Contents

0.1 With particle-hole symmetric interaction

The four-Fermi interaction we are considering is of the form

$$\mathcal{H}_I = \sum_{k,q,\sigma_i} u c^\dagger_{k\sigma_1} c^\dagger_{d\sigma_2} c_{q\sigma_3} c_{d\sigma_4} \delta_{(\sigma_1+\sigma_2=\sigma_3+\sigma_4)} \tag{0.1}$$

The u in general depends on the spin and the momenta. Expanding the summation by using the delta gives

$$\mathcal{H}_I = \underbrace{\sum_{k,q,\sigma,\sigma'} u_1 c^\dagger_{k\sigma} c^\dagger_{d\sigma'} c_{q\sigma} c_{d\sigma'}}_{\text{spin-preserving scattering}} + \overbrace{\sum_{k,q,\sigma} u_2 c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} u_2 c_{q\overline{\sigma}} c_{d\sigma}}^{\text{spin-flip scattering}} \tag{0.2}$$

At this point, we drop the dependence of u on the momenta and assume it depends only on the spin transfer. The first term (attached with u_1) involves no spin-flip between the scattering momenta or the scattering impurity electrons ($k\sigma \to q\sigma, d\sigma' \to d\sigma'$). We label this coupling as u_P. The other coupling involves a spin-flip scattering, so we label that as u_A.

$$\mathcal{H}_I = \sum_{k,q,\sigma,\sigma'} u_P c^\dagger_{k\sigma} c^\dagger_{d\sigma'} c_{q\sigma} c_{d\sigma'} + \sum_{k,q,\sigma} u_A c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} u_2 c_{q\overline{\sigma}} c_{d\sigma} \tag{0.3}$$

The anti-parallel part of the interaction is not particle-hole symmetric in this form, as is seen by performing a particle-hole transformation $c_k \to c^\dagger_k, c_d \to -c^\dagger_d$:

$$\sum_{k,q,\sigma,\sigma'} c^\dagger_{k\sigma} c^\dagger_{d\sigma'} c_{q\sigma} c_{d\sigma'} \to \sum_{k,q,\sigma,\sigma'} c_{k\sigma} c_{d\sigma'} c^\dagger_{q\sigma} c^\dagger_{d\sigma'} \tag{0.4}$$

The anti-parallel part is on the other hand invariant under this change:

$$\begin{aligned}\sum_{k,q,\sigma} c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} c_{q\overline{\sigma}} c_{d\sigma} \to &\sum_{k,q,\sigma} c_{k\sigma} c_{d\overline{\sigma}} c^\dagger_{q\overline{\sigma}} c^\dagger_{d\sigma} \\ &= \sum_{k,q,\sigma} c^\dagger_{q\overline{\sigma}} c^\dagger_{d\sigma} c_{k\sigma} c_{d\overline{\sigma}} \\ &= \sum_{k,q,\sigma} c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} c_{q\overline{\sigma}} c_{d\sigma}\end{aligned} \tag{0.5}$$

To make the parallel interaction particle-hole symmetric, we add its particle-hole transformed partner:

$$\mathcal{H}_I = \sum_{k,q,\sigma} u_A c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} c_{q\overline{\sigma}} c_{d\sigma} + \sum_{k,q,\sigma,\sigma'} \frac{u_P}{2} \left[c^\dagger_{k\sigma} c^\dagger_{d\sigma'} c_{q\sigma} c_{d\sigma'} + c_{k\sigma} c_{d\sigma'} c^\dagger_{q\sigma} c^\dagger_{d\sigma'} \right] \tag{0.6}$$

Since this term is now invariant under the particle-hole transformation, we can see how the other impurity terms transform:

$$\epsilon_d \sum_\sigma \hat{n}_{d\sigma} + \sum_{k\sigma} V_k \left(c^\dagger_{k\sigma} c_{d\sigma} + c^\dagger_{q\sigma} c_{k\sigma} \right) + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$

$$\downarrow$$

$$\begin{aligned} &\epsilon_d \sum_\sigma (1 - \hat{n}_{d\sigma}) - \sum_{k\sigma} V_k \left(c_{k\sigma} c^\dagger_{d\sigma} + c_{q\sigma} c^\dagger_{k\sigma} \right) + U (1 - \hat{n}_{d\uparrow})(1 - \hat{n}_{d\downarrow}) \\ &= (-U - \epsilon_d) \sum_\sigma \hat{n}_{d\sigma} + \sum_{k\sigma} V_k \left(c^\dagger_{k\sigma} c_{d\sigma} + c^\dagger_{q\sigma} c_{k\sigma} \right) + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} \end{aligned}$$

