Hierarchical structure and topological content of entanglement of free fermions

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Introduction

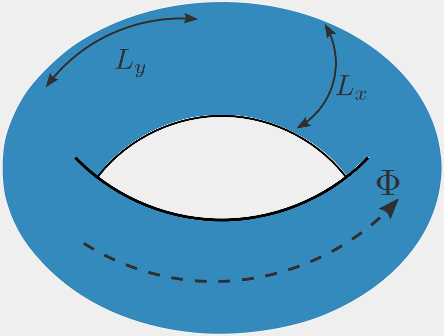
The system

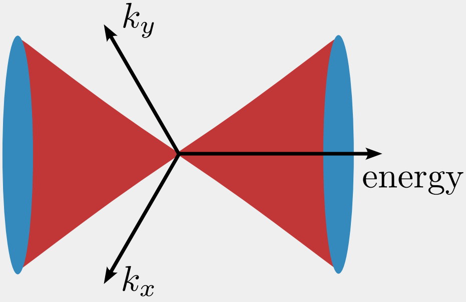
Massless Dirac fermions on a 2-torus

$$L = i\overline{\psi}\gamma_\mu \partial_\mu \psi$$

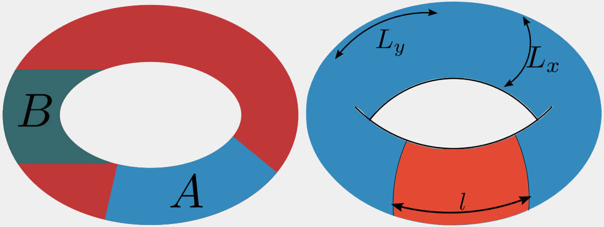
In presence of an Aharonov-Bohm flux

$$L = \overline{\psi}\left(i\gamma_\mu + eA_\mu\right)\partial_\mu \psi$$





$\rho = |\Psi\rangle\langle\Psi| \longrightarrow$ density matrix



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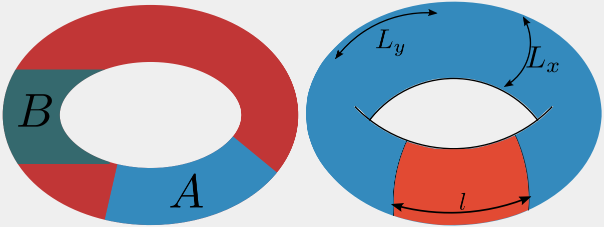
ρ_A = partial trace over system A
$\longrightarrow$ reduced DM



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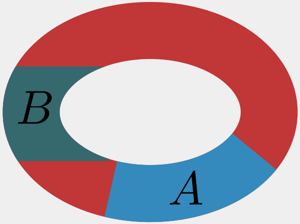

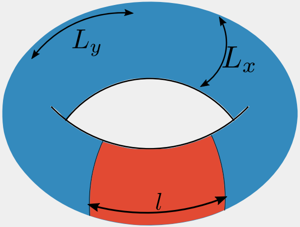

$$S(A) = -\mathrm{Tr}\left[\rho_A \ln \rho_A\right] \longrightarrow \text{entanglement entropy of A}$$

$\longrightarrow$ quantifies information shared between A and rest

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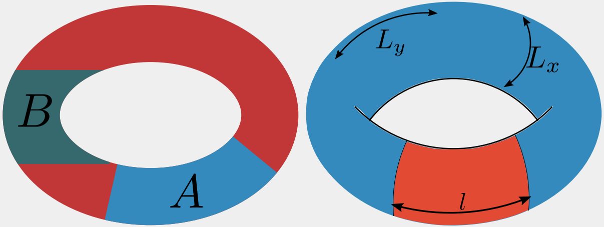

$I(A:B) = S(A) + S(B) - S(A \cup B) \longrightarrow$ mutual information between A and B

