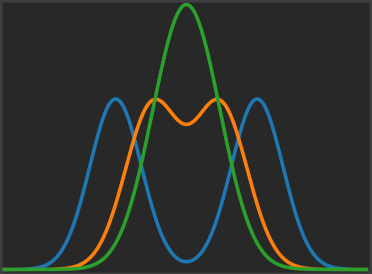
Unveiling the Kondo cloud: unitary RG study of the Kondo model



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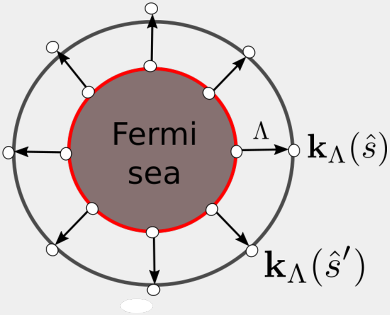
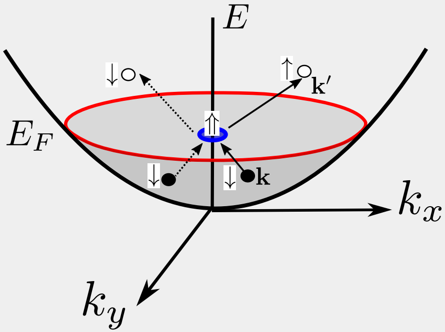
The Model

The Model

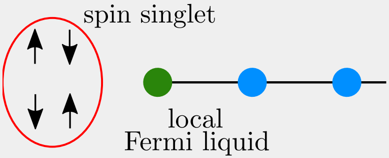
$$\mathcal{H} = \sum_{k\sigma} \epsilon_k \hat{n}_{k\sigma} + J\vec{S}_d \cdot \vec{s}, \qquad \vec{s} \equiv \sum_{kk',\alpha,\beta} \vec{\sigma}_{\alpha\beta} c^\dagger_{k\alpha} c_{k'\beta}$$





The Model

- Kondo coupling J renormalises to infinity



Anderson and Yuval 1969; Anderson 1970; Wilson 1975; Andrei, Furuya, and Lowenstein 1983a; Andrei, Furuya, and Lowenstein 1983b; Wiegmann 1981.

The Model

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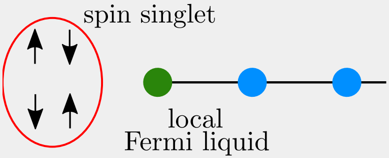

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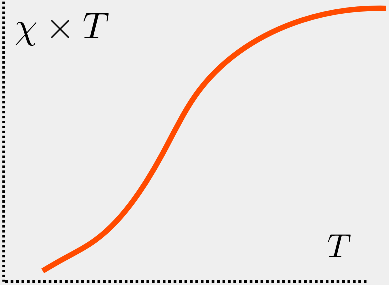

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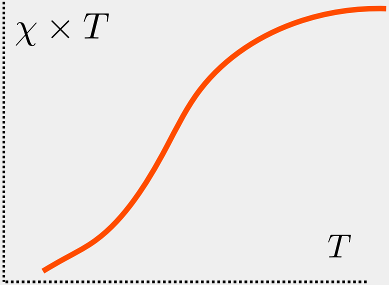
- low energy phase of metal is local Fermi liquid
- χ constant at low temperatures, C_v linear
- thermal quantities functions of single scale T/T_K



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- Finite J effective Hamiltonian at fixed point
- Hamiltonian for the itinerant electrons forming the **macroscopic singlet**
- Nature of correlations inside the Kondo cloud: **Fermi liquid vs off-diagonal**
- Behaviour of **many-particle entanglement** and many-particle correlation under RG flow

The Unitary Renormalization Group Method

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The General Idea

- Apply unitary many-body transformations to the Hamiltonian

$$\begin{array}{c} H_j \\ \Big\downarrow{\scriptstyle U_j} \\ H_{j-1} \end{array}$$

Mukherjee and Lal 2020a; Mukherjee and Lal 2020b.