In the last step I dropped a constant term. The particle-hole symmetry condition is thus

$$-U - \epsilon_d = \epsilon_d \implies U + 2\epsilon_d = 0 \tag{0.7}$$

The total Hamiltonian can be written as

$$\begin{aligned} \mathcal{H} = &\sum_k \left(\epsilon_k \hat{n}_{k\sigma} + V_k c^\dagger_{k\sigma} c_{d\sigma} + \text{h.c.} \right) + \epsilon_d \sum_\sigma \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \\ &\sum_{k,q,\sigma} u_A c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} c_{q\overline{\sigma}} c_{d\sigma} + \sum_{k,q,\sigma,\sigma'} \frac{u_P}{2} \left[c^\dagger_{k\sigma} c^\dagger_{d\sigma'} c_{q\sigma} c_{d\sigma'} + c_{k\sigma} c_{d\sigma'} c^\dagger_{q\sigma} c^\dagger_{d\sigma'} \right] \end{aligned} \tag{0.8}$$

The Hamiltonian with a single electron $q\beta$ on the N^{th} shell is

$$\begin{aligned} \mathcal{H}_N = &H_{N-1} + H_{imp} + \epsilon_q \hat{n}_{q\beta} + V_k c^\dagger_{q\beta} c_{d\beta} + \text{h.c.} \\ &+ \sum_{k<\Lambda_N} u_A \left(c^\dagger_{q\beta} c^\dagger_{d\overline{\beta}} c_{k\overline{\beta}} c_{d\beta} + c^\dagger_{k\overline{\beta}} c^\dagger_{d\beta} c_{q\beta} c_{d\overline{\beta}} \right) \\ &+ \sum_{k<\Lambda_N \sigma} \frac{u_P}{2} \left(c^\dagger_{q\beta} c^\dagger_{d\sigma} c_{k\beta} c_{d\sigma} + c^\dagger_{k\beta} c^\dagger_{d\sigma} c_{q\beta} c_{d\sigma} + c_{k\beta} c_{d\sigma} c^\dagger_{q\beta} c^\dagger_{d\sigma} + c_{q\beta} c_{d\sigma} c^\dagger_{k\beta} c^\dagger_{d\sigma} \right) \\ &- \sum_\sigma \frac{u_P}{2} \left[\hat{n}_{q\beta} \hat{n}_{d\sigma} + \left(1 - \hat{n}_{q\beta} \right) (1 - \hat{n}_{d\sigma}) \right] \\ = &H_{N-1} + H_{imp} + \epsilon_q \hat{n}_{q\beta} + V_k c^\dagger_{q\beta} c_{d\beta} + \text{h.c.} + u_A \sum_{k<\Lambda_N} \left(c^\dagger_{q\beta} c^\dagger_{d\overline{\beta}} c_{k\overline{\beta}} c_{d\beta} + c^\dagger_{k\overline{\beta}} c^\dagger_{d\beta} c_{q\beta} c_{d\overline{\beta}} \right) \\ &+ u_P \sum_{k<\Lambda_N \sigma} \left(\tau_{d\sigma} c_{k\beta} c^\dagger_{q\beta} + \tau_{d\sigma} c_{q\beta} c^\dagger_{k\beta} \right) - u_P \tau_{q\beta} \sum_\sigma \hat{n}_{d\sigma} - u_P \left(1 - \hat{n}_{q\beta} \right) \end{aligned} \tag{0.9}$$

We see that the interaction adds to the already-present dispersions; the energy of a hole state is now $-u_P$, and the impurity site energy is now $\epsilon_d - u_P \tau_{k\sigma}$. The total diagonal part is

$$\mathcal{H}^D_N = \begin{cases} H^D_{N-1} + \epsilon^\pm_q + \left(\epsilon_d - \frac{1}{2} u_P \right) \sum_\sigma \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} & \text{particle state} \\ H^D_{N-1} - u_P + \left(\epsilon_d + \frac{1}{2} u_P \right) \sum_\sigma \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} & \text{hole state} \end{cases} \tag{0.10}$$

H_{N-1}^D has the dispersions for the electrons on the lower shells and the $u_P \tau_{k\sigma'} \sum_\sigma \hat{n}_{d\sigma}$-type term for $k < \Lambda_N$. The $\epsilon_q^\pm$ denotes the kinetic energy for states above ($\epsilon_q^+ = |\epsilon_q|$) or below ($\epsilon_q^- = -|\epsilon_q|$) the Fermi level.

