0.2.4 Scaling equations (with both interactions)

$$
\begin{aligned}
\mathcal{H} &= \sum_k \left(\epsilon_k \hat{n}_{k\sigma} + V_k c^\dagger_{k\sigma} c_{d\sigma} + \text{h.c.}\right) + \epsilon_d \sum_\sigma \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow}\hat{n}_{d\downarrow} + \overbrace{\sum_{k,q,\sigma} u_A c^\dagger_{k\sigma} c^\dagger_{d\overline{\sigma}} c_{q\overline{\sigma}} c_{d\sigma}}^{\text{spin-flip}} \\
&\quad + \underbrace{\sum_{k,q,\sigma,\sigma'} u_P c^\dagger_{k\sigma} c^\dagger_{d\sigma'} c_{q\sigma} c_{d\sigma'}}_{\text{spin-preserving}}
\end{aligned}
$$

$$
\begin{aligned}
\Delta\epsilon_d &= \sum_q \left(\frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d + u_P} + \frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - u_P} - \frac{2|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d - u_P} \right) \\
\Delta U &= \sum_q 2 \left(\frac{|V_q^1|^2}{\omega - \epsilon_q^+ + \epsilon_d + U + u_P} - \frac{|V_q^0|^2}{\omega - \epsilon_q^+ + \epsilon_d + u_P} - \frac{|V_q^1|^2}{\omega + \epsilon_q^- - \epsilon_d - U - u_P} \right. \\
&\qquad \left. + \frac{|V_q^0|^2}{\omega + \epsilon_q^- - \epsilon_d - u_P} \right) \\
\Delta V_k^1 &= \sum_q V_q^1 \left[\frac{2u_P}{\omega + \epsilon_q^- - \epsilon_k^+} + \frac{u_A - u_P}{\omega - \epsilon_q^+ + \epsilon_d + U + u_P} \right] \\
\Delta V_k^{1^*} &= \sum_q V_q^{1^*} \left[\frac{u_A - u_P}{\omega + \epsilon_k^- - \epsilon_q^+} + \frac{2u_P}{\omega + \epsilon_q^- - \epsilon_d - U - u_P} \right] \\
\Delta V_k^0 &= \sum_q V_q^0 \frac{u_P + u_A}{\omega + \epsilon_q^- - \epsilon_k^+} \\
\Delta V_k^{0^*} &= \sum_q V_q^{0^*} \frac{u_P + u_A}{\omega + \epsilon_q^- - \epsilon_d - u_P} \\
\Delta u_P &= u_P^2 \left(\frac{1}{\omega - \epsilon_q^+ + \epsilon_{k'}^-} - \frac{1}{\omega - \epsilon_q^+ + \epsilon_k^-} \right) \\
\Delta u_A &= 2u_P u_A \left(\frac{1}{\omega - \epsilon_q^+ + \epsilon_{k'}^-} - \frac{1}{\omega - \epsilon_q^+ + \epsilon_k^-} \right)
\end{aligned}
\tag{0.53}
$$

Note: The reason for the lack of a u_P or u_A in the denominators of the last two equations is the following: The dispersion of the conduction electrons in the presence of the 4-Fermi scattering term is $\epsilon_k - u_P n_d$, n_d being the number of impurity electrons. The scattering processes that give rise to the last two RG equations involve a $c_k^\dagger c_q$ or its h.c. in front of the propagator. Such a process creates a conduction electron by destroying another. The net change in energy in this process is $(\epsilon_q - u_P n_d) - (\epsilon_k - u_P n_d)$. It is clear that u_P will vanish from such a difference.