Research Progress Report: 2023 - 2024

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Publications and Ongoing projects

Currently in progress

- Development of auxiliary model-based method for interacting electronics.
- Studies of the plateau-to-plateau transition in integer quantum hall systems.

Published

- 2023 New J. Phys. 25 113011
- 2024 J. Phys. A: Math. Theor. 57 275401
- 2022 Phys. Rev. B 105, 085119
- 2023 J. Phys.: Condens. Matter 35 315601

Ongoing collaborations

- Breakdown of Kondo screening in presence of magnetic field
- Quantum critical Mott MIT in a three-orbital impurity model
- Universal features of Kondo breakdown in quantum impurity models

Ongoing collaborations

Breakdown of Kondo Screening in Presence of Magnetic Field

Analysis of effect of impurity magnetic field on Kondo screening
(D Debata, A Mukherjee, S Lal) [*in preparation*]

- Impurity undergoes localisation **transition** at large B, crit. point has non-Fermi liquid excitations.
- Models the effects of measurement on quantum system + **fermionic bath**

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Quantum critical Mott MIT in a Three-Orbital Impurity Model

Search for impurity model with a quantum critical phase intercepting a local MIT (Aashish Kumar, D Debata, A Mukherjee, N. S. Vidhyadhiraja and S Lal) [*in preparation*]

- **QC phase** indeed obtained with composite excitations and pseudogapped spectral function.
- **Self-energy** exponents characterise the non-Fermi liquid

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Universal Features of Kondo Breakdown in Impurity Models

A unified framework for Kondo breakdown, in terms of entanglement measures (D Debata, A Mukherjee, S Lal) [*in preparation*]

- Universal signatures of Kondo breakdown include **partial magnetisation** and phase shift.
- Entanglement within the **Kondo cloud** also suffers at Kondo breakdown.

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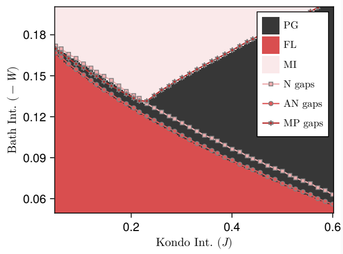
Project I: A New Auxiliary Model Approach to Systems of Interacting Electrons

Broad Objectives

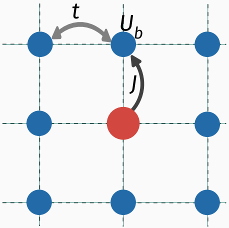
- Designing a **new method** by which to leverage quantum impurity models towards studying lattice models of interacting electrons
- Using such a method to go after the **Mott-Hubbard MIT** on the 2D square lattice
- Capturing the enhanced effects of $k-$**space anisotropy** (due to the square lattice) on signatures near the transition
- Studying the (presumably) **non-Fermi liquid behaviour** in the excitations close to and at the transition

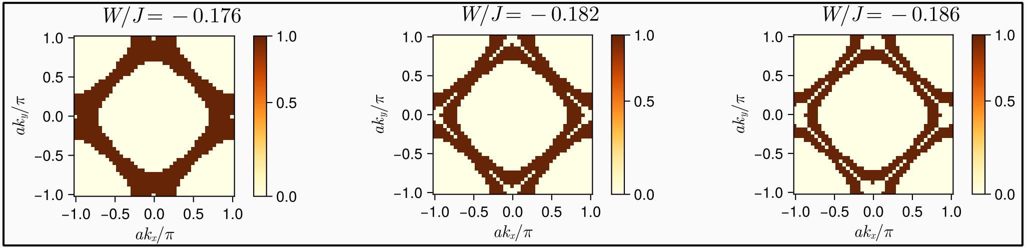
Momentum-Resolved Renormalisation Group Flows



Hamiltonian RG equations of
embedded e-SIAM

$$\Delta J^{(j)}_{\mathbf{k}_1,\mathbf{k}_2} = -\sum_{\mathbf{q}\in \text{PS}} \frac{J^{(j)}_{\mathbf{k}_2,\mathbf{q}} J^{(j)}_{\mathbf{q},\mathbf{k}_1} + 4 J^{(j)}_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}}}{\omega - \frac{1}{2}|\varepsilon_j| + J^{(j)}_{\mathbf{q}}/4 + W_{\mathbf{q}}/2}$$