0.1.1 Particle sector

For the particle sector, the diagonal part becomes

$$\mathcal{H}_N^D = H_{N-1}^D + \epsilon_q^+ + \left(\epsilon_d - \frac{1}{2}u_P\right)\sum_\sigma \hat{n}_{d\sigma} + U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow} \tag{0.11}$$

The + on the ϵ_q signifies that it is the energy of a particle excitation, as compared to that of a hole excitation while will be denoted by ϵ_q^-. The change is

$$\Delta^+\mathcal{H}_N = \sum_{q\beta}\left[V_q^* c_{d\beta}^\dagger c_{q\beta} + u_P \sum_{k<\Lambda_N \sigma} \tau_{d\sigma} c_{q\beta} c_{k\beta}^\dagger + u_A \sum_{k<\Lambda_N} c_{k\overline{\beta}}^\dagger c_{d\beta}^\dagger c_{q\beta} c_{d\overline{\beta}}\right]$$
$$\frac{1}{\hat{\omega} - H_{N-1}^D - \epsilon_q^+ - \left(\epsilon_d - \frac{1}{2}u_P\right)\sum_\sigma \hat{n}_{d\sigma} - U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow}}$$
$$\left[V_q c_{q\beta}^\dagger c_{d\beta} + u_P \sum_{k<\Lambda_N \sigma} \tau_{d\sigma} c_{k\beta} c_{q\beta}^\dagger + u_A \sum_{k<\Lambda_N} c_{q\beta}^\dagger c_{d\overline{\beta}}^\dagger c_{k\overline{\beta}} c_{d\beta}\right]$$

1.

$$\Delta_1^+\mathcal{H}_N = \sum_{q\beta} V_q^* c_{d\beta}^\dagger c_{q\beta} \frac{1}{\hat{\omega} - H_{N-1}^D - \epsilon_q^+ - \left(\epsilon_d - \frac{1}{2}u_P\right)\sum_\sigma \hat{n}_{d\sigma} - U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow}} V_q c_{q\beta}^\dagger c_{d\beta} \tag{0.12}$$

Inside the propagator, $\hat{n}_{d\beta} = 0$ and $\hat{n}_{q\beta} = 1$, so

$$\begin{aligned}\Delta_1^+\mathcal{H}_N &= \sum_{q\beta} |V_q|^2 c_{d\beta}^\dagger c_{q\beta} \frac{1}{\hat{\omega} - H_{N-1}^D - \epsilon_q^+ - \left(\epsilon_d - \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}}} V_q c_{q\beta}^\dagger c_{d\beta} \\ &= \sum_{q\beta} |V_q|^2 \hat{n}_{d\beta}\left(1 - \hat{n}_{q\beta}\right) \frac{1}{\hat{\omega} - H_{N-1}^D - \epsilon_q^+ - \left(\epsilon_d - \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}}}\end{aligned} \tag{0.13}$$

The energy for the initial state on which $\Delta_1^+\mathcal{H}_N$ acts is

$$H_G = H_{N-1}^D + \left(\epsilon_d + \frac{1}{2}u_P\right)\sum_\beta \hat{n}_{d\beta} + U\hat{n}_{d\beta}\hat{n}_{d\overline{\beta}} - u_P \tag{0.14}$$

This allows us to write the denominator in terms of this ground state energy:

$$\hat{\omega} - \epsilon_q^+ - \left(\epsilon_d - \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}} - H_{N-1}^D = \hat{\omega} - H_G - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P + u_P\hat{n}_{d\overline{\beta}} + U\hat{n}_{d\overline{\beta}} \tag{0.15}$$

If we measuring the quantum fluctuation $\hat{\omega}$ from the initial state energy H_G, we can set $H_G = 0$ and replace $\hat{\omega}$ with its eigenvalue ω.

$$\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P + u_P\hat{n}_{d\overline{\beta}} + U\hat{n}_{d\overline{\beta}} \tag{0.16}$$

