URG on Kondo Model

April 4, 2021

Contents

1 Questions

- What is the motivation behind the choice of the initial condition $\tau_{q\uparrow} = -\tau_{q\downarrow} = \frac{1}{2}$? Does that choice not violate the SU(2) symmetry of the model? Why not take a more symmetric choice like $\tau_{q\beta} = \tau_{q\bar{\beta}} = \frac{1}{2}$ for $q\beta$ below fermi level and $-1/2$ for above it?

- If we follow your notes and try to derive the equations with just the initial configuration $\tau_{q\uparrow} = -\tau_{q\downarrow} = \frac{1}{2}$, we end up with a field-like term αS_d^z in ΔH, which violates SU(2). However, if we also add the ΔH from the initial state with $\uparrow$ and $\downarrow$ flipped, then we lose the field term.

- On setting $J_z = J_t$, we do not get $\Delta J_z = \Delta J_t$.

2 Formulation

$$
\begin{aligned}
H &= \sum_{k\alpha} \epsilon_k \tau_{k\alpha} + J_z S_d^z \sum_{kk'} \left(c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k\downarrow} c_{k'\downarrow} \right) \\
&\quad + J_t \sum_{kk'} \left(S_d^+ c^\dagger_{k\downarrow} c_{k'\uparrow} + S_d^- c^\dagger_{k\uparrow} c_{k'\downarrow} \right) \\
&= H^D + H^i + H^I
\end{aligned} \tag{2.1}
$$

$$
H^D = \sum_{k\alpha} \epsilon_k \tau_{k\alpha} + J_z S_d^z \sum_{k\beta} \beta \tau_{k\beta} \tag{2.2}
$$

$$
\begin{aligned}
H^i &= J_z S_d^z \sum_{kk' \neq q} \beta \left(c^\dagger_{k\beta} c_{k'\beta} - c^\dagger_{k\bar{\beta}} c_{k'\bar{\beta}} \right) (1 - \delta_{kk'}) \\
&\quad + J_t \sum_{k' \neq q, k} \left(c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k\bar{\beta}} c_{k'\beta} + c^\dagger_{d\bar{\beta}} c_{d\beta} c^\dagger_{k'\beta} c_{k\bar{\beta}} \right)
\end{aligned} \tag{2.3}
$$

$$
\begin{aligned}
H^I &= J_t \sum_{k \neq q} \left(c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k\bar{\beta}} c_{q\beta} + c^\dagger_{d\bar{\beta}} c_{d\beta} c^\dagger_{q\beta} c_{k\bar{\beta}} \right) \\
&\quad + J_z S_d^z \beta \sum_{k \neq q} \left(c^\dagger_{k\beta} c_{q\beta} + c^\dagger_{q\beta} c_{k\beta} \right) \\
&= c^\dagger_{q\beta} T_{q\beta} + T^\dagger_{q\beta} c_{q\beta}
\end{aligned} \tag{2.4}
$$

where

$$
T_{q\beta} = J_z S_d^z \beta \sum_{k \neq q} c_{k\beta} + J_t c^\dagger_{d\bar{\beta}} c_{d\beta} \sum_{k \neq q} c_{k\bar{\beta}} \tag{2.5}
$$

The transformed hamiltonian is

$$
U H U^\dagger = H^D + H^i + \underbrace{c^\dagger_{q\beta} T_{q\beta} \eta}_{\text{Particle}} + \underbrace{\eta_0 c^\dagger_{q\beta} T_{q\beta}}_{\text{Hole}} \tag{2.6}
$$

where $\eta_0 = -\eta$

3 Particle, hole sectors (Left GFs)

For simpler calculations, take H^D in the green's functions of η, η_0 as

$$
H^D = \epsilon_q \tau_{q\beta} + \beta J_z S_d^z \left(\tau_{q\beta} - \tau_{q\bar{\beta}} \right) \tag{3.1}
$$

