## Kondo Model appendix, Equation 9.61 of thesis

$$\Delta \hat{\mathcal{H}}_{(j)} = \sum_{\substack{m=1,\ eta=\uparrow/ota,\ eta=\uparrow/ota,}}^{n_j} rac{(J^{(j)})^2}{2} rac{ au_{j,\hat{\mathbf{s}}_m,eta}}{(2\omega au_{j,\hat{\mathbf{s}}_m,eta}-\epsilon_{j,l} au_{j,\hat{\mathbf{s}}_m,eta}-J^{(j)}S^z s_{j,\hat{\mathbf{s}}_m}^z)$$

$$imes \left[ S^a S^b \sigma^a_{\alpha\beta} \sigma^b_{\beta\gamma} \sum_{(j_1,j_2 < j),} c^\dagger_{j_1,\hat{\mathbf{s}}_n,\alpha} c_{j_2,\hat{\mathbf{s}}_o,\gamma} (\mathbf{1} - \hat{\pmb{n}}_{j,\hat{\mathbf{s}}_m,\beta}) + ... 
ight]$$

$$(j_1,j_2< j),$$

$$n_j \qquad (J^{(j)})^2 \qquad \qquad [a \land a \lor a \lor b \lor b]$$

$$+\sum_{m=1,}^{n_j}rac{(J^{(j)})^2}{2(2\omega au_{j,\hat{\mathbf{s}}_m,eta}-\epsilon_{j,l} au_{j,\hat{\mathbf{s}}_m,eta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)}igg[S^xS^y\sigma_{lphaeta}^x\sigma_{etalpha}^yc_{j,\hat{\mathbf{s}}_m,lpha}^tc_{j,\hat{\mathbf{s}}_m,eta}c_{j,\hat{\mathbf{s}}_m,eta}c_{j,\hat{\mathbf{s}}_m,lpha}+...$$

$$\Delta \hat{\mathcal{H}}_{(j)} = \sum_{\substack{m=1,\ eta=\uparrow/\downarrow}}^{n_j} rac{(J^{(j)})^2}{2} rac{ au_{j,\hat{\mathbf{s}}_m,eta}}{(2\omega au_{j,\hat{\mathbf{s}}_m,eta}-\epsilon_{j,l} au_{j,\hat{\mathbf{s}}_m,eta}-J^{(j)}S^z S^z_{j,\hat{\mathbf{s}}_m})$$

 $\times \left| S^a S^b \sigma^a_{\alpha\beta} \sigma^b_{\beta\gamma} \sum_{(j_1,\hat{\mathbf{s}}_n,\alpha} c_{j_2,\hat{\mathbf{s}}_o,\gamma} (1-\hat{\mathbf{n}}_{j,\hat{\mathbf{s}}_m,\beta}) + \ldots \right|$ 

$$+\sum_{m=1,}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)} \bigg[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{\mathbf{s}}_m,\alpha}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\beta}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta} + \dots \bigg]$$

▶ The  $\tau$  should not be there in numerator i presume?

$$\Delta \hat{\mathcal{H}}_{(j)} = \sum_{\substack{m=1,\ eta=\uparrow/ar{j}}}^{n_j} rac{(J^{(j)})^2}{2} rac{ au_{j,\hat{\mathbf{s}}_m,eta}}{(2\omega au_{j,\hat{\mathbf{s}}_m,eta}-\epsilon_{j,l} au_{j,\hat{\mathbf{s}}_m,eta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)}$$

$$imes \left[S^aS^b\sigma^a_{lphaeta}\sigma^b_{eta\gamma}\sum_{\substack{(j_1,j_2< j),\ n,o}}c^\dagger_{j_1,\hat{\mathbf{s}}_n,lpha}c_{j_2,\hat{\mathbf{s}}_o,\gamma}(\mathsf{1}-\hat{\pmb{n}}_{j,\hat{\mathbf{s}}_m,eta})+...
ight.$$

$$+\sum_{m=1,}^{n_j}\frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)}\bigg[S^xS^y\sigma_{\alpha\beta}^x\sigma_{\beta\alpha}^yc_{j,\hat{\mathbf{s}}_m,\alpha}^{\dagger}c_{j,\hat{\mathbf{s}}_m,\beta}c_{j,\hat{\mathbf{s}}_m,\beta}^{\dagger}c_{j,\hat{\mathbf{s}}_m,\beta}+\ldots$$

