Shifting the chemical potential to $\mu = \frac{1}{2}J_c$ is equivalent to replacing $\epsilon_q \to \epsilon_q - \frac{1}{2}J_c$.

$$\Delta\epsilon_d = \sum_q \Bigg[\frac{|V_q^0|^2}{\omega^+ - \epsilon_q + \frac{1}{2}J_c + \epsilon_d} + \frac{|V_q^1|^2}{\omega^- - \epsilon_q + \epsilon_d + U - \frac{1}{2}J_c} - \frac{2|V_q^0|^2}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z}$$
$$+ \sum_k \left(\frac{J_t^2 + \frac{1}{4}J_z^2}{\omega^- - \epsilon_q - \frac{1}{2}J_z} + \frac{\frac{1}{4}J_z^2}{\omega^- - \epsilon_q + \frac{1}{2}J_z}\right)\Bigg]$$

$$\Delta U = \sum_q 2\Bigg[\frac{|V_q^1|^2}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z} - \frac{|V_q^0|^2}{\omega^+ - \epsilon_q + \frac{1}{2}J_c + \epsilon_d} + \frac{|V_q^0|^2}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z}$$
$$- \frac{|V_q^1|^2}{\omega^- - \epsilon_q + \epsilon_d + U - \frac{1}{2}J_c} - \sum_k \left(\frac{J_t^2 + \frac{1}{4}J_z^2}{\omega^- - \epsilon_q - \frac{1}{2}J_z} + \frac{\frac{1}{4}J_z^2}{\omega^- - \epsilon_q + \frac{1}{2}J_z}\right)\Bigg]$$

$$\Delta V_1 = -\sum_q V_1(q)\left(\frac{\frac{1}{2}J_z + J_t}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z}\right)$$

$$\Delta V_1^* = -\sum_q V_1^*(q)\left(\frac{\frac{1}{2}J_z + J_t}{\omega^+ - \epsilon_q + \frac{1}{2}J_z}\right)$$

$$\Delta V_0 = -\sum_q V_0(q)\frac{\frac{1}{2}J_z + J_t}{\omega^- - \epsilon_q - \frac{1}{2}J_z}$$

$$\Delta V_0^* = -\sum_q V_0(q)^*\frac{\frac{1}{2}J_z + J_t}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z}$$

$$\Delta J_c = -J_t^2\sum_q\left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{2}J_z} - \frac{1}{\omega^- - \epsilon_q - \frac{1}{2}J_z}\right)$$

$$\Delta J_z = -J_t^2\sum_q\left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{2}J_z} + \frac{1}{\omega^- - \epsilon_q - \frac{1}{2}J_z}\right)$$

$$\Delta J_t = -J_zJ_t\sum_q\left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{2}J_z} + \frac{1}{\omega^- - \epsilon_q - \frac{1}{2}J_z}\right)$$

1.4 Marginality of J_c

The second fraction in ΔJ_c is in the hole sector, so we need to change $J_z \to -J_z$:

$$\Delta J_c = -J_t^2\sum_q\left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{2}J_z} - \frac{1}{\omega^- - \epsilon_q + \frac{1}{2}J_z}\right) = 0 \tag{0.102}$$

This ensures that if there is no off-diagonal term of the form $\hat{n}_d\sum_{kk'\sigma}c_{k\sigma}^\dagger c_{k'\sigma}$ in the bare Hamiltonian, it will not be generated along the flow.

1.5 Particle-hole symmetry

The particle-hole asymmetry parameter RG equation is

$$\Delta\left(\epsilon_d + \frac{1}{2}U\right) = \sum_q \left[\frac{|V_q^1|^2}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z} - \frac{|V_q^0|^2}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z}\right] \tag{0.103}$$

Again making the change $\epsilon_d, J_z \to -\epsilon_d, -J_z$ for the hole term and setting $|V^1|^2 = |V^0|^2$ for a particle-hole symmetric Hamiltonian, we get

$$\Delta\left(\epsilon_d + \frac{1}{2}U\right) = \sum_q |V_q|^2 \left[\frac{1}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z} - \frac{1}{\omega^- - \epsilon_q - \epsilon_d + \frac{1}{2}J_z}\right] \tag{0.104}$$

This becomes zero when $\epsilon_d = -\epsilon_d - U$.

