# MAST90138 Week 3 Lab

- Recall that if $Z_1, \dots, Z_n$ are independent $N(0,1)$ then
$$X = \sum_{k=1}^{n} Z_k^2 \sim \chi_n^2$$
is a chi-square random variable with $n$ degrees of freedom.

- Recall that if $M$ is an $p \times n$ matrix whose columns are independent and all have a $N_p(0, \Sigma)$ distribution, then
$$\mathcal{Y} = MM^T \sim W_p(\Sigma, n)\,. \tag{1}$$

## Problems

1. In R, generate a sample $X_1 = (X_{11}, X_{21}), \dots, X_n = (X_{1n}, X_{2n})$ of size $n = 200$ from a $N_2(\mu, \Sigma)$ distribution and draw the scatterplot of the pairs $(X_{1i}, X_{2i})$, for $i = 1, \dots, n$, using a red $*$ as the symbol to represent the data points. Do this for the following four different matrices $\Sigma$:
$$\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \Sigma = \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}, \Sigma = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}, \Sigma = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}.$$
You can use any $\mu$ of your choice. Plot the four scatterplots on a single graph window showing 4 scatterplots presented as the first two scatterplots on the first row and the last two scatterplots on the second row. Label the axes as $X1$ and $X2$ and add the title `Scatterplot 1`,..., `Scatterplot 4` to the first,..., fourth scatterplot. What do you notice about the cloud of points when you compare the four cases? Explain what is going on.

2. We know if $X$ is a $p$-random-vector with covariance matrix $\Sigma_X$, then $Y = AX$ is a $q$-random-vector with covariance matrix $\Sigma_Y = A\Sigma_X A^T$ for a $q \times p$ matrix $A$. Use this fact to prove that $\Sigma_X$ must be positive semidefinite. (And hence, any covariance matrix must be positive semidefinite.)

3. Show that if $\mathcal{Y}$ is defined at (1) and $B$ is a $q \times p$ matrix then
$$B\mathcal{Y}B^T \sim W_q(B\Sigma B^T, n)\,.$$

4. Show thatif $\mathcal{Y}$ is defined at (1) and $a$ is a $p \times 1$ vector such that $a^T\Sigma a \neq 0$, then
$$a^T\mathcal{Y}a / a^T\Sigma a \sim \chi_n^2\,.$$