$\longrightarrow$ quantifies information shared between A and B

Diagonal in k-space $\longrightarrow$ Vanishing entanglement in momentum space

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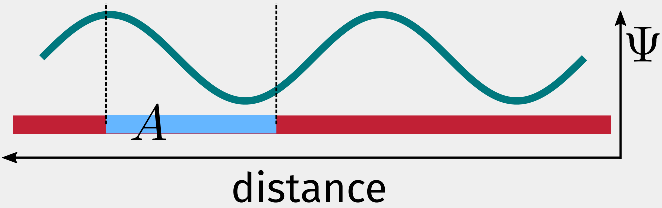
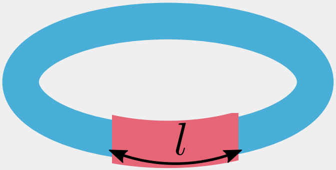
Off-diagonal in r-space $\longrightarrow$ Fluctuations exist in real space
$\longrightarrow$ leads to entanglement in real space



Entanglement of free fermions

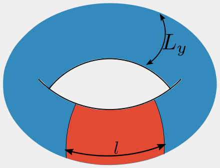
1D-ring of massless fermions: $\frac{2}{3}\ln\left(\frac{L}{\pi a}\sin\frac{\pi l}{L}\right)$



1D-line of massless fermions: $\frac{1}{3}\ln\left(\frac{2L}{\pi a}\sin\frac{\pi l}{L}\right)$

1D-line of relativistic fermions: $-\frac{1}{3}\ln(ma)$

2D-torus of massless fermions: $\alpha\frac{L_y}{\epsilon}$



What are we going after?

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- Effect of a magnetic flux on the entanglement
- Distribution of the entanglement among subsystems of various sizes
- Emergent space generated by the transformations between these subsystems
- Curvature and related quantities of this space

Reduction to $(1+1)$–D systems

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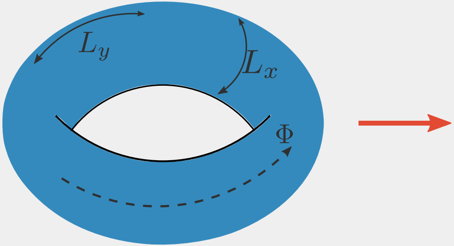
In presence of flux: $L = \int dxdy \quad \overline{\Psi}(x)\left(i\gamma_\mu + eA_\mu\right)\partial_\mu\Psi(x)$

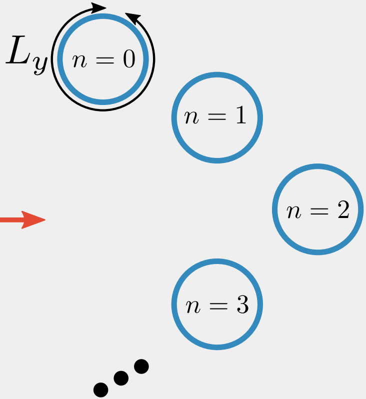
Periodic boundary conditions along $\vec{x}$: $k_x^n = \frac{2\pi n}{L_x}, \quad n \in \mathbb{Z}$

Introduce Fourier modes: $\Psi(x) = \sum_{n=-\infty}^{\infty} e^{ixk_x^n}\Psi(k_x^n)$

Reduction to (1 + 1)–D systems

Decouples into 1D modes: $L = \sum_n \int dy\, \overline{\Psi}(k_x, y)\left(i\gamma_\mu \partial_\mu - M\right)\Psi(k_x, y)$





Reduction to (1 + 1)–D systems

$2D$ system is described by sum over $1D$ modes.

$\downarrow$

Modes do not couple - no inter-mode entanglement.

$\downarrow$

Total entanglement is sum of each part: $S = \sum_n S_n$

$$S_n(\phi) = -c \ln\left(\epsilon \frac{2\pi|n+\phi|}{L_x}\right)$$

References I