Entanglement features in the Kondo Model

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Abstract

abstract..

I. INTRODUCTION

Literature survey...

Motivation for the work In the antiferromagnetic side a Kondo cloud is formed via the entanglement between the impurity spin and conduction electrons. On the otherhand in the ferromagnetic side the impurity spin disentangles from the conduction electrons. In the present work we want to study the interplay between electronic correlation in the Fermi surface neighbourhood and fermion exchange signs in shaping the entanglement between the Kondo cloud and the impurity spin. For the study we will use a unitary renormalization group (URG) theory for the Kondo model. Using a combination of entangled based and Green function based measures we will unravel the quantum correlation of Kondo cloud.

II. THE MODEL

The Kondo model[1, 2] describes the coupling between a magnetic quantum impurity localized in real space with a conduction electron bath,

$$\hat{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}\sigma} + \frac{J}{2} \sum_{\mathbf{k},\mathbf{k}'} \mathbf{S} \cdot c^{\dagger}_{\mathbf{k}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} \ . \tag{1}$$

Here we consider a 2d electronic bath $\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y)$ with the Fermi energy $E_F = \mu$. J is the Kondo scattering coupling between the impurity and the conduction electrons. An important feature of the Kondo coupling is the two different classes of scattering processes, one involving spin flip $c^{\dagger}_{\mathbf{k}\uparrow} c_{\mathbf{k}'\downarrow} + h.c.$ another not involving spin flip. In the antiferromagnetic regime $J > 0$ the spin flip scattering processes generates quantum entanglement between the impurity spin and the Kondo cloud resultingly screening the impurity.

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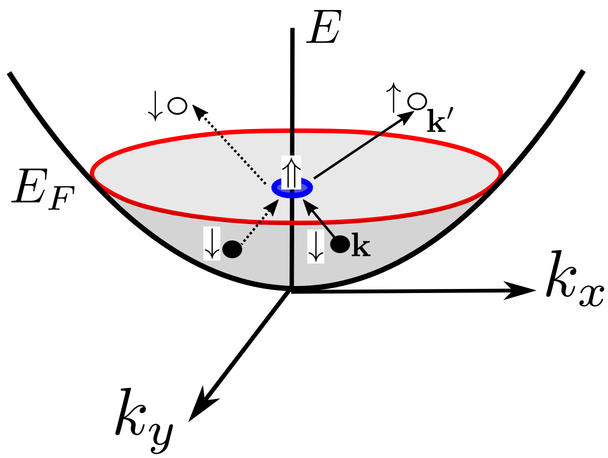

FIG. 1. The Kondo model is composed of a two dimensional conduction electron bath (Fermi liquid) coupled to a magnetic impurity via spin flip(solid)/non spin flip(dashed) scattering processes.

III. URG THEORY FOR THE KONDO MODEL

We construct a unitary RG theory for the Kondo model, electronic states from the bath are stepwise disentangled starting with the highest energy electrons eventually scaling towards the Fermi surface. The electronic states are labelled in terms of the normal distance Λ from the Fermi surface and the orientation unit vectors (Fig.??)$\hat{s}$: $\mathbf{k}_{\Lambda\hat{s}} = \mathbf{k}_F(\hat{s}) + \Lambda\hat{s}$ where $\hat{s} = \frac{\nabla\epsilon_{\mathbf{k}}}{|\nabla\epsilon_{\mathbf{k}}|}|_{\epsilon_{\mathbf{k}}=E_F}$. The states are labelled as $|j,l,\sigma\rangle = |\mathbf{k}_{\Lambda_j\hat{s}},\sigma\rangle, l := (\hat{s}_m,\sigma)$.

The Λ's are arranged as follows: $\Lambda_N > \Lambda_{N-1} > \ldots > 0$, where the electronic states farthest from Fermi surface Λ_N is disentangled first, eventually scaling towards the Fermi surface. This leads to the Hamiltonian flow equation,

$$H_{(j-1)} = U_{(j)}H_{(j)}U^\dagger_{(j)}\ . \tag{2}$$

where the unitary operation $U_{(j)}$ is the unitary map. $U_{(j)}$ disentangles all the electronic states $\mathbf{k}_{\Lambda_j\hat{s}_m},\sigma$ on the isogeometric curve and has the form[3],

$$U_{(j)} = \prod_l U_{j,l}, U_{j,l} = \frac{1}{\sqrt{2}}[1+\eta_{j,l}-\eta^\dagger_{j,l}], \tag{3}$$

where $\eta_{j,l}$ are electron hole transition operators following the algebra,

$$\{\eta_{j,l},\eta^\dagger_{j,l}\} = 1, \left[\eta_{j,l},\eta^\dagger_{j,l}\right] = 1\ . \tag{4}$$

The transition operator in terms of the diagonal H^D and off-diagonal parts of the Hamilto-

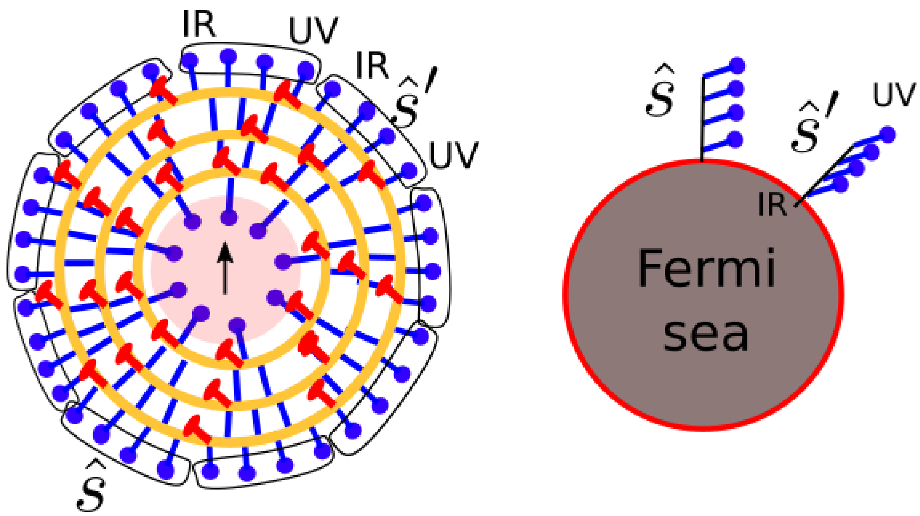

FIG. 2. Figure in the left shows a tensor network representation of the Kondo RG with 8 Fermi surface patches. The 32 blue nodes represents the 8 (for 8 $\hat{s}$) sets of 4(along a given normal $\hat{s}$) electronic states (qubits). arranged from UV to IR (see figure in the right). The yellow circular block represents the unitary gate that disentangles at each RG step 8 electronic qubits at UV. The red nodes represents the disentangled qubits comprising the bulk of the tensor network. At the IR bulk of the tensor network the pink circle represents the Kondo cloud coupling with the spin vector (up arrow).