Since coupling is  $\frac{J}{2}$ , shouldn't the thing be  $\frac{J^2}{4}$  instead of  $\frac{J^2}{2}$ ?

$$egin{aligned} \Delta \hat{H}_{(j)} &= \sum_{\substack{m=1,\ eta=\uparrow/\downarrow}}^{n_j} rac{(J^{(j)})^2}{2} rac{ au_{j,\hat{\mathbf{s}}_m,eta}}{(2\omega au_{j,\hat{\mathbf{s}}_m,eta}-\epsilon_{j,l} au_{j,\hat{\mathbf{s}}_m,eta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z) \ & imes \left[S^aS^b\sigma^a_{lphaeta}\sigma^b_{eta\gamma}\sum_{(j_1,j_2\leq j),}c^\dagger_{j_1,\hat{\mathbf{s}}_n,lpha}c_{j_2,\hat{\mathbf{s}}_o,\gamma}(1-\hat{n}_{j,\hat{\mathbf{s}}_m,eta})+... 
ight. \end{aligned}$$

$$+\sum_{\substack{m=1,\\\alpha=1,\beta}}^{n_j}\frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)}\bigg[S^xS^y\sigma_{\alpha\beta}^x\sigma_{\beta\alpha}^yc_{j,\hat{\mathbf{s}}_m,\alpha}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta}c_{j,\hat{\mathbf{s}}_m,\beta}^\dagger c_{j,\hat{\mathbf{s}}_m,\alpha}+...$$

➤ You mentioned the following in the google document- "interchange sigma\_a and sigma\_b (you get -1 sign)". But these are matrix elements (numbers). So why the minus sign?

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1,\\\beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{\mathbf{s}}_m,\beta}}{(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta} - J^{(j)}S^zS^z_{j,\hat{\mathbf{s}}_m})}$$

$$imes \left[ S^a S^b \sigma^a_{lphaeta} \sigma^b_{eta\gamma} \sum_{\substack{(j_1,j_2 < j), \ n,o}} c^\dagger_{j_1,\hat{\mathbf{s}}_n,lpha} c_{j_2,\hat{\mathbf{s}}_o,\gamma} (1-\hat{n}_{j,\hat{\mathbf{s}}_m,eta}) + ... 
ight.$$

$$+\sum_{m=1,}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta}-J^{(j)}S^z\mathbf{s}_{j,\hat{\mathbf{s}}_m}^z)} \bigg[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{\mathbf{s}}_m,\alpha}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\beta}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta} + \dots \bigg]$$

▶ How do you combine the product of two sigmas ( $\sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma}$ ) into a single  $\sigma^c_{\alpha\gamma}$ ?

Kondo URG coupling equation for J (equation 9.65):

$$\Delta J^{(j)} = n_j (J^{(j)})^2 \left[ \omega - \frac{\epsilon_{j,l}}{2} \right] \left[ \left( \frac{\epsilon_{j,l}}{2} - \omega \right)^2 - \frac{\left( J^{(j)} \right)^2}{16} \right]^{-1}$$

One-loop form (after setting  $\omega = \epsilon_{i,l}$ ):

$$\Delta J^{(j)} = \frac{n_j(J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2\frac{n_j(J^{(j)})^2}{\epsilon_{j,l}} \rightarrow \frac{2\rho|\Delta D|J^2}{D} \quad [n_j, \rho \rightarrow \mathsf{DOS} \; \mathsf{per} \; \mathsf{spin}]$$

One-loop form in Coleman (Introduction to Many-Body Physics) ( $\tilde{J} = J/2$ ):

$$\Delta ilde{J} = rac{2
ho |\Delta D| ilde{J}^2}{D} \implies \Delta J = rac{
ho |\Delta D| J^2}{D}$$

Is there any reason for this difference?

