Unitary Renormalization Group Approach to the Single-Impurity Anderson model

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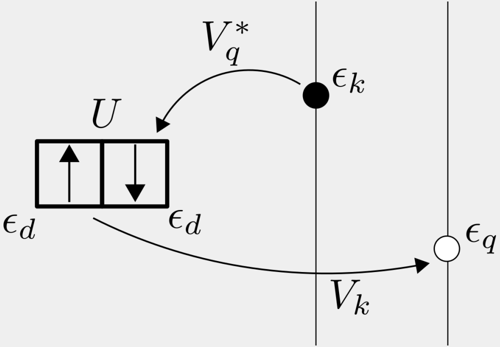
Outline

- The model
- Motivation
- Unitary Renormalization Group (URG) formalism
- Results

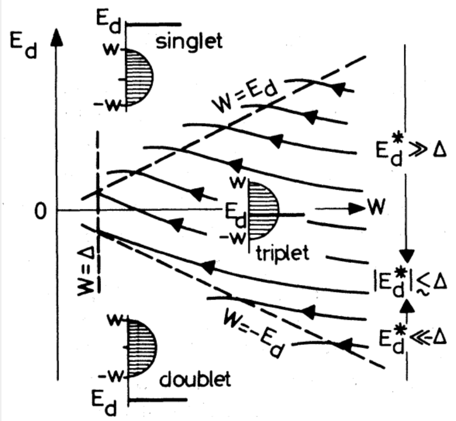
$$\mathcal{H}_{\text{siam}} = \sum_{k\sigma} \epsilon_k \hat{n}_{k\sigma} + \sum_{k\sigma} \left[V(k) c^\dagger_{k\sigma} c_{d\sigma} + \text{h.c.} \right] + \epsilon_d \sum_\sigma \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$



Poor Man's Scaling Results

For large U, Haldane and Jefferson find[1] three low energy theories:

- the **frozen impururity fixed point** ($\langle n_d \rangle = 0$)
- the **local moment fixed point** ($\langle n_d \rangle = 1$), and
- the **valence fluctuation fixed point** ($\langle n_d \rangle \sim \frac{1}{2}$).

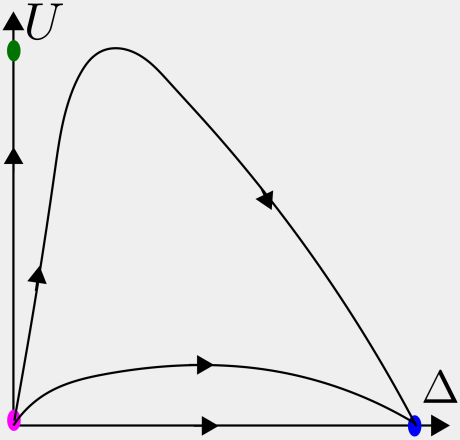


[1]Haldane-1978, Jefferson-1977, Hewson, A. C.-1993-The Kondo Problem to Heavy Fermions

NRG Results - Symmetric Model

For the symmetric Anderson model[1]:

- the **free-orbital** fixed point ($U = \Delta = 0$) - unstable
- the **local moment** fixed point ($U = \infty, \Delta = 0$) - saddle point, and
- the **strong-coupling** fixed point ($\Delta = \infty, U =$ finite) - stable.



[1] Krishna-murthy et al, 1980

NRG Results - Asymmetric Model

Two more fixed points exist -

- the **valence fluctuation** fixed point ($\epsilon_d = V = 0, U = \infty$)
- the **frozen impurity** fixed point ($U = V = 0, \epsilon_d = \infty$)

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- Is it possible to show the **transfer of spectral weight** along the flow, possibly by tracking the bath spectral function or the many-particle entanglement?
- How does NRG obtain the local moment in the **absence of hybridisation**?
- Are there any interesting **topological aspects** of the fixed points?

The Short Version

Apply *unitary many-body transformations* to the Hamiltonian so as to successively *decouple* high energy states and hence obtain scaling equations.