nian H^X has the form,

$$\eta_{j,l} = Tr_{j,l}(c^\dagger_{j,l}H_{j,l})c_{j,l}\frac{1}{\hat{\omega}_{j,l} - Tr_{j,l}(H^D_{j,l}\hat{n}_{j,l})\hat{n}_{j,l}}. \tag{5}$$

In the numerator the operator $Tr_{j,l}(c^\dagger_{j,l}H_{j,l})c_{j,l} + h.c.$ is composed of all possible scattering vertices that modify the configuration of the electronic state $|j,l\rangle$. The generic forms of $H^D_{j,l}$, $H^X_{j,l}$ are as follows,

$$\begin{aligned} H^D_{j,l} &= \sum_{\Lambda\hat{s},\sigma}\epsilon^{j,l}\hat{n}_{\mathbf{k}_{\Lambda\hat{s}},\sigma} + \sum_\alpha \Gamma^{4,(j,l)}_\alpha \hat{n}_{\mathbf{k}\sigma}\hat{n}_{\mathbf{k}'\sigma'} + \sum_\beta \Gamma^{8,(j,l)}_\beta \hat{n}_{\mathbf{k}\sigma}\hat{n}_{\mathbf{k}'\sigma'}(1-\hat{n}_{\mathbf{k}''\sigma''}) + \ldots \\ H^X_{j,l} &= \sum_\alpha \Gamma^2_\alpha c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}'\sigma'} + \sum_\beta \Gamma^4_\beta c^\dagger_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}'\sigma'}c_{\mathbf{k}'_1\sigma'_1}c_{\mathbf{k}_1\sigma_1} + \sum_\beta \Gamma^6_\beta c^\dagger_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}'\sigma'}c_{\mathbf{k}'_1\sigma'_1}c_{\mathbf{k}_1\sigma_1}\ldots \end{aligned} \tag{6}$$

The operator $\hat{\omega}_{j,l}$ accounts for the quantum fluctuation arising from the non commutivity between different parts of the renormalized Hamiltonian and has the form,[4]

$$\hat{\omega}_{j,l} = H^D_{j,l} + H^X_{j,l} - H^X_{j,l-1}\ . \tag{7}$$

Upon disentangling electronic states $\hat{s},\sigma$ along a isogeometric curve at distance Λ_j the effective Hamiltonians $H_{j,l}$ are generated,

$$H_{j,l} = \prod_{m=1}^{l} U_{j,m} H_{(j)} [\prod_{m=1}^{l} U_{j,l}]^{\dagger} \ . \tag{8}$$

After having disentangled all the electronic states $2n_j$ on the isogeometric curve the effective Hamiltonian $H_{j,2n_j+1} = H_{(j-1)}$ for the next RG step is obtained. Disentangling multiple qubits succesively in a given momentum shell at distance Λ_j from FS leads to renormalized contribution from one and higher particle correlated tangential scattering process. Accounting for the leading tangential scattering processes and other momentum transfer processes along normal $\hat{s}$ the renormalized Hamiltonian has the form,

$$H_{(j-1)} = Tr_{j,(1,\ldots,2n_j)}(H_{(j)}) + \sum_{l=1}^{2n_j} \{c^{\dagger}_{j,l} Tr_{j,l}(H_{(j)} c_{j,l}), \eta_{j,l}\} \tau_{j,l} \ . \tag{9}$$

Here $2n_j$ are the number of electronic states on the curve at distance Λ_j.

IV. RESULTS

The unitary RG process generates the effective Hamiltonian's $\hat{H}_{(j)}(\omega)$'s for various eigen directions $|\Phi(\omega)\rangle$ of the $\hat{\omega}$ operator. Note the associated eigenvalue ω identifies a subspectrum in the interacting many body eigenspace. The form of $\hat{H}_{(j)}(\omega)$ is given by,

$$\hat{H}_{(j)}(\omega) = \sum_{j,l,\sigma} \epsilon_{j,l} \hat{n}_{j,l} + \frac{J^{(j)}(\omega)}{2} \sum_{\substack{j_1,j_2<j,\\ m,m'}} \mathbf{S} \cdot c^{\dagger}_{j_1,\hat{s}_m,\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{j_2,\hat{s}_{m'},\beta} + \sum_{\substack{a=N,\\ m=1}}^{j,n_j} J^{(a)} S^z s^z_{a,\hat{s},m}, \tag{10}$$

where $s^z_{l,\hat{s},m} = \frac{1}{2}(\hat{n}_{l,\hat{s}_m,\uparrow} - \hat{n}_{l,\hat{s}_m,\downarrow})$. The Kondo coupling RG equation for the RG steps(AppendixA) has the form,

$$\frac{\Delta J^{(j)}(\omega)}{\Delta \log \frac{\Lambda j}{\Lambda_0}} = \frac{2n_j (J^{(j)})^2 \left[(\frac{\epsilon_j}{2} - \omega) \right]}{(\frac{\epsilon_j}{2} - \omega)^2 - \frac{\left(J^{(j)}\right)^2}{16}} \tag{11}$$

Note the denominator $\Delta \log \frac{\Lambda j}{\Lambda_0} = 1$ for the RG scale parameterization $\Lambda_j = \Lambda_0 \exp(-j)$. We redefine Kondo coupling as a dimensionless parameter,

$$K^{(j)} = \frac{J^{(j)}}{\frac{\epsilon_j}{2} - \omega} \ , \tag{12}$$

We operate in the regime $\frac{\epsilon_j}{2} > \omega$. In order to obtain a continuum RG equation we approximate the dispersion $\epsilon_j = (2m)^{-1}(\hbar k_F + v_F\Lambda)^2 - \hbar^2 k_F^2 \approx \hbar v_F \Lambda$ for $\Lambda < \Lambda_0$,

$$\frac{dK}{d\log\frac{\Lambda}{\Lambda_0}} = \left(1 + \frac{\omega}{\hbar v_F \Lambda} - \omega\right) K + \frac{n(\Lambda)K^2}{1 - \frac{K^2}{16}} \tag{13}$$

