Entanglement properties in the Kondo Model

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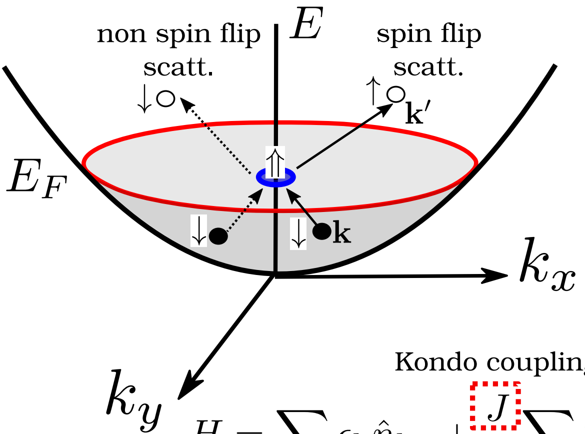
Kondo Model



Kondo coupling

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}\sigma} + \frac{\boxed{J}}{2} \sum_{\mathbf{k}\mathbf{k}'} \mathbf{S} \cdot c_{\mathbf{k}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta}$$

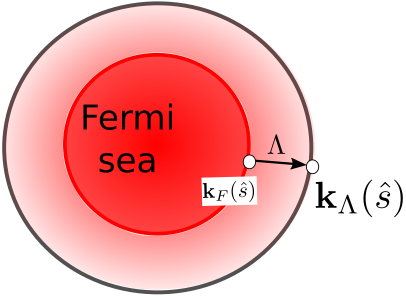
Motivation for the work

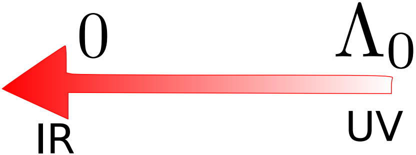
In the antiferromagnetic side a Kondo cloud is formed via the entanglement between the impurity spin and conduction electrons. On the otherhand in the ferromagnetic side the impurity spin disentangles from the conduction electrons.

- How does local electronic correlation and fermion exchange signs interplay in shaping the entanglement properties of the Kondo model?

- How does the physics of the local metal around the impurity differ across the critical point? Can we understand the distinction on the basis of entanglement based witness and green function based measures?

Unitary RG Algorithm

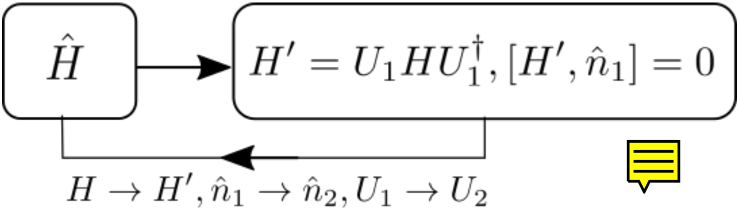




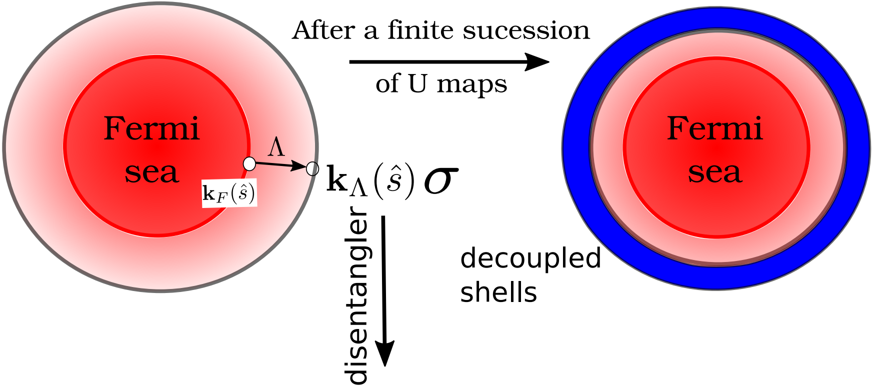
1. Effective H's are generated from UV to IR.