$$\mathcal{H}(D)$$

$$\Downarrow$$

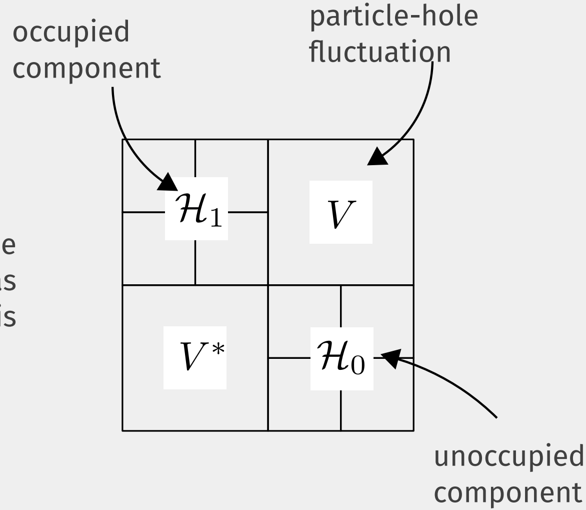
$$U\mathcal{H}(D)U^\dagger$$

$$\Downarrow$$

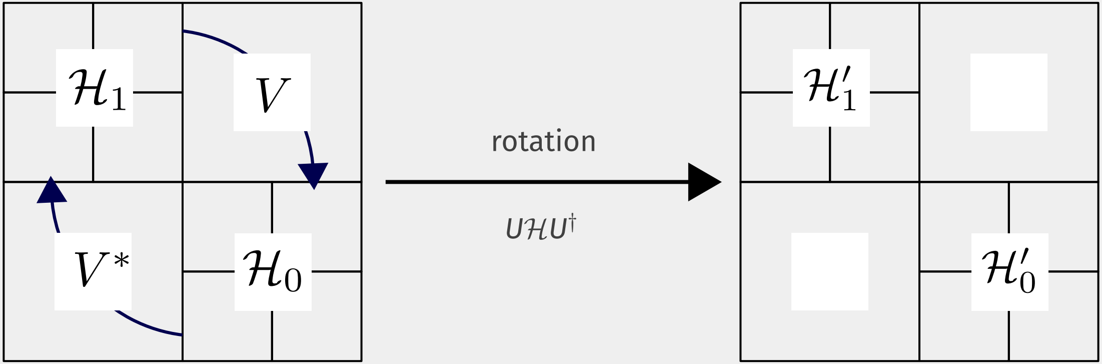
$$H(D) + \mathcal{H}'(D - \delta D)$$

Step 1:

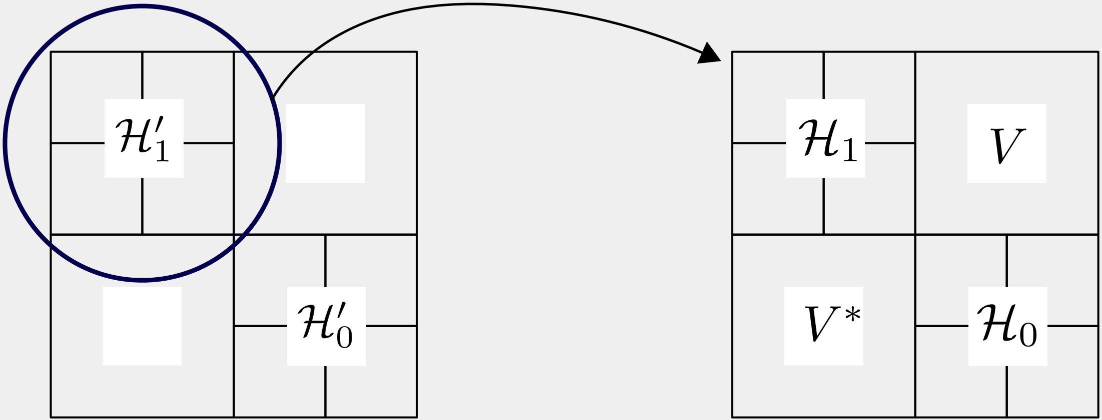
Start with the electrons farthest from the Fermi surface. Write the Hamiltonian as *diagonal and off-diagonal terms* in this basis.



Step 2:
Rotate the Hamiltonian to kill the off-diagonal blocks.



Step 3:
Repeat the process with the new blocks.



Some Characteristic features of the URG

- Presence of the quantum fluctuation energy scale ω
- Presence of finite-valued fixed points
- Spectrum-preserving transformations
- Tractable low-energy effective Hamiltonians

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RG Equations

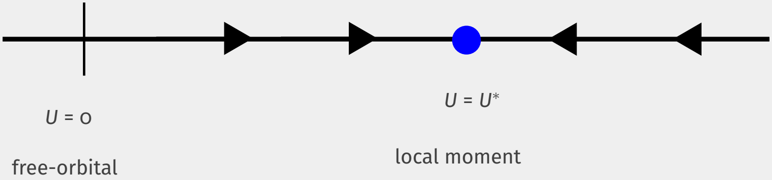
$$\Delta U_n = \left(U + \frac{1}{2}J\right) \sum_{|q|=\Lambda_n} \frac{|V(q)|^2}{(\omega - \epsilon_q - \frac{1}{2}U - \frac{1}{4}J)(\omega - \epsilon_q)}$$

$$\Delta V(q)_n = -\frac{3}{4}J \sum_{|q|=\Lambda_n} \frac{V(q)}{\omega - \epsilon_q - \frac{1}{2}U - \frac{1}{4}J}$$

$$\Delta J_n = -\frac{1}{4}J^2 \sum_{\substack{|q|=\Lambda_n \\ k<\Lambda_n}} \frac{1}{\omega - \epsilon_q - \frac{1}{2}U - \frac{1}{4}J}$$

The case of $J = 0$

$$\Delta U_n = U \sum_{|q|=\Lambda_n} \frac{|V(q)|^2}{(\omega - \epsilon_q - \frac{1}{2}U)(\omega - \epsilon_q)}$$



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- **No separatrix**[1] for the flows to the local moment.

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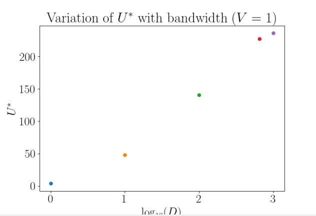
- **No separatrix**[1] for the flows to the local moment.
- Local moment forms at finite U.



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