1.6 Hermiticity

The equations in consideration are those of ΔV_1 and ΔV_1^*. The superscript 1 signifies that $d\overline{\beta}$ is filled. For the moment, we label the ω^+ in ΔV_1^* as ω^{+^*} - the quantum fluctuation energy for the process $\hat{n}_{d\overline{\beta}} c^\dagger_{d\beta} c_k$ - to distinguish it from the ω^+ that characterizes the process $\hat{n}_{d\overline{\beta}} c^\dagger_k c_{d\beta}$. In other words, ω^+ is the fluctuation energy scale for the singly-occupied state, while ω^{+^*} is the fluctuation energy scale for the doubly-occupied state. The difference between the two scales is $\epsilon_d + U$, so we can write $\omega^{+^*} = \omega^+ + \epsilon_d + U$. Assuming $V_1 = V_1^*$ in the bare model, the two RG equations now becomes

$$\Delta V_1 = -\sum_q V_1(q) \left(\frac{\frac{1}{2}J_z + J_t}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z}\right) = \Delta V_1^* \tag{0.105}$$

Similarly, if we take the RG equations for ΔV_0 and ΔV_0^*, the two quantum fluctuation scales ω^- and ω^{-^*} correspond to those of the singly-occupied and empty states respectively. Since the difference between these states is ϵ_d, we can write $\omega^- - \omega^{-^*} = \epsilon_d$.

$$\Delta V_0 = -\sum_q V_0(q) \frac{\frac{1}{2}J_z + J_t}{\omega^- * -\epsilon_q + \epsilon_d - \frac{1}{2}J_z} = \Delta V_0^* \tag{0.106}$$

1.7 Scaling equations that satisfy all checks (with appropriate shifts and sign changes)

$$\Delta\epsilon_d = \sum_q \left[\frac{|V_q^0|^2}{\omega - \epsilon_q + \frac{1}{2}J_c + \epsilon_d} + \frac{|V_q^1|^2}{\omega - \epsilon_q - \epsilon_d - U + \frac{1}{2}J_c} - \frac{2|V_q^0|^2}{\omega - \epsilon_q - \epsilon_d + \frac{1}{2}J_z}\right.$$
$$\left. + \sum_k \left(\frac{J_t^2 + \frac{1}{4}J_z^2}{\omega - \epsilon_q + \frac{1}{2}J_z} + \frac{\frac{1}{4}J_z^2}{\omega - \epsilon_q - \frac{1}{2}J_z}\right)\right]$$

$$\Delta U = \sum_q 2\left[\frac{|V_q^1|^2}{\omega - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z} - \frac{|V_q^0|^2}{\omega - \epsilon_q + \frac{1}{2}J_c + \epsilon_d} + \frac{|V_q^0|^2}{\omega - \epsilon_q - \epsilon_d + \frac{1}{2}J_z}\right.$$

$$\left. - \frac{|V_q^1|^2}{\omega - \epsilon_q - \epsilon_d - U + \frac{1}{2}J_c} - \sum_k \left(\frac{J_t^2 + \frac{1}{4}J_z^2}{\omega - \epsilon_q + \frac{1}{2}J_z} + \frac{\frac{1}{4}J_z^2}{\omega - \epsilon_q - \frac{1}{2}J_z}\right)\right]$$

$$\Delta V_1 = -\sum_q V_1(q)\left(\frac{\frac{1}{2}J_z + J_t}{\omega - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z}\right)$$

$$\Delta V_0 = -\sum_q V_0(q)\frac{\frac{1}{2}J_z + J_t}{\omega - \epsilon_q + \epsilon_d - \frac{1}{2}J_z}$$

$$\Delta J_z = -J_t^2 \sum_q \left(\frac{1}{\omega - \epsilon_q + \frac{1}{2}J_z} + \frac{1}{\omega - \epsilon_q - \frac{1}{2}J_z}\right)$$

$$\Delta J_t = -J_z J_t \sum_q \left(\frac{1}{\omega - \epsilon_q + \frac{1}{2}J_z} + \frac{1}{\omega - \epsilon_q - \frac{1}{2}J_z}\right)$$