

Question 1

Section 2.2, Equation 2.18 of thesis

$$\frac{1}{H' - H_e \hat{n}_N} c_N^\dagger T = c_N^\dagger T \frac{1}{H' - H_h (1 - \hat{n}_N)}$$
$$\implies H_e \hat{n}_N c_N^\dagger T = c_N^\dagger T H_h (1 - \hat{n}_N)$$

This seems to **require** H' **commuting with** T , because

$$c_N^\dagger T H' - c_N^\dagger T H_h (1 - \hat{n}_N) = H' c_N^\dagger T - H_e \hat{n}_N c_N^\dagger T$$

Why should H' commute with T ?

(where $H_e = \text{Tr}(H \hat{n}_N)$, $H_h = \text{Tr}[H(1 - \hat{n}_N)]$ and $T = \text{Tr}(H c_N)$)

Question 2

Section 2.2, Equation 2.19 of thesis

$$\eta_N H \eta_N^\dagger = H_h (1 - n_N)$$

If I try to derive this using the result on the previous slide:

$$\begin{aligned} \eta H \eta^\dagger &= \eta H_e \eta^\dagger = \eta H_e c^\dagger T G = \eta c^\dagger T H_h G \\ &= \eta c^\dagger T G H_h = \eta \eta^\dagger H_h = H_h (1 - \hat{n}) \end{aligned}$$

That required $[G, H_h] = 0$. How does that work out?

(where $H_e = \text{Tr}(H \hat{n}_N)$, $H_h = \text{Tr}[H(1 - \hat{n}_N)]$ and $T = \text{Tr}(H c_N)$)

Question 3

Kondo Model appendix, Equation 9.61 of thesis

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 \leq j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 \leq j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

► The τ should **not** be there in numerator i presume?

Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

► Since coupling is $\frac{J}{2}$, shouldn't the thing be $\frac{J^2}{4}$ instead of $\frac{J^2}{2}$?

Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega_{\tau_{j,\hat{s}_m,\beta}} - \epsilon_{j,l\tau_{j,\hat{s}_m,\beta}} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega_{\tau_{j,\hat{s}_m,\beta}} - \epsilon_{j,l\tau_{j,\hat{s}_m,\beta}} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j, \hat{s}_m, \alpha}^\dagger c_{j, \hat{s}_m, \beta} c_{j, \hat{s}_m, \beta}^\dagger c_{j, \hat{s}_m, \alpha} + \dots \right] \right]
 \end{aligned}$$

- You mentioned the following in the google document- "*interchange sigma_a and sigma_b (you get -1 sign)*". But these are matrix elements (numbers). So **why the minus sign?**

Question 3

$$\begin{aligned}
 \Delta \hat{H}_{(j)} = & \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2} \frac{\tau_{j,\hat{s}_m,\beta}}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \\
 & \times \left[S^a S^b \sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b \sum_{\substack{(j_1, j_2 < j), \\ n, o}} c_{j_1, \hat{s}_n, \alpha}^\dagger c_{j_2, \hat{s}_o, \gamma} (1 - \hat{n}_{j, \hat{s}_m, \beta}) + \dots \right. \\
 & \left. + \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{2(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)}S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} + \dots \right] \right]
 \end{aligned}$$

- How do you combine the product of two sigmas ($\sigma_{\alpha\beta}^a \sigma_{\beta\gamma}^b$) into a single $\sigma_{\alpha\gamma}^c$?

Question 4

Kondo URG coupling equation for J (equation 9.65):

$$\Delta J^{(j)} = n_j (J^{(j)})^2 \left[\omega - \frac{\epsilon_{j,l}}{2} \right] \left[\left(\frac{\epsilon_{j,l}}{2} - \omega \right)^2 - \frac{(J^{(j)})^2}{16} \right]^{-1}$$

One-loop form (after setting $\omega = \epsilon_{j,l}$):

$$\Delta J^{(j)} = \frac{n_j (J^{(j)})^2}{\omega - \frac{\epsilon_{j,l}}{2}} = 2 \frac{n_j (J^{(j)})^2}{\epsilon_{j,l}} \rightarrow \frac{2\rho |\Delta D| J^2}{D} \quad [n_j, \rho \rightarrow \text{DOS per spin}]$$

One-loop form in Coleman (Introduction to Many-Body Physics) ($\tilde{J} = J/2$):

$$\Delta \tilde{J} = \frac{2\rho |\Delta D| \tilde{J}^2}{D} \implies \Delta J = \frac{\rho |\Delta D| J^2}{D}$$

Is there any reason for this difference?