3.1 Particle Sector

$$
\begin{aligned}
c^\dagger_{q\beta} T_{q\beta}\eta &= \frac{1}{\omega - H^D} c^\dagger_{q\beta} T_{q\beta} T^\dagger_{q\beta} c_{q\beta} \\
&= \frac{1}{\omega - H^D} \sum_{kk'\neq q} c^\dagger_{q\beta} \left(J_z S^z_d \beta c_{k\beta} + J_t c^\dagger_{d\bar{\beta}} c_{d\beta} c_{k\bar{\beta}} \right) \left(J_z S^z_d \beta c^\dagger_{k'\beta} + J_t c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k'\bar{\beta}} \right) c_{q\beta}
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
\left(c^\dagger_{q\beta} T_{q\beta} \eta \right)_1 &= J_z^2 \frac{1}{\omega - H^D} \sum_{kk'\neq q} \beta S^z_d c^\dagger_{q\beta} c_{k\beta} \beta S^z_d c^\dagger_{k'\beta} c_{q\beta} \\
&= \frac{1}{4} J_z^2 \frac{1}{\omega - \frac{1}{2}\epsilon_q - \beta J_z S^z_d \left(\tau_{q\beta} - \tau_{q\bar{\beta}} \right)} \sum_{kk'\neq q} c_{k\beta} c^\dagger_{k'\beta} n_{q\beta}
\end{aligned}
\tag{3.3a}
$$

$$
\begin{aligned}
\left(c^\dagger_{q\beta} T_{q\beta} \eta \right)_2 &= J_z J_t \frac{1}{\omega - H^D} \sum_{kk'\neq q} \beta S^z_d c^\dagger_{q\beta} c_{k\beta} c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k'\bar{\beta}} c_{q\beta} \\
&= \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2}\epsilon_q - \frac{1}{2} J_z \left(\tau_{q\beta} - \tau_{q\bar{\beta}} \right)} c^\dagger_{d\beta} c_{d\bar{\beta}} \sum_{kk'\neq q} c_{k\beta} c^\dagger_{k'\bar{\beta}} n_{q\beta}
\end{aligned}
\tag{3.3b}
$$

$$
\begin{aligned}
\left(c^\dagger_{q\beta} T_{q\beta} \eta \right)_3 &= J_z J_t \frac{1}{\omega - H^D} \sum_{kk'\neq q} c^\dagger_{d\bar{\beta}} c_{d\beta} c^\dagger_{q\beta} c_{k\bar{\beta}} \beta S^z_d c^\dagger_{k'\beta} c_{q\beta} \\
&= \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2}\epsilon_q + \frac{1}{2} J_z \left(\tau_{q\beta} - \tau_{q\bar{\beta}} \right)} c^\dagger_{d\bar{\beta}} c_{d\beta} \sum_{kk'\neq q} c_{k\bar{\beta}} c^\dagger_{k'\beta} n_{q\beta}
\end{aligned}
\tag{3.3c}
$$

$$
\begin{aligned}
\left(c^\dagger_{q\beta} T_{q\beta} \eta \right)_4 &= J_t^2 \frac{1}{\omega - H^D} \sum_{kk'\neq q} c^\dagger_{d\bar{\beta}} c_{d\beta} c^\dagger_{q\beta} c_{k\bar{\beta}} c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k'\bar{\beta}} c_{q\beta} \\
&= J_t^2 \frac{1}{\omega - \frac{1}{2}\epsilon_q + \frac{1}{2} J_z \left(\tau_{q\beta} - \tau_{q\bar{\beta}} \right)} \left(\frac{1}{2} + \bar{\beta} S^z_d \right) \sum_{kk'\neq q} c_{k\bar{\beta}} c^\dagger_{k'\bar{\beta}} n_{q\beta}
\end{aligned}
\tag{3.3d}
$$

3.2 Hole Sector

$$
\begin{aligned}
\eta_0 c^\dagger_{q\beta} T_{q\beta} &= \frac{1}{\omega' - H^D} T^\dagger_{q\beta} c_{q\beta} c^\dagger_{q\beta} T_{q\beta} \\
&= \frac{1}{\omega' - H^D} \sum_{kk'\neq q} \left(J_z S^z_d \beta c^\dagger_{k'\beta} + J_t c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k'\bar{\beta}} \right) c_{q\beta} c^\dagger_{q\beta} \left(J_z S^z_d \beta c_{k\beta} + J_t c^\dagger_{d\bar{\beta}} c_{d\beta} c_{k\bar{\beta}} \right)
\end{aligned}
\tag{3.4}
$$

