URG Analysis of Electron in a Periodic Potential

Role of the center of mass

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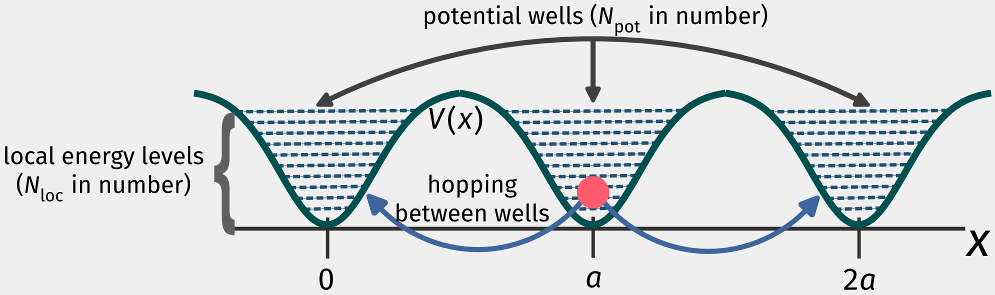
- We conclude by connecting this problem to that of the **IQHE**.

The Problem of a Particle in a Periodic Potential (PPP)

$$H = \int_{-\infty}^{\infty} dx\, c^\dagger(x)\left[\hat{p}^2/2m + V(x)\right]c(x), \quad V(x+a) = V(x)$$



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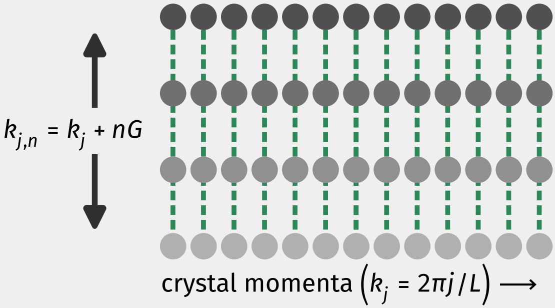
Potential only connects momentum states separated by a reciprocal lattice vector.

$$\langle k + q|V|k\rangle = \delta_{q,G}V(G)$$

Leads to conserved **crystal momenta**: $\{k_j < G\}$



The PPP as a Particle on a Circle

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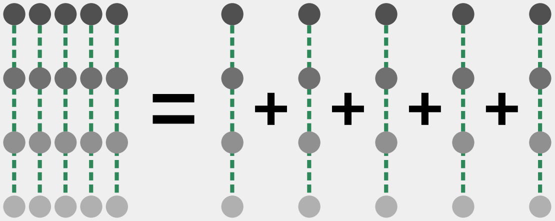
The conserved crystal momenta leads to a block-diagonal form of the Hamiltonian.

$$H = \sum_k H(k), \quad H(k) \sim \left(-i\hbar \frac{\partial}{\partial x'} + \hbar k\right)^2 + V(x')$$

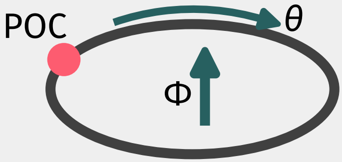


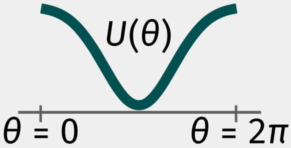
The PPP as a Particle on a Circle

Define dimensionless position and momentum.

$$H(k) = \frac{\hbar^2}{2ma^2}\left(\hat{Q} + \Phi/2\pi\right)^2 + U(\theta)$$

Hamiltonian is that of a **particle on a circle**. Flux is $\Phi = ka$.





URG Analysis of the POC

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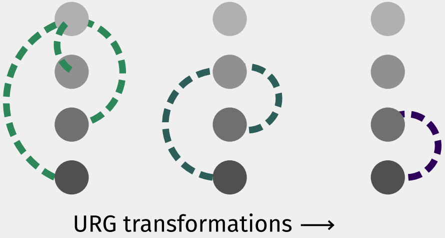
Resolve fluctuations in angular momentum states by applying unitary transformations.

$$\Delta U_{ij}^{(l)}(\omega) = \frac{U_{il}U_{lj}}{\omega - \varepsilon(Q_l + \Phi/2\pi)}, \quad U_{ij} = U(Q_i - Q_j)$$

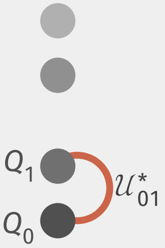


Appearance of band gaps

Effective Hamiltonian for the final two states:

$$H^*_{01} = \varepsilon^*(Q_0)\,|Q_0\rangle\langle Q_0| + \varepsilon^*(Q_1)\,|Q_1\rangle\langle Q_1| + \left(U^*_{01}\,|Q_1\rangle\langle Q_0| + \text{h.c.}\right)$$

Diagonalise the final Hamiltonian: $E_\pm = \varepsilon^* \pm |U^*_{01}|$



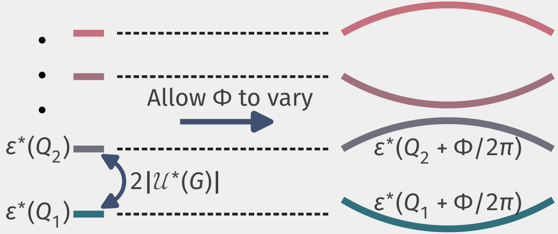
Gives the **shifts in energies**:

$$\Delta\varepsilon^* \simeq \frac{|U^*_{01}|^2}{\varepsilon^* \pm |U^*_{01}| - \varepsilon^*} \simeq \pm|U^*_{01}|$$

Allow the flux Φ to vary:

$$\varepsilon^*(\Phi) - |U^*_{01}|; \Phi = ak$$

Creates the **first band**!



THANK YOU.