

# URG ON KONDO MODEL

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# 1 Questions

- What is the motivation behind the choice of the initial condition  $\tau_{q\uparrow} = -\tau_{q\downarrow} = \frac{1}{2}$ ? Does that choice not violate the SU(2) symmetry of the model? Why not take a more symmetric choice like  $\tau_{q\beta} = \tau_{q\bar{\beta}} = \frac{1}{2}$  for  $q\beta$  below fermi level and  $-1/2$  for above it?
- If we follow your notes and try to derive the equations with just the initial configuration  $\tau_{q\uparrow} = -\tau_{q\downarrow} = \frac{1}{2}$ , we end up with a field-like term  $\alpha S_d^z$  in  $\Delta H$ , which violates SU(2). However, if we also add the  $\Delta H$  from the initial state with  $\uparrow$  and  $\downarrow$  flipped, then we lose the field term.
- On setting  $J_z = J_t$ , we do not get  $\Delta J_z = \Delta J_t$ .

## 2 Formulation

$$\begin{aligned}
H &= \sum_{k\alpha} \epsilon_k \tau_{k\alpha} + J_z S_d^z \sum_{kk'} \left( c_{k\uparrow}^\dagger c_{k'\uparrow} - c_{k\downarrow}^\dagger c_{k'\downarrow} \right) \\
&\quad + J_t \sum_{kk'} \left( S_d^+ c_{k\downarrow}^\dagger c_{k'\uparrow} + S_d^- c_{k\uparrow}^\dagger c_{k'\downarrow} \right) \\
&= H^D + H^i + H^I
\end{aligned} \tag{2.1}$$

$$H^D = \sum_{k\alpha} \epsilon_k \tau_{k\alpha} + J_z S_d^z \sum_{k\beta} \beta \tau_{k\beta} \tag{2.2}$$

$$\begin{aligned}
H^i &= J_z S_d^z \sum_{kk' \neq q} \beta \left( c_{k\beta}^\dagger c_{k'\beta} - c_{k\bar{\beta}}^\dagger c_{k'\bar{\beta}} \right) (1 - \delta_{kk'}) \\
&\quad + J_t \sum_{k' \neq q, k} \left( c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k\bar{\beta}}^\dagger c_{k'\beta} + c_{d\bar{\beta}}^\dagger c_{d\beta} c_{k'\beta}^\dagger c_{k\bar{\beta}} \right)
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
H^I &= J_t \sum_{k \neq q} \left( c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k\bar{\beta}}^\dagger c_{q\beta} + c_{d\bar{\beta}}^\dagger c_{d\beta} c_{q\beta}^\dagger c_{k\bar{\beta}} \right) \\
&\quad + J_z S_d^z \beta \sum_{k \neq q} \left( c_{k\beta}^\dagger c_{q\beta} + c_{q\beta}^\dagger c_{k\beta} \right) \\
&= c_{q\beta}^\dagger T_{q\beta} + T_{q\beta}^\dagger c_{q\beta}
\end{aligned} \tag{2.4}$$

where

$$T_{q\beta} = J_z S_d^z \beta \sum_{k \neq q} c_{k\beta} + J_t c_{d\bar{\beta}}^\dagger c_{d\beta} \sum_{k \neq q} c_{k\bar{\beta}} \tag{2.5}$$

The transformed hamiltonian is

$$U H U^\dagger = H^D + H^i + \underbrace{c_{q\beta}^\dagger T_{q\beta} \eta}_{\text{Particle}} + \underbrace{\eta_0 c_{q\beta}^\dagger T_{q\beta}}_{\text{Hole}} \tag{2.6}$$

where  $\eta_0 = -\eta$

## 3 Particle, hole sectors (Left GFs)

For simpler calculations, take  $H^D$  in the green's functions of  $\eta$ ,  $\eta_0$  as

$$H^D = \epsilon_q \tau_{q\beta} + \beta J_z S_d^z (\tau_{q\beta} - \tau_{q\bar{\beta}}) \tag{3.1}$$