Upon approaching the Fermi surface $\Lambda_j \to 0$ therefore $\left(1 - \frac{\omega}{\omega - \hbar v_F \Lambda}\right) \to 0$ for $\omega \to 0-$ and $n(\Lambda)$ can be replaced by no. of states on the Fermi surface $n(0)$, leading to

$$\frac{dK}{d\log\frac{\Lambda}{\Lambda_0}} = \frac{n(0)K^2}{1 - \frac{K^2}{16}} \tag{14}$$

We observe two important aspects of the RG equation: for $K << 1$ the RG equation

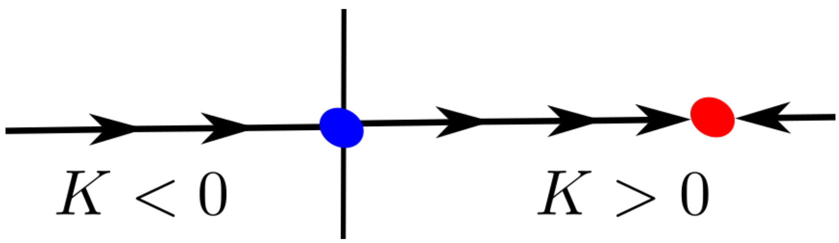

FIG. 3. Schematic RG phase diagram. Red dot represents intermediate coupling fixed point. Blue dot is the critical fixed point.

reduces to the one loop form, $\frac{dK}{d\log\frac{\Lambda}{\Lambda_0}} = K^2$[2]. On the otherhand the nonperturbative form of flow equations shows the presence of intermediate coupling fixed points $K^* = 4$ in the

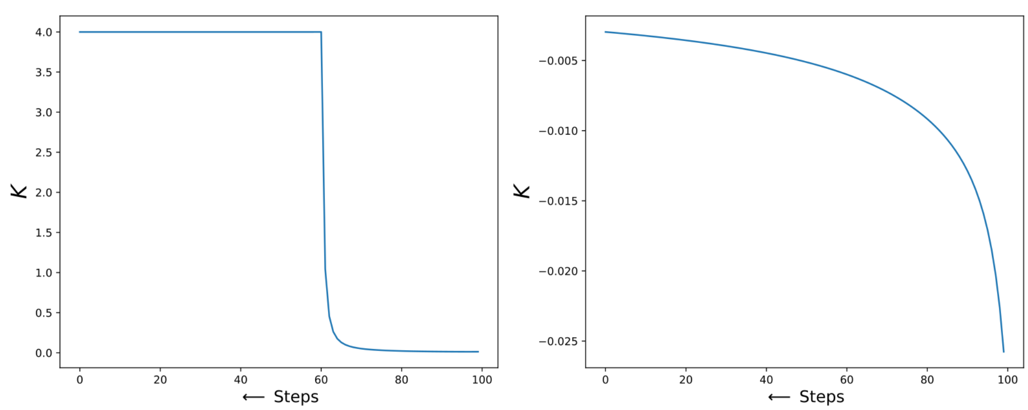

FIG. 4. Renormalized dimensionless Kondo coupling K with RG steps ($\log \Lambda_j/\Lambda_0$), left panel:$K > 0$, right panel: $K < 0$.

antiferromagnetic regime $K > 0$. Upon integrating the RG equation and using the fixed point value $K^* = 4$ we obtain the energy scale,

$$\begin{aligned}
&\frac{1}{K_0} - \frac{1}{2} + \frac{K_0}{16} = -n(0)\log\frac{\Lambda}{\Lambda_0} \\
&\Lambda = \Lambda_0 \exp\left(\frac{1}{2n(0)} - \frac{1}{n(0)K_0} - \frac{K_0}{n(0)16}\right)
\end{aligned} \tag{15}$$

At the IR fixed point in the AF regime the effective Hamiltonian is given by,

$$H^* = \sum_{|\Lambda|<\Lambda^*} \hbar v_F \Lambda \hat{n}_{\Lambda,\hat{s},\sigma} + \frac{J^*}{2} \sum_{\substack{j_1,j_2<j^*,\\ m,m'}} \mathbf{S}\cdot c^\dagger_{j_1,\hat{s}_m,\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{j_2,\hat{s}_{m'},\beta} + \sum_{j'=N,m=1}^{j^*,n_{j'}} J^{j'} S^z s^z_{j',m} \tag{16}$$

We can now extract a zero mode from the above Hamiltonian that captures the low energy theory near the Fermi surface,

$$\begin{aligned}
H^*_K &= \frac{1}{N}\sum_{|\Lambda|<\Lambda^*} \hbar v_F \Lambda \sum_{|\Lambda|<\Lambda^*} \hat{n}_{\Lambda,\hat{s},\sigma} + \frac{J^*}{2} \sum_{\substack{j_1,j_2<j^*,\\ m,m'}} \mathbf{S}\cdot c^\dagger_{j_1,\hat{s}_m,\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{j_2,\hat{s}_{m'},\beta} + \sum_{j'=N,m=1}^{j,n_{j'}} J^{j'} S^z s^z_{j',m} \\
&= \frac{J^*}{2} \sum_{\substack{j_1,j_2<j^*,\\ m,m'}} \mathbf{S}\cdot c^\dagger_{j_1,\hat{s}_m,\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{j_2,\hat{s}_{m'},\beta} + \sum_{j'=N,m=1}^{j^*,n_{j'}} J^{j'} S^z s^z_{j',m} \ .
\end{aligned} \tag{17}$$

Indeed we observe that the zero mode Hamiltonian at the IR fixed point is responsible for the formation of the singlet ground state.

$$|\Psi*\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle \sum_{\Lambda,\hat{s}} |1_{\Lambda,\hat{s},\downarrow}\rangle \otimes_{\Lambda'\neq\Lambda,\hat{s}'\neq\hat{s}} |\Lambda',\hat{s}'\rangle - |\downarrow\rangle \sum_{\Lambda,\hat{s}} |1_{\Lambda,\hat{s},\uparrow}\rangle \otimes_{\Lambda'\neq\Lambda,\hat{s}'\neq\hat{s}} |\Lambda',\hat{s}'\rangle\right] \tag{18}$$

A. Variation of the Kondo cloud size ξ and effective Kondo coupling J^* as function of bare coupling J_0

All plots below are obtained for momentum space grid 100×100 and with RG scale factor ($b = 0.9999$) $\Lambda_j = b\Lambda_{j+1}$. The E_F for the 2d tight binding band $-W/2 < E_k = -2t(\cos k_x + \cos k_y) < W/2$ ($W = 4t$) is chosen at $E_F = -3.8t$.