2. At each step a shell in UV at distance Λ is disentangled by the unitary map U.

$$\mathbf{k}_\Lambda(\hat{s}) = \mathbf{k}_F(\hat{s}) + \Lambda\hat{s}$$



Unitary RG Algorithm



$$U_\Lambda = \prod_{\hat{s},\sigma} U_{\Lambda,\hat{s},\sigma}$$

Hamiltonian RG flow equation

Hamiltonian flow $$H_{(j)} \xrightarrow{U_{(j)}} H_{(j-1)}$$

Structure of the unitary disentangler $$U_{(j)} = \prod_{j,l} \boxed{U_{j,l}}$$

disentangles electronic state

where $l = \hat{s}, \sigma$ $\quad |\mathbf{k}_{\Lambda_j}(\hat{s}), \sigma\rangle = |j, l\rangle$

$$U_{j,l} = \frac{1}{\sqrt{2}}\left[1 + \eta_{j,l} - \eta^{\dagger}_{j,l}\right] = \exp\frac{\pi}{4}(\eta_{j,l} - \eta^{\dagger}_{j,l})$$

$\{\eta_{j,l}, \eta^{\dagger}_{j,l}\} = 1, [\eta_{j,l}, \eta^{\dagger}_{j,l}] = 1 - 2\hat{n}_{j,l}$ algebra of the operators

Hamiltonian RG flow equation

Definition of electron-hole transition operator

$$\eta_{j,l} = Tr(c^{\dagger}_{j,l} H_{j,l}) c_{j,l} \frac{1}{\hat{\omega}_{j,l} - Tr_{j,l}(H^{D}_{j,l} \hat{n}_{j,l}) \hat{n}_{j,l}}$$

off-diagonal scattering operation between e-h configuration

Quantum fluctuation operator

$$\hat{\omega}_{j,l} = \boxed{H^{D}_{j,l}} + \boxed{H^{X}_{j,l} - H^{X}_{j,l-1}}$$

diag. part of H

renormalized off diag. part of H

Hamiltonian RG flow equation

H flow eqn.

$$\Delta H_{(j)} = \sum_{l=1}^{2n_j}\{c^{\dagger}_{j,l} Tr_{j,l}(H_{(j)}c_{j,l}), \eta_{j,l}\}$$

ignored higher order correlated tangential scattering

Kondo coupling flow

$$\frac{\Delta J^{(j)}(\omega)}{\Delta \log \frac{\Lambda j}{\Lambda_0}} = \frac{n_j (J^{(j)})^2 \left[(\omega - \frac{\hbar v_F \Lambda_j}{2}) \right]}{(\omega - \frac{\hbar v_F \Lambda_j}{2})^2 - \frac{\left(J^{(j)}\right)^2}{16}}$$

Assumption

Circular Fermi surface(at low filling in 2d TB model).

Hamiltonian RG flow equation

H flow eqn.

$$\Delta H_{(j)} = \sum_{l=1}^{2n_j}\{c^{\dagger}_{j,l} Tr_{j,l}(H_{(j)} c_{j,l}), \eta_{j,l}\}$$

ignored higher order correlated tangential scattering

Kondo coupling flow

no. of states on F

$$\frac{\Delta J^{(j)}(\omega)}{\Delta \log \frac{\Lambda j}{\Lambda_0}} = \frac{n_j (J^{(j)})^2 \left[(\omega - \frac{\hbar v_F \Lambda_j}{2})\right]}{(\omega - \frac{\hbar v_F \Lambda_j}{2})^2 - \frac{\left(J^{(j)}\right)^2}{16}} \Rightarrow \frac{dK}{d \log \frac{\Lambda}{\Lambda_0}} = \frac{n(0) K^2}{1 - \frac{K^2}{16}}$$

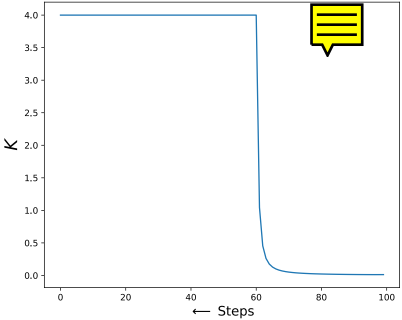
continuum RG flow eqn.

