

EMERGENCE IN FREE AND CORRELATED FERMIONS: FROM IMPURITY MODELS TO THE BULK

RPC PRESENTATION 2021-22

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SUMMARY OF WORK

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1. 1-channel Kondo problem: *as second author*, published in Phys. Rev. B **Phys. Rev. B 105, 085119**, arXiv:2111.10580v3
2. Multi-channel Kondo problem: *as second author*, under review at Phys. Rev. B **arXiv:2205.00790**
3. Generalised Anderson impurity model: manuscript **in preparation**
4. Entanglement scaling in free fermions: manuscript **in preparation**
5. New auxiliary model approach to correlated systems: **ongoing project**

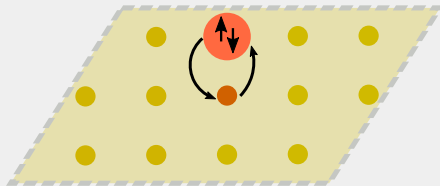
SINGLE-CHANNEL KONDO PROBLEM

Phys. Rev. B 105, 085119

Anirban Mukherjee, **Abhirup Mukherjee**, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal

Model of impurity interacting with conduction electrons through spin-flips

1. Computation of the impurity spectral function
2. Emergence of a local Fermi liquid, and orthogonality catastrophe between local moment and singlet states
3. Calculating of thermal entropy



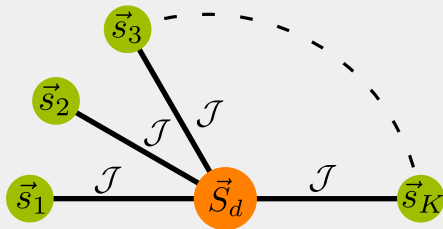
MULTI-CHANNEL KONDO PROBLEM

arXiv:2205.00790

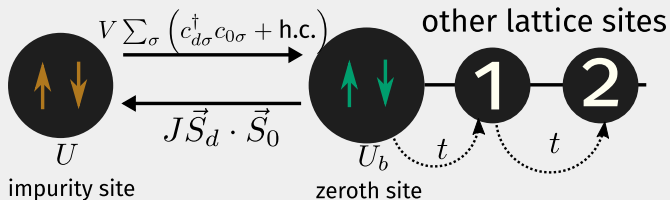
Siddhartha Patra, **Abhirup Mukherjee**, Anirban Mukherjee, N. S. Vidhyadhiraja, A. Taraphder,
Siddhartha Lal

Model of impurity interacting with multiple conduction electron channels

1. Obtaining RG fixed point Hamiltonian
2. Analytical forms for degree of compensation, magnetization and susceptibility
3. Presence of a local marginal Fermi liquid
4. Dualities of the MCK model

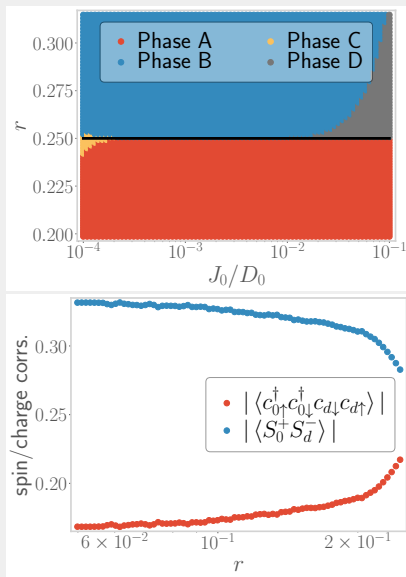


LOCAL METAL-INSULATOR TRANSITION IN EXTENDED ANDERSON IMPURITY MODEL

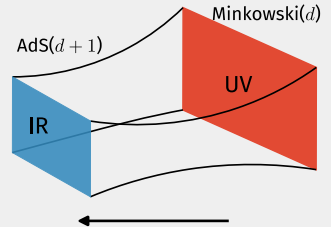
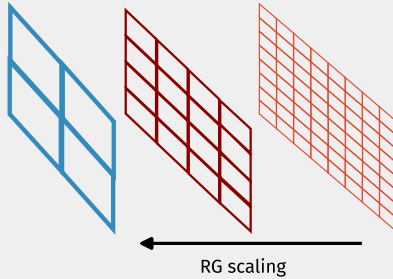
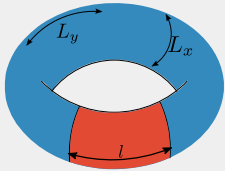


LOCAL MIT IN AN EXTENDED ANDERSON IMPURITY MODEL

- Competition between J and U_b leads to phase transition from screened singlet phase at $|U_b| \leq 4J$ to unscreened local moment phase at $|U_b| > 4J$.
- Impurity spectral function becomes gapped beyond the critical point.
- Decoupling the impurity model leads to an effective model with the zeroth site as the correlated impurity, demonstrating the symmetry between the impurity and zeroth site.
- Geometric entanglement and mutual information track the transition by vanishing beyond the critical point.
- Subdominant pairing tendencies are observed near the quantum critical point.

