Kondo Model appendix, Equation 9.61 of thesis

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)}$$

$$\times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1,j_2<j), \\ n,o}} c_{j_1,\hat{s}_n,\alpha}^\dagger c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + ... \right.$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} + ... \right.$$

$$\Delta\hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}$$

$$\times \Bigg[S^a S^b \sigma^a_{\alpha\beta} \sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j), \\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + ...$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})} \Bigg[S^x S^y \sigma^x_{\alpha\beta} \sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + ...$$

- **The τ should not be there in numerator i presume?**

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)}$$

$$\times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1,j_2<j), \\ n,o}} c_{j_1,\hat{s}_n,\alpha}^\dagger c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + ... \right.$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} + ... \right.$$

▶ **Since coupling is $\frac{J}{2}$, shouldn't the thing be $\frac{J^2}{4}$ instead of $\frac{J^2}{2}$?**

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j), \\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + ... \right.$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})} \left[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + ... \right.$$

- ► **You mentioned the following in the google document- "*interchange sigma_a and sigma_b (you get -1 sign)*". But these are matrix elements (numbers). So why the minus sign?**

$$\Delta \hat{H}_{(j)} = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})}$$

$$\times \left[S^a S^b \sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma} \sum_{\substack{(j_1,j_2<j), \\ n,o}} c^\dagger_{j_1,\hat{s}_n,\alpha} c_{j_2,\hat{s}_o,\gamma}(1 - \hat{n}_{j,\hat{s}_m,\beta}) + ... \right.$$

$$+ \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s^z_{j,\hat{s}_m})} \left[S^x S^y \sigma^x_{\alpha\beta}\sigma^y_{\beta\alpha} c^\dagger_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\beta} c^\dagger_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\alpha} + ... \right.$$

▶ **How do you combine the product of two sigmas ($\sigma^a_{\alpha\beta}\sigma^b_{\beta\gamma}$) into a single $\sigma^c_{\alpha\gamma}$?**

Kondo URG coupling equation for J (equation 9.65):

$$\Delta J^{(j)} = n_j (J^{(j)})^2 \left[\omega - \frac{\epsilon_{j,l}}{2} \right] \left[(\frac{\epsilon_{j,l}}{2} - \omega)^2 - \frac{(J^{(j)})^2}{16} \right]^{-1}$$

One-loop form (after setting $\omega = \epsilon_{j,l}$):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2 \frac{n_j (J^{(j)})^2}{\epsilon_{j,l}} \to \frac{2\rho |\Delta D| J^2}{D} \quad [n_j, \rho \to \text{DOS per spin}]$$

One-loop form in Coleman (Introduction to Many-Body Physics) ($\tilde{J} = J/2$):

$$\Delta \tilde{J} = \frac{2\rho |\Delta D| \tilde{J}^2}{D} \implies \Delta J = \frac{\rho |\Delta D| J^2}{D}$$

Is there any reason for this difference?

▶ In the Kondo URG, are you considering two electrons on the shell Λ_N, one that we are decoupling ($q\beta$) and another with the same momentum but opposite spin ($q\overline{\beta}$)?

▶ If so, why does that kinetic energy piece ($\epsilon_q \tau_{q\overline{\beta}}$) not come down in the denominator?

▶ Is that what gives rise to the second RG equation and hence the $S^z s^z$ term in the effective Hamiltonian?

$$\Delta H_{(j)}^2 = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} \right.$$

$$\left. + S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} \right]$$

$$= \sum_{\substack{m=1, \\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} S^z \frac{\sigma_{\alpha\alpha}^z}{2} \left[\hat{n}_{j,\hat{s}_m,\alpha}(1 - \hat{n}_{j,\hat{s}_m,\beta}) - \right.$$

$$\left. \hat{n}_{j,\hat{s}_m,\beta}(1 - \hat{n}_{j,\hat{s}_m,\alpha}) \right]$$

$$\Delta H_{(j)}^2 = \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \Bigg[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha}$$

$$+ S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} \Bigg]$$

$$= \sum_{\substack{m=1, \\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} S^z \frac{\sigma_{\alpha\alpha}^z}{2} \Bigg[\hat{n}_{j,\hat{s}_m,\alpha}(1-\hat{n}_{j,\hat{s}_m,\beta}) - ...$$

What I got:

$$S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} = i^2 S^z \sigma_{\alpha\alpha}^z \hat{n}_{j,\hat{s}_m,\alpha} \left(1 - \hat{n}_{j,\hat{s}_m,\beta}\right)$$

Section 2.2, Equation 2.18 of thesis

$$\frac{1}{H' - H_e \hat{n}_N} c_N^\dagger T = c_N^\dagger T \frac{1}{H' - H_h(1 - \hat{n}_N)}$$
$$\implies H_e \hat{n}_N c_N^\dagger T = c_N^\dagger T H_h(1 - \hat{n}_N)$$

This seems to **require** H' **commuting with** T, because

$$c_N^\dagger T H' - c_N^\dagger T H_h(1 - \hat{n}_N) = H' c_N^\dagger T - H_e \hat{n}_N c_N^\dagger T$$

Why should H' commute with T?

$$(\text{where } H_e = Tr\,(H\hat{n}_N)\,,\ H_h = Tr\,[H\,(1 - \hat{n}_N)]\ \text{ and }\ T = Tr\,(Hc_N))$$

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^\dagger = H_h (1 - n_N)$$

If I try to derive this using the result on the previous slide:

$$\eta H \eta^\dagger = \eta H_e \eta^\dagger = \eta H_e c^\dagger T G = \eta c^\dagger T H_h G$$
$$= \eta c^\dagger T G H_h = \eta \eta^\dagger H_h = H_h (1 - \hat{n})$$

That required $[G, H_h] = 0$. How does that work out?

(where $H_e = Tr(H \hat{n}_N)$, $H_h = Tr[H(1 - \hat{n}_N)]$ and $T = Tr(H c_N)$)

In eq. 2.21 of thesis,

$$UHU^\dagger = \frac{1}{2} Tr(H) + \tau Tr(H\tau) + \tau\{c^\dagger T, \eta\}$$

so the renormalization is

$$\tau\{c^\dagger T, \eta\} = \frac{1}{2} \left[\overbrace{c^\dagger T \eta}^{\text{particle sector}} - \underbrace{\eta c^\dagger T}_{\text{hole sector}} \right] = \text{difference of the 2 sectors}$$

Yet in most RG equations ($\triangle H_F$ of 2d Hubbard, $\triangle H_j$ of Kondo), you have *added* the two sectors. How/Why?

In the Kondo URG, you simplify the $\hat{\omega}$ as

$$\hat{\omega} = \omega\tau$$

What is the formal way of doing this? Shouldn't it be

$$\hat{\omega} = \omega_1\hat{n} + \omega_1(1 - \hat{n})$$

Is this just an assumption?

In the RG equation for BCS instability (eq. 8.130 of thesis), you use

$$G^{-1} = \omega - \epsilon_1\tau_1 - \epsilon_2\tau_2$$

How is this choice of $\hat{\omega}$ consistent with what was done in Kondo URG?