Entanglement features of the Kondo Cloud

Motivation

The key feature of the Kondo screening cloud is the entanglement content between a magnetic impurity and conduction electrons in the vicinity of the Fermi surface. A recent work Phys. Rev. Lett. 120, 146801 (2018) shows that the entanglement content is related to electronic conductivity. In many body systems entanglement entropy scaling shows distinct features for gapped as against gapless phases Phys. Rev. Lett. (2007), arxiv:2003.06118 (2020). Are there any observable entanglement RG scaling features with regards to formation of the Kondo cloud? Our URG procedure mitigates fermion exchange signatures , i.e. it functions as a decoder circuit comprising a error correcting code leading to an emergent subspace where an electronic cloud entangles with the Kondo spin. In this work we want to study the interplay fermion exchange signatures, many particle entanglement, and quantum transport observables like conductivity, shot noise, spectral function etc.

List of things we can do

1. We can start with the Heisenberg Kondo Hamiltonian with isotropic Fermi surface of the Fermi liquid and obtain the URG flow in the space of Hamiltonians arranged from UV to IR. From the IR fixed points obtained in the antiferromagnetic side of the Kondo model we can compute the effective Hamiltonian and the eigenstates.

2. Our experience suggests that the effective Hamiltonian in the strong coupling regime on the antiferromagnetic side will be of the pseudospin kind. By reversing the RG flow we can tomographically create the many body states at UV, by re-entangling the high energy electronic states with their IR counterparts. This allows realization of an entanglement renormalization group and altogether comprise the construction of the EHM tensor network.

3. In the construction of the entanglement RG flow we can study the effect of fermion exchange signatures in the entanglement entropy, mutual information (MI) flow. We can also study the Ryu-Takayanagi entropy bound, emergent holographic spacetime generated from MI. Can this entanglement features witness the entanglement phase transition between the ferromagnetic and antiferromagnetic side of the Kondo model?

4. We can extract the reduced density matrix comprising the HIlbert space associated with the Fermi surface (FS) and the Kondo Impurity (KI). We can study the RG dynamics of MI content between the FS and the KI, does this show the formation of the Kondo cloud? Does the fermion exchange signs have observable effects in the entanglement scaling flow towards the Kondo cloud?

5. In the case when the Kondo cloud is formed can we confirm Martin's sum rule, i.e. the reduction in Luttinger's sum by the no. of electronic states added to the KI. This implies that the ferromagnetic to antiferromagnetic transition is a topological transition. How does this coincide with our understanding of the entanglement phase transition?

6. Finally we can study the holographic renormalization of the quantum geometric tensor for the Fermi surface and KI HIlbert space, this will surely be a witness to the formation of the Kondo cloud.

7. Show quantum advantage in the kondo cloud for error correction. But it requires a bit more study of the current plan. Especially the entanglement scaling features fermion sign issues. Then we can understand how they get error corrected upon scaling, and eventually form the cloud, i.e., How fermion sign issues are resolved resulting in the formation of Kondo cloud. This could be used in a proposal for quantum error correction and an equivalent machine learning protocol.

8. Can we perform a gauge theoretic construction of the local quantum liquid generated by isolating the Kondo impurity via tracing out its degree of freedom?

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Abstract

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$$\hat{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}\sigma} + \frac{J}{2} \sum_{\mathbf{k},\mathbf{k}'} \mathbf{S} \cdot c^{\dagger}_{\mathbf{k}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} \tag{1}$$

$$\mathbf{k}_{\Lambda\hat{s}} = \mathbf{k}_F(\hat{s}) + \Lambda\hat{s}, \hat{s} = \frac{\nabla\epsilon_{\mathbf{k}}}{|\nabla\epsilon_{\mathbf{k}}|}|_{\epsilon_{\mathbf{k}}=E_F} \tag{2}$$

$$|j, l, \sigma\rangle = |\mathbf{k}_{\Lambda_j\hat{s}}, \sigma\rangle, l := (\hat{s}_m, \sigma) \tag{3}$$

If there are n_j normal directions then l ranges from 1 to $2n_j$ as follows: $1 = (\hat{s}_1, \uparrow)$, $2 = (\hat{s}_1, \downarrow)$, $3 = (\hat{s}_2, \uparrow)$, ...

$$H_{(j-1)} = U_{(j)} H_{(j)} U^{\dagger}_{(j)} \tag{4}$$

$$\begin{aligned}
U_{(j)} &= \prod_l U_{j,l} U_{j,l}, \\
U_{j,l} &= \frac{1}{\sqrt{2}}[1 + \eta_{j,l} - \eta^{\dagger}_{j,l}], \\
\eta_{j,l} &= Tr_{j,l}(c^{\dagger}_{j,l} H_{j,l}) c_{j,l} \frac{1}{\hat{\omega}_{j,l} - Tr_{j,l}(H^D_{j,l}\hat{n}_{j,l})\hat{n}_{j,l}}, \\
\hat{\omega}_{j,l} = H^D_{j,l} + H^X_{j,l} - H^X_{j,l-1} H_{j,l} &= \prod_{m=1}^{l} U_{j,m} H_{(j)} [\prod_{m=1}^{l} U_{j,m,\sigma}]^{\dagger}
\end{aligned} \tag{5}$$

Note $H_{j,2n_j+1} = H_{(j-1)}$.

$$H_{j,l+1} = Tr_{j,l}(H_{(j,l)}) + \{c^{\dagger}_{j,l} Tr_{j,l}(H_{(j,l)} c_{j,l}), \eta_{j,l}\} \tau_{j,l} \tag{6}$$

Ignoring the higher order correlated tangential scattering processes on a given high energy shell (constituting states being

$$\hat{H}_{(j-1)}(\omega) = \sum_{j,l,\sigma} \epsilon_{j,l} \hat{n}_{j,l} + \frac{J^{(j-1)}(\omega)}{2} \sum_{\substack{j_1, j_2 < j-1, \\ \hat{s}_1, \hat{s}_2}} \mathbf{S} \cdot c^{\dagger}_{j_1,\hat{s}_1,\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{j_2,\hat{s}_2,\beta} (1 + \sum_{l=1}^{j} \tau_l + \sum_{l,l'=1}^{j} \tau_l \tau_{l'} + \ldots) \; . \tag{7}$$

$$J^{(j-1)} - J^{(j)} = \sum_{m=1}^{n_j} \left[\frac{2(J^{(j)})^2 \tau_{j,\hat{s}_m,\uparrow}}{\omega - \epsilon_{j,m}\tau_{j,\hat{s}_m,\uparrow} - \frac{J^{(j)}}{2} S^z(\tau_{j,\hat{s}_m,\uparrow} - \tau_{j,\hat{s}_m,\downarrow})} + \frac{2(J^{(j)})^2 \tau_{j,\hat{s}_m,\downarrow}}{\omega - \epsilon_{j,m}\tau_{j,\hat{s}_m,\downarrow} - \frac{J^{(j)}}{2} S^z(\tau_{j,\hat{s}_m,\uparrow} - \tau_{j,\hat{s}_m,\downarrow})} \right] \tag{8}$$

$$\begin{aligned}
\frac{1}{a}\left(1 - \frac{b}{a} S^z\right)^{-1} &= \frac{1}{a}\left(1 + \frac{b}{a} S^z + \frac{b^2}{4a^2} + \frac{b^3}{4a^3} S^z + \ldots\right) \\
&= \frac{1}{a}\left[\frac{1}{1 - \frac{b^2}{4a^2}} + \ldots\right]
\end{aligned} \tag{9}$$