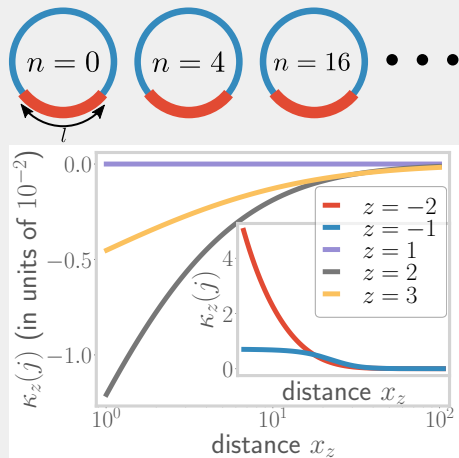


ENTANGLEMENT SCALING IN FREE FERMIONS



ENTANGLEMENT SCALING IN FREE FERMIONS

- Under coarse-graining or fine-graining in k -space, entanglement of free Dirac fermions shows hierarchical onion-like structure.
- Entanglement scaling can be used to define distances, leads to additional spatial dimension \rightarrow holography
- Emergent distances and curvature can be related to RG beta function; the sign of the curvature is topological.
- Pole structure of the entanglement tracks the Luttinger volume - invariant under the scaling transformations



FUTURE PROSPECTS

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- Better model can be obtained by taking multiple impurities and general impurity filling
- novel auxiliary model method can be used for studying other models of strong-correlations as well as topologically active or flat band systems
- The URG can be applied to heavy-fermion materials towards a study of phase diagram and unconventional superconductivity, as well as Kondo insulators
- Interacting systems in a magnetic field is also a potential area of study, specifically fractional Chern insulators (e.g. the fractional quantum hall effects)

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FURTHER DETAILS

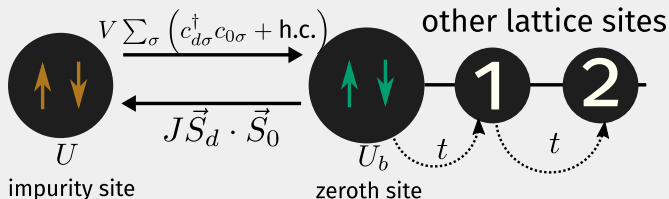
LOCAL METAL-INSULATOR TRANSITION IN EXTENDED ANDERSON IMPURITY MODEL

EXTENDED ANDERSON IMPURITY MODEL

Standard Anderson impurity model \rightarrow only one stable phase (strong-coupling)

no possibility of phase transition \rightarrow Introduce additional correlation

- spin-flip correlation between impurity and bath: J
- local correlation in the bath: U_b

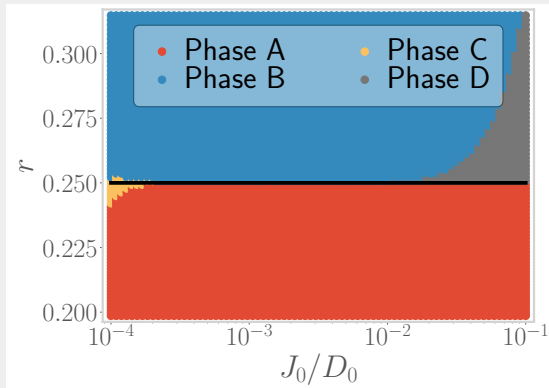


RG PHASE DIAGRAM

RG equations reveal critical point where J, V **become irrelevant**

1. orange phase: J is relevant:
strong-coupling
2. blue phase: J is irrelevant: local
moment
3. yellow phase: spin+charge liquid
4. gray phase: all couplings irrelevant

$$r = -U_b/J$$



PRESENCE OF A PHASE TRANSITION

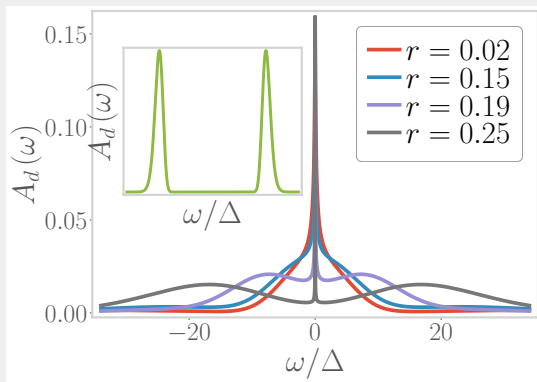
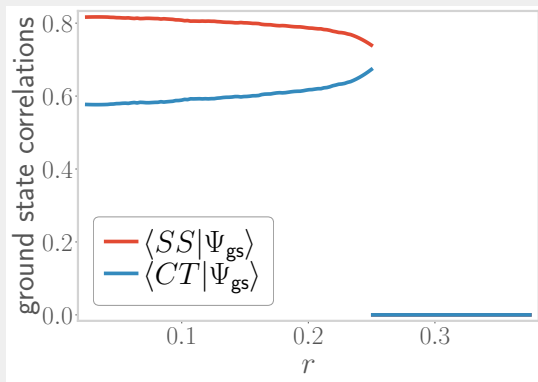
singlet \longrightarrow spin+charge liquid \longrightarrow local moment

