Emergence in correlated fermions: From impurity models to the bulk

Abhirup Mukherjee

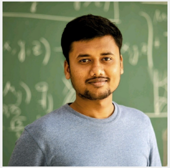
RPC Presentation 2022-2023

Emergent Phenomena in Quantum Matter Group
Department of Physical Sciences, IISER Kolkata

July 11, 2023

Siddhartha Lal
IISER K

Anirban Mukherjee
IISER K (Graduated)



Siddhartha Patra
IISER K (Graduated)

A huge thanks to all my collaborators!

Thanks to IISER K and SERB for financial support.

Arghya Taraphder
IIT Kharagpur

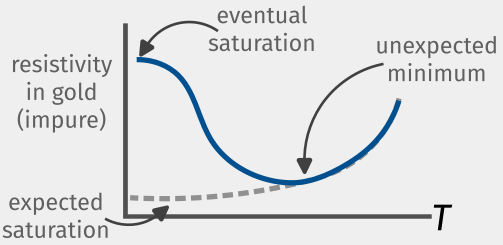
N. S. Vidhyadhiraja
JNCASR Bangalore

Introducing the Kondo effect

Where it all began

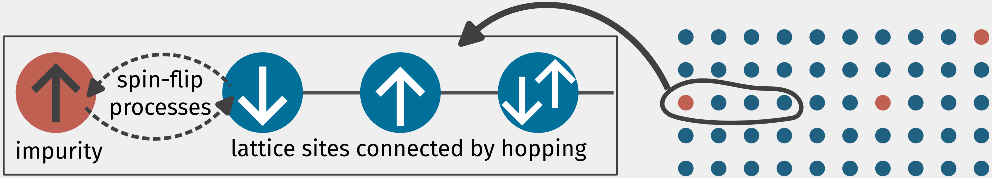
What is the Kondo effect?

Dilute metallic alloys show anomalous resistivity **minimum** and eventual saturation.



Can be explained using the **Kondo model**:

$$H_{\text{Kondo}} = KE_{\text{bath}} + J\vec{S}_{\text{imp}} \cdot \vec{S}_{\text{bath}}$$



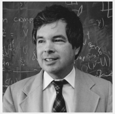
Second order perturbation theory explains resistivity minimum.
However, solution **diverges** at $T \to 0$!

How to explain the resistance minimum & eventual saturation?

Breakdown of perturbation theory indicates a **change in ground state**!

Obtaining T = 0 ground state requires more **powerful methods**

Numerical RG



(K. G. Wilson)

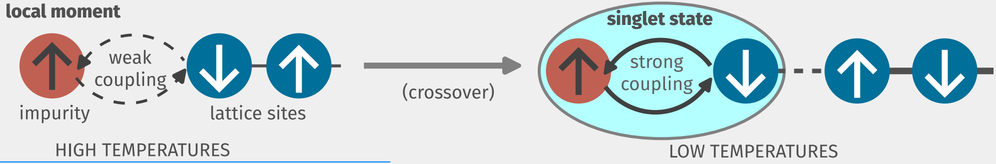
Bethe ansatz

(Natan Andrei)

Conf. field theory

(Ian Affleck)

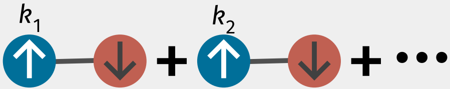
- impurity becomes **strongly coupled** at low temperatures
- local moment crosses over into **nonmagnetic** singlet

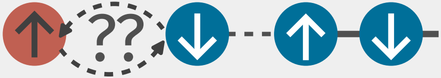


Wilson 1975; Andrei 1980; Wiegmann 1981; Affleck 1995.

Some important questions

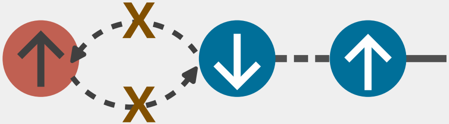
How do we describe the dynamics of the electrons that screen the impurity (the so-called **Kondo cloud**)?
[Mukherjee et al 2023 Phys. Rev. B 105, 085119]





What kind of physics can **disturb the Kondo screening** effect and distort the singlet state?
[Patra et al 2023 J. Phys.: Condens. Matter 35 315601]

What is the simplest impurity model that completely destroys the Kondo effect and leads to a **phase transition**?
[Mukherjee et al 2023 arXiv:2302.02328]



The single-channel Kondo problem: Anatomy of the Kondo cloud

PHYSICAL REVIEW B
covering condensed matter and materials physics

Unveiling the Kondo cloud: Unitary renormalization-group study of the Kondo model

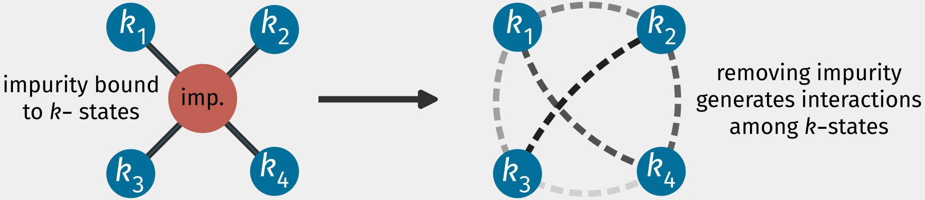
Anirban Mukherjee, Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal

Phys. Rev. B **105**, 085119 – Published 14 February 2022

Effective Hamiltonian for the Kondo cloud

We first applied the **unitary RG** to obtain a low energy fixed point theory.

