## IR fixed point theory

Low energy fixed point Hamiltonian for  $J_0 > 0$ 

$$H^*(\omega) = \sum_{|\Lambda| < \Lambda^*} \hbar v_F \Lambda \hat{n}_{\Lambda, \hat{s}, \sigma} + \frac{J^*(\omega)}{2} \sum_{\Lambda, \Lambda' < \Lambda^*, \hat{s} \hat{s}'} \mathbf{S} \cdot c^{\dagger}_{\mathbf{k}_{\Lambda} \hat{s}, \alpha} \boldsymbol{\sigma}_{\alpha \beta} c^{\dagger}_{\mathbf{k}_{\Lambda'} \hat{s}', \alpha}$$

## Hamiltonian containing only zero mode

$$H_0^*(\omega) = \frac{\hbar v_F}{N(\Lambda^*)} \sum_{|\Lambda| < \Lambda^*, \hat{s}} \Lambda \sum_{\Lambda, \hat{s}} \hat{n}_{\mathbf{k}_{\Lambda}\hat{s}\sigma} + \frac{J^*(\omega)}{2} \sum_{\Lambda, \Lambda' < \Lambda^*, \hat{s}\hat{s}'} \mathbf{S} \cdot c^{\dagger}_{\mathbf{k}_{\Lambda}\hat{s}, \alpha} \boldsymbol{\sigma}_{\alpha\beta} c^{\dagger}_{\mathbf{k}_{\Lambda'}\hat{s}', \alpha}$$

This term is zero, due to equal and opposite energy contribution from between inside and outside of FS.