no. of e states on the shell

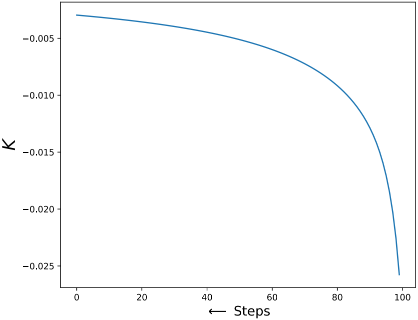
$$K^{(j)} = \frac{J^{(j)}}{\omega - \frac{\hbar v_F}{2}\Lambda_j}$$ (dimensionless coupling)

$$\omega > \frac{\hbar v_F}{2}\Lambda_0$$

Coupling RG phase diagram

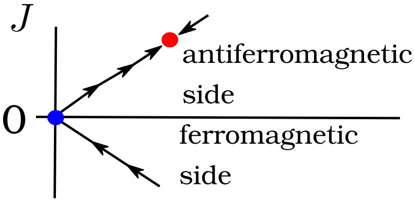


$J_0 > 0$



$J_0 < 0$

$$\frac{dK}{d\log\frac{\Lambda}{\Lambda_0}} = \frac{n(0)K^2}{1-\frac{K^2}{16}}$$



- intermediate coupling fixed point
- critical fixed point

IR fixed point theory

Low energy fixed point Hamiltonian for $J_0 > 0$

$$H^*(\omega) = \sum_{|\Lambda|<\Lambda^*} \hbar v_F \Lambda \hat{n}_{\Lambda,\hat{s},\sigma} + \frac{J^*(\omega)}{2} \sum_{\Lambda,\Lambda'<\Lambda^*,\hat{s}\hat{s}'} \mathbf{S}\cdot c^\dagger_{\mathbf{k}_{\Lambda\hat{s}},\alpha} \boldsymbol{\sigma}_{\alpha\beta} c^\dagger_{\mathbf{k}_{\Lambda'\hat{s}'},\alpha}$$

Hamiltonian containing only zero mode

$$H_0^*(\omega) = \boxed{\frac{\hbar v_F}{N(\Lambda^*)} \sum_{|\Lambda|<\Lambda^*,\hat{s}} \Lambda \sum_{\Lambda,\hat{s}} \hat{n}_{\mathbf{k}_{\Lambda\hat{s}}\sigma}} + \frac{J^*(\omega)}{2} \sum_{\Lambda,\Lambda'<\Lambda^*,\hat{s}\hat{s}'} \mathbf{S}\cdot c^\dagger_{\mathbf{k}_{\Lambda\hat{s}},\alpha} \boldsymbol{\sigma}_{\alpha\beta} c^\dagger_{\mathbf{k}_{\Lambda'\hat{s}'},\alpha}$$

This term is zero, due to equal and opposite energy contribution from between inside and outside of FS.

IR fixed point theory

Low energy fixed point Hamiltonian for $J_0 > 0$

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Hamiltonian containing only zero mode $J_0 > 0$

$$H_0^*(\omega) = \boxed{\frac{\hbar v_F}{N(\Lambda^*)} \sum_{|\Lambda|<\Lambda^*,\hat{s}} \Lambda \sum_{\Lambda,\hat{s}} \hat{n}_{\mathbf{k}_{\Lambda\hat{s}}\sigma}} + \frac{J^*(\omega)}{2} \sum_{\Lambda,\Lambda'<\Lambda^*,\hat{s}\hat{s}'} \mathbf{S}\cdot c^{\dagger}_{\mathbf{k}_{\Lambda\hat{s}},\alpha} \boldsymbol{\sigma}_{\alpha\beta} c^{\dagger}_{\mathbf{k}_{\Lambda'\hat{s}'},\alpha}$$

- Zero mode accounts for the low enery physics near FS, and is responsible for the singlet ground state.

- The other non-zero mode are sources of excitation around the ground state.

Singlet ground state in the AF regime

$$H_0^*(\omega) = \frac{J^*(\omega)}{2} \sum_{\Lambda,\Lambda'<\Lambda^*,\hat{s}\hat{s}'} \mathbf{S}\cdot c^\dagger_{\mathbf{k}_{\Lambda\hat{s}},\alpha}\boldsymbol{\sigma}_{\alpha\beta}c^\dagger_{\mathbf{k}_{\Lambda'\hat{s}'},\alpha}$$

