$$\mathcal{H} = t\left(\hat{n}_\pi - \hat{n}_0\right) + \frac{U}{2} n_\uparrow n_\downarrow + \frac{U}{2} \prod_\sigma \left[c^\dagger_{0,\sigma} c_{\pi,\sigma} + c^\dagger_{\pi,\sigma} c_{0,\sigma} \right] \tag{1}$$

$$\mathcal{H}_{\pi\uparrow} = \frac{1}{2} \mathrm{Tr}_{\pi\uparrow} \mathcal{H} + \tau_{\pi\uparrow} \mathrm{Tr}_{\pi\uparrow} \left[\mathcal{H} \tau_{\pi\uparrow} \right] + \tau_{\pi\uparrow} \left\{ c^\dagger_{\pi\uparrow} \mathrm{Tr}_{\pi\uparrow} \left[\mathcal{H} c_{\pi\uparrow} \right], \eta_{\pi\uparrow} \right\} \tag{2}$$

$$\eta_{\pi\uparrow} = \frac{1}{\hat{\omega} - \mathrm{Tr}_{\pi\uparrow} \left[\mathcal{H} \left(1 - \hat{n}_{\pi\uparrow} \right) \right] \left(1 - \hat{n}_{\pi\uparrow} \right)} \mathrm{Tr}_{\pi\uparrow} \left[c^\dagger_{\pi\uparrow} \mathcal{H} \right] c_{\pi\uparrow} \tag{3}$$

$$= \frac{1}{\hat{\omega} - \left[t\left(\hat{n}_{\pi\downarrow} - \hat{n}_0 \right) + U n_{0\uparrow} n_\downarrow \right] \left(1 - \hat{n}_{\pi\uparrow} \right)} \frac{U}{2} \left[c^\dagger_{0,\downarrow} c_{\pi,\downarrow} + c^\dagger_{\pi,\downarrow} c_{0,\downarrow} \right] c^\dagger_{0\uparrow} c_{\pi\uparrow} \tag{4}$$

$$\eta^\dagger_{\pi\uparrow} = \frac{1}{\hat{\omega} - \mathrm{Tr}_{\pi\uparrow} \left[\mathcal{H} \hat{n}_{\pi\uparrow} \right] \hat{n}_{\pi\uparrow}} \mathrm{Tr}_{\pi\uparrow} \left[c^\dagger_{\pi\uparrow} \mathcal{H} \right] c_{\pi\uparrow} \tag{5}$$

$$= \frac{1}{\hat{\omega} - \left[t\left(1 + \hat{n}_{\pi\downarrow} - \hat{n}_0 \right) + U n_{0\uparrow} n_\downarrow \right] \left(1 - \hat{n}_{\pi\uparrow} \right)} \frac{U}{2} \left[c^\dagger_{0,\downarrow} c_{\pi,\downarrow} + c^\dagger_{\pi,\downarrow} c_{0,\downarrow} \right] c^\dagger_{0\uparrow} c_{\pi\uparrow} \tag{6}$$

$$c^\dagger_{\pi\uparrow} \mathrm{Tr}_{\pi\uparrow} \left[\mathcal{H} c_{\pi\uparrow} \right] = \frac{U}{2} \left[c^\dagger_{0,\downarrow} c_{\pi,\downarrow} + c^\dagger_{\pi,\downarrow} c_{0,\downarrow} \right] c^\dagger_{\pi\uparrow} c_{0\uparrow} \tag{7}$$