▶ In the Kondo URG, are you considering two electrons on the shell  $\Lambda_N$ , one that we are decoupling  $(q\beta)$  and another with the same momentum but opposite spin  $(q\overline{\beta})$ ?

▶ If so, why does that kinetic energy piece  $(\epsilon_q \tau_{q\overline{\beta}})$  not come down in the denominator?

▶ Is that what gives rise to the second RG equation and hence the S<sup>z</sup>s<sup>z</sup> term in the effective Hamiltonian?

$$\Delta H_{(j)}^{2} = \sum_{\substack{m=1,\\\beta=\pm/1}}^{\infty} \frac{(J^{(j)})^{2}}{(2\omega\tau_{j,\hat{\mathbf{s}}_{m},\beta} - \epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_{m},\beta} - J^{(j)}S^{z}S_{j,\hat{\mathbf{s}}_{m}}^{z})} \left[ S^{x}S^{y}\sigma_{\alpha\beta}^{x}\sigma_{\beta\alpha}^{y}c_{j,\hat{\mathbf{s}}_{m},\alpha}^{\dagger}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\alpha}^{\dagger}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\alpha}^{\dagger}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{\mathbf{s}}_{m},\alpha}^{\dagger}c_{j,\hat{\mathbf{s}}_{m},\beta}c_{j,\hat{$$

$$\begin{array}{ll}
& \sum_{\substack{m=1,\\\beta=\uparrow/\downarrow}} (2\omega\tau_{j,\hat{\mathbf{s}}_{m},\beta} - \epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_{m},\beta} - J^{0})S^{2}S_{j,\hat{\mathbf{s}}_{m}}^{2}) \left[ + S^{y}S^{x}\sigma_{\alpha}^{x}\sigma_{\alpha}^{y}G_{\alpha}^{\dagger} - G_{\alpha}^{\dagger}\hat{\mathbf{s}}_{\alpha}G_{\alpha}^{\dagger} - G_{\alpha}^{\dagger}\hat{\mathbf{s}}_{\alpha}G_{\alpha}^{\dagger} - G_{\alpha}^{\dagger}\hat{\mathbf{s}}_{\alpha}G_{\alpha}^{\dagger} \right]$$

 $\left.\hat{n}_{j,\hat{\mathbf{s}}_m,eta}(\mathbf{1}-\hat{n}_{j,\hat{\mathbf{s}}_m,lpha})\right|$ 

 $+\left.S^{y}S^{x}\sigma_{\alpha\beta}^{x}\sigma_{etalpha}^{y}c_{j,\hat{\mathbf{s}}_{m},eta}^{\dagger}c_{j,\hat{\mathbf{s}}_{m},lpha}c_{j,\hat{\mathbf{s}}_{m},lpha}^{\dagger}c_{j,\hat{\mathbf{s}}_{m},eta}
ight|$ 

 $=\sum_{m=1,}^{n_j}\frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)}S^z\frac{\sigma_{\alpha\alpha}^z}{2}\bigg[\hat{\mathbf{n}}_{j,\hat{\mathbf{s}}_m,\alpha}(1-\hat{\mathbf{n}}_{j,\hat{\mathbf{s}}_m,\beta})-$ 

 $\Delta H_{(j)}^2 = \sum_{m=1, \frac{1}{2}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta} - J^{(j)}S^z s_{j,\hat{\mathbf{s}}_m}^z)} \bigg[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{\mathbf{s}}_m,\alpha}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf$ 

$$\begin{split} \Delta \mathcal{H}_{(j)}^2 &= \sum_{\substack{m=1,\\\beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta} - J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)} \bigg[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{\mathbf{s}}_m,\alpha}^\dagger c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\beta} c_{j,\hat{\mathbf{s}}_m,\alpha} \\ &+ S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{\mathbf{s}}_m,\beta}^\dagger c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\alpha} c_{j,\hat{\mathbf{s}}_m,\beta} \bigg] \end{split}$$

$$=\sum_{\substack{m=1,\\ \alpha}}^{n_j}\frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{\mathbf{s}}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{\mathbf{s}}_m,\beta}-J^{(j)}S^zs_{j,\hat{\mathbf{s}}_m}^z)}S^z\frac{\sigma_{\alpha\alpha}^z}{2}\Bigg[\hat{\mathbf{n}}_{j,\hat{\mathbf{s}}_m,\alpha}(1-\hat{\mathbf{n}}_{j,\hat{\mathbf{s}}_m,\beta})-...$$

