

## Question 1

Section 2.2, Equation 2.18 of thesis

$$\frac{1}{H' - H_e \hat{n}_N} c_N^\dagger T = c_N^\dagger T \frac{1}{H' - H_h (1 - \hat{n}_N)}$$
$$\implies H_e \hat{n}_N c_N^\dagger T = c_N^\dagger T H_h (1 - \hat{n}_N)$$

This seems to **require**  $H'$  **commuting with**  $T$ , because

$$c_N^\dagger T H' - c_N^\dagger T H_h (1 - \hat{n}_N) = H' c_N^\dagger T - H_e \hat{n}_N c_N^\dagger T$$

**Why should  $H'$  commute with  $T$ ?**

(where  $H_e = \text{Tr}(H \hat{n}_N)$ ,  $H_h = \text{Tr}[H(1 - \hat{n}_N)]$  and  $T = \text{Tr}(H c_N)$ )

## Question 2

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^\dagger = H_h (1 - n_N)$$

If I try to derive this using the result on the previous slide:

$$\begin{aligned} \eta H \eta^\dagger &= \eta H_e \eta^\dagger = \eta H_e c^\dagger T G = \eta c^\dagger T H_h G \\ &= \eta c^\dagger T G H_h = \eta \eta^\dagger H_h = H_h (1 - \hat{n}) \end{aligned}$$

**That required  $[G, H_h] = 0$ . How does that work out?**

(where  $H_e = \text{Tr}(H \hat{n}_N)$ ,  $H_h = \text{Tr}[H(1 - \hat{n}_N)]$  and  $T = \text{Tr}(H c_N)$ )

### Question 3

Kondo Model appendix, Equation 9.61 of thesis

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[ S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 \leq j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} + \dots \right] \right]
 \end{aligned}$$

### Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[ S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

► The  $\tau$  should **not** be there in numerator i presume?

### Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[ S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

► Since coupling is  $\frac{J}{2}$ , shouldn't the thing be  $\frac{J^2}{4}$  instead of  $\frac{J^2}{2}$ ?

### Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j, \hat{s}_m, \beta}}{(2\omega_{\tau_{j, \hat{s}_m, \beta}} - \epsilon_{j, l\tau_{j, \hat{s}_m, \beta}} - J^{(j)} S^z s_{j, \hat{s}_m}^z)} \\
 & \times \left[ S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega_{\tau_{j, \hat{s}_m, \beta}} - \epsilon_{j, l\tau_{j, \hat{s}_m, \beta}} - J^{(j)} S^z s_{j, \hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

- You mentioned the following in the google document- "*interchange sigma\_a and sigma\_b (you get -1 sign)*". But these are matrix elements (numbers). So **why the minus sign?**

### Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[ S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} + \dots \right] \right]
 \end{aligned}$$

- How do you combine the product of two sigmas (  $\sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b$  ) into a single  $\sigma_{\alpha\gamma}^c$  ?

#### Question 4

Kondo URG coupling equation for  $J$  (equation 9.65):

$$\Delta J^{(j)} = n_j (J^{(j)})^2 \left[ \omega - \frac{\epsilon_{j,l}}{2} \right] \left[ \left( \frac{\epsilon_{j,l}}{2} - \omega \right)^2 - \frac{(J^{(j)})^2}{16} \right]^{-1}$$

One-loop form (after setting  $\omega = \epsilon_{j,l}$ ):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2 \frac{n_j (J^{(j)})^2}{\epsilon_{j,l}} \rightarrow \frac{2\rho |\Delta D| J^2}{D} \quad [n_j, \rho \rightarrow \text{DOS per spin}]$$

One-loop form in Coleman (Introduction to Many-Body Physics) ( $\tilde{J} = J/2$ ):

$$\Delta \tilde{J} = \frac{2\rho |\Delta D| \tilde{J}^2}{D} \implies \Delta J = \frac{\rho |\Delta D| J^2}{D}$$

**Is there any reason for this difference?**



## Question 5

- ▶ In the Kondo URG, are you considering **two electrons** on the shell  $\Lambda_N$ , one that we are decoupling ( $q\beta$ ) and another with the same momentum but **opposite spin** ( $q\bar{\beta}$ )?
- ▶ If so, why does that kinetic energy piece ( $\epsilon_{q\tau_{q\bar{\beta}}}$ ) not come down in the denominator?
- ▶ Is that what gives rise to the second RG equation and hence the  **$S^z s^z$  term** in the effective Hamiltonian?

## Question 6

$$\begin{aligned}
 \Delta H_{(j)}^2 &= \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} \right. \\
 &\quad \left. + S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} \right] \\
 &= \sum_{\substack{m=1, \\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} S^z \frac{\sigma_{\alpha\alpha}^z}{2} \left[ \hat{n}_{j,\hat{s}_m,\alpha} (1 - \hat{n}_{j,\hat{s}_m,\beta}) - \right. \\
 &\quad \left. \hat{n}_{j,\hat{s}_m,\beta} (1 - \hat{n}_{j,\hat{s}_m,\alpha}) \right]
 \end{aligned}$$

## Question 6

$$\begin{aligned}
 \Delta H_{(j)}^2 &= \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} \left[ S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} \right. \\
 &\quad \left. + S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} \right] \\
 &= \sum_{\substack{m=1, \\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} S^z \frac{\sigma_{\alpha\alpha}^z}{2} \left[ \hat{n}_{j,\hat{s}_m,\alpha} (1 - \hat{n}_{j,\hat{s}_m,\beta}) - \dots \right]
 \end{aligned}$$

What I got:

$$S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} = i^2 S^z \sigma_{\alpha\alpha}^z \hat{n}_{j,\hat{s}_m,\alpha} (1 - \hat{n}_{j,\hat{s}_m,\beta})$$

## Question 7

In eq. 2.21 of thesis,

$$UHU^\dagger = \frac{1}{2} \text{Tr}(H) + \tau \text{Tr}(H\tau) + \tau\{\mathbf{c}^\dagger T, \eta\}$$

so the renormalization is

$$\tau\{\mathbf{c}^\dagger T, \eta\} = \frac{1}{2} \left[ \overbrace{\mathbf{c}^\dagger T \eta}^{\text{particle sector}} - \underbrace{\eta \mathbf{c}^\dagger T}_{\text{hole sector}} \right] = \text{difference of the 2 sectors}$$

**Yet in most RG equations ( $\Delta H_F$  of 2d Hubbard,  $\Delta H_j$  of Kondo), you have *added* the two sectors. How/Why?**

## Question 8

In the Kondo URG, you simplify the  $\hat{\omega}$  as

$$\hat{\omega} = \omega \tau$$

**What is the formal way of doing this?** Shouldn't it be

$$\hat{\omega} = \omega_1 \hat{n} + \omega_1 (1 - \hat{n})$$

Is this just an assumption?

In the RG equation for BCS instability (eq. 8.130 of thesis), you use

$$G^{-1} = \omega - \epsilon_1 \tau_1 - \epsilon_2 \tau_2$$

**How is this choice of  $\hat{\omega}$  consistent with what was done in Kondo URG?**