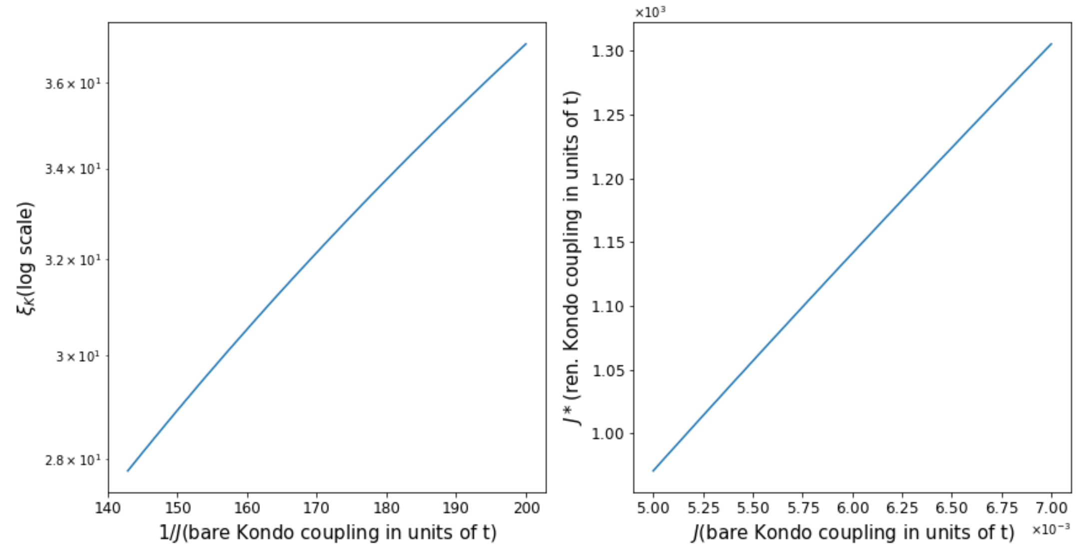

FIG. 5. Left panel: Kondo cloud length ξ in log scale (y-axis) vs. $1/J_0$ (x-axis), right panel: renormalized Kondo coupling J^* vs. J_0, for $5\times 10^{-3}t < J_0 < 7\times 10^{-3}t$.

V. IMPURITY SPIN SUSCEPTIBILITY AND SPECIFIC HEAT CALCULATION FOR THE KONDO HAMILTONIAN

The effective Hamiltonian obtained at the IR fixed point eq.(16) commutes with the z-component of the net spin vector,

$$S^z_{tot} = S^z + \sum_{\substack{\Lambda,\Lambda'<\Lambda^*,\\ m,m'\\ \alpha,\beta}} c^\dagger_{\Lambda,\hat{s}_m,\alpha}\frac{\sigma^z_{\alpha\beta}}{2}c_{\Lambda',\hat{s}_{m'},\beta} + \sum_{\Lambda\geq\Lambda^*,m} S^z s^z_{\Lambda,m}, \quad s^z_{\Lambda,m} = \frac{1}{2}\left(\hat{n}_{\Lambda,\hat{s}_m,\uparrow} - \hat{n}_{\Lambda,\hat{s}_m,\downarrow}\right) . \tag{19}$$

Let us defined the fourier transform of the electronic states for $\Lambda < \Lambda^*$,

$$c^\dagger_{\Lambda,\hat{s}_m,\sigma} = \frac{1}{\sqrt{L^*}}\sum_{\mathbf{r}} e^{i\mathbf{k}_{\Lambda\hat{s}_m}\cdot\mathbf{r}} d^\dagger_{\mathbf{r}\sigma} \tag{20}$$

$$\sum_{\Lambda,\Lambda',m,m'} c^\dagger_{\Lambda,\hat{s}_m,\alpha}\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}c_{\Lambda',\hat{s}_{m'},\beta} = \sum_{\mathbf{r},\mathbf{r}'} e^{i\mathbf{k}_{\Lambda\hat{s}_m}\cdot\mathbf{r}-i\mathbf{k}_{\Lambda'\hat{s}_{m'}}\cdot\mathbf{r}'} d^\dagger_{\mathbf{r}\alpha}\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}\mathbf{d}_{\mathbf{r}'\beta} = d^\dagger_{0,\alpha}\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}d_{0,\beta} \tag{21}$$

$$\left[\sum_{\mathbf{k}\sigma'}\epsilon_{\mathbf{k}}\hat{n}_{\mathbf{k}\sigma'}, \sum_{\mathbf{k}_1,\mathbf{k}_2}\sigma c^\dagger_{\mathbf{k}_1\sigma}c_{\mathbf{k}_2\sigma}\right] = \delta_{\sigma\sigma'}\delta_{\mathbf{k}_1,\mathbf{k}}\sigma\left(\left[\hat{n}_{\mathbf{k}\sigma}, c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}'\sigma}\right] + \left[\hat{n}_{\mathbf{k}\sigma}, c^\dagger_{\mathbf{k}'\sigma}c_{\mathbf{k}\sigma}\right]\right)$$
$$c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}'\sigma}(1-\hat{n}_{\mathbf{k}\sigma}) - \tag{22}$$