To obtain a theory for the Kondo cloud, we **trace out impurity** from fixed point Hamiltonian.

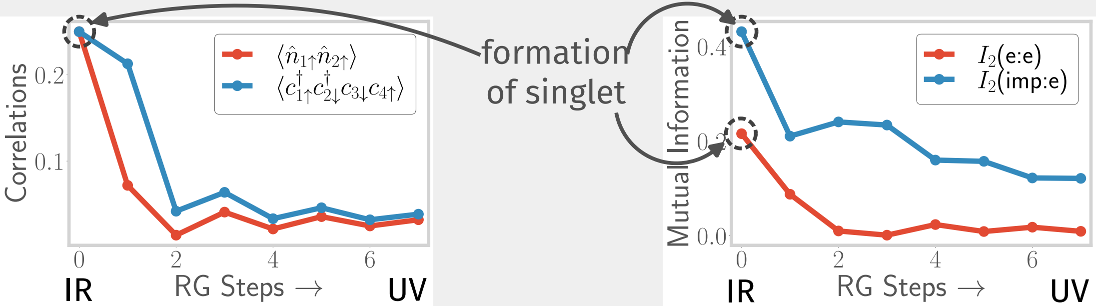


$$KE + J^* \vec{S}_d \cdot \sum_{kk'} \vec{s}_{kk'} \longrightarrow KE + \sum_{k_1,k_2} \frac{2\varepsilon(k_1)\varepsilon(k_2)}{J^*} \hat{n}_{k_1} \hat{n}_{k_2} + \sum_{\{k_i\}} V_{1234} c^\dagger_{k_4\uparrow} c^\dagger_{k_3\downarrow} c_{k_2\downarrow} c^\dagger_{k_1\uparrow}$$

- all-to-all interactions between momentum states, **large entanglement**
- 2-particle interaction terms **not** present in Fermi liquid, are **responsible for screening**

Mukherjee et al. 2022; Mukherjee et al. 2020a; Mukherjee et al. 2020b.

In order to demonstrate formation of Kondo cloud, we study the **variation of entanglement** and correlations under RG transformations.



- Both entanglement and k–space correlations **increase** as RG proceeds from UV to IR.
- This shows the formation of the **Kondo singlet** and the growth of two-particle correlations in the **Kondo cloud**.

Mukherjee et al. 2022.

Distorting the Kondo singlet: The multi-channel Kondo Problem

IOPscience

Journal of Physics: Condensed Matter

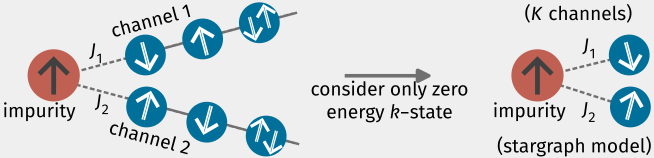
Frustration shapes multi-channel Kondo physics: a star graph perspective

Siddhartha Patra[1], Abhirup Mukherjee[1], Anirban Mukherjee[1], N S Vidhyadhiraja[2], A Taraphder[3] and Siddhartha Lal[4,1]

WHAT IS THE MULTICHANNEL KONDO PROBLEM?

Single impurity interacting with **multiple channels** in the bath

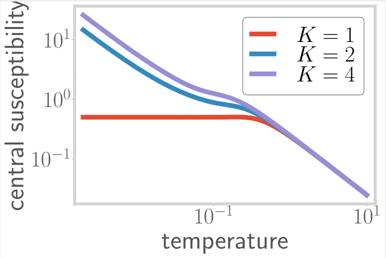
$$H_{\text{Kondo}} = KE_{\text{bath}} + \sum_l J_l \vec{S}_{\text{imp}} \cdot \vec{s}^{(l)}$$



Known to display divergent T = 0 impurity susceptibility (incomplete screening), and orthogonality catastrophe, **non-Fermi liquid** excitations.

Zero bandwidth limit is (analytically) solvable: $\{|S^z_{\text{tot}}\rangle\}$

- Ground state degeneracy for K > 1 explains **orthogonality catastrophe**
- $S^z_{\text{tot}} \neq 0$ in ground states shows incomplete screening
- Excitations shows **non-Fermi liquid** physics in



How to destroy the Kondo cloud: Effect of local interactions in the bath

arXiv > cond-mat > arXiv:2302.02328

Condensed Matter > Strongly Correlated Electrons

[Submitted on 5 Feb 2023]

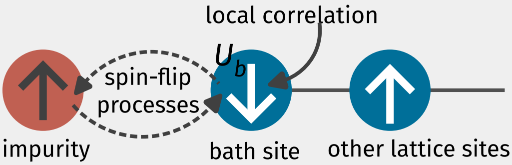
Kondo frustration via charge fluctuations: a route to Mott localisation

Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

What is the new physics ingredient?

