- In the Kondo URG, are you considering two electrons on the shell $\Lambda_N$, one that we are decoupling ($q\beta$) and another with the same momentum but opposite spin ($q\overline{\beta}$)?

- If so, why does that kinetic energy piece ($\epsilon_q \tau_{q\overline{\beta}}$) not come down in the denominator?

- Is that what gives rise to the second RG equation and hence the $S^z s^z$ term in the effective Hamiltonian?

$$\Delta H_{(j)}^2 = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \Big[ S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha}$$

$$+ S^y S^x \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} \Big]$$

$$= \sum_{\substack{m=1,\\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} S^z \frac{\sigma^z_{\alpha\alpha}}{2} \Big[ \hat{n}_{j,\hat{s}_m,\alpha}(1 - \hat{n}_{j,\hat{s}_m,\beta}) - \ldots$$

## **What you used:**

$$S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} = S^z \frac{\sigma^z_{\alpha\alpha}}{2} \hat{n}_{j,\hat{s}_m,\alpha}\left(1 - \hat{n}_{j,\hat{s}_m,\beta}\right)$$

## **What I got:**

$$S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} = i^2 S^z \frac{\sigma^z_{\alpha\alpha}}{2} \hat{n}_{j,\hat{s}_m,\alpha}\left(1 - \hat{n}_{j,\hat{s}_m,\beta}\right)$$

In the Kondo URG, you simplify the $\hat{\omega}$ as

$$\hat{\omega} = \omega\tau$$

**What is the formal way of doing this?** Shouldn't it be

$$\hat{\omega} = \omega_1\hat{n} + \omega_1(1 - \hat{n})$$

Is this just an assumption?

In the RG equation for BCS instability (eq. 8.130 of thesis), you use

$$G^{-1} = \omega - \epsilon_1\tau_1 - \epsilon_2\tau_2$$

**How is this choice of $\hat{\omega}$ consistent with what was done in Kondo URG?**

While calculating the impurity susceptibility of the Kondo model, you took the following Hamiltonian and definition of susceptibility:

$$H = J\vec{S}_d \cdot \vec{s}$$

$$H(B, J) = J\vec{S}_d \cdot \vec{s} + BS_d^z,$$

$$\chi_{\text{imp}} = \lim_{B \to 0} \frac{\partial \ln Z(B)}{\partial B}$$

Wilson's definition was

$$\chi_{\text{imp}} = \chi(J) - \chi(J = 0) + \frac{1}{4} = \lim_{B \to 0} \frac{\partial}{\partial B} \ln \frac{Z(B, J)}{Z(B, J = 0)} + \frac{1}{4}$$

which would require

$$H(B) = \sum_k \epsilon_k \hat{n}_{k\sigma} + J\vec{S}_d \cdot \vec{s} + \tilde{J} S_d^z s^z + BS_d^z + B\mu_B \left(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow}\right)$$

Section 2.2, Equation 2.18 of thesis

$$\frac{1}{H' - H_e \hat{n}_N} c_N^\dagger T = c_N^\dagger T \frac{1}{H' - H_h(1 - \hat{n}_N)}$$
$$\implies H_e \hat{n}_N c_N^\dagger T = c_N^\dagger T H_h(1 - \hat{n}_N)$$

This seems to **require** $H'$ **commuting with** $T$, because

$$c_N^\dagger T H' - c_N^\dagger T H_h(1 - \hat{n}_N) = H' c_N^\dagger T - H_e \hat{n}_N c_N^\dagger T$$

**Why should $H'$ commute with $T$?**

(where $H_e = Tr\left(H\hat{n}_N\right)$, $H_h = Tr\left[H\left(1 - \hat{n}_N\right)\right]$ and $T = Tr\left(Hc_N\right)$)

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^\dagger = H_h (1 - n_N)$$

If I try to derive this using the result on the previous slide:

$$\begin{aligned}\eta H \eta^\dagger = \eta H_e \eta^\dagger = \eta H_e c^\dagger T G = \eta c^\dagger T H_h G \\ = \eta c^\dagger T G H_h = \eta \eta^\dagger H_h = H_h (1 - \hat{n})\end{aligned}$$

