1.3 Scaling equations

$$\Delta\epsilon_d = \sum_q \Bigg[\frac{|V_q^0|^2}{\omega^+ - \epsilon_q + \epsilon_d} + \frac{|V_q^1|^2}{\omega^- - \epsilon_q + \epsilon_d + U} - \frac{2|V_q^0|^2}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z}$$
$$+ \sum_{qk} \left(\frac{J_t^2 + \frac{1}{4}J_z^2}{\omega^- - \epsilon_q - \frac{1}{2}J_z} + \frac{\frac{1}{4}J_z^2}{\omega - \epsilon_q + \frac{1}{2}J_z} \right) \Bigg]$$
$$\Delta U = \sum_q 2 \Bigg[\frac{|V_q^1|^2}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z} - \frac{|V_q^0|^2}{\omega^+ - \epsilon_q + \epsilon_d} + \frac{|V_q^0|^2}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z} - \frac{|V_q^1|^2}{\omega^- - \epsilon_q + \epsilon_d + U}$$
$$-2 \sum_{qk} \left(\frac{J_t^2 + \frac{1}{4}J_z^2}{\omega^- - \epsilon_q - \frac{1}{2}J_z} + \frac{\frac{1}{4}J_z^2}{\omega - \epsilon_q + \frac{1}{2}J_z} \right) \Bigg]$$
$$\Delta J_z = -J_t^2 \sum_q \left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{2}J_z} + \frac{1}{\omega^- - \epsilon_q - \frac{1}{2}J_z} \right)$$
$$\Delta J_t = -J_z J_t \sum_q \left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{2}J_z} + \frac{1}{\omega^- - \epsilon_q - \frac{1}{2}J_z} \right)$$

1.4 SU(2) invariance and Kondo one-loop form

Setting $J_z = J_t = \frac{1}{2}J$ makes the interaction $SU(2)$ symmetric; the last two RG equations can then be written in the common form:

$$2\Delta J_z = 2\Delta J_t = \Delta J = -\frac{1}{2}J^2 \sum_q \left(\frac{1}{\omega^+ - \epsilon_q + \frac{1}{4}J} + \frac{1}{\omega^- - \epsilon_q - \frac{1}{4}J} \right) \tag{0.63}$$

If we now consider low energy excitations ($\omega^\pm - \epsilon_q \approx -\epsilon_q$) and expand the denominator in powers of J and keep only the lowest order, we get

$$\Delta J = -\frac{1}{2}J^2 \sum_q \frac{2}{-\epsilon_q} \tag{0.64}$$

For an isotropic dispersion, we can use $\epsilon_q = D$, where D is the current(running) bandwidth. The sum can then be evaluated as

$$\sum_q = \rho(D)\Delta D \tag{0.65}$$

where $\rho(D)$ is the single-spin density of states at the energy D and $|\Delta D|$ is the thickness of the band that we disentangled at this step. The flow equation of J becomes

$$\Delta J = J^2 \rho(D) \frac{|\Delta D|}{D} \tag{0.66}$$

This is the familiar one-loop Kondo flow equation obtained from Poor man's scaling. To get the continuum version, we must note that since we are decreasing the bandwidth, we have to set $\Delta D = -|\Delta D|$. Therefore,

$$\frac{\mathrm{d}J}{\mathrm{d}\ln D} = -J^2\rho(D) \tag{0.67}$$

1.5 Particle-hole symmetry of impurity levels and Anderson model one-loop form

The terms of order J^2 in $\Delta\epsilon_d$ and ΔU already satisfy $\Delta\epsilon_d + \frac{1}{2}\Delta U = 0$. They are not relevant to the one-loop form either, because the lowest order is J. So we can ignore those terms in this discussion. The RG equation for the asymmetry factor $(\epsilon_d + \frac{1}{2}U)$ becomes (after making some obvious cancellations)

$$\Delta\epsilon_d + \frac{1}{2}\Delta U = \sum_q \left[-\frac{|V_q^0|^2}{\omega^- - \epsilon_q + \epsilon_d - \frac{1}{2}J_z} + \frac{|V_q^1|^2}{\omega^+ - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z}\right] \tag{0.68}$$

For a particle-hole symmetric model, we have $\omega^+ = \omega^- = \omega$ and $|V_q^0|^2 = |V_q^1|^2 = |V_q|^2$. Also, in the URG formalism, the hole contribution comes with an additional minus sign on the excited energy, so we need to invert that sign to compare the particle and hole terms. This involves, for the first term, taking $\epsilon_d \to -\epsilon_d$ and $J_z \to -J_z$. These give

$$\Delta\epsilon_d + \frac{1}{2}\Delta U = \sum_q |V_q|^2 \left[-\frac{1}{\omega - \epsilon_q - \epsilon_d + \frac{1}{2}J_z} + \frac{1}{\omega - \epsilon_q + \epsilon_d + U + \frac{1}{2}J_z}\right] \tag{0.69}$$

We can now use the particle-hole symmetry condition $\epsilon_d + U = -\epsilon_d$ to see that the two terms cancel and we get $\Delta\epsilon_d + \frac{1}{2}\Delta U = 0$.