The Hamiltonian is represented in term of ee and hh pairs in momentum space,

$$\begin{aligned}
\hat{H}^* &= \frac{\bar{U}_0}{N} \sum_{\Lambda\Lambda', \hat{s}} A^+_{\Lambda\hat{s}} A^-_{\Lambda' -\hat{s}} = \frac{\bar{U}_0}{N} \sum_{\Lambda\Lambda', \hat{s}} c^\dagger_{\mathbf{k}_{\Lambda\hat{s}}\sigma} c^\dagger_{\mathbf{k}_{-\Lambda T\hat{s}} -\sigma} c_{\mathbf{k}_{-\Lambda' -T\hat{s}} -\sigma} c_{\mathbf{k}_{\Lambda' -\hat{s}}\sigma} \\
&= \frac{\bar{U}_0}{N} \sum_{\Lambda\Lambda', \hat{s}} \sum_{\substack{\mathbf{r}_1, \mathbf{r}_2, \\ \mathbf{r}_3, \mathbf{r}_4}} e^{i(\mathbf{k}_{\Lambda\hat{s}} \cdot \mathbf{r}_1 + \mathbf{k}_{-\Lambda T\hat{s}} \cdot \mathbf{r}_2 - \mathbf{k}_{-\Lambda' T\hat{s}} \cdot \mathbf{r}_3 - \mathbf{k}_{-\Lambda'\hat{s}} \cdot \mathbf{r}_4)} c^\dagger_{\mathbf{r}_1\sigma} c^\dagger_{\mathbf{r}_2 -\sigma} c_{\mathbf{r}_3 -\sigma} c_{\mathbf{r}_4\sigma} (1)
\end{aligned}$$

Using the following relations,

$$\begin{aligned}
\mathbf{k}_{\Lambda\hat{s}} \cdot \mathbf{r}_1 + \mathbf{k}_{-\Lambda T\hat{s}} \cdot \mathbf{r}_2 &= \frac{1}{2}(\mathbf{k}_{\Lambda\hat{s}} + \mathbf{k}_{-\Lambda T\hat{s}}) \cdot (\\
\mathbf{k}_{\Lambda\hat{s}} + \mathbf{k}_{-\Lambda T\hat{s}} &= -(\mathbf{k}_{\Lambda -\hat{s}} + \mathbf{k}_{-\Lambda -T\hat{s}} \\
(\pi, \pi) \cdot (\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4) + \frac{1}{2}(\mathbf{k}_{\Lambda\hat{s}} - \mathbf{k}_{-\Lambda T\hat{s}} - \mathbf{k}_{-\Lambda' T\hat{s}} + \mathbf{k}_{-\Lambda'\hat{s}}) \cdot (\mathbf{r}_1 - \mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4) &= \mathbf{k}_{\Lambda\hat{s}} \cdot \mathbf{r}_1 + \mathbf{k}_{-\Lambda T\hat{s}} \cdot
\end{aligned}$$

the Hamiltonian has the form,

$$H^* = \frac{\bar{U}_0}{N} \sum_{\mathbf{r}_1, \mathbf{r}_2, \delta} V_{\hat{s}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{a}) c^\dagger_{\mathbf{r}_1\sigma} c^\dagger_{\mathbf{r}_2 -\sigma} c_{\mathbf{r}_2 + \mathbf{a} -\sigma} c_{\mathbf{r}_1 - \mathbf{a}\sigma} \qquad (3)$$