Emergence in free and correlated fermions: from impurity models to the bulk

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Summary of Work

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1. 1-channel Kondo problem: *as second author*, **published** in Phys. Rev. B
2. Multi-channel Kondo problem: *as second author*, **under review** at Phys. Rev. B
3. Generalised Anderson impurity model: manuscript **in preparation**
4. Entanglement scaling in free fermions: manuscript **in preparation**
5. New auxiliary model approach to correlated systems: **ongoing project**

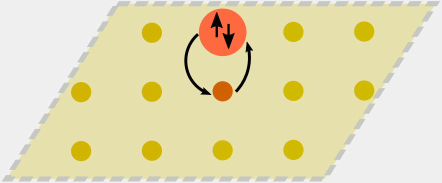
Single-channel Kondo problem

Model of impurity interacting with conduction electrons through spin-flips

1. Computation of the impurity spectral function

2. Emergence of a local Fermi liquid, and orthogonality catastrophe between local moment and singlet states

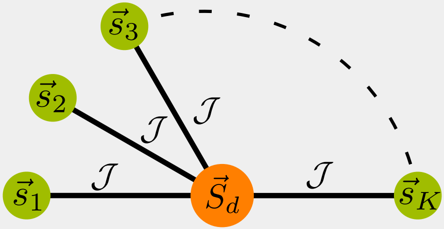
3. Calculating of thermal entropy



Multi-channel Kondo problem

Model of impurity interacting with multiple conduction electron channels

1. Obtaining RG fixed point Hamiltonian
2. Analytical forms for degree of compensation, magnetization and susceptibility
3. Presence of a local marginal Fermi liquid
4. Dualities of the MCK model



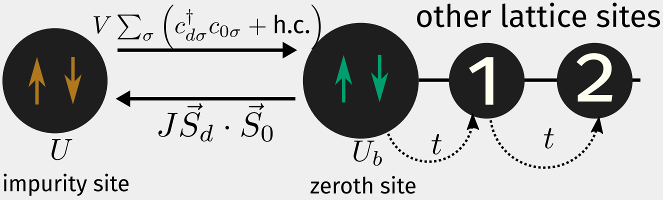
Local metal-insulator transition in extended Anderson impurity model

EXTENDED ANDERSON IMPURITY MODEL

Standard Anderson impurity model $\longrightarrow$ only one stable phase (strong-coupling)

no possibility of phase transition $\longrightarrow$ Introduce additional correlation

- spin-flip correlation between impurity and bath: J
- local correlation in the bath: U_b

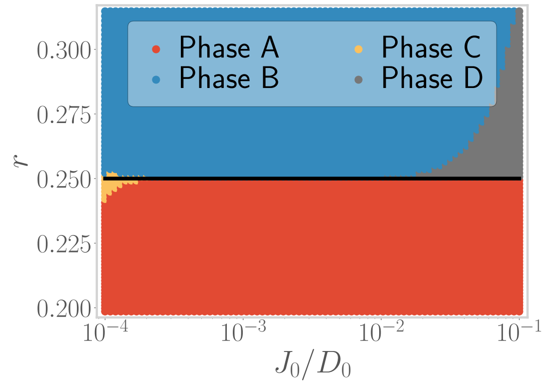


RG Phase Diagram

RG equations reveal critical point where J, V **become irrelevant**

1. orange phase: J is relevant: strong-coupling
2. blue phase: J is irrelevant: local moment
3. yellow phase: spin+charge liquid
4. gray phase: all couplings irrelevant