**That required $[G, H_h] = 0$. How does that work out?**

(where $H_e = Tr\left(H \hat{n}_N\right)$, $H_h = Tr\left[H\left(1 - \hat{n}_N\right)\right]$ and $T = Tr\left(H c_N\right)$)

In eq. 2.21 of thesis,

$$UHU^\dagger = \frac{1}{2}Tr(H) + \tau Tr(H\tau) + \tau\{c^\dagger T, \eta\}$$

so the renormalization is

$$\tau\{c^\dagger T, \eta\} = \frac{1}{2}\left[\overbrace{c^\dagger T\eta}^{\text{particle sector}} - \underbrace{\eta c^\dagger T}_{\text{hole sector}}\right] = \text{difference of the 2 sectors}$$

**Yet in most RG equations ($\Delta H_F$ of 2d Hubbard, $\Delta H_j$ of Kondo), you have *added* the two sectors. How/Why?**

Say we have a Hamiltonian (with circular isoenergetic shells)

$$H = \sum_k \epsilon_k S^z_k - V \sum_{k \neq k'} \left[ S^+_k S^-_{k'} + \text{h.c.} \right] + U \sum_{k \neq k'} S^z_k S^z_{k'}$$

*This is a simplified form of reduced BCS (courtesy of Siddhartha da).* We are decoupling $q \in \Lambda_N$. To include $U$ in the denominator, we also consider another spin $q' \in \Lambda_N$. Then,

$$\Delta H \sim \sum_{kk'} S^+_k S^-_q \frac{V^2}{\omega - \epsilon_q S^z_q - U S^z_q S^z_{q'}} S^+_q S^-_{k'}$$

- Should we or should we not include $\epsilon_{q'}$ in the denominator?
- Suppose we take the initial configuration $S^z_q = S^z_{q'} = -\frac{1}{2}$. Should the intermediate configuration be $S^z_{q'} = -\frac{1}{2}$ or $S^z_{q'} = +\frac{1}{2}$?

## Kondo Model appendix, Equation 9.61 of thesis

$$\Delta\hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}$$

$$\times \Bigg[ S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1-\hat{n}_{j,\hat{s}_m,\beta}) + \dots$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})} \Big[ S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \dots$$

- **The $\tau$ should not be there in numerator i presume?**

Kondo Model appendix, Equation 9.61 of thesis

$$\Delta\hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[ S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1-\hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right.$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})} \left[ S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots \right.$$

- **Since coupling is $\frac{J}{2}$, shouldn't the thing be $\frac{J^2}{4}$ instead of $\frac{J^2}{2}$?**

Kondo Model appendix, Equation 9.61 of thesis

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[ S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j), \\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right.$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \left[ S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots \right.$$

- **You mentioned the following in the google document- "*interchange sigma_a and sigma_b (you get -1 sign)*". But these are matrix elements (numbers). So why the minus sign?**

Kondo Model appendix, Equation 9.61 of thesis

$$\Delta\hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \Big[ S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1-\hat{n}_{j,\hat{s}_m,\beta}) + \ldots$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \Big[ S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots$$

- **How do you combine the product of two sigmas ( $\sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma}$ ) into a single $\sigma^c_{\alpha\gamma}$?**

Kondo URG coupling equation for $J$ (equation 9.65):

$$\Delta J^{(j)} = n_j (J^{(j)})^2 \left[\omega - \frac{\epsilon_{j,l}}{2}\right] \left[ (\frac{\epsilon_{j,l}}{2} - \omega)^2 - \frac{\left(J^{(j)}\right)^2}{16} \right]^{-1}$$

One-loop form (after setting $\omega = \epsilon_{j,l}$):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2 \frac{n_j (J^{(j)})^2}{\epsilon_{j,l}} \rightarrow \frac{2\rho |\Delta D| J^2}{D} \quad [n_j, \rho \rightarrow \text{DOS per spin}]$$

One-loop form in Coleman (Introduction to Many-Body Physics) ($\tilde{J} = J/2$):

$$\Delta \tilde{J} = \frac{2\rho |\Delta D| \tilde{J}^2}{D} \implies \Delta J = \frac{\rho |\Delta D| J^2}{D}$$

**Is there any reason for this difference?**