zero mode IR theory

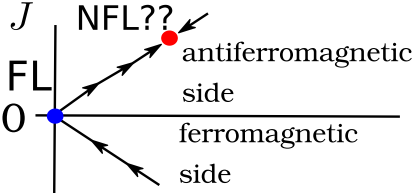
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle \sum_{\Lambda\hat{s}} |1_{\mathbf{k}_{\Lambda\hat{s},\downarrow}}\rangle \otimes_{\Lambda'\neq\Lambda,\hat{s}'\neq\hat{s}} |\Lambda'\hat{s}',\sigma\rangle - |\downarrow\rangle \sum_{\Lambda\hat{s}} |1_{\mathbf{k}_{\Lambda\hat{s},\uparrow}}\rangle \otimes_{\Lambda'\neq\Lambda,\hat{s}'\neq\hat{s}} |\Lambda'\hat{s}',\sigma\rangle\right]$$

ground state wavefunction

A electronic local quantum liquid (NFL) couples with the impurity spin in AF side

Note: in Ferromagnetic side the electronic state is a Fermi liquid

Could this be a local non Fermi Liquid?



Note: Nozeires had shown that the remnant EL not coupled with impurity is a Fermi

Studying the electronic quantum liquid in AF Kondo regime

In order to study its properties we reverse the RG procedure, thus generating states at UV from IR fixed point.

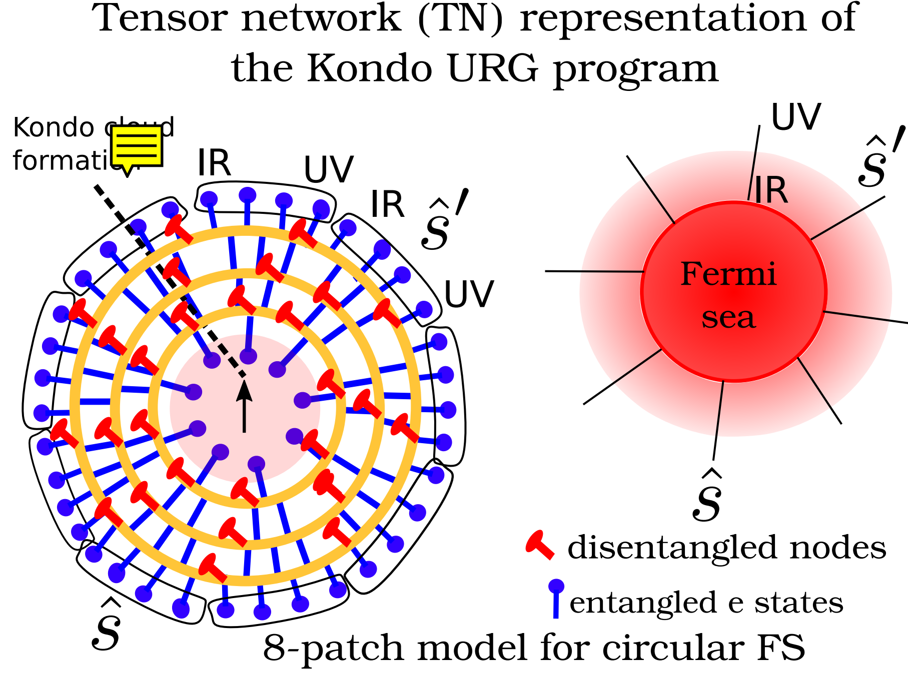
$$\begin{pmatrix} H_{(j)}(\omega^1_{(j)}) \\ \vdots \\ H_{(j)}(\omega) \\ \vdots \\ H_{(j)}(\omega^{2^N}_{(j)}) \end{pmatrix} \xrightarrow{U_{(j)}} \begin{pmatrix} H_{(j-1)}(\omega^1_{(j-1)}) \\ \vdots \\ H_{(j-1)}(\omega) \\ \vdots \\ H_{(j)}(\omega^{2^N}_{(j-1)}) \end{pmatrix} \to \dots \xrightarrow{U_{(j*)}} \begin{pmatrix} H_{(j*)}(\omega^1_{(j*)}) \\ \vdots \\ H^*(\omega) \\ \vdots \\ H_{(j)}(\omega^{2^N}_{(j*)}) \end{pmatrix}$$

$$|\Psi_{(j)}\rangle \xrightarrow{U_{(j)}} |\Psi_{(j-1)}\rangle \to \dots \xrightarrow{U_{(j^*+1)}} |\Psi_{(j^*+1)}\rangle \xrightarrow{U_{(j^*)}} \boxed{|\Psi_{(j^*)}\rangle}$$