The renormalization in $\mathcal{H}_N$ coming from the particle sector due to the first scattering is thus

$$\begin{aligned}\Delta_1^+\mathcal{H}_N &= \sum_{q\beta}|V_q|^2\hat{n}_{d\beta}\left(1-\hat{n}_{q\beta}\right)\frac{1}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P + u_P\hat{n}_{d\overline{\beta}} + U\hat{n}_{d\overline{\beta}}} \\ &= \sum_{q\beta}|V_q|^2\hat{n}_{d\beta}\left(1-\hat{n}_{q\beta}\right)\left[\frac{\hat{n}_{d\overline{\beta}}}{\omega - \epsilon_q^+ + \epsilon_d + U + \frac{1}{2}u_P} + \frac{1-\hat{n}_{d\overline{\beta}}}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P}\right]\end{aligned} \tag{0.17}$$

Assuming $\hat{n}_{q\beta} = 0$ initially (states at the upper edge of the band should be empty in the ground state),

$$\begin{aligned}&\Delta_1^+\mathcal{H}_N \\ &= \sum_{q\beta}|V_q|^2\hat{n}_{d\beta}\left[\frac{\hat{n}_{d\overline{\beta}}}{\omega - \epsilon_q^+ + \epsilon_d + U + \frac{1}{2}u_P} + \frac{1-\hat{n}_{d\overline{\beta}}}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P}\right] \\ &= \sum_{q\beta}\hat{n}_{d\beta}\left[\frac{|V_q^1|^2\hat{n}_{d\overline{\beta}}}{\omega - \epsilon_q^+ + \epsilon_d + U + \frac{1}{2}u_P} + \frac{|V_q^0|^2\left(1-\hat{n}_{d\overline{\beta}}\right)}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P}\right] \\ &= \sum_{q\beta}|V_q|^2\hat{n}_{d\beta}\left[\frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} + \hat{n}_{d\overline{\beta}}\left(\frac{|V_q^1|^2}{\omega - \epsilon_q^+ + \epsilon_d + U + \frac{1}{2}u_P} - \frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P}\right)\right]\end{aligned} \tag{0.18}$$

0.1.2 Hole sector

For the hole sector, the diagonal part becomes

$$\mathcal{H}_N^D = H_{N-1}^D - u_P + \left(\epsilon_d + \frac{1}{2}u_P\right)\sum_\sigma \hat{n}_{d\sigma} + U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow} \tag{0.19}$$

The change in the Hamiltonian is

$$\begin{aligned}\Delta_1^-\mathcal{H}_N &= \sum_{q\beta}V_q c_{q\beta}^\dagger c_{d\beta}\frac{1}{\hat{\omega} - H_{N-1}^D + u_P - \left(\epsilon_d + \frac{1}{2}u_P\right)\sum_\sigma \hat{n}_{d\sigma} - U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow}}V_q^* c_{d\beta}^\dagger c_{q\beta} \\ &= \sum_{q\beta}|V_q|^2 c_{q\beta}^\dagger c_{d\beta}\frac{1}{\hat{\omega} - H_{N-1}^D + u_P - \left(\epsilon_d + \frac{1}{2}u_P\right) - \left(\epsilon_d + \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}} - U\hat{n}_{d\overline{\beta}}}c_{d\beta}^\dagger c_{q\beta} \\ &= \sum_{q\beta}|V_q|^2\hat{n}_{q\beta}\left(1-\hat{n}_{d\beta}\right)\frac{1}{\hat{\omega} - H_{N-1}^D + u_P - \left(\epsilon_d + \frac{1}{2}u_P\right) - \left(\epsilon_d + \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}} - U\hat{n}_{d\overline{\beta}}}\end{aligned} \tag{0.20}$$

The initial state energy in this case is

$$
\begin{aligned}
H_G &= H_{N-1}^D + \epsilon_q^- + \left(\epsilon_d - \frac{1}{2}u_P\right)\sum_\sigma \hat{n}_{d\sigma} + U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow} \\
&= H_{N-1}^D + \epsilon_q^- + \left(\epsilon_d - \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}} \\
&= H_{N-1}^D + \epsilon_q^- + \left(\epsilon_d + \frac{1}{2}u_P\right)\hat{n}_{d\overline{\beta}} - u_P\hat{n}_{d\overline{\beta}}
\end{aligned}
\tag{0.21}
$$