impurity spectral function gaps out

$$r = -U_b/J$$

$$|SS\rangle = |\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle$$

$$|CT\rangle = |2, 0\rangle + |0, 2\rangle$$



- Decoupling the impurity site leads to an Anderson impurity model

$$H_{0+\text{rest}} = \underbrace{- (U_0 + U_b) (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2}_{\text{new correlated impurity}} \underbrace{- t \sum_{\substack{j \in \text{n.n. of } 0, \\ \sigma}} (c_{0\sigma}^\dagger c_{j\sigma} + \text{h.c.})}_{\text{hopping between new impurity \& new bath}} \underbrace{- t \sum_{\langle i,j \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.})}_{\text{K.E. of new bath}}$$

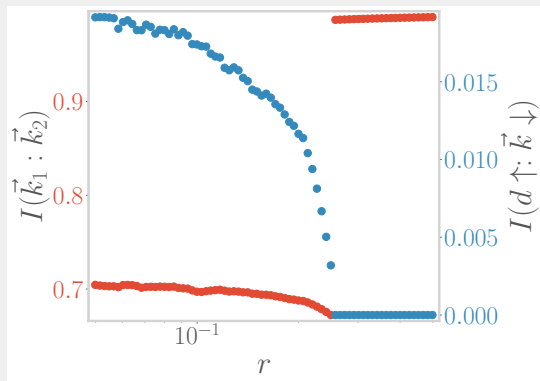
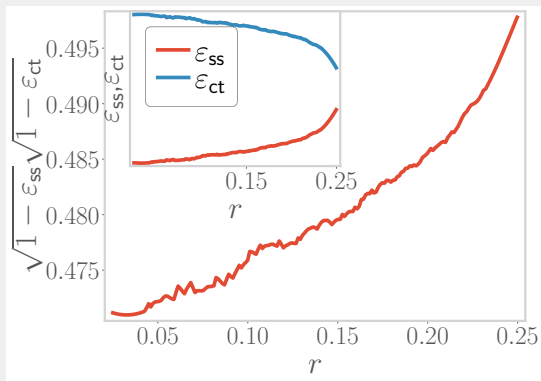
- correlated, dominant spin-flip processes lead to repulsive $U_{\text{eff}} = U_0 + U_b \sim J^*/8$
- J symmetrises the two sites, leading to similar spectral functions \rightarrow essence of self-consistency

ENTANGLEMENT AS A PROBE FOR THE TRANSITION

Geometric entanglement: $\varepsilon(\psi_1, \psi_2) = 1 - |\langle \psi_1 | \psi_2 \rangle|^2$

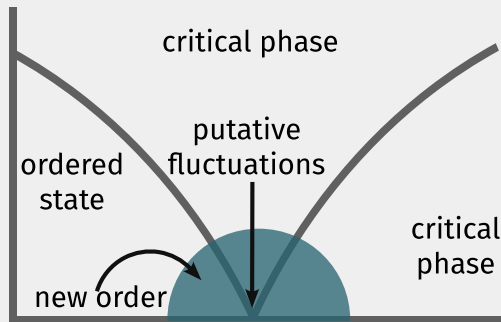
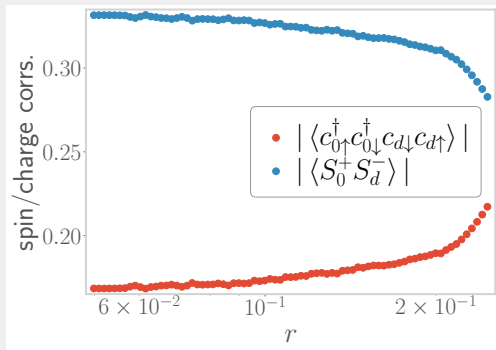
$\rightarrow \sqrt{1 - \varepsilon_{SS}} \sqrt{1 - \varepsilon_{CT}}$ is maximised, then vanishes

Mutual information between impurity and cloud vanishes



PRESENCE OF SUBDOMINANT PAIR FLUCTUATIONS

- **pairing tendencies** observed near the quantum critical point
- might lead to **superconductivity** with doping
- seen in cuprates, heavy-fermions materials, pnictides, etc