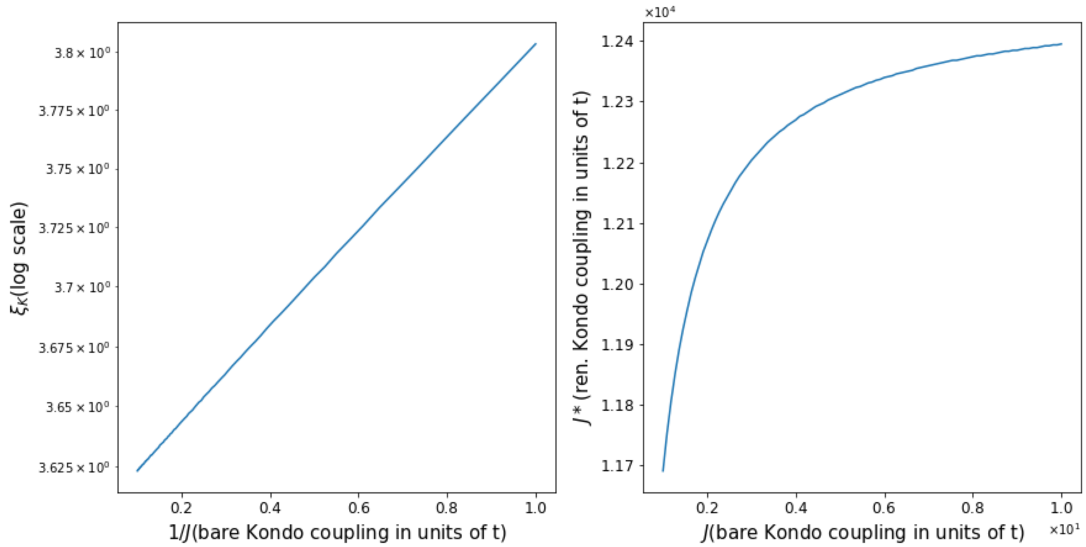

FIG. 6. Left panel: Kondo cloud length ξ in log scale (y-axis) vs. $1/J_0$ (x-axis), right panel: renormalized Kondo coupling J^* vs. J_0, for $t < J_0 < 7\times 10^{-3}t$.

$$[H^*, S^z_{tot}] = 0 \tag{23}$$

The spin susceptibility originating purely from the impurity at the IR fixed originating simply from the impurity can be defined as,

$$\chi(T) = \frac{1}{kT}\left[\frac{Tr((S^z_{tot})^2\exp(-H^*(J^*)/kT)}{Tr(\exp(-H^*(J^*)/kT))} - \left(\frac{Tr((S^z_{tot})^2\exp(-H^*(0)/kT)}{Tr(\exp(-H^*(0)/kT))} - \frac{1}{4}\right)\right] \tag{24}$$

Since the eigenvalues of S^z_{tot} are good quantum nos for the eigenstates of H^* we can use those states to obtain these quantity. Similar thing can be done for the specific heat. The second term in the above expression cancels of the contribution coming to susceptibility from conduction electrons at $J^* = 0$. The final term $1/4$ cancels of the impurity susceptibility from $J^* = 0$ term. This prescription is due to Wilson [5]. The eigenstates and eigenvalues of H^* is given by,

VI. TENSOR NETWORK REPRESENTATION OF THE KONDO URG PROGRAM

In this section we present the tensor network representation of the URG program[4, 6]. The yellow blocks represent the complete U transformation $U_{(j)}$ for a given RG step

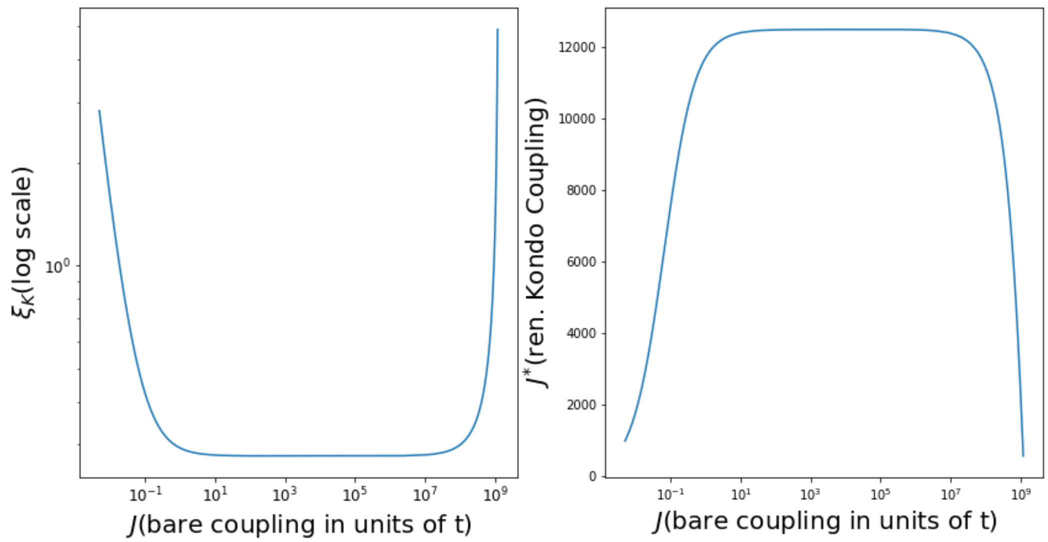

FIG. 7. Left panel: Kondo cloud length ξ vs. J_0 (x-axis in log scale) in , right panel: renormalized Kondo coupling J^* vs. J_0. J_0 is chosen as $4.9 \times 10^{-3} \times (1.02656)^n$ and $1 < n < 1000$.