$$\left(\eta_0 c^\dagger_{q\beta} T_{q\beta}\right)_1 = J_z^2 \frac{1}{\omega' - H^D} \sum_{kk' \neq q} \beta S_d^z c^\dagger_{k'\beta} c_{q\beta} \beta S_d^z c^\dagger_{q\beta} c_{k\beta}$$
$$= \frac{1}{4} J_z^2 \frac{1}{\omega' + \frac{1}{2}\epsilon_q - \beta J_z S_d^z \left(\tau_{q\beta} - \tau_{q\bar{\beta}}\right)} \sum_{kk' \neq q} c^\dagger_{k'\beta} c_{k\beta} \left(1 - n_{q\beta}\right) \tag{3.5a}$$
$$\left(\eta_0 c^\dagger_{q\beta} T_{q\beta}\right)_2 = J_z J_t \frac{1}{\omega' - H^D} \sum_{kk' \neq q} \beta S_d^z c^\dagger_{k'\beta} c_{q\beta} c^\dagger_{d\bar{\beta}} c_{d\beta} c^\dagger_{q\beta} c_{k\bar{\beta}}$$
$$= -\frac{1}{2} J_z J_t \frac{1}{\omega' + \frac{1}{2}\epsilon_q + \frac{1}{2} J_z \left(\tau_{q\beta} - \tau_{q\bar{\beta}}\right)} c^\dagger_{d\bar{\beta}} c_{d\beta} \sum_{kk' \neq q} c^\dagger_{k'\beta} c_{k\bar{\beta}} \left(1 - n_{q\beta}\right) \tag{3.5b}$$
$$\left(\eta_0 c^\dagger_{q\beta} T_{q\beta}\right)_3 = J_z J_t \frac{1}{\omega' - H^D} \sum_{kk' \neq q} c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k'\bar{\beta}} c_{q\beta} \beta S_d^z c^\dagger_{q\beta} c_{k\beta}$$
$$= -\frac{1}{2} J_z J_t \frac{1}{\omega' + \frac{1}{2}\epsilon_q - \frac{1}{2} J_z \left(\tau_{q\beta} - \tau_{q\bar{\beta}}\right)} c^\dagger_{d\beta} c_{d\bar{\beta}} \sum_{kk' \neq q} c^\dagger_{k'\bar{\beta}} c_{k\beta} \left(1 - n_{q\beta}\right) \tag{3.5c}$$
$$\left(\eta_0 c^\dagger_{q\beta} T_{q\beta}\right)_4 = J_t^2 \frac{1}{\omega' - H^D} \sum_{kk' \neq q} c^\dagger_{d\beta} c_{d\bar{\beta}} c^\dagger_{k'\bar{\beta}} c_{q\beta} c^\dagger_{d\bar{\beta}} c_{d\beta} c^\dagger_{q\beta} c_{k\bar{\beta}}$$
$$= J_t^2 \frac{1}{\omega' + \frac{1}{2}\epsilon_q - \frac{1}{2} J_z \left(\tau_{q\beta} - \tau_{q\bar{\beta}}\right)} \left(\frac{1}{2} - \bar{\beta} S_d^z\right) \sum_{kk' \neq q} c^\dagger_{k'\bar{\beta}} c_{k\bar{\beta}} \left(1 - n_{q\beta}\right) \tag{3.5d}$$

4 Decoupling $q\beta$, $q\bar{\beta}$

We consider the decoupling for the initial condition $\tau_{q\beta} = -\tau_{q\bar{\beta}} = \frac{1}{2}$ with the ansatz $\omega = -\omega'$

What is the motivation behind the initial condition $\tau_{q\beta} = -\tau_{q\bar{\beta}} = \frac{1}{2}$? Can't we take $\tau_{q\beta} = \tau_{q\bar{\beta}} = \frac{1}{2}$ for $q\beta$ below fermi level and $-1/2$ for above it?