ENTANGLEMENT SCALING IN FREE FERMIONS

Free Dirac fermions on torus: $k_x^n = \frac{2\pi}{L_x}n$, $n \in \mathbb{Z}$; define **sparsity** = $\Delta n = 1$

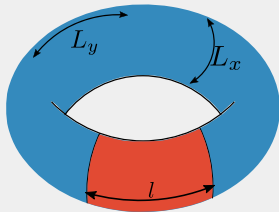
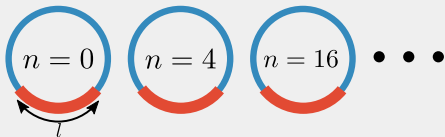
Simplest choice: the entire set

sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \dots, -1, 0, 1, \dots, N-2, N-1, N\}$

Coarser choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \dots, -2, 0, 2, \dots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \dots, -4, 0, 4, \dots, N-8, N-4, N\}$



$$S_{A_z(j)} = f_z(j) c \alpha L_x - c \log |2 \sin(\pi f_z(j) \phi)|$$

$$i < j, S_{i \cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$



- presents a **hierarchy** of entanglement \rightarrow EE distributed across RG steps
RG transformation \rightarrow reveals entanglement
- distribution of entanglement also present in **multipartite** entanglement

MUTUAL INFORMATION = DISTANCE

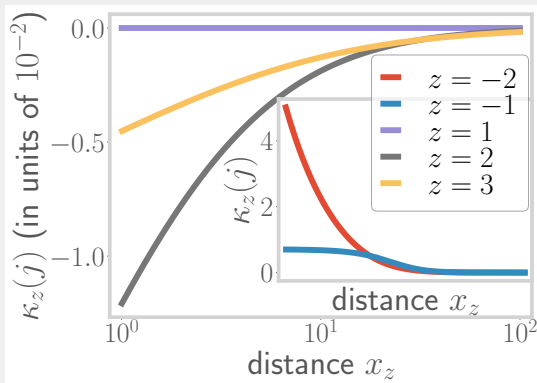
Mutual information: $I^2(A : B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j) / \Delta x_z(j), \quad v' = \Delta v_z(j) / \Delta x_z(j)$$

$$\text{Curvature as well: } \kappa_z(j) = \frac{v'_z(j)}{[1 + v_z(j)^2]^{\frac{3}{2}}}$$



- Distances and curvature can be related to an RG **beta function**
- Amounts to an **explicit demonstration** of the holographic principle
- Sign of curvature is **topological**, can be written in terms of winding numbers

$$S_{A_z(j)} = f_z(j)c\alpha L_x - \underbrace{c \log |2 \sin(\pi f_z(j)\phi)|}_{=Q(\phi), \text{ geometry-independent term}}$$

- $Q(\phi)$ is periodic in the flux ϕ , $\phi = 1$ transports a charge across Fermi surface
- pole structure of $\left(\sin \frac{\pi}{4} - |\sin(\pi f_z(j)\phi)|\right)^{-1}$ counts number of states \rightarrow tracks Luttinger volume
- Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers

FUTURE PROSPECTS

IMPROVEMENTS TO THE AUXILIARY MODEL

- Better model can be obtained by using multiple impurities
- Allows entangled liquid-like insulating phases
- Might also provide k -space resolution
 - ▶ partial gapping of Fermi surface?
 - ▶ pseudogap phases
- Introducing general impurity filling
 - ▶ new phases?
 - ▶ dominant pair fluctuations?

- Using local impurity models to create bulk lattice models (Bloch's theorem)

$$H_{\text{bulk}} = \sum_i H_{\text{local}}(i), \quad \psi_{\text{bulk}}(\vec{k}) \sim \sum_i e^{i\vec{k} \cdot \vec{r}_i} \psi_{\text{local}}(i)$$

- Relates bulk correlation functions to those of the auxiliary model
- phase transition in the extended AIM \longrightarrow phase transition in the bulk model, **metal-insulator transition** in Hubbard-Heisenberg model

- Should be useful for studying other models of strong-correlations
 - ▶ periodic Anderson/Kondo models
 - ▶ Heisenberg models
- Another potential application: topologically active systems:
 - ▶ Fractional quantum hall systems
- Extend the formalism towards higher order Greens functions
 - ▶ two-particle Greens functions, doublon-holon correlations
 - ▶ can provide more info on the MIT

- Materials with very high quasiparticle masses
- Outstanding questions exist about the nature of phases and phase transitions
 - ▶ microscopic justification of certain phases
 - ▶ theory for the strange metal excitations
 - ▶ microscopic justification for the origin of unconventional superconductivity
- the URG, MERG and auxiliary model methods should prove useful