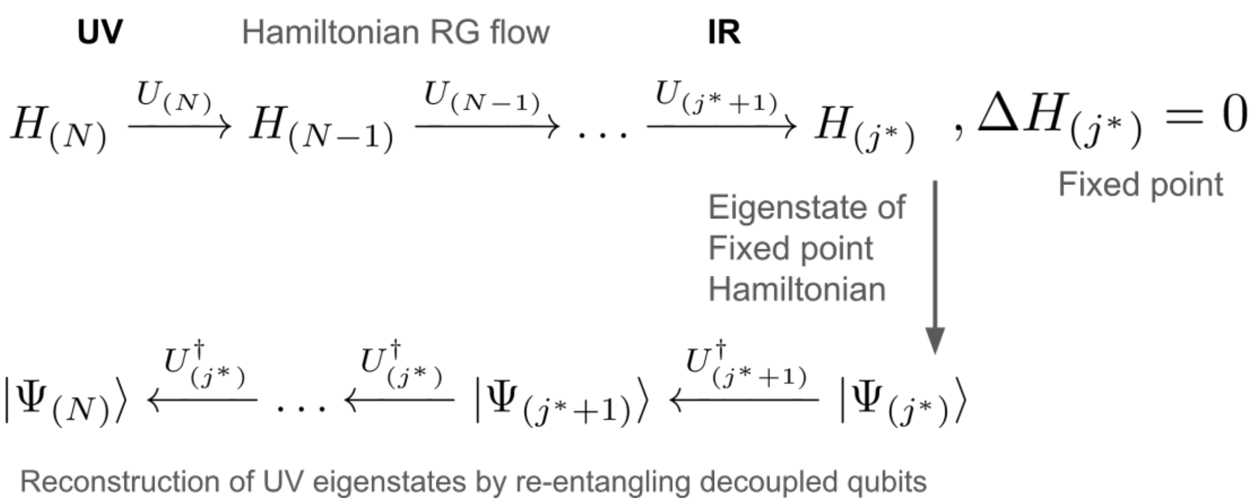

FIG. 8. The first line represent the Hamiltonian RG flow via the unitary maps. Upon reaching the Kondo IR fixed point, reverse RG will re-entangled decoupled electronic states with the Kondo singlet. This will result in generation of the many body eigenstates at UV.

that is composed of $U_{j,l}$ here individual $U_{j,l}$ disentangles one electronic state $\mathbf{k}_{\Lambda\hat{s}}, \sigma$. The wavefunction $|\Psi^*\rangle$ comprises the IR bulk of the tensor network. The yellow layers are arranged along the RG direction. The tensor network Fig.2 is composed of yellow layers that lead to a holographic arrangement of eigenstates from UV to IR. From the singlet state obtained at RG fixed point we can perform the inverse unitary map to regenerate the entanglement with UV degrees of freedom Fig.(8). This enables udsd to obtain the complete form of the two point retarded green function along the RG flow direction (RG time)$\tau_j = 1/v_F\Lambda_j$,

$$G(\mathbf{k}\sigma,\tau;\mathbf{k}'\sigma',\tau') = \Theta(\tau-\tau')\langle\Psi_0|\{c_{\mathbf{k}\sigma}(\tau)c^\dagger_{\mathbf{k}'\sigma'}(\tau')\}|\Psi_0\rangle \tag{25}$$

where $|\Psi_0\rangle = U_N^\dagger\ldots U_{j^*}^\dagger|\Psi_{(j^*)}\rangle$ is the many body state residing at the UV boumdary of the tensor network. The many body rotated $c^\dagger$, c operators are given by,

$$c^\dagger_{\mathbf{k}'\sigma'}(\tau') = [U_j\ldots U_N]c^\dagger_{\mathbf{k}'\sigma'}[U_j\ldots U_N]^\dagger \tag{26}$$

From equal time green function $G(\mathbf{k}\sigma,\tau;\mathbf{k}'\sigma',\tau)$ we can obtain the complete one particle self energy and compute the spectral function, lifetime, quasoparticle residue. On the otherhand this quantity is related to mutual information or entanglement content between pair of electronic states. This quantity will enable a dual probe study of the quantum liquid composing the Kondo cloud.

Appendix A: Calculation for effective Hamiltonian RG

Starting from the Kondo Hamiltonian eq.(1) and using the URG based Hamiltonian RG equation eq.(9) we obtain,

$$\begin{aligned}\Delta\hat{H}_{(j)} = &\sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j}\frac{(J^{(j)})^2\tau_{j,\hat{s}_m,\beta}}{2(2\omega\tau_{j,\hat{s}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{s}_m,\beta}-J^{(j)}S^z s^z_{j,\hat{s}_m})}\\ &\times\Bigg[S^aS^b\sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma}\sum_{\substack{(j_1,j_2<j),\\ n,o}}c^\dagger_{j_1,\hat{s}_n,\alpha}c_{j_2,\hat{s}_o,\gamma}(1-\hat{n}_{j,\hat{s}_m,\beta})+S^bS^a\sigma^b_{\beta\gamma}\sigma^a_{\alpha\beta}\sum_{\substack{(j_1,j_2<j),\\ n,o}}c_{j_2,\hat{s}_o,\gamma}c^\dagger_{j_1,\hat{s}_n,\alpha}\hat{n}_{j,\hat{s}_m,\beta}\Bigg]\\ &+\sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j}\frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{s}_m,\beta}-J^{(j)}S^z s^z_{j,\hat{s}_m})}\Bigg[S^xS^y\sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha}c^\dagger_{j,\hat{s}_m,\alpha}c_{j,\hat{s}_m,\beta}c^\dagger_{j,\hat{s}_m,\beta}c_{j,\hat{s}_m,\alpha}\\ &+S^yS^x\sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha}c^\dagger_{j,\hat{s}_m,\beta}c_{j,\hat{s}_m,\alpha}c^\dagger_{j,\hat{s}_m,\alpha}c_{j,\hat{s}_m,\beta}\Bigg]\end{aligned} \tag{A1}$$

The first term corresponds to the renormalization of the Kondo coupling and describes the sd-exchange interactions for the entangled degrees of freedom,

$$\begin{aligned}
\Delta H^1_{(j)} &= \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{s}_m,\beta}-J^{(j)}S^z s^z_{j,\hat{s}_m})}\mathbf{S}\cdot c^\dagger_{j,\hat{s}_m,\alpha}\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}c_{j,\hat{s}_m,\beta} \\
&= \frac{1}{2}\sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2\left[(\frac{\omega}{2}-\frac{\epsilon_{j,l}}{4})+\frac{J^{(j)}}{2}S^z\tau_{j,\hat{s}_m,\beta}(\tau_{j,\hat{s}_m,\uparrow}-\tau_{j,\hat{s}_m,\downarrow}))\right]}{(\omega-\frac{\epsilon_{j,l}}{2})^2-\frac{\left(J^{(j)}\right)^2}{16}} \\
&= \frac{n_j(J^{(j)})^2\left[(\omega-\frac{\epsilon_j}{2})\right]}{(\omega-\frac{\epsilon_{j,l}}{2})^2-\frac{\left(J^{(j)}\right)^2}{16}}\mathbf{S}\cdot\sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha}\frac{\boldsymbol{\sigma}_{\alpha\gamma}}{2}c_{j_2,\hat{s}_o,\gamma}
\end{aligned} \tag{A2}$$