### 3.1 Particle Sector

$$\begin{aligned}
c_{q\beta}^\dagger T_{q\beta} \eta &= \frac{1}{\omega - H^D} c_{q\beta}^\dagger T_{q\beta} T_{q\beta}^\dagger c_{q\beta} \\
&= \frac{1}{\omega - H^D} \sum_{kk' \neq q} c_{q\beta}^\dagger \left( J_z S_d^z \beta c_{k\beta} + J_t c_{d\bar{\beta}}^\dagger c_{d\beta} c_{k\bar{\beta}} \right) \left( J_z S_d^z \beta c_{k'\beta}^\dagger + J_t c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k'\bar{\beta}}^\dagger \right) c_{q\beta}
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_1 &= J_z^2 \frac{1}{\omega - H^D} \sum_{kk' \neq q} \beta S_d^z c_{q\beta}^\dagger c_{k\beta} \beta S_d^z c_{k'\beta}^\dagger c_{q\beta} \\
&= \frac{1}{4} J_z^2 \frac{1}{\omega - \frac{1}{2}\epsilon_q - \beta J_z S_d^z (\tau_{q\beta} - \tau_{q\bar{\beta}})} \sum_{kk' \neq q} c_{k\beta} c_{k'\beta}^\dagger n_{q\beta}
\end{aligned} \tag{3.3a}$$

$$\begin{aligned}
\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_2 &= J_z J_t \frac{1}{\omega - H^D} \sum_{kk' \neq q} \beta S_d^z c_{q\beta}^\dagger c_{k\beta} c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k'\bar{\beta}}^\dagger c_{q\beta} \\
&= \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2}\epsilon_q - \frac{1}{2} J_z (\tau_{q\beta} - \tau_{q\bar{\beta}})} c_{d\beta}^\dagger c_{d\bar{\beta}} \sum_{kk' \neq q} c_{k\beta} c_{k'\bar{\beta}}^\dagger n_{q\beta}
\end{aligned} \tag{3.3b}$$

$$\begin{aligned}
\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_3 &= J_z J_t \frac{1}{\omega - H^D} \sum_{kk' \neq q} c_{d\bar{\beta}}^\dagger c_{d\beta} c_{q\beta}^\dagger c_{k\bar{\beta}} \beta S_d^z c_{k'\beta}^\dagger c_{q\beta} \\
&= \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2}\epsilon_q + \frac{1}{2} J_z (\tau_{q\beta} - \tau_{q\bar{\beta}})} c_{d\bar{\beta}}^\dagger c_{d\beta} \sum_{kk' \neq q} c_{k\bar{\beta}} c_{k'\beta}^\dagger n_{q\beta}
\end{aligned} \tag{3.3c}$$

$$\begin{aligned}
\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_4 &= J_t^2 \frac{1}{\omega - H^D} \sum_{kk' \neq q} c_{d\bar{\beta}}^\dagger c_{d\beta} c_{q\beta}^\dagger c_{k\bar{\beta}} c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k'\bar{\beta}}^\dagger c_{q\beta} \\
&= J_t^2 \frac{1}{\omega - \frac{1}{2}\epsilon_q + \frac{1}{2} J_z (\tau_{q\beta} - \tau_{q\bar{\beta}})} \left( \frac{1}{2} + \bar{\beta} S_d^z \right) \sum_{kk' \neq q} c_{k\bar{\beta}} c_{k'\bar{\beta}}^\dagger n_{q\beta}
\end{aligned} \tag{3.3d}$$

### 3.2 Hole Sector

$$\begin{aligned}
\eta_0 c_{q\beta}^\dagger T_{q\beta} &= \frac{1}{\omega' - H^D} T_{q\beta}^\dagger c_{q\beta} c_{q\beta}^\dagger T_{q\beta} \\
&= \frac{1}{\omega' - H^D} \sum_{kk' \neq q} \left( J_z S_d^z \beta c_{k'\beta}^\dagger + J_t c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k'\bar{\beta}}^\dagger \right) c_{q\beta} c_{q\beta}^\dagger \left( J_z S_d^z \beta c_{k\beta} + J_t c_{d\bar{\beta}}^\dagger c_{d\beta} c_{k\bar{\beta}} \right)
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
\left(\eta_0 c_{q\beta}^\dagger T_{q\beta}\right)_1 &= J_z^2 \frac{1}{\omega' - H^D} \sum_{kk' \neq q} \beta S_d^z c_{k'\beta}^\dagger c_{q\beta} \beta S_d^z c_{q\beta}^\dagger c_{k\beta} \\
&= \frac{1}{4} J_z^2 \frac{1}{\omega' + \frac{1}{2}\epsilon_q - \beta J_z S_d^z (\tau_{q\beta} - \tau_{q\bar{\beta}})} \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\beta} (1 - n_{q\beta}) \quad (3.5a)
\end{aligned}$$

