Research Progress Report: 2023 - 2024

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Publications and Ongoing projects

Currently in progress

- Development of a new auxiliary model-based method for studying systems of interacting electronics.
- Studies of the plateau-to-plateau transition in integer quantum hall systems.

Published

- Abhirup Mukherjee et al 2023 New J. Phys. 25 113011
- Abhirup Mukherjee et al 2024 J. Phys. A: Math. Theor. 57 275401

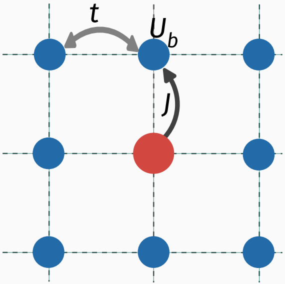
- Anirban Mukherjee et al 2022 Phys. Rev. B 105, 085119
- Siddhartha Patra et al 2023 J. Phys.: Condens. Matter 35 315601

Project I: A New Auxiliary Model Approach to Systems of Interacting Electrons

Broad Objectives

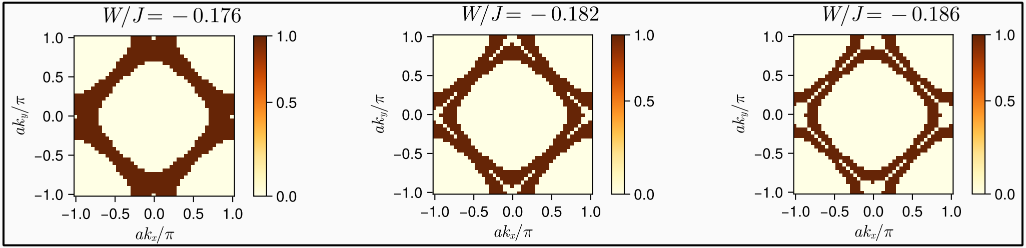
- Designing a **new method** by which to leverage quantum impurity models towards studying lattice models of interacting electrons
- Using such a method to go after the **Mott-Hubbard MIT** on the 2D square lattice
- Capturing the enhanced effects of $k-$**space anisotropy** (due to the square lattice) on signatures near the transition
- Studying the (presumably) **non-Fermi liquid behaviour** in the excitations close to and at the transition

Momentum-Resolved Renormalisation Group Flows



Hamiltonian RG equations of **embedded e-SIAM**

$$\Delta J^{(j)}_{\mathbf{k}_1,\mathbf{k}_2} = -\sum_{\mathbf{q}\in \mathrm{PS}} \frac{J^{(j)}_{\mathbf{k}_2,\mathbf{q}} J^{(j)}_{\mathbf{q},\mathbf{k}_1} + 4 J^{(j)}_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}}}{\omega - \frac{1}{2}|\varepsilon_j| + J^{(j)}_{\mathbf{q}}/4 + W_{\mathbf{q}}/2}$$



'Periodising' the Hamiltonian and Eigenstates

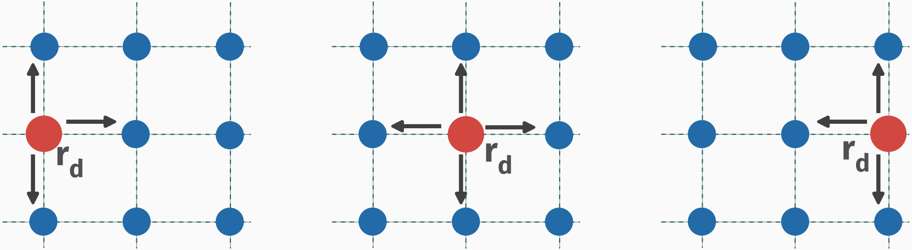
Periodising the Hamiltonian creates a **Hubbard-Heisenberg** model:

$$H_{\text{tiled}} = \sum_{\mathbf{r}} T^{\dagger}(\mathbf{r} - \mathbf{r}_d) H_{\text{aux}}(\mathbf{r}_d) T(\mathbf{r} - \mathbf{r}_d)$$

$$= H_{\text{Hub.}} + \frac{\tilde{J}}{Z} \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle} \mathbf{S}_{\mathbf{r}_i} \cdot \mathbf{S}_{\mathbf{r}_j}$$

