Everything is in interaction picture. Hamiltonian is $H = H_0 + V$.

$$
\begin{aligned}
U(t) &= e^{iH_0 t}e^{-iHt} \\
\psi(t) &= S(t,t')\psi(t') \\
S(t,t') &= U(t)U^\dagger(t') \\
i\frac{dS(t,t')}{dt} &= V(t)S(t,t') \\
S(t,t') &= \hat{T}\exp\left(-i\int_{t'}^{t} d\tau V(\tau)\right) \\
&= \sum_n \frac{(-i)^n}{n!}\int_{-\infty}^{\infty} d\tau_1 ... d\tau_n \hat{T}\left[V(\tau_1)...V(\tau_n)\right] \\
G(t-t') &= -i\left\langle 0\right| \hat{T}c(t)c^\dagger(t')\left|0\right\rangle \rightarrow \text{1-particle G.F.} \\
G(t_1...t_n;t'_1...t'_n) &= (-i)^n\left\langle 0\right| \hat{T}c(t_1)...c(t_n)c^\dagger(t'_1)...c^\dagger(t'_n)\left|0\right\rangle \rightarrow \text{n-particle G.F.} \\
&= \sum_{\{P'_i\}} G(t_1-P'_1)...G(t_n-P'_n) \rightarrow \text{Wick's theorem} \\
\left\langle \phi_0\right| S\left|\phi_0\right\rangle &= \sum_n \frac{(-i)^n}{n!}\int_{-\infty}^{\infty} d\tau_1 ... d\tau_n \left\langle \phi_0\right| \hat{T}\left[V(\tau_1)...V(\tau_n)\right]\left|\phi_0\right\rangle \\
&= \sum_n \frac{(-i)^n}{n!}\int_{-\infty}^{\infty} d\tau_1 ... d\tau_n \sum_{\text{contradictions}} G_1G_2...G_n
\end{aligned}
\tag{1}
$$