Question 1

Section 2.2, Equation 2.18 of thesis

$$\frac{1}{H' - H_e \hat{n}_N} c_N^\dagger T = c_N^\dagger T \frac{1}{H' - H_h(1 - \hat{n}_N)}$$

$$\implies H_e \hat{n}_N c_N^\dagger T = c_N^\dagger T H_h (1 - \hat{n}_N)$$

This seems to **require H' commuting with T**, because

$$c_N^\dagger T H' - c_N^\dagger T H_h (1 - \hat{n}_N) = H' c_N^\dagger T - H_e \hat{n}_N c_N^\dagger T$$

Why should H' commute with T?

(where $H_e = Tr\left(H\hat{n}_N\right)$, $H_h = Tr\left[H\left(1 - \hat{n}_N\right)\right]$ and $T = Tr\left(Hc_N\right)$)

Question 2

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^\dagger = H_h \left(1 - n_N\right)$$

If I try to derive this using the result on the previous slide:

$$\eta H \eta^\dagger = \eta H_e \eta^\dagger = \eta H_e c^\dagger T G = \eta c^\dagger T H_h G$$

$$= \eta c^\dagger T G H_h = \eta \eta^\dagger H_h = H_h \left(1 - \hat{n}\right)$$

That required $[G, H_h] = 0$. How does that work out?

(where $H_e = Tr\left(H \hat{n}_N\right)$, $H_h = Tr\left[H\left(1 - \hat{n}_N\right)\right]$ and $T = Tr\left(H c_N\right)$)

Question 3

Kondo Model appendix, Equation 9.61 of thesis

$$
\begin{aligned}
\Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \\
& \times \left[S^a S^b \sigma^a_{\alpha\beta} \sigma^b_{\beta\gamma} \sum_{\substack{(j_1, j_2 < j), \\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma} (1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right. \\
& + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \\
& \left[S^x S^y \sigma^x_{\alpha\beta} \sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + S^y S^x \sigma^x_{\alpha\beta} \sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\beta} \cdots \right.
\end{aligned}
$$

Question 3

$$\Delta\hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right.$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\left[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + S^y S^x \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\beta} \cdots \right.$$

- The τ should not be there in numerator i presume?

Question 3

$$
\begin{aligned}
\Delta \hat{H}_{(j)} = &\sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \\
&\times \left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1-\hat{n}_{j,\hat{s}_m,\beta}) + \dots \right. \\
&+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \\
&\left[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + S^y S^x \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\beta} \dots \right.
\end{aligned}
$$

- Since coupling is $\frac{J}{2}$, shouldn't the thing be $\frac{J^2}{4}$ instead of $\frac{J^2}{2}$?

Question 4

URG coupling equation for J (equation 9.65):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2 \left[\omega - \frac{\epsilon_{j,l}}{2}\right]}{(\frac{\epsilon_{j,l}}{2} - \omega)^2 - \frac{\left(J^{(j)}\right)^2}{16}} .$$

One-loop form (after setting $\omega = \epsilon_{j,l}$):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2\frac{n_j (J^{(j)})^2}{\epsilon_{j,l}} = \frac{2\rho|\Delta D|J^2}{D}$$

One-loop form in Coleman (Introduction to Many-Body Physics) ($\tilde{J} = J/2$):

$$\Delta \tilde{J} = \frac{2\rho|\Delta D|\tilde{J}^2}{D} \implies \Delta J = \frac{\rho|\Delta D|J^2}{D}$$