Question 5

- ▶ In the Kondo URG, are you considering **two electrons** on the shell Λ_N , one that we are decoupling ($q\beta$) and another with the same momentum but **opposite spin** ($q\bar{\beta}$)?
- ▶ If so, why does that kinetic energy piece ($\epsilon_{q\tau_{q\bar{\beta}}}$) not come down in the denominator?
- ▶ Is that what gives rise to the second RG equation and hence the **$S^z s^z$ term** in the effective Hamiltonian?

Question 6

$$\begin{aligned}
 \Delta H_{(j)}^2 &= \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} \right. \\
 &\quad \left. + S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} \right] \\
 &= \sum_{\substack{m=1, \\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} S^z \frac{\sigma_{\alpha\alpha}^z}{2} \left[\hat{n}_{j,\hat{s}_m,\alpha} (1 - \hat{n}_{j,\hat{s}_m,\beta}) - \right. \\
 &\quad \left. \hat{n}_{j,\hat{s}_m,\beta} (1 - \hat{n}_{j,\hat{s}_m,\alpha}) \right]
 \end{aligned}$$

Question 6

$$\begin{aligned}
 \Delta H_{(j)}^2 &= \sum_{\substack{m=1, \\ \beta=\uparrow/\downarrow}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} \left[S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} \right. \\
 &\quad \left. + S^y S^x \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} \right] \\
 &= \sum_{\substack{m=1, \\ \beta}}^{n_j} \frac{(J^{(j)})^2}{(2\omega\tau_{j,\hat{s}_m,\beta} - \epsilon_{j,l}\tau_{j,\hat{s}_m,\beta} - J^{(j)} S^z s_{j,\hat{s}_m}^z)} S^z \frac{\sigma_{\alpha\alpha}^z}{2} \left[\hat{n}_{j,\hat{s}_m,\alpha} (1 - \hat{n}_{j,\hat{s}_m,\beta}) - \dots \right]
 \end{aligned}$$

What I got:

$$S^x S^y \sigma_{\alpha\beta}^x \sigma_{\beta\alpha}^y c_{j,\hat{s}_m,\alpha}^\dagger c_{j,\hat{s}_m,\beta} c_{j,\hat{s}_m,\beta}^\dagger c_{j,\hat{s}_m,\alpha} = i^2 S^z \sigma_{\alpha\alpha}^z \hat{n}_{j,\hat{s}_m,\alpha} (1 - \hat{n}_{j,\hat{s}_m,\beta})$$

Question 7

In eq. 2.21 of thesis,

$$UHU^\dagger = \frac{1}{2} \text{Tr}(H) + \tau \text{Tr}(H\tau) + \tau\{\mathbf{c}^\dagger T, \eta\}$$

so the renormalization is

$$\tau\{\mathbf{c}^\dagger T, \eta\} = \frac{1}{2} \left[\overbrace{\mathbf{c}^\dagger T \eta}^{\text{particle sector}} - \underbrace{\eta \mathbf{c}^\dagger T}_{\text{hole sector}} \right] = \text{difference of the 2 sectors}$$

Yet in most RG equations (ΔH_F of 2d Hubbard, ΔH_j of Kondo), you have *added* the two sectors. How/Why?

Question 8

In the Kondo URG, you simplify the $\hat{\omega}$ as

$$\hat{\omega} = \omega \tau$$

What is the formal way of doing this? Shouldn't it be

$$\hat{\omega} = \omega_1 \hat{n} + \omega_1 (1 - \hat{n})$$

Is this just an assumption?

In the RG equation for BCS instability (eq. 8.130 of thesis), you use

$$G^{-1} = \omega - \epsilon_1 \tau_1 - \epsilon_2 \tau_2$$

How is this choice of $\hat{\omega}$ consistent with what was done in Kondo URG?