In obtaining the last step of the calculation we have assumed $\epsilon_{j,l}=\epsilon_j$ for a circular Fermi surface geometry. The renormalization of the number diagonal Hamiltonian for the immediately disentangled electronic states $|j,\hat{s}_m,\sigma\rangle$ has the form,

$$\begin{aligned}
\Delta H^2_{(j)} &= \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{s}_m,\beta}-J^{(j)}S^z s^z_{j,\hat{s}_m})}\Big[S^xS^y\sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha}c^\dagger_{j,\hat{s}_m,\alpha}c_{j,\hat{s}_m,\beta}c^\dagger_{j,\hat{s}_m,\beta}c_{j,\hat{s}_m,\alpha} \\
&\quad + S^yS^x\sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha}c^\dagger_{j,\hat{s}_m,\beta}c_{j,\hat{s}_m,\alpha}c^\dagger_{j,\hat{s}_m,\alpha}c_{j,\hat{s}_m,\beta}\Big] \\
&= \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{s}_m,\beta}-J^{(j)}S^z s^z_{j,\hat{s}_m})}S^z\frac{\sigma^z_{\alpha\alpha}}{2}\Big[\hat{n}_{j,\hat{s}_m,\alpha}(1-\hat{n}_{j,\hat{s}_m,\beta})-\hat{n}_{j,\hat{s}_m,\beta}(1-\hat{n}_{j,\hat{s}_m,\alpha})\Big] \\
&= \sum_{m=1}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta}-\epsilon_{j,l}\tau_{j,\hat{s}_m,\beta}-J^{(j)}S^z s^z_{j,\hat{s}_m})}S^z s^z_{j,\hat{s}_m}
\end{aligned} \tag{A3}$$

in the last step we have used $\hat{n}_{j,\hat{s}_m,\alpha}(1-\hat{n}_{j,\hat{s}_m,\beta})-\hat{n}_{j,\hat{s}_m,\beta}(1-\hat{n}_{j,\hat{s}_m,\alpha})=\hat{n}_{j,\hat{s}_m,\alpha}-\hat{n}_{j,\hat{s}_m,\beta}$and the spin density for the state $|j,\hat{s}_m\rangle$ is given by $s^z_{j,\hat{s}_m}=\frac{1}{2}(\hat{n}_{j,\hat{s}_m,\uparrow}-\hat{n}_{j,\hat{s}_m,\downarrow})$.

In obtaining the above RG equation we have replaced $\hat{\omega}_{(j)}=2\omega\tau_{j,\hat{s}_m,\beta}$. We set the electronic configuration $\tau_{j,\hat{s}_m,\uparrow}=+\tau_{j,\hat{s}_m,\downarrow}=-\frac{1}{2}$ to account for the spin scattering between the Kondo impurity and the fermionic bath. The operator $\hat{\omega}_{(j)}$ (eq.(7)) for RG step j is determined by the occupation number diagonal piece of the Hamiltonian $H^D_{(j-1)}$ attained at the next RG step $j-1$, this demands a self consistents treatment of the RG equation to determine the ω. In this fashion two particle and higher order quantum fluctuations autamatically get encoded into the RG dynamics of $\hat{\omega}$. In the present work we restrict our study by ignoring the RG contribution in ω. The electron/hole configuration $(|1_{j,\hat{s}_m,\beta}\rangle/|0_{j,\hat{s}_m,\beta}\rangle)$ of the disentangled electronic state and associated with $\pm\epsilon_{j,l}$ energy is accounted by $\pm\omega$ fluctuation energy scales. From the above Hamiltonian RG equations eq.(A2) we can obtain the form of the Kondo coupling RG equations,

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2 \left[(\frac{\epsilon_{j,l}}{2} - \omega) \right]}{(\frac{\epsilon_{j,l}}{2} - \omega)^2 - \frac{\left(J^{(j)}\right)^2}{16}} . \tag{A4}$$

Appendix B: Computing spin susceptibility

$$H_K^* = J^* \mathbf{S} \cdot \mathbf{s} + \sum_{m=1}^{n_l} J^* S^z s^z_{*,m} + \sum_{l=N,m=1,\alpha}^{j^*,n_l} \epsilon_{l,m} \hat{n}_{l,m,\alpha} + BS^z, J^{j^*} = J^* \tag{B1}$$

$$s^z_{l,m} = \frac{1}{2}(\hat{n}_{l,m,\uparrow} - \hat{n}_{l,m,\downarrow}) \tag{B2}$$

$$S^z_{tot} = S^z + s^z + \sum_{l=N,m=1}^{j^*,n_l} s^z_{l,m} \tag{B3}$$

$$[H_K^*, S^z + s^z] = 0, \left[H_K^*, s^z_{l,m}\right] = 0 \tag{B4}$$

$$\begin{aligned}
&\exp(-\beta H) \\
&= \prod_{\substack{l=j^*+1, \\ m=1, \\ \alpha=\uparrow/\downarrow}}^{N,n_l} \exp(-\beta \epsilon_l \hat{n}_{l,m,\alpha}) \times \\
&\Bigg(\sum_{\substack{n=0,1, \\ m=1}}^{n_l} \exp(-2n\beta \epsilon_{j^*,m}) \Big[|\uparrow,\uparrow,n,n\rangle \exp(-\beta \frac{J^*}{4} - \beta \frac{B}{2}) \langle \uparrow,\uparrow,n,n| + |\downarrow,\downarrow,n,n\rangle \exp(-\beta \frac{J^*}{4} + \beta \frac{B}{2}) \langle \downarrow,\downarrow,n,n \\
&+ |s,n,n\rangle \exp(3\beta \frac{J^*}{4}) \cosh(\beta \frac{B}{2}) \langle s,n,n| + |t,n,n\rangle \exp(-\beta \frac{J^*}{4}) \cosh(\beta \frac{B}{2}) \langle t,n,n| \Big] \\
&+ \sum_{\substack{n=0,1, \\ m=1}}^{n_l} \exp(-\beta \epsilon_{j^*,m}) \Big[|\uparrow,\uparrow,n,1-n\rangle \exp(-\beta \frac{J^*}{4} - \beta \frac{B}{2}) \langle \uparrow,\uparrow,n,1-n| + |\downarrow,\downarrow,n,1-n\rangle \exp(-\beta \frac{J^*}{4} + \beta \frac{B}{2}) \langle \\
&+ |\psi_1,n,1-n\rangle \exp(-\beta \lambda_+) \langle \psi_1,n,1-n| + |\psi_2,n,1-n\rangle \exp(-\beta \lambda_-) \langle \psi_2,n,1-n| \Big]
\end{aligned}$$