Therefore, the denominator can be written as

$$
\hat{\omega} - H_G + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P - \left(U + u_p\right)\hat{n}_{d\overline{\beta}} = \omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P - \left(U + u_p\right)\hat{n}_{d\overline{\beta}}
$$

$$
\Delta_1^-\mathcal{H}_N = \sum_{q\beta}|V_q|^2\hat{n}_{q\beta}\left(1-\hat{n}_{d\beta}\right)\frac{1}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P - \left(U + u_p\right)\hat{n}_{d\overline{\beta}}}
\tag{0.22}
$$

We can set $\hat{n}_{q\beta} = 1$ because the state at the lower edge of the band will be occupied.

$$
\begin{aligned}
\Delta_1^-\mathcal{H}_N &= \sum_{q\beta}|V_q|^2\left(1-\hat{n}_{d\beta}\right)\left[\frac{\hat{n}_{d\overline{\beta}}}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P} + \frac{1-\hat{n}_{d\overline{\beta}}}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P}\right] \\
&= \sum_{q\beta}\left(1-\hat{n}_{d\beta}\right)\left[\frac{|V_q^1|^2\hat{n}_{d\overline{\beta}}}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P} + \frac{|V_q^0|^2\left(1-\hat{n}_{d\overline{\beta}}\right)}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P}\right] \\
&= \sum_{q\beta}|V_q|^2\left(1-\hat{n}_{d\beta}\right)\Bigg[\hat{n}_{d\overline{\beta}}\left(\frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P} - \frac{|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P}\right) \\
&\qquad + \frac{|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P}\Bigg] \\
&= \sum_{q\beta}\Bigg[\hat{n}_{d\overline{\beta}}\left(\frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P} - \frac{2|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P}\right) \\
&\qquad + \hat{n}_{d\beta}\hat{n}_{d\overline{\beta}}\left(\frac{|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P} - \frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P}\right)\Bigg]
\end{aligned}
\tag{0.23}
$$

0.1.3 Scaling equations

1.

$$
\Delta\epsilon_d = \sum_q\left[\frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} + \frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P} - \frac{2|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P}\right]
\tag{0.24}
$$

2.

$$\Delta U = \sum_q \left[\frac{|V_q^1|^2}{\omega - \epsilon_q^+ + \epsilon_d + U + \frac{1}{2}u_P} - \frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} + \frac{|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d + \frac{1}{2}u_P} \right. \\ \left. - \frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - \frac{1}{2}u_P} \right] \tag{0.25}$$

0.1.4 Particle-hole symmetry

On imposing the constraint $2\epsilon_d + U = 0$ and invoking the relation $\epsilon_q^+ = -\epsilon_q^-$, the equations become

$$\begin{aligned} \Delta\epsilon_d &= 2\sum_q \left[\frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} - \frac{|V_q^0|^2}{\omega - \epsilon_q^+ - \epsilon_d + \frac{1}{2}u_P} \right] \\ \frac{1}{2}\Delta U &= \sum_q \left[\frac{|V_q^1|^2 + |V_q^0|^2}{\omega - \epsilon_q^+ - \epsilon_d + \frac{1}{2}u_P} - \frac{|V_q^1|^2 + |V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} \right] \end{aligned} \tag{0.26}$$

We hence see that the quantity $\epsilon_d + \frac{1}{2}U$ is invariant under the RG if we started with a particle-hole symmetric Hamiltonian:

$$\begin{aligned} \Delta\left(\epsilon_d + \frac{1}{2}U\right) &= \sum_q \left[\frac{|V_q^1|^2 - |V_q^0|^2}{\omega - \epsilon_q^+ - \epsilon_d + \frac{1}{2}u_P} - \frac{|V_q^1|^2 - |V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} \right] \\ &= \sum_q \left[|V_q^1|^2 - |V_q^0|^2 \right] \left[\frac{1}{\omega - \epsilon_q^+ - \epsilon_d + \frac{1}{2}u_P} - \frac{1}{\omega - \epsilon_q^+ + \epsilon_d - \frac{1}{2}u_P} \right] \end{aligned} \tag{0.27}$$

If we had started with particle-hole symmetry, that is, had $|V_q^1|^2 = |V_q^0|^2$, then this change will be 0.