Comparison between PMS, Kondo URG and SIAM URG

The model:

$$\mathcal{H} = J\vec{S}_d \cdot \vec{s} \tag{0.1}$$

Poor Man's Scaling (Phillip Phillips)

Discrete equation:

$$\delta J = -\frac{1}{2}J^2 n(D)\frac{2}{\omega - D} \tag{0.2}$$

Continuum equation:

$$[\omega = 0] \longrightarrow \quad \frac{\mathrm{d}J}{\mathrm{d}\ln D} = -\rho(D)J^2 \tag{0.3}$$

Kondo Model URG

Discrete equation:

$$\delta J = J^2 n(D)\frac{\omega - \frac{1}{2}D}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{J^2}{16}} \tag{0.4}$$

One-loop equation:

$$\begin{aligned} [\omega = D] &\longrightarrow \quad \frac{\mathrm{d}J}{\mathrm{d}\ln D} = -2\rho(D)J^2 \quad \longrightarrow \text{matches PMS up to a factor of } 2 \\ [\omega = 0] &\longrightarrow \quad \frac{\mathrm{d}J}{\mathrm{d}\ln D} = 2\rho(D)J^2 \end{aligned} \tag{0.5}$$

SIAM URG

Discrete equation:

$$\delta J = -J^2 n(D)\frac{\omega - D}{(\omega - D)^2 - \frac{J^2}{16}} \tag{0.6}$$

One-loop equation:

$$[\omega = 0] \longrightarrow \quad \frac{\mathrm{d}J}{\mathrm{d}\ln D} = -\rho(D)J^2 \quad \longrightarrow \text{matches PMS exactly} \tag{0.7}$$