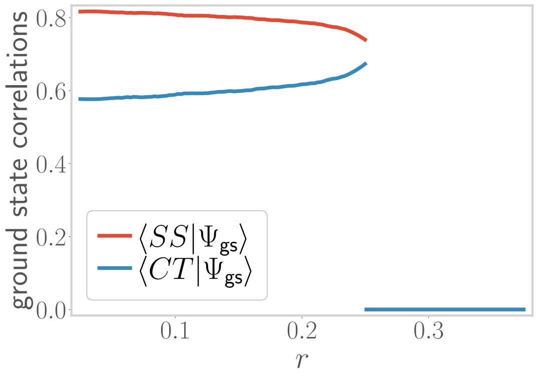
$r = U_b / J$

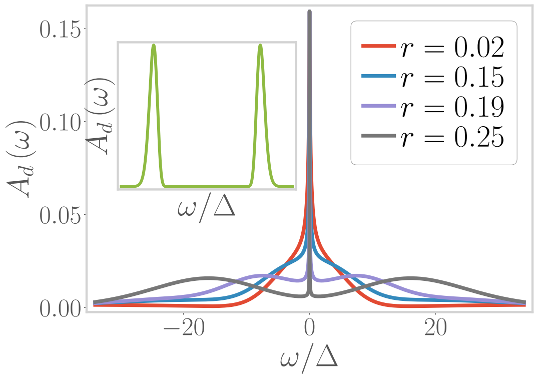


Presence of a phase transition

singlet ⟶ spin+charge liquid ⟶ local moment

impurity spectral function gaps out

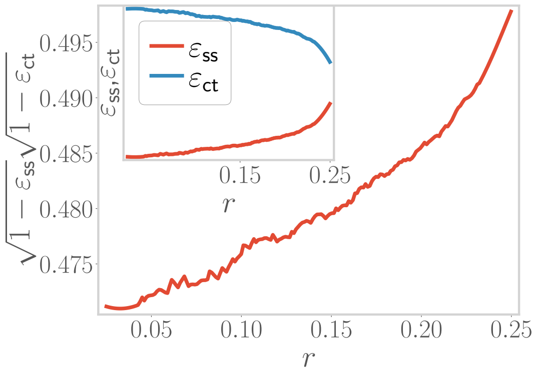


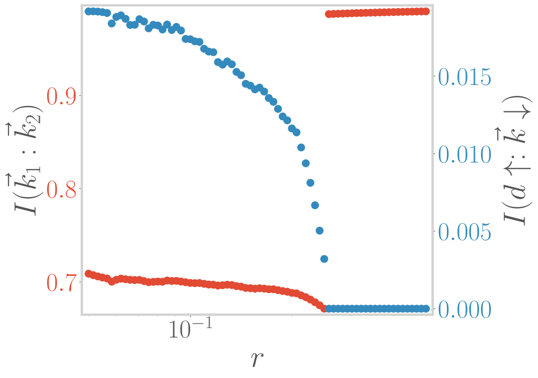


Geometric entanglement: $\varepsilon(\psi_1, \psi_2) = 1 - |\langle \psi_1 | \psi_2 \rangle|^2$

$\longrightarrow \sqrt{1 - \varepsilon_{SS}}\sqrt{1 - \varepsilon_{CT}}$ is maximised, then vanishes

Mutual information between impurity and cloud vanishes





Entanglement scaling in free fermions

Free Dirac fermions on torus: $k_x^n = \dfrac{2\pi}{L_x} n, \quad n \in \mathbb{Z};$ define **sparsity** $= \Delta n = 1$

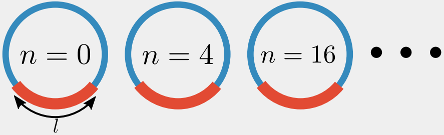
Simplest choice: the entire set

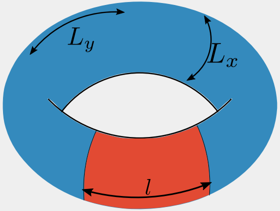
sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \dots, -1, 0, 1, \dots, N-2, N-1, N\}$

Coarser choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \dots, -2, 0, 2, \dots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \dots, -4, 0, 4, \dots, N-8, N-4, N\}$





$$S_{A_z(j)} = f_z(j)c\alpha L_x - c\log\left|2\sin\left(\pi f_z(j)\phi\right)\right|$$

$$i < j, \quad S_{i\cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$

- presents a **hierarchy** of entanglement $\longrightarrow$ EE distributed across RG steps
  RG transformation $\longrightarrow$ reveals entanglement

- distribution of entanglement also present in **multipartite** entanglement

Calabrese et al. 2004; Casini et al. 2005; Arias et al. 2015; Chen et al. 2017; Murciano et al. 2020.

Mutual information = distance

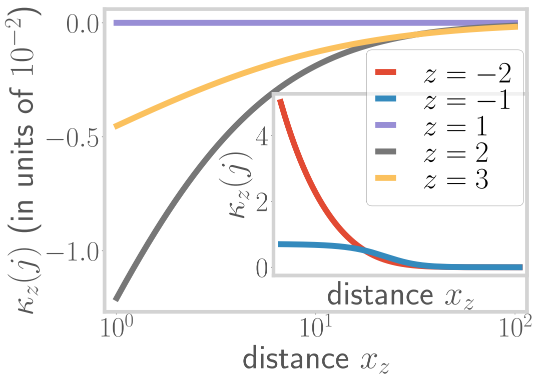
Mutual information: $I^2(A:B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j)/\Delta x_z(j), \quad v' = \Delta v_z(j)/\Delta x_z(j)$$

Curvature as well: $\kappa_z(j) = \dfrac{v_z'(j)}{\left[1 + v_z(j)^2\right]^{\frac{3}{2}}}$



Van Raamsdonk 2010; Lee et al. 2016; Mukherjee et al. 2022; Lee 2010; Lee 2014; Qi 2013; Lee et al. 2016; Mukherjee et al. 2020a; Mukherjee et al. 2020b; Ryu et al. 2006b; Ryu et al. 2006a; Nozaki et al. 2012.

RG evolution = emergent distance

- Distances and curvature can be related to an RG **beta function**
- Amounts to an **explicit demonstration** of the holographic principle
- Sign of curvature is **topological**, can be written in terms of winding numbers

Topological nature of geometry-independent term

$$S_{A_z(j)} = f_z(j) c \alpha L_x - \underbrace{c \log \left| 2 \sin\left(\pi f_z(j) \phi\right) \right|}_{=Q(\phi), \text{geometry-independent term}}$$

- $Q(\phi)$ is periodic in the flux ϕ, $\phi = 1$ transports a charge across Fermi surface
- pole structure of $\left(\sin\frac{\pi}{4} - |\sin\left(\pi f_z(j)\right)\phi|\right)^{-1}$ counts number of states $\longrightarrow$ tracks Luttinger volume
- Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers