Define $x \equiv \exp\left(\frac{T^*}{T}\right) = \exp\left(\frac{\beta}{\beta^*}\right)$. The given expression for susceptibility in Haldane's paper is

$$\chi = \frac{\beta}{2(x+2)} + \chi_0 \tag{1}$$

This is a partial attempt to derive this expression. The impurity level is fixed at $k_B T^*$. So, under a magnetic field,

$$\begin{aligned} \epsilon_\uparrow &= k_B T^* - h \\ \epsilon_\downarrow &= k_B T^* + h \end{aligned} \tag{2}$$

The magnetisation is

$$\begin{aligned} m = n_\uparrow - n_\downarrow &= n(\epsilon_\uparrow) - n(\epsilon_\downarrow) \\ &= \frac{1}{\exp\left(\beta\epsilon_\uparrow\right)+1} - \frac{1}{\exp\left(\beta\epsilon_\downarrow\right)+1} \\ &= \frac{1}{\exp\left(\frac{\beta}{\beta^*} - \beta h\right)+1} - \frac{1}{\exp\left(\frac{\beta}{\beta^*} + \beta h\right)+1} \end{aligned} \tag{3}$$

Then,

$$\frac{\partial m}{\partial h} = -\frac{\exp\left(\frac{\beta}{\beta^*} - \beta h\right)(-\beta)}{\left[\exp\left(\frac{\beta}{\beta^*} - \beta h\right)+1\right]^2} + \frac{\exp\left(\frac{\beta}{\beta^*} + \beta h\right)\beta}{\left[\exp\left(\frac{\beta}{\beta^*} + \beta h\right)+1\right]^2} \tag{4}$$

Susceptibility is

$$\begin{aligned} \chi = \lim_{h\to 0}\frac{\partial m}{\partial h} &= -\frac{\exp\left(\frac{\beta}{\beta^*}\right)(-\beta)}{\left[\exp\left(\frac{\beta}{\beta^*}\right)+1\right]^2} + \frac{\exp\left(\frac{\beta}{\beta^*}\right)\beta}{\left[\exp\left(\frac{\beta}{\beta^*}\right)+1\right]^2} \\ &= \beta\frac{2x}{(1+x)^2} \\ &= \frac{2\beta}{x+2+x^{-1}} \end{aligned} \tag{5}$$

Its quite close. Any ideas on improving this? This is a test line.