$$
\begin{aligned}
Z &= Tr(\exp(-\beta H)) = z_1 z_2, \\
z_1 &= \sum_{l=j^*+1}^{N} n(l)(1+\exp(-\beta\epsilon_l))^2 \\
z_2 &= (1+\exp(-2\beta\epsilon_{j^*}))\left[3\exp(-\beta\frac{J^*}{4})\cosh(\beta\frac{B}{2}) + \exp(3\beta\frac{J^*}{4})\cosh(\beta\frac{B}{2})\right] \\
&\quad + \exp(-\beta\epsilon_{j^*})(\exp(-\beta\lambda_+) + \exp(-\beta\lambda_-) + 2\exp(-\beta\frac{J^*}{4})\cosh(\beta\frac{B}{2}))
\end{aligned} \tag{B6}
$$

$$
\begin{pmatrix} \frac{B}{2} + \frac{J_{j^*-1}}{4} - \frac{J^*}{4} & \frac{J^*}{2} \\ \frac{J^*}{2} & -\frac{B}{2} - \frac{J_{j^*-1}}{4} - \frac{J^*}{4} \end{pmatrix} \tag{B7}
$$

$$
\begin{aligned}
|\psi_1\rangle &= \cos\frac{\theta}{2}|\uparrow\downarrow\rangle + \sin\frac{\theta}{2}|\downarrow\uparrow\rangle, \cos\theta = \frac{\frac{J^*}{2}}{\sqrt{\frac{(J^*)^2}{4}+\frac{(J^*)^2}{4}}} = \frac{1}{\sqrt{2}}, \\
|\psi_2\rangle &= -\sin\frac{\theta}{2}|\uparrow\downarrow\rangle + \cos\frac{\theta}{2}|\downarrow\uparrow\rangle
\end{aligned} \tag{B8}
$$

$$
(\frac{B}{2}-\lambda)(-\frac{B}{2}-\frac{J^*}{2}-\lambda) = \frac{(J^*)^2}{4} ab - (a+b)\lambda + \lambda^2 = c^2, \lambda^2 - (a+b)\lambda + ab - c^2 = 0 \tag{B9}
$$

$a = \frac{B}{2}$, $b = -\frac{B}{2} - \frac{J^*}{2}$, $c^2 = \frac{(J^*)^2}{4}$

$$
\lambda_\pm = \frac{(a+b) \pm \sqrt{(a+b)^2 - 4(ab-c^2)}}{2} \tag{B10}
$$

Appendix C: Construction of the local Fermi liquid formed around the Kondo cloud

$$
H_{LFL} = \sum_{l,m} \epsilon_l(\hat{n}_{l,m,\uparrow} + \hat{n}_{l,m,\downarrow}) - \sum_{l,l',m,m'} f_{ll'} s^z_{l,m} s^z_{l,m'}, f_{ll'} = \frac{J_l J_{l'}}{J^*}, s^z_{l,m} = \frac{1}{2}(\hat{n}_{l,m,\uparrow} - \hat{n}_{l,m,\downarrow}) \tag{C1}
$$

$$
\mathcal{E} = \mathcal{E}_0 + \sum_{l,m,\sigma} \epsilon_l \delta n_{l,m,\sigma} - \sum_{\substack{l,l',m,m',\\ \sigma\sigma'}} \frac{J_l J_{l'}}{J^*} \sigma\sigma' \delta n_{l,m,\sigma} \delta n_{l',m',\sigma'} \tag{C2}
$$

$$
\begin{aligned}
\frac{\delta\mathcal{E}}{\delta n_{l,m,\sigma}} &= \epsilon_l + \sum_{l',m',\sigma'} \sigma\sigma' \frac{J_l J_{l'}}{J^*} \delta n_{l',m',\sigma'} \\
&= \epsilon_l + \frac{2t\pi(k_{Fx}+k_{Fy})}{L} \sum_{l',m',\sigma'} \sigma\sigma' \frac{L J_l J_{l'}}{J^* 2t\pi(k_{Fx}+k_{Fy})} \delta n_{l',m',\sigma'}
\end{aligned} \tag{C3}
$$

$E_k - E_F = -2t\cos(k_{Fx}+\Lambda) - 2t\cos(k_{Fy}+\Lambda) + 2t\cos k_{Fx} + 2t\cos k_{Fy} = 2t(\sin k_{Fx} + \sin k_{Fy})\Lambda \approx 2t(k_{Fx}+k_{Fy})\Lambda$, $\Delta E_k = 2t(k_F + k_{Fy})\Delta\Lambda$, $\Delta\Lambda = \frac{2\pi}{L}$

$$\delta_\sigma(n_{l',m',-\sigma},\epsilon_l) = \sum_{l',m',\sigma'} \frac{LJ_lJ_{l'}}{J^* 2t\pi(k_{Fx}+k_{Fy})}\delta n_{l',m',-\sigma} \tag{C4}$$

$$\frac{\delta_\sigma(n_{l',m',\sigma'},\epsilon_l+\Delta\mu) - \delta_\sigma(n_{l',m',\sigma'},\epsilon_l)}{\Delta\mu} = \frac{\alpha(\epsilon_l)}{\pi} \tag{C5}$$

Here α is the quasiparticle density of states. In terms of this the low temperature specific heat is given by,

$$\gamma = 2\frac{\pi^2 k_B^2}{3}\frac{\alpha(E_F)}{\pi} \tag{C6}$$

We can also obtain the impurity magnetization by inserting a small B magnetic field and then taking $B \to 0$ limit,

$$M = \frac{\delta_\uparrow(E_F) - \delta_\downarrow(E_F)}{\pi} \tag{C7}$$

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