Unitary Renormalization Group Approach to the Single-Impurity Anderson model

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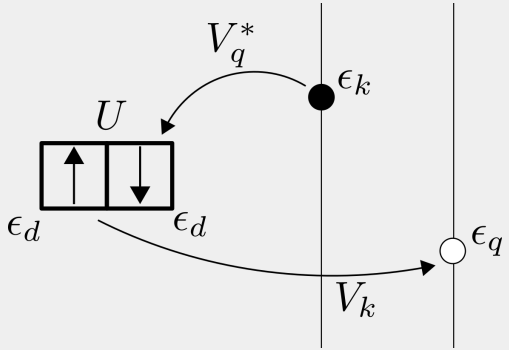
Outline

- The model
- Motivation
- Unitary Renormalization Group (URG) formalism
- Results

The Single-Impurity Anderson Model

$$\mathcal{H}_{\text{siam}} = \sum_{k\sigma} \epsilon_k \hat{n}_{k\sigma} + \sum_{k\sigma} \left[V(k) c^\dagger_{k\sigma} c_{d\sigma} + \text{h.c.} \right] + \epsilon_d \sum_\sigma \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$



Motivation

- "Poor man's" scaling[1] is *perturbative* and fails at large values - cannot show strong-coupling (SC) fixed point.

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- Instead, one needs to flow to large value of U, do a Schrieffer -Wolff transformation and then flow to the SC fixed point.
- **It would be nice to get a single set of equations that show the crossover to the strong-coupling fixed point.**

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- NRG cannot show *how the Hamiltonians and many-body wavefunctions vary along the flow* - projective in nature.
- **It would be enlightening to see the flow into SC regime by tracking the change in entanglement** - hence we need wavefunctions.

The Short Version

Apply *unitary many-body transformations* to the Hamiltonian so as to successively *decouple* high energy states and hence obtain scaling equations.

$$\mathcal{H}(D)$$

$$\Downarrow$$

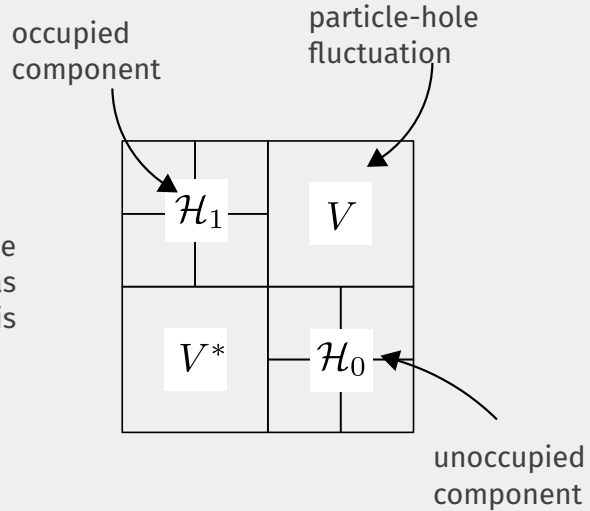
$$U\mathcal{H}(D)U^{\dagger}$$

$$\Downarrow$$

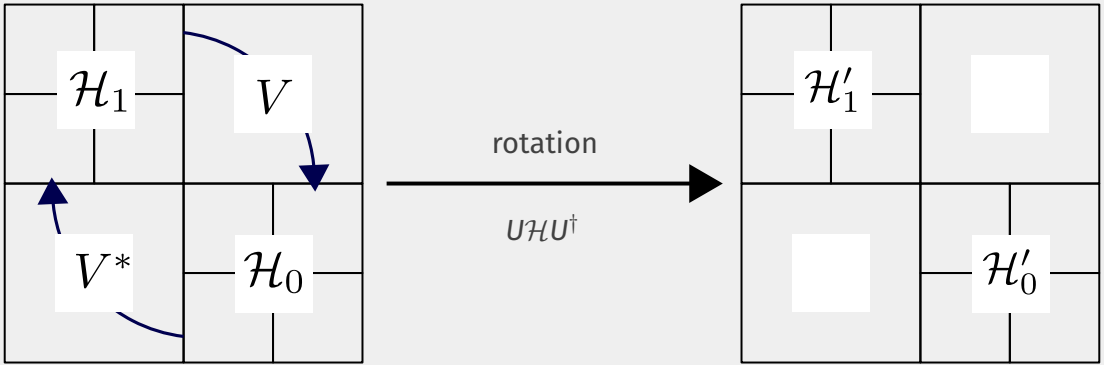
$$H(D) + \mathcal{H}'(D - \delta D)$$

Step 1:

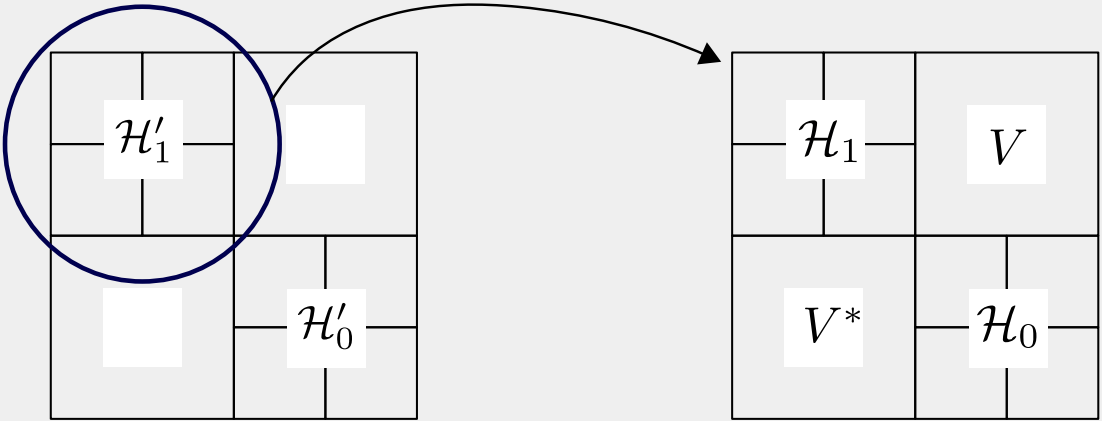
Start with the electrons farthest from the Fermi surface. Write the Hamiltonian as *diagonal and off-diagonal terms* in this basis.



Step 2:
Rotate the Hamiltonian to kill the off-diagonal blocks.



Step 3:
Repeat the process with the new blocks.



Results

$$\mathcal{H} = \sum_{k\sigma} \epsilon_k \hat{n}_{k\sigma} + \sum_{k\sigma} \left[V(k) c^{\dagger}_{k\sigma} c_{d\sigma} + \text{h.c.} \right] + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + J \vec{S_d} \cdot \sum_{kq\alpha\beta} \vec{\sigma}_{\alpha,\beta} c^{\dagger}_{k\alpha} c_{q\beta}$$