ is a vector of big-M coefficients such that when $z_i = 1$, inequality (5c) is redundant.

Bibliography

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