‘Periodising’ the Hamiltonian and Eigenstates

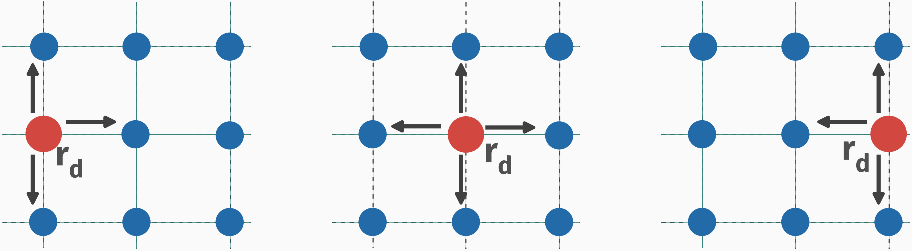
Periodising the Hamiltonian creates a **Hubbard-Heisenberg** model:

$$H_{\text{tiled}} = \sum_{\mathbf{r}} T^{\dagger}(\mathbf{r} - \mathbf{r}_d) H_{\text{aux}}(\mathbf{r}_d) T(\mathbf{r} - \mathbf{r}_d)$$

Wavefunctions can be related using a many-body **Bloch's theorem** :

$$|\Psi_{\text{gs}}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}_d} e^{i\mathbf{k}\cdot\mathbf{r}_d} |\psi_{\text{gs}}(\mathbf{r}_d)\rangle$$

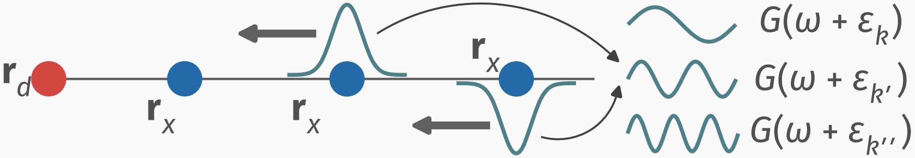
$$H_{\text{tiled}} = -\frac{\tilde{t}}{\sqrt{Z}} \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle ; \sigma} \left(c^{\dagger}_{\mathbf{r}_i,\sigma} c_{\mathbf{r}_j,\sigma} + \text{h.c.} \right) + \frac{\tilde{J}}{Z} \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle} \mathbf{S}_{\mathbf{r}_i} \cdot \mathbf{S}_{\mathbf{r}_j} - \frac{\tilde{U}}{2} \sum_{\mathbf{r}} \left(\hat{n}_{\mathbf{r},\uparrow} - \hat{n}_{\mathbf{r},\downarrow} \right)^2$$



Periodising the Greens Functions

Greens function = sum of 1-particle **k-space** Greens functions starting from **all sites** in impurity model.

$$\tilde{G}(\mathbf{r};\tilde{\omega}) = \frac{1}{N}\sum_{\mathbf{k},\mathbf{r}_x}\Big[e^{i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_p\left(\mathbf{r}_x;\omega+\varepsilon_{\mathbf{k}}\right) + e^{-i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_h\left(\mathbf{r}_x;\omega-\varepsilon_{\mathbf{k}}\right)\Big]$$



$$\tilde{A}(\mathbf{K};\omega) = -\frac{1}{\pi}\mathrm{Im}\left[\tilde{G}(\mathbf{K};\tilde{\omega})\right]$$

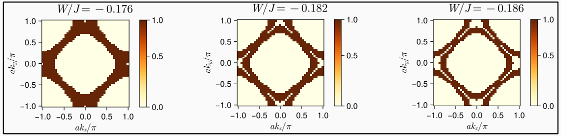
$$\tilde{\Sigma}(\mathbf{K};\omega) = \left(\tilde{G}^{(0)}(\mathbf{K};\tilde{\omega})\right)^{-1} - \left(\tilde{G}(\mathbf{K};\tilde{\omega})\right)^{-1}$$

Subsequently allows periodising spectral functions and self-energies

Periodising Correlation Functions and Entanglement Measures

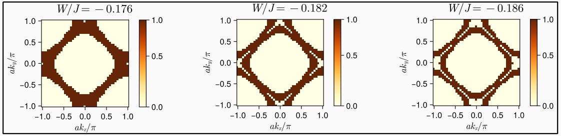
k-space spin-spin correlation

$$\tilde{S}_{\text{flip}}(\mathbf{K}_1, \mathbf{K}_2) = \frac{1}{2}\left[\sqrt{\langle S^+(\mathbf{d})\, S^-(\mathbf{K}_2)\rangle \langle S^-(\mathbf{d})\, S^+(\mathbf{K}_1)\rangle} + \text{h.c.}\right]$$



k-space reduced density matrix

$$\overline{\rho}_{\mathbf{K},\sigma} = \frac{1}{2}\left[c^\dagger_{\mathbf{K},\sigma}\rho_{\text{gs}}(\mathbf{r}_c)c_{\mathbf{r}_c,\sigma} + c^\dagger_{\mathbf{r}_c,\sigma}\rho_{\text{gs}}(\mathbf{r}_c)c_{\mathbf{K},\sigma}\right] + \text{h.c.}$$



