- $\mathcal{P}_1 = |1010\rangle\langle 1010|$ (up-polarized) : $\mathcal{H}_{N=2,up} = 0$

- $\mathcal{P}_2 = |0101\rangle\langle 0101|$ (down-polarized) : $\mathcal{H}_{N=2,down} = 0$

- $\mathcal{P}_3 = |1100\rangle\langle 1100| + |1001\rangle\langle 1001|$: $\mathcal{H}_{N=2,mix} = \begin{pmatrix} U & 2t & & \\ 2t & 0 & & \\ & & U & \\ & & & 0 \end{pmatrix}$

$$U^\dagger \mathcal{H}_{N=2,mix} U = \begin{pmatrix} \frac{U+\sqrt{U^2+16t^2}}{2} & & & \\ & \frac{U-\sqrt{U^2+16t^2}}{2} & & \\ & & U & \\ & & & 0 \end{pmatrix} \tag{1}$$

Combining,

$$\mathcal{H}^{rot}_{N=2} = \begin{pmatrix} 0 & & & & & \\ & \frac{U+\sqrt{U^2+16t^2}}{2} & & & & \\ & & \frac{U-\sqrt{U^2+16t^2}}{2} & & & \\ & & & U & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix} \simeq \text{(large U)} \begin{pmatrix} 0 & & & & & \\ & U + \frac{4t^2}{U} & & & & \\ & & -\frac{4t^2}{U} & & & \\ & & & U & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix} \tag{2}$$

$$\mathcal{H}_{heis} = \begin{pmatrix} 0 & & & \\ & -2J & J & \\ & J & -2J & \\ & & & 0 \end{pmatrix} = \begin{pmatrix} 0 & & & \\ & 2\frac{t^2}{U} & \frac{-t^2}{U} & \\ & \frac{-t^2}{U} & 2\frac{t^2}{U} & \\ & & & 0 \end{pmatrix} \rightarrow_{\text{(diagonalising)}} \begin{pmatrix} 0 & & & \\ & -4\frac{t^2}{U} & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \tag{3}$$