

# UNITARY RENORMALIZATION GROUP APPROACH TO THE SINGLE-IMPURITY ANDERSON MODEL

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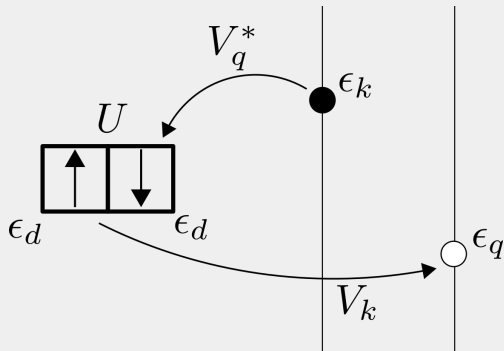
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JANUARY 6, 2021

- The model
- Motivation
- Unitary Renormalization Group (URG) formalism
- Results

# THE SINGLE-IMPURITY ANDERSON MODEL

$$\mathcal{H}_{\text{siam}} = \sum_{k\sigma} \epsilon_k \hat{n}_{k\sigma} + \sum_{k\sigma} \left[ V(k) c_{k\sigma}^\dagger c_{d\sigma} + \text{h.c.} \right] + \epsilon_d \sum_{\sigma} \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$



- "Poor man's" scaling<sup>1</sup> is *perturbative* and fails at large values - cannot show strong-coupling (SC) fixed point.

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- Instead, one needs to flow to large value of  $U$ , do a Schrieffer -Wolff transformation and then flow to the SC fixed point.
- **It would be nice to get a single set of equations that show the crossover to the strong-coupling fixed point.**

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- NRG cannot show *how the Hamiltonians and many-body wavefunctions vary along the flow* - projective in nature.
- **It would be enlightening to see the flow into SC regime by tracking the change in entanglement** - hence we need wavefunctions.

## The Short Version

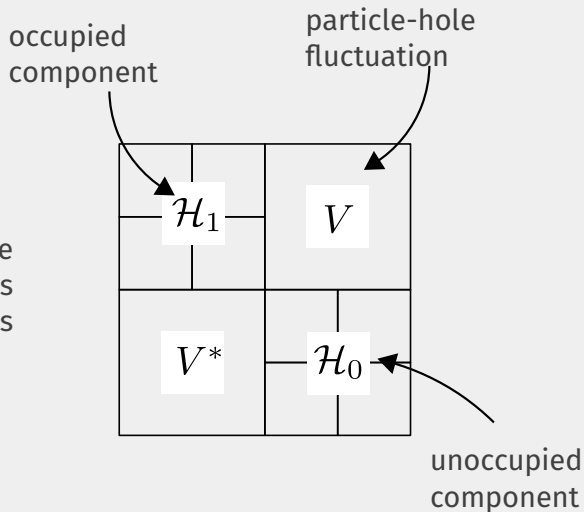
Apply *unitary many-body transformations* to the Hamiltonian so as to successively *decouple* high energy states and hence obtain scaling equations.

$$\begin{array}{c} \mathcal{H}(D) \\ \Downarrow \\ U\mathcal{H}(D)U^\dagger \\ \Downarrow \\ H(D) + \mathcal{H}'(D - \delta D) \end{array}$$

# UNITARY RENORMALIZATION GROUP FORMALISM

## Step 1:

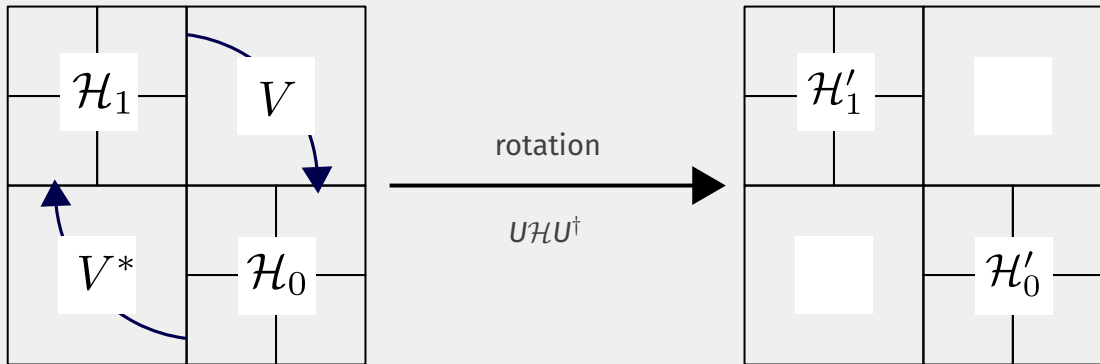
Start with the electrons farthest from the Fermi surface. Write the Hamiltonian as *diagonal and off-diagonal terms* in this basis.



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## Step 2:

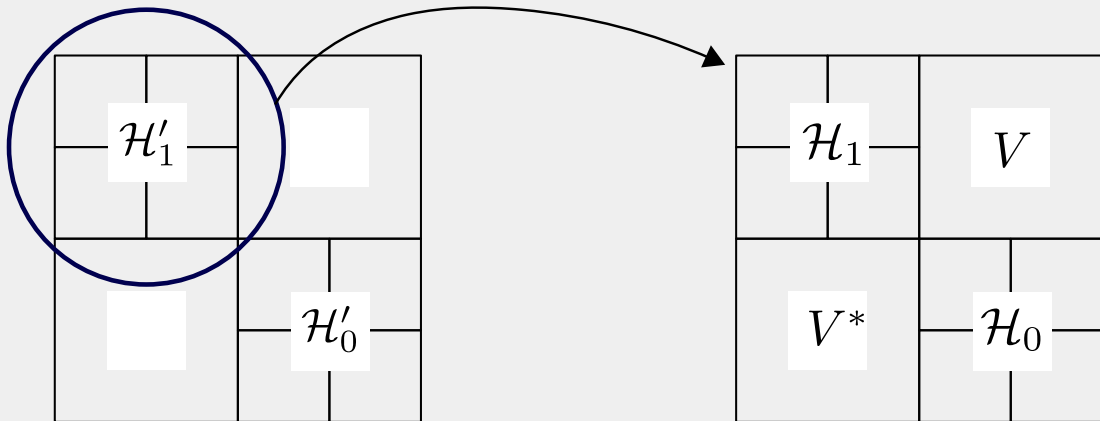
Rotate the Hamiltonian to kill the off-diagonal blocks.



# UNITARY RENORMALIZATION GROUP FORMALISM

## Step 3:

Repeat the process with the new blocks.



$$\mathcal{H} = \sum_{k\sigma} \epsilon_k \hat{n}_{k\sigma} + \sum_{k\sigma} \left[ v(k) c_{k\sigma}^\dagger c_{d\sigma} + \text{h.c.} \right] + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + J \vec{S}_d \cdot \vec{s}$$