Outstanding Questions

- A better understanding of the mechanism of the pseudogap phase diagram
- Calculation of spectral functions and self-energies
- Characterisation of non-Fermi liquid behaviour in the pseudogapped region

Project II: Search for Punctured-Chern Topology at IQHE Transitions

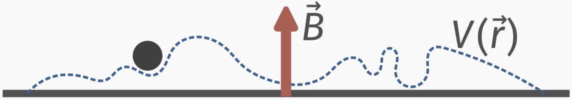
Broad Objectives

- Obtaining the **IQHE phase diagram** from a model of 2D lattice electrons
- Characterising the plateau-to-plateau transition **critical point** through a topological invariant
- Checking the robustness of our conclusions to the addition of **disorder**

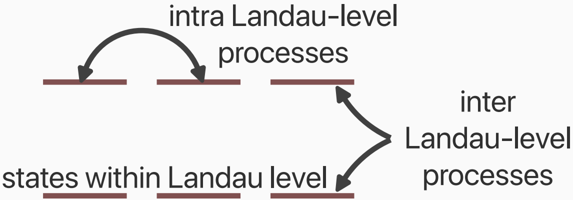
The Model

Non-interacting electrons, magnetic field, one-particle potential

$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 + V(\mathbf{r})$$



In the absence of $V(\mathbf{r})$, produces decoupled Landau levels with large degeneracy.

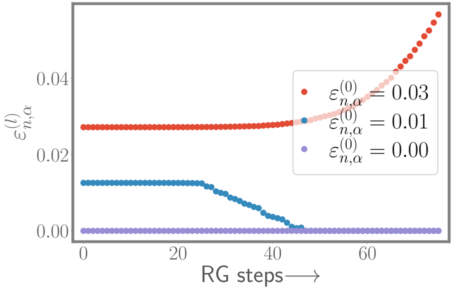


$V(\mathbf{r})$ leads to scattering among these states.

URG Analysis of Intra Landau-level Processes

$$H_n^* = \sum_{\varepsilon_{n,\alpha}\sim 0} \varepsilon_{n,\alpha} c^\dagger_{n,\alpha} c_{n,\alpha} + \sum_{|\varepsilon^*_{n,\alpha}|>\Delta^*} \varepsilon^*_{n,\alpha} c^\dagger_{n,\alpha} c_{n,\alpha} + \sum_{\varepsilon_{n,\alpha_1},\varepsilon_{n,\alpha_2}\sim 0} L^*_{\alpha_1,\alpha_2}(n)\left(c^\dagger_{n,\alpha_1} c_{n,\alpha_2} + \text{h.c.}\right)$$

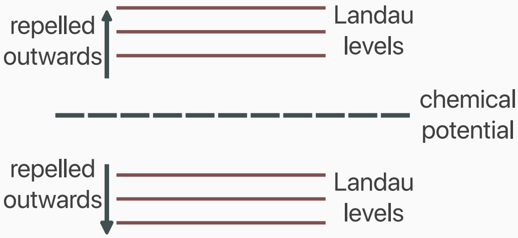
States within a window are attracted towards central state, with relevant forward scattering among them.



URG Analysis of Inter Landau-level Processes

Landau levels are repelled away from chemical potential (**stability**).

LLs below chemical potential are de-coupled and filled.



- **Insulating phase** (chemical potential between two LLs):
  No longitudinal transport. Central state allows transverse transport.
- **Critical point** (chemical potential placed at a LL):
  Marginal scattering processes at chemical potential lead to longitudinal resistivity.

Outstanding Questions

- Characterisation of the low-lying excitations at the critical point.
- Topological invariant at the critical point.

Future Plans

Future Plans

- Finish the embedded eSIAM project and the IQHE projects.
- Study heavy-fermion physics using auxiliary mode approach (simple extension of the embedded eSIAM project).

Thank you!