$$\begin{aligned}
\left(\eta_0 c_{q\beta}^\dagger T_{q\beta}\right)_2 &= J_z J_t \frac{1}{\omega' - H^D} \sum_{kk' \neq q} \beta S_d^z c_{k'\beta}^\dagger c_{q\beta} c_{d\bar{\beta}}^\dagger c_{d\beta} c_{q\beta}^\dagger c_{k\bar{\beta}} \\
&= -\frac{1}{2} J_z J_t \frac{1}{\omega' + \frac{1}{2}\epsilon_q + \frac{1}{2} J_z (\tau_{q\beta} - \tau_{q\bar{\beta}})} c_{d\bar{\beta}}^\dagger c_{d\beta} \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\bar{\beta}} (1 - n_{q\beta}) \quad (3.5b)
\end{aligned}$$

$$\begin{aligned}
\left(\eta_0 c_{q\beta}^\dagger T_{q\beta}\right)_3 &= J_z J_t \frac{1}{\omega' - H^D} \sum_{kk' \neq q} c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k'\bar{\beta}}^\dagger c_{q\beta} \beta S_d^z c_{q\beta}^\dagger c_{k\beta} \\
&= -\frac{1}{2} J_z J_t \frac{1}{\omega' + \frac{1}{2}\epsilon_q - \frac{1}{2} J_z (\tau_{q\beta} - \tau_{q\bar{\beta}})} c_{d\beta}^\dagger c_{d\bar{\beta}} \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\beta} (1 - n_{q\beta}) \quad (3.5c)
\end{aligned}$$

$$\begin{aligned}
\left(\eta_0 c_{q\beta}^\dagger T_{q\beta}\right)_4 &= J_t^2 \frac{1}{\omega' - H^D} \sum_{kk' \neq q} c_{d\beta}^\dagger c_{d\bar{\beta}} c_{k'\bar{\beta}}^\dagger c_{q\beta} c_{d\bar{\beta}}^\dagger c_{d\beta} c_{q\beta}^\dagger c_{k\bar{\beta}} \\
&= J_t^2 \frac{1}{\omega' + \frac{1}{2}\epsilon_q - \frac{1}{2} J_z (\tau_{q\beta} - \tau_{q\bar{\beta}})} \left( \frac{1}{2} - \bar{\beta} S_d^z \right) \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\bar{\beta}} (1 - n_{q\beta}) \quad (3.5d)
\end{aligned}$$

## 4 Decoupling $q\beta$ , $q\bar{\beta}$

We consider the decoupling for the initial condition  $\tau_{q\beta} = -\tau_{q\bar{\beta}} = \frac{1}{2}$  with the ansatz  $\omega = -\omega'$

What is the motivation behind the initial condition  $\tau_{q\beta} = -\tau_{q\bar{\beta}} = \frac{1}{2}$ ? Can't we take  $\tau_{q\beta} = \tau_{q\bar{\beta}} = \frac{1}{2}$  for  $q\beta$  below fermi level and  $-1/2$  for above it?

$$\begin{aligned}
c_{q\beta}^\dagger T_{q\beta} \eta &= \frac{1}{4} J_z^2 \frac{1}{\omega - \frac{1}{2}\epsilon_q - \beta J_z S_d^z} \sum_{kk' \neq q} c_{k\beta} c_{k'\beta}^\dagger \\
&\quad + \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2}\epsilon_q - \frac{1}{2} J_z} c_{d\beta}^\dagger c_{d\bar{\beta}} \sum_{kk' \neq q} c_{k\beta} c_{k'\bar{\beta}}^\dagger
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2} \epsilon_q + \frac{1}{2} J_z} c_{d\bar{\beta}}^\dagger c_{d\beta} \sum_{kk' \neq q} c_{k\bar{\beta}} c_{k'\beta}^\dagger \\
& + J_t^2 \frac{1}{\omega - \frac{1}{2} \epsilon_q + \frac{1}{2} J_z} \left( \frac{1}{2} + \bar{\beta} S_d^z \right) \sum_{kk' \neq q} c_{k\bar{\beta}} c_{k'\bar{\beta}}^\dagger
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
\eta_0 c_{q\bar{\beta}}^\dagger T_{q\bar{\beta}} &= -\frac{1}{4} J_z^2 \frac{1}{\omega - \frac{1}{2} \epsilon_q - \bar{\beta} J_z S_d^z} \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\bar{\beta}} \\
& + \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2} \epsilon_q + \frac{1}{2} J_z} c_{d\bar{\beta}}^\dagger c_{d\bar{\beta}} \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\beta} \\
& + \frac{1}{2} J_z J_t \frac{1}{\omega - \frac{1}{2} \epsilon_q - \frac{1}{2} J_z} c_{d\bar{\beta}}^\dagger c_{d\beta} \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\bar{\beta}} \\
& - J_t^2 \frac{1}{\omega - \frac{1}{2} \epsilon_q - \frac{1}{2} J_z} \left( \frac{1}{2} - \beta S_d^z \right) \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\beta}
\end{aligned} \tag{4.2}$$