$$c^\dagger_{q\beta} T_{q\beta} \eta = \frac{1}{4} J_z^2 \frac{1}{\omega - \frac{1}{2}\epsilon_q - \beta J_z S_d^z} \sum_{kk' \neq q} c_{k\beta} c^\dagger_{k'\beta}$$
$$+ \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2}\epsilon_q - \frac{1}{2} J_z} c^\dagger_{d\beta} c_{d\bar{\beta}} \sum_{kk' \neq q} c_{k\beta} c^\dagger_{k'\bar{\beta}}$$

$$
\begin{aligned}
&+\frac{1}{2}J_zJ_t\frac{1}{\omega-\frac{1}{2}\epsilon_q+\frac{1}{2}J_z}c^\dagger_{d\bar\beta}c_{d\beta}\sum_{kk'\neq q}c_{k\bar\beta}c^\dagger_{k'\beta}\\
&+J_t^2\frac{1}{\omega-\frac{1}{2}\epsilon_q+\frac{1}{2}J_z}\left(\frac{1}{2}+\bar\beta S_d^z\right)\sum_{kk'\neq q}c_{k\bar\beta}c^\dagger_{k'\bar\beta}
\end{aligned}
\tag{4.1}
$$

$$
\begin{aligned}
\eta_0c^\dagger_{q\bar\beta}T_{q\bar\beta}=&-\frac{1}{4}J_z^2\frac{1}{\omega-\frac{1}{2}\epsilon_q-\bar\beta J_zS_d^z}\sum_{kk'\neq q}c^\dagger_{k'\bar\beta}c_{k\bar\beta}\\
&+\frac{1}{2}J_zJ_t\frac{1}{\omega-\frac{1}{2}\epsilon_q+\frac{1}{2}J_z}c^\dagger_{d\beta}c_{d\bar\beta}\sum_{kk'\neq q}c^\dagger_{k'\bar\beta}c_{k\beta}\\
&+\frac{1}{2}J_zJ_t\frac{1}{\omega-\frac{1}{2}\epsilon_q-\frac{1}{2}J_z}c^\dagger_{d\bar\beta}c_{d\beta}\sum_{kk'\neq q}c^\dagger_{k'\beta}c_{k\bar\beta}\\
&-J_t^2\frac{1}{\omega-\frac{1}{2}\epsilon_q-\frac{1}{2}J_z}\left(\frac{1}{2}-\beta S_d^z\right)\sum_{kk'\neq q}c^\dagger_{k'\beta}c_{k\beta}
\end{aligned}
\tag{4.2}
$$

$$
\left(c^\dagger_{q\beta}T_{q\beta}\eta\right)_2+\left(\eta_0c^\dagger_{q\bar\beta}T_{q\bar\beta}\right)_2=J_zJ_t\frac{\left(\omega-\frac{1}{2}\epsilon_q\right)}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}c^\dagger_{d\beta}c_{d\bar\beta}\sum_{kk'\neq q}c^\dagger_{k'\bar\beta}c_{k\beta}
\tag{4.3a}
$$

$$
\left(c^\dagger_{q\beta}T_{q\beta}\eta\right)_3+\left(\eta_0c^\dagger_{q\bar\beta}T_{q\bar\beta}\right)_3=J_zJ_t\frac{\left(\omega-\frac{1}{2}\epsilon_q\right)}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}c^\dagger_{d\bar\beta}c_{d\beta}\sum_{kk'\neq q}c^\dagger_{k'\beta}c_{k\bar\beta}
\tag{4.3b}
$$

$$
\begin{aligned}
\left(c^\dagger_{q\beta}T_{q\beta}\eta\right)_1+\left(\eta_0c^\dagger_{q\bar\beta}T_{q\bar\beta}\right)_4=&\frac{J_t^2\left(\omega-\frac{1}{2}\epsilon_q\right)+\frac{1}{2}J_z\left(J_t^2-\frac{1}{2}J_z^2\right)}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}\beta S_d^z\sum_{kk'\neq q}c^\dagger_{k'\beta}c_{k\beta}\\
&-\frac{1}{2}\frac{\left(\omega-\frac{1}{2}\epsilon_q\right)\left(J_t^2+\frac{1}{2}J_z^2\right)+\frac{1}{2}J_zJ_t^2}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}\sum_{kk'\neq q}c^\dagger_{k'\beta}c_{k\beta}\\
&+\frac{1}{4}J_z^2\frac{\left(\omega+\frac{1}{2}\epsilon_q+\beta\mathbf{J_zS_d^z}\right)}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}\sum_{k\neq q}1
\end{aligned}
\tag{4.3c}
$$