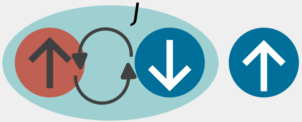
Add **local correlation** on bath (zeroth) site coupled to impurity

$$KE_{\text{bath}} + J\vec{S}_{\text{imp}}\cdot\vec{S}_{\text{bath}} - U_b\left(\vec{S}_{\text{bath}}\right)^2$$



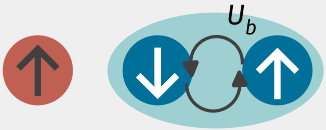
URG equations show that an **attractive** U_b frustrates the zeroth site.

$$\Delta J \sim J^2 + 4U_b J \implies \textbf{phase transition} \text{ at } J = -4U_b$$



singlet state (favour J)

~~OR~~



decoupled local moment (favour U_b)

Such a model sheds light on the Mott MIT in ∞-dimensions (as seen from DMFT).

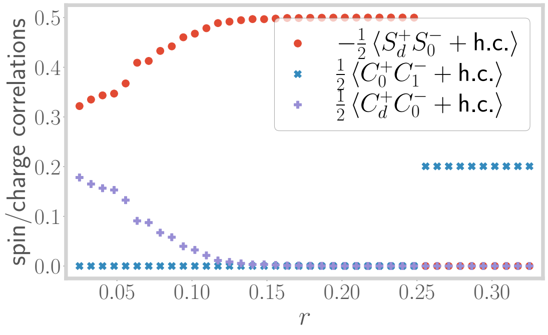
Georges et al. 1996.

Nature of the transition

Across the transition,

- impurity correlations vanish
- bath correlations become non-zero

Shows that **pairing correlations** in the bath are responsible for the transition.

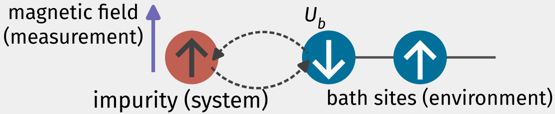


The state **precisely at the transition** is special:

- non-Fermi liquid excitations
- **fractional** impurity magnetisation and occupancy

Concluding remarks

- Our analyses often link entanglement measures with correlations, providing bridges between the worlds of condensed matter and quantum information.
- Models of Kondo breakdown can be used to study the effects of measurement on a system coupled to a bath.



- The Kondo model with attractive U_b term has applications in studying the physics of Mott transitions.


Thank you, all of you.

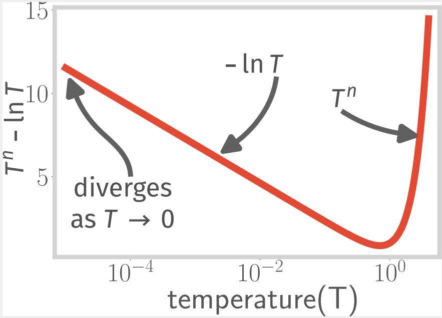
How to explain the resistance minimum & eventual saturation?

Second order perturbation theory in J gives:

$$\rho \sim T^n - \ln T$$

(Jun Kondo)

Explains the **non-monotonic** behaviour!

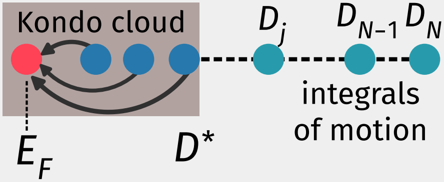


However, solution **diverges** at $T \to 0$!

Kondo 1964.

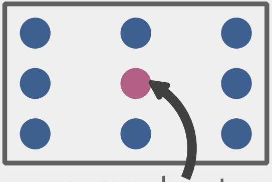
Unitary RG approach to impurity models

- Integrate out **high energy fluctuations** to reach strong-coupling low-energy theory
- Leads to **singlet ground state** and decoupled high-energy k-states
- Decoupling is carried out through **unitary transformations**



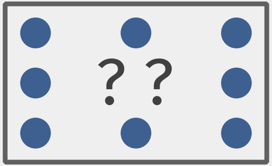
Mukherjee et al. 2020a; Mukherjee et al. 2020b.

- Entanglement entropy $S(A) \Longrightarrow$ quantifies how much **information is gained** about the rest of the system by measuring A



measure subsystem A

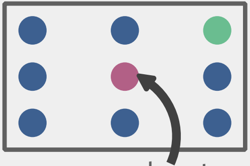
$$S(A) = \mathrm{Trace}\left(\rho_A \ln \rho_A\right)$$



gain information about rest

- Entanglement entropy $S(A) \Longrightarrow$ quantifies how much **information is gained** about the rest of the system by measuring A

- Mutual information $I_2(A : B) \Longrightarrow$ quantifies how much **information about subsystem A** is gained by measuring B



measure subsystem A

$$I_2(A : B) = S(A) + S(B) - S(A \cup B)$$

gain information about B