$$U^\dagger_{(j^*+1)} \qquad U^\dagger_{(j^*)}$$

UV

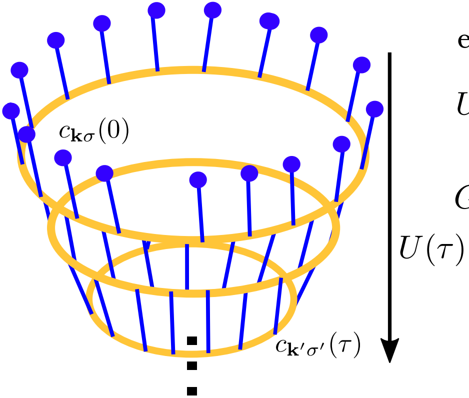
IR



Tensor network (TN) representation of the Kondo URG program
Kondo cloud formation
IR
UV
IR
$\hat{S}'$
UV
$\hat{S}$
UV
$\hat{S}'$
IR
Fermi sea
$\hat{S}$
disentangled nodes
entangled e states
8-patch model for circular FS

Studying EQL using TN

Properties of TN



Green Function based

$$c_{\mathbf{k}\sigma}(\tau) = U^{\dagger}(\tau)c_{\mathbf{k}\sigma}U(\tau)$$

equivalent RG time evolution

$$U(\tau) = \prod_{j=N}^{l} U_{(j)}, \tau = \frac{1}{v_F\Lambda_l}$$

$$G(\mathbf{k}\sigma,\tau) = \langle c_{\mathbf{k}\sigma}(\tau)c^{\dagger}_{\mathbf{k}\sigma}(\tau)\rangle,$$

1 electron green func.

Entanglement based

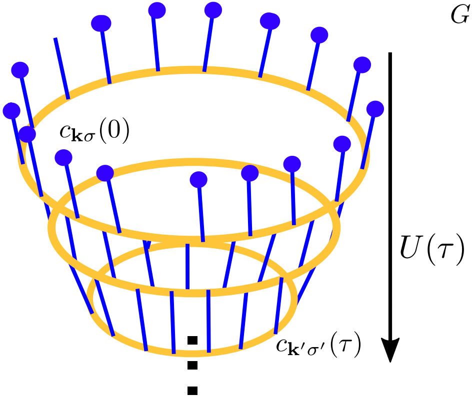
$$S_{\mathbf{k}\sigma}(\tau) = -Tr(\rho_{\mathbf{k}\sigma}(\tau)\ln\rho_{\mathbf{k}\sigma}(\tau)),$$

$$\rho_{\mathbf{k}\sigma} = Tr_{\bar{\mathbf{k}\sigma}}(|\Psi(\tau)\rangle\langle\Psi(\tau)|)$$

Entanglement entropy

Studying EQL using TN

Properties of TN



Green Function based

$$G(\mathbf{k}\sigma,\tau;\mathbf{k}'\sigma',\tau') = \langle c_{\mathbf{k}\sigma}(\tau)c^{\dagger}_{\mathbf{k}'\sigma'}(\tau')\rangle$$

off diagonal green function

Entanglement based

$$I(\mathbf{k}\sigma : \mathbf{k}'\sigma', \tau)$$

tual Information

entangledness
between pair of
e states

A dual probe for the EQL

From the organization of eigenstates in the TN from UV to IR we can obtain the complete 1e Greens function

$$G(\tau) = \begin{pmatrix} G(k\sigma) & G(k\sigma, k'\sigma') & \dots \\ G(k\sigma, k'\sigma') & G(k'\sigma') & \dots \\ \vdots & \ddots & \end{pmatrix}$$

And therefore obtain the Self energy matrix $\Sigma(\tau) = G^{-1}(\tau) - G_0^{-1}(\tau)$

$$|\Psi(\tau)\rangle = \sum_{\phi} \lambda_{\phi} |a_{\phi}\rangle |b_{\phi}\rangle$$

From the Schmidt decomposition of the TN states we can obtain the entanglement features

References

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3.Anderson, P. W. Journal of Physics C: Solid State Physics 3.12 (1970): 2436

Thank you