$$
\begin{aligned}
\left(c^\dagger_{q\beta}T_{q\beta}\eta\right)_4+\left(\eta_0c^\dagger_{q\bar\beta}T_{q\bar\beta}\right)_1=&-\frac{J_t^2\left(\omega-\frac{1}{2}\epsilon_q\right)-\frac{1}{2}J_z\left(J_t^2-\frac{1}{2}J_z^2\right)}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}\bar\beta S_d^z\sum_{kk'\neq q}c^\dagger_{k'\bar\beta}c_{k\bar\beta}\\
&-\frac{1}{2}\frac{\left(\omega-\frac{1}{2}\epsilon_q\right)\left(J_t^2+\frac{1}{2}J_z^2\right)-\frac{1}{2}J_zJ_t^2}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}\sum_{kk'\neq q}c^\dagger_{k'\bar\beta}c_{k\bar\beta}\\
&+J_t^2\frac{\left(\omega+\frac{1}{2}\epsilon_q-\frac{1}{2}J_z\right)}{\left(\omega-\frac{1}{2}\epsilon_q\right)^2-\left(\frac{1}{2}J_z\right)^2}\sum_{k\neq q}1
\end{aligned}
\tag{4.3d}
$$

Field terms arise in $UHU^\dagger$ if we don't sum Eqs. (4.3) over β

4.1 Scaling Equations

(Summing over β)

$$\Delta J_t = 2J_z J_t \frac{\left(\omega - \frac{1}{2}\epsilon_q\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2} \tag{4.4}$$

$$\Delta J_z = \frac{J_z\left(J_t^2 - \frac{1}{2}J_z^2\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2} \tag{4.5}$$

ΔJ_z doesn't have the same form as ΔJ_t.

Question 1

Section 2.2, Equation 2.18 of thesis

$$\frac{1}{H' - H_e \hat{n}_N} c_N^\dagger T = c_N^\dagger T \frac{1}{H' - H_h(1 - \hat{n}_N)}$$
$$\implies H_e \hat{n}_N c_N^\dagger T = c_N^\dagger T H_h(1 - \hat{n}_N)$$

This seems to **require** H' **commuting with** T, because

$$c_N^\dagger T H' - c_N^\dagger T H_h(1 - \hat{n}_N) = H' c_N^\dagger T - H_e \hat{n}_N c_N^\dagger T$$

Why should H' commute with T?

(where $H_e = Tr\left(H\hat{n}_N\right)$, $H_h = Tr\left[H\left(1 - \hat{n}_N\right)\right]$ and $T = Tr\left(Hc_N\right)$)

Question 2

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^\dagger = H_h \left(1 - n_N\right)$$

If I try to derive this using the result on the previous slide:

$$\begin{aligned} \eta H \eta^\dagger = \eta H_e \eta^\dagger = \eta H_e c^\dagger T G &= \eta c^\dagger T H_h G \\ &= \eta c^\dagger T G H_h = \eta \eta^\dagger H_h = H_h \left(1 - \hat{n}\right) \end{aligned}$$

That required $[G, H_h] = 0$. How does that work out?

$$\text{(where } H_e = Tr\left(H \hat{n}_N\right), H_h = Tr\left[H\left(1 - \hat{n}_N\right)\right] \text{ and } T = Tr\left(H c_N\right)\text{)}$$

Question 3

Kondo Model appendix, Equation 9.61 of thesis

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[S^a S^b \sigma^a_{\alpha\beta} \sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma} (1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right.$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \left[S^x S^y \sigma^x_{\alpha\beta} \sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots \right.$$

Question 3

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right.$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \left[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots \right.$$

- **The τ should not be there in numerator i presume?**

Question 3

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \Bigg[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1, j_2 < j), \\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma} (1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \Big[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots$$

- **Since coupling is $\frac{J}{2}$, shouldn't the thing be $\frac{J^2}{4}$ instead of $\frac{J^2}{2}$?**

Question 3

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + \ldots \right.$$

$$+ \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \left[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots \right.$$

- **You mentioned the following in the google document- "*interchange sigma_a and sigma_b (you get -1 sign)*". But these are matrix elements (numbers). So why the minus sign?**