Wavefunctions can be related using a many-body **Bloch's theorem** :

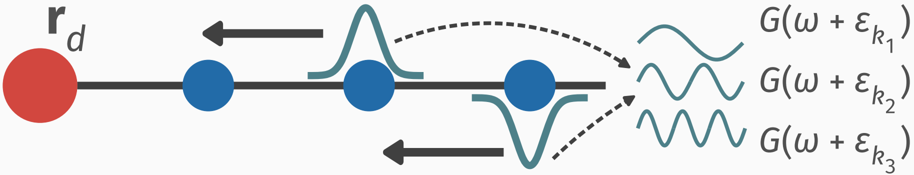
$$|\Psi_{\text{gs}}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}_d} e^{i\mathbf{k}\cdot\mathbf{r}_d} |\psi_{\text{gs}}(\mathbf{r}_d)\rangle$$



'Periodising' the Greens Functions

Greens function = sum of 1-particle **k-space** Greens functions starting from **all sites** in impurity model.

$$\tilde{G}(\mathbf{r};\tilde{\omega}) = \frac{1}{N}\sum_{\mathbf{k},\mathbf{r}_x}\Big[e^{i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_p\left(\mathbf{r}_x;\omega+\varepsilon_{\mathbf{k}}\right) + e^{-i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_h\left(\mathbf{r}_x;\omega-\varepsilon_{\mathbf{k}}\right)\Big]$$



Results

We use impurity model eigenstates (Hamiltonians) and Bloch's theorem to reconstruct full eigenstates (Hamiltonian):

$$|\Psi_{\vec{k}}\rangle \sim \sum_{\vec{R}_i} e^{i\vec{k}\cdot\vec{R}_i}\,|\psi_{\text{aux}}(\vec{R}_i)\rangle\,, \quad H \sim \sum_{\vec{R}_i} H_{\text{aux}}(\vec{R}_i)$$

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Allow us to relate corresponding objects between the impurity model and the lattice model

- Greens functions and self-energies
- Two-particle correlation functions
- entanglement measures

Results: Momentum space spectral function

Results: Momentum space spin correlations

Search for punctured-Chern topology at IQHE transitions

Broad questions

- Obtaining the IQHE phase diagram from a model of 2D lattice electrons

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- Understanding the topology of the ground state precisely at a transition
- Extending this to systems with disorder and interactions.

Preliminary results

Emergence of Landau levels in a magnetic field is similar to the formation of bands in a periodic potential.

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We first studied the simpler problem of particle in a periodic potential.

- Can understand the formation of bands under RG
- Obtained insights regarding the effective center of mass degrees of freedom
- Needs to be extended by incorporating a magnetic field

Summary

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- Development of auxiliary model-based method for studying bulk correlated systems.
- Studies of the plateau-to-plateau transition in integer quantum hall systems.

- 2022 Phys. Rev. B 105, 085119.
  A Mukherjee, Abhirup Mukherjee, ..., S. Lal
- 2023 J. Phys.: Condens. Matter 35 315601.
  S Patra, Abhirup Mukherjee, ..., S. Lal
- 2023 arXiv:2302.02328.
  Abhirup Mukherjee, ..., S. Lal
- 2023 arXiv:2302.10590.
  Abhirup Mukherjee, ..., S. Lal

Future plans

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Lattice models of impurities

- either directly or through the auxiliary model approach
- phase diagrams: strange metals and QCPs
- unconventional superconductivity

Keimer et al. 2015; Qin et al. 2022.

Future plans

Fractional Chern insulators

- microscopic understanding of the FQHE ground states
- emergence of composite degrees of freedom and topological theories

Future plans

Classification of RG flows in fermionic models

- growth of multipartite entanglement towards stable fixed points
- extending this to impurity models
- connections with the URG noise operator

THANK YOU.