The Unitary Renormalization Group Method

The General Idea

- Apply unitary many-body transformations to the Hamiltonian
- Successively decouple high energy states

$$H_j \xrightarrow{U_j} H_{j-1}$$

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The Unitary Renormalization Group Method

The General Idea

- Apply unitary many-body transformations to the Hamiltonian
- Successively decouple high energy states
- Obtain sequence of Hamiltonians and hence scaling equations

$$\begin{tikzcd} H_j \arrow[d, "U_j"] \\ H_{j-1} \end{tikzcd}$$

Mukherjee and Lal 2020a; Mukherjee and Lal 2020b.

Select a UV-IR Scheme

UV shell

$\vec{k}_N$ (zeroth RG step)

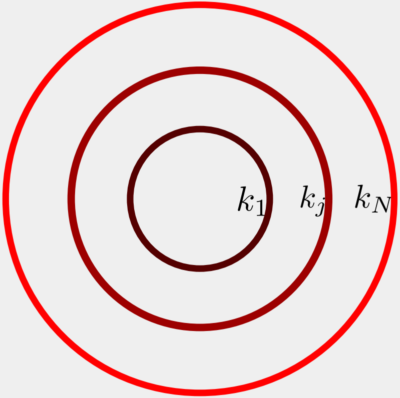
$\vdots$

$\vec{k}_j$ (j^{th} RG step)

$\vdots$

$\vec{k}_1$ (Fermi surface)

IR shell

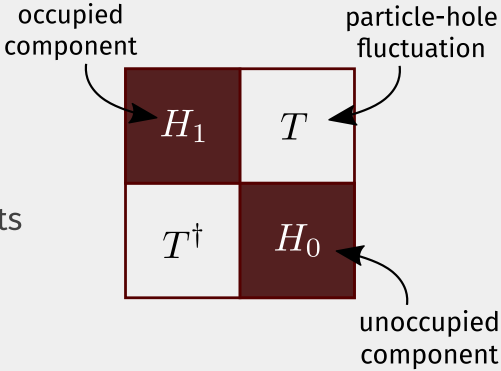


Mukherjee and Lal 2020a; Mukherjee and Lal 2020b.

Write Hamiltonian in the basis of $\vec{k}_j$

$$H_{(j)} = H_1\hat{n}_j + H_0\left(1 - \hat{n}_j\right) + c_j^\dagger T + T^\dagger c_j$$

$$2^{j-1}\text{-dim.} \longrightarrow \begin{cases} H_1, H_0 \longrightarrow \text{diagonal parts} \\ V \longrightarrow \text{off-diagonal part} \end{cases}$$



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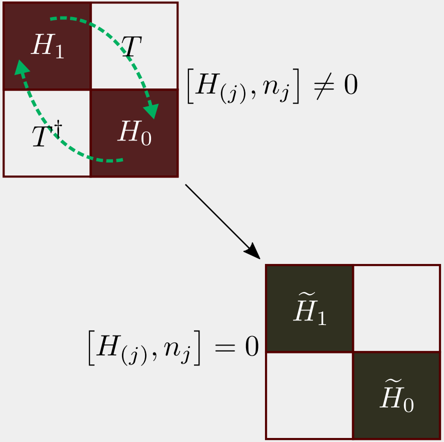
Rotate Hamiltonian and kill off-diagonal blocks

$$H_{(j-1)} = U_{(j)} H_{(j)} U^{\dagger}_{(j)}$$

$$U_{(j)} = \frac{1}{\sqrt{2}}\left(1 - \eta_{(j)} + \eta^{\dagger}_{(j)}\right)$$

$$\left.\eta^{\dagger}_{(j)} = \frac{1}{\hat{\omega}_{(j)} - H_D} c^{\dagger}_j T \right\} \rightarrow \text{many-particle rotation}$$



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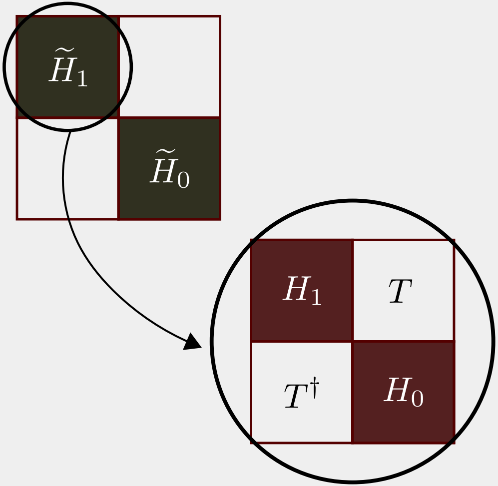
Repeat with renormalised Hamiltonian



$$H_{(j-1)} = \widetilde{H}_1 \hat{n}_j + \widetilde{H}_0 \left(1 - \hat{n}_j\right)$$

$$\widetilde{H}_1 = H_1 \hat{n}_{j-1} + H_0 \left(1 - \hat{n}_{j-1}\right) + c^\dagger_{j-1} T + T^\dagger c_{j-1}$$

Mukherjee and Lal 2020a; Mukherjee and Lal 2020b.