## What I got:

$$S^x S^y \sigma^x_{lphaeta} \sigma^y_{etalpha} c^\dagger_{j,\hat{\hat{\mathbf{s}}}_m,lpha} c_{j,\hat{\hat{\mathbf{s}}}_m,eta} c^\dagger_{j,\hat{\hat{\mathbf{s}}}_m,eta} c_{j,\hat{\hat{\mathbf{s}}}_m,eta} = i^2 S^z \sigma^z_{lphalpha} \hat{\pmb{\eta}}_{j,\hat{\hat{\mathbf{s}}}_m,lpha} \left(1-\hat{\pmb{\eta}}_{j,\hat{\hat{\mathbf{s}}}_m,eta}
ight)$$

Section 2.2, Equation 2.18 of thesis

$$egin{aligned} rac{1}{H'-H_e\hat{n}_N}c_N^\dagger T &= c_N^\dagger T rac{1}{H'-H_h(1-\hat{n}_N)} \ &\Longrightarrow H_e\hat{n}_N c_N^\dagger T &= c_N^\dagger T H_h(1-\hat{n}_N) \end{aligned}$$

This seems to **require** H' **commuting with** T, because

$$oldsymbol{c}_{N}^{\dagger} T H' - oldsymbol{c}_{N}^{\dagger} T H_{h} (1-\hat{n}_{N}) = H' oldsymbol{c}_{N}^{\dagger} T - H_{e} \hat{n}_{N} oldsymbol{c}_{N}^{\dagger} T$$

Why should H' commute with T?

(where 
$$H_e = Tr(H\hat{n}_N)$$
,  $H_h = Tr[H(1 - \hat{n}_N)]$  and  $T = Tr(Hc_N)$ )

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^{\dagger} = H_h (1 - n_N)$$

If I try to derive this using the result on the previous slide:

$$egin{aligned} \eta H \eta^\dagger &= \eta H_{ extsf{e}} \sigma^\dagger T G = \eta \sigma^\dagger T H_{ extsf{h}} G \ &= \eta \sigma^\dagger T G H_{ extsf{h}} = \eta \eta^\dagger H_{ extsf{h}} = H_{ extsf{h}} (1-\hat{p}) \end{aligned}$$

That required  $[G, H_h] = 0$ . How does that work out?

(where 
$$H_e = Tr(H\hat{n}_N)$$
,  $H_h = Tr[H(1 - \hat{n}_N)]$  and  $T = Tr(Hc_N)$ )

In eq. 2.21 of thesis,

$$UHU^{\dagger} = \frac{1}{2}Tr(H) + \tau Tr(H\tau) + \tau \{c^{\dagger}T, \eta\}$$

so the renormalization is

$$au\{c^{\dagger}T,\eta\} = rac{1}{2} \left[ \overbrace{c^{\dagger}T\eta}^{ ext{particle sector}} - \underbrace{\eta c^{\dagger}T}_{ ext{hole sector}} 
ight] = ext{difference of the 2 sectors}$$

Yet in most RG equations ( $\triangle H_F$  of 2d Hubbard,  $\triangle H_j$  of Kondo), you have added the two sectors. How/Why?

In the Kondo URG, you simplify the  $\hat{\omega}$  as

$$\hat{\omega} = \omega \tau$$

What is the formal way of doing this? Shouldn't it be

$$\hat{\omega} = \omega_1 \hat{n} + \omega_1 (1 - \hat{n})$$

Is this just an assumption?

In the RG equation for BCS instability (eq. 8.130 of thesis), you use

$$G^{-1} = \omega - \epsilon_1 \tau_1 - \epsilon_2 \tau_2$$

How is this choice of  $\hat{\omega}$  consistent with what was done in Kondo URG?