Wherever you have derived the URG formalism, you have written down the expression for the renormalization as

$$\Delta H = \boldsymbol{\tau}\left\{\boldsymbol{c^\dagger T}, \boldsymbol{\eta}\right\} \tag{1}$$

This can be written as

$$\Delta H = \frac{1}{2}\left(c^\dagger T\eta - \eta c^\dagger T\right) = \boldsymbol{\frac{1}{2}\left(c^\dagger T\frac{1}{\omega - H_d}T^\dagger c - \frac{1}{\omega - H_d}T^\dagger c c^\dagger T\right)} \tag{2}$$

There are two features here.

- This is a difference between a particle-type term ($c^\dagger c$) and a hole-type term ($cc^\dagger$).
- The Greens function is to the left in one of the terms (NOT sandwiched between the off-diagonal terms).

However, when I look at the various RG equations in your thesis and other published works, they seem to be a sum of terms, instead of a difference, and the Greens function is usually sandwiched in between the off-diagonal terms (for eg., in your thesis, eq. 4.56 $\Delta H^F_{(j)}$ Hubbard, or eq. 8.130 BCS instab., or eq. 9.61 Kondo).

In reference to these apparent differences, I have the following questions:

- Is eq. 1 (and hence eq. 2) the one you use for calculating all renormalizations, or is there some other operational equation that you use?
- If yes, how do you convert the difference form to a sum? Is it by absorbing the sign change into a new ω, in the following manner?

$$c^\dagger T\frac{1}{\boldsymbol{\omega} - H_d}T^\dagger c - \frac{1}{\boldsymbol{\omega} - H_d}T^\dagger c c^\dagger T = c^\dagger T\frac{1}{\boldsymbol{\omega} - H_d}T^\dagger c + \frac{1}{\boldsymbol{\omega'} - H_d}T^\dagger c c^\dagger T \tag{3}$$

- If so, how do you relate the new ω' with the old ω?
- Is eq. 1 a complete equation by which I can get both particle and hole sector contributions, simply by choosing appropriate configurations of the number operators of the electrons I am decoupling?

$$\begin{aligned} \text{particle sector contribution } &= \frac{1}{2}c^\dagger T\frac{1}{\omega - H_d}T^\dagger c \\ \text{particle sector contribution } &= -\frac{1}{2}\frac{1}{\omega - H_d}T^\dagger c c^\dagger T \end{aligned} \tag{4}$$

Or is it that eq. 1 gives the contributions only from the particle sector, and we require a separate formula for calculating the hole contributions (possibly by switching η and $\eta^\dagger$ in the expression)?

To illustrate my confusion in an actual problem, if I take the stargraph Hamiltonian $H = \sum_i \epsilon_i S_i^z + J\sum_{i=1}^N \vec{S}_0 \cdot \vec{S}_i$. The RG equation I get by following eq. 2 is

$$\begin{aligned}\Delta H &= \frac{J^2}{4} S_N^+ S_0^- \frac{1}{\omega - H_d} S_N^- S_0^+ - \frac{J^2}{4}\frac{1}{\omega - H_d} S_N^- S_0^+ S_N^+ S_0^- \\ &= \underbrace{\frac{J^2}{4} S_N^+ S_0^- \frac{1}{\omega + \frac{1}{2}\epsilon_N - \frac{1}{2}\epsilon_0 + \frac{1}{4}J} S_N^- S_0^+}_{\text{particle sector}} \underbrace{- \frac{1}{\omega + \frac{1}{2}\epsilon_N - \frac{1}{2}\epsilon_0 + \frac{1}{4}J} S_N^- S_0^+ S_N^+ S_0^-}_{\text{hole sector}}\end{aligned} \tag{5}$$

Meanwhile, Siddhartha da writes

$$\begin{aligned}\text{particle sector} &= \frac{J^2}{4} S_N^+ S_0^- \frac{1}{\omega - H_d} S_N^- S_0^+ = \frac{J^2}{4} S_N^+ S_0^- \frac{1}{\omega + \frac{1}{2}\epsilon_N - \frac{1}{2}\epsilon_0 + \frac{1}{4}J} S_N^- S_0^+ \\ \text{hole sector} &= \frac{J^2}{4} S_N^- S_0^+ \frac{1}{\omega - H_d} S_N^+ S_0^- = \frac{J^2}{4} S_N^+ S_0^- \frac{1}{\omega - \frac{1}{2}\epsilon_N + \frac{1}{2}\epsilon_0 + \frac{1}{4}J} S_N^- S_0^+\end{aligned} \tag{6}$$

Here, Siddhartha da always keeps the Greens function sandwiched between the two off-diagonal terms and keeps the same sign for both the sectors - both these facts are at conflict with what I am getting.