$$\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_2 + \left( \eta_0 c_{q\bar{\beta}}^\dagger T_{q\bar{\beta}} \right)_2 = J_z J_t \frac{(\omega - \frac{1}{2} \epsilon_q)}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} c_{d\bar{\beta}}^\dagger c_{d\bar{\beta}} \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\beta} \tag{4.3a}$$

$$\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_3 + \left( \eta_0 c_{q\bar{\beta}}^\dagger T_{q\bar{\beta}} \right)_3 = J_z J_t \frac{(\omega - \frac{1}{2} \epsilon_q)}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} c_{d\bar{\beta}}^\dagger c_{d\beta} \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\bar{\beta}} \tag{4.3b}$$

$$\begin{aligned}
\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_1 + \left( \eta_0 c_{q\bar{\beta}}^\dagger T_{q\bar{\beta}} \right)_4 &= \frac{J_t^2 (\omega - \frac{1}{2} \epsilon_q) + \frac{1}{2} J_z (J_t^2 - \frac{1}{2} J_z^2)}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} \beta S_d^z \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\beta} \\
& - \frac{1}{2} \frac{(\omega - \frac{1}{2} \epsilon_q) (J_t^2 + \frac{1}{2} J_z^2) + \frac{1}{2} J_z J_t^2}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} \sum_{kk' \neq q} c_{k'\beta}^\dagger c_{k\beta} \\
& + \frac{1}{4} J_z^2 \frac{(\omega + \frac{1}{2} \epsilon_q + \beta \mathbf{J}_z \mathbf{S}_d^z)}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} \sum_{k \neq q} 1
\end{aligned} \tag{4.3c}$$

$$\begin{aligned}
\left( c_{q\beta}^\dagger T_{q\beta} \eta \right)_4 + \left( \eta_0 c_{q\bar{\beta}}^\dagger T_{q\bar{\beta}} \right)_1 &= -\frac{J_t^2 (\omega - \frac{1}{2} \epsilon_q) - \frac{1}{2} J_z (J_t^2 - \frac{1}{2} J_z^2)}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} \bar{\beta} S_d^z \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\bar{\beta}} \\
& - \frac{1}{2} \frac{(\omega - \frac{1}{2} \epsilon_q) (J_t^2 + \frac{1}{2} J_z^2) - \frac{1}{2} J_z J_t^2}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} \sum_{kk' \neq q} c_{k'\bar{\beta}}^\dagger c_{k\bar{\beta}} \\
& + J_t^2 \frac{(\omega + \frac{1}{2} \epsilon_q - \frac{1}{2} J_z)}{(\omega - \frac{1}{2} \epsilon_q)^2 - (\frac{1}{2} J_z)^2} \sum_{k \neq q} 1
\end{aligned} \tag{4.3d}$$

Field terms arise in  $UHU^\dagger$  if we don't sum Eqs. (4.3) over  $\beta$

## 4.1 Scaling Equations

Without summing over  $\beta$

$$\begin{aligned}\Delta J_{z\uparrow} &= \frac{J_t^2 \left(\omega - \frac{1}{2}\epsilon_q\right) + \frac{1}{2}J_z \left(J_t^2 - \frac{1}{2}J_z^2\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2} \\ \Delta J_{z\downarrow} &= -\frac{J_t^2 \left(\omega - \frac{1}{2}\epsilon_q\right) - \frac{1}{2}J_z \left(J_t^2 - \frac{1}{2}J_z^2\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2} \\ \Delta J_t &= J_z J_t \frac{\left(\omega - \frac{1}{2}\epsilon_q\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2}\end{aligned}\tag{4.4}$$

Field term:

$$\frac{1}{4}J_z^2 \frac{\beta J_z S_d^z}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2} \sum_{k \neq q} 1\tag{4.5}$$

On summing over  $\beta$

$$\Delta J_t = 2J_z J_t \frac{\left(\omega - \frac{1}{2}\epsilon_q\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2}\tag{4.6}$$

$$\Delta J_z = \frac{J_z \left(J_t^2 - \frac{1}{2}J_z^2\right)}{\left(\omega - \frac{1}{2}\epsilon_q\right)^2 - \left(\frac{1}{2}J_z\right)^2}\tag{4.7}$$

$\Delta J_z$  doesn't have the same form as  $\Delta J_t$ .