RG Equations and Denominator Fixed Point

$$\Delta H_{(j)} = \left(\hat{n}_j - \frac{1}{2}\right)\left\{c_j^\dagger T, \eta_{(j)}\right\}$$

$$\eta_{(j)}^\dagger = \frac{1}{\hat{\omega}_{(j)} - H_D} c_j^\dagger T$$

Fixed point: $\hat{\omega}_{(j^*)} - (H_D)^* = 0$

Mukherjee and Lal 2020a; Mukherjee and Lal 2020b.

Novel Features of the Method

- Quantum fluctuation energy scale ω

$$H_{(j-1)} = U_{(j)} H_{(j)} U_{(j)}^\dagger$$

$$U_{(j)} = \frac{1}{\sqrt{2}} \left(1 - \eta_{(j)} + \eta_{(j)}^\dagger\right)$$

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Novel Features of the Method

- Quantum fluctuation energy scale ω
- Finite-valued fixed points for finite systems
- Spectrum-preserving unitary transformations
- Tractable low-energy effective Hamiltonians

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URG of the Kondo Model

RG Equation, Fixed Point Hamiltonian & Phase Diagram

Assumption: isotropic energy surfaces: $\epsilon_{\vec{k}_j} \equiv D_j$

$$\Delta J_{(j)} = \frac{n_j \, J^2_{(j)} \left(\omega_{(j)} - \frac{D_j}{2}\right)}{\left(\omega_{(j)} - \frac{D_j}{2}\right)^2 - \frac{1}{16} J^2_{(j)}}$$

$$J^* = 4\left(\omega^* - \frac{1}{2} D^*\right)$$

$D^* \longrightarrow$ emergent window

For $J_{(j)} \ll D_j$, we recover weak-coupling form: $\frac{\Delta J_{(j)}}{\Delta \ln D_j} \sim n_j \, J^2_{(j)}$

Anderson 1970.

RG Equation, Fixed Point Hamiltonian & Phase Diagram

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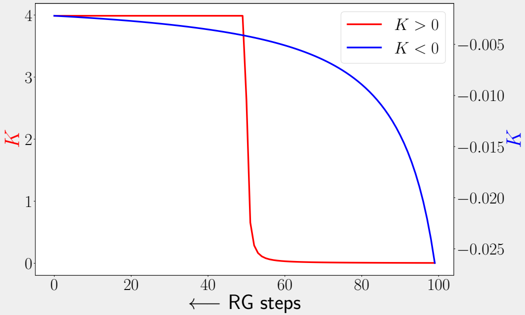
$$\omega_{(j)} > \frac{D_j}{2}$$

$$K_{(j)} = J_{(j)}\left(\omega^* - \frac{1}{2}D^*\right)^{-1}$$



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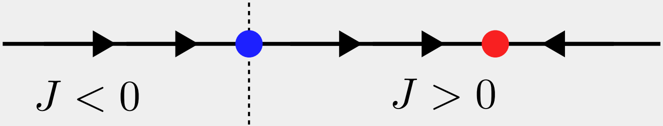

- Decay towards FM fixed point for $J < 0$
- Attractive flow towards AFM fixed point for $J > 0$

RG Equation, Fixed Point Hamiltonian & Phase Diagram

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$$\omega_{(j)} > \frac{D_j}{2}$$

$$J^* = 4\left(\omega^* - \frac{1}{2} D^*\right)$$

$$D^* \longrightarrow \text{ emergent window}$$

$$H^* = \underbrace{\sum_{k,\sigma} \epsilon_k \hat{n}_{k\sigma} + J^* \vec{S} \cdot \vec{s}_<}_{\text{emergent window}} + \underbrace{\sum_{j=j^*}^{N} J^j S^z s_j^z}_{\text{integrals of motion}}$$