Question 3

$$\Delta\hat{H}_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}$$

$$\times\left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j),\\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1-\hat{n}_{j,\hat{s}_m,\beta}) + \ldots\right.$$

$$\left. + \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}\right[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + \ldots$$

- **How do you combine the product of two sigmas ($\sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma}$) into a single $\sigma^c_{\alpha\gamma}$?**

Question 4

Kondo URG coupling equation for J (equation 9.65):

$$\Delta J^{(j)} = n_j (J^{(j)})^2 \left[\omega - \frac{\epsilon_{j,l}}{2}\right] \left[(\frac{\epsilon_{j,l}}{2} - \omega)^2 - \frac{\left(J^{(j)}\right)^2}{16}\right]^{-1}$$

One-loop form (after setting $\omega = \epsilon_{j,l}$):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2\frac{n_j (J^{(j)})^2}{\epsilon_{j,l}} \to \frac{2\rho|\Delta D|J^2}{D} \quad [n_j, \rho \to \text{DOS per spin}]$$

One-loop form in Coleman (Introduction to Many-Body Physics) ($\tilde{J} = J/2$):

$$\Delta\tilde{J} = \frac{2\rho|\Delta D|\tilde{J}^2}{D} \implies \Delta J = \frac{\rho|\Delta D|J^2}{D}$$

Is there any reason for this difference?

Question 5

- In the Kondo URG, are you considering two electrons on the shell Λ_N, one that we are decoupling ($q\beta$) and another with the same momentum but opposite spin ($q\overline{\beta}$)?
- If so, why does that kinetic energy piece ($\epsilon_q \tau_{q\overline{\beta}}$) not come down in the denominator?
- Is that what gives rise to the second RG equation and hence the $S^z s^z$ term in the effective Hamiltonian?

Question 6

$$
\begin{aligned}
\Delta H^2_{(j)} &= \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \Big[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} \\
&\quad + S^y S^x \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} \Big] \\
&= \sum_{\substack{m=1,\\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} S^z \frac{\sigma^z_{\alpha\alpha}}{2} \Big[\hat{n}_{j,\hat{s}_m,\alpha}(1 - \hat{n}_{j,\hat{s}_m,\beta}) - \\
&\qquad \hat{n}_{j,\hat{s}_m,\beta}(1 - \hat{n}_{j,\hat{s}_m,\alpha}) \Big]
\end{aligned}
$$

Question 6

$$\Delta H^2_{(j)} = \sum_{\substack{m=1,\\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} \Big[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha}$$

$$+ S^y S^x \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} \Big]$$

$$= \sum_{\substack{m=1,\\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s^z_{j,\hat{s}_m})} S^z \frac{\sigma^z_{\alpha\alpha}}{2} \Big[\hat{n}_{j,\hat{s}_m,\alpha}(1 - \hat{n}_{j,\hat{s}_m,\beta}) - \dots$$

What I got:

$$S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} = i^2 S^z \sigma^z_{\alpha\alpha} \hat{n}_{j,\hat{s}_m,\alpha} \left(1 - \hat{n}_{j,\hat{s}_m,\beta}\right)$$

Question 7

In eq. 2.21 of thesis,

$$UHU^\dagger = \frac{1}{2}Tr(H) + \tau Tr(H\tau) + \tau\{c^\dagger T, \eta\}$$

so the renormalization is

$$\tau\{c^\dagger T, \eta\} = \frac{1}{2}\left[\overbrace{c^\dagger T\eta}^{\text{particle sector}} - \underbrace{\eta c^\dagger T}_{\text{hole sector}}\right] = \text{difference of the 2 sectors}$$

Yet in most RG equations (ΔH_F of 2d Hubbard, ΔH_j of Kondo), you have *added* the two sectors. How/Why?

Question 8

In the Kondo URG, you simplify the $\hat{\omega}$ as

$$\hat{\omega} = \omega\tau$$

What is the formal way of doing this? Shouldn't it be

$$\hat{\omega} = \omega_1 \hat{n} + \omega_1 (1 - \hat{n})$$

Is this just an assumption?

In the RG equation for BCS instability (eq. 8.130 of thesis), you use

$$G^{-1} = \omega - \epsilon_1 \tau_1 - \epsilon_2 \tau_2$$

How is this choice of $\hat{\omega}$ consistent with what was done in Kondo URG?