Emergence in free and correlated fermions: from impurity models to the bulk

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Abhirup Mukherjee
Supervisor: Dr. Siddhartha Lal

Department of Physical Sciences, IISER Kolkata, Mohanpur

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Summary of Work

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1. 1-channel Kondo problem: *as second author*, published in Phys. Rev. B
   Phys. Rev. B 105, 085119

2. Multi-channel Kondo problem: *as second author*, under review at Phys. Rev. B
   arXiv:2205.00790

3. Generalised Anderson impurity model: manuscript **in preparation**

4. Entanglement scaling in free fermions: manuscript **in preparation**

5. New auxiliary model approach to correlated systems: **ongoing project**

Single-channel Kondo problem

Phys. Rev. B 105, 085119

Anirban Mukherjee, **Abhirup Mukherjee**, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal

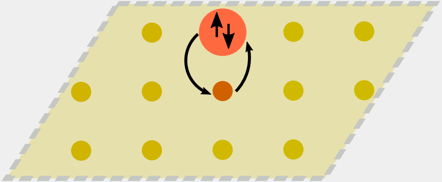
Single-channel Kondo problem

Model of impurity interacting with conduction electrons through spin-flips

1. Computation of the impurity spectral function

2. Emergence of a local Fermi liquid, and orthogonality catastrophe between local moment and singlet states

3. Calculating of thermal entropy



Kondo 1964; Wilson 1975; Andrei et al. 1983; Hewson 1993; Nozieres 1974; Anderson 1970; Tsvelick et al. 1983; Affleck et al. 1993; Goldhaber-Gordon et al. 1998; V. Borzenets et al. 2020; Sakai et al. 1989; Costi et al. 1990; Nozaki et al. 2012; Affleck 1995.

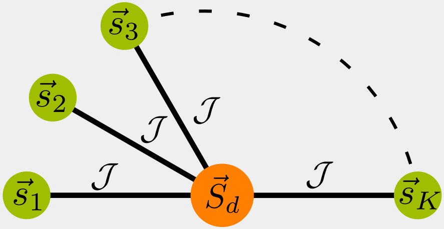
Multi-channel Kondo problem

arXiv:2205.00790

Siddhartha Patra, **Abhirup Mukherjee**, Anirban Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

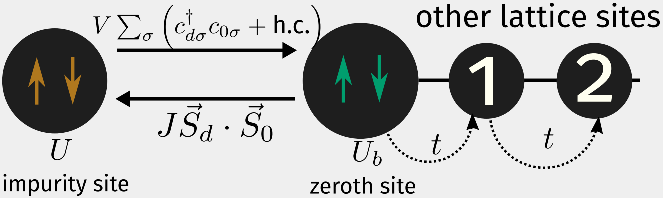
Model of impurity interacting with multiple conduction electron channels

1. Obtaining RG fixed point Hamiltonian
2. Analytical forms for degree of compensation, magnetization and susceptibility
3. Presence of a local marginal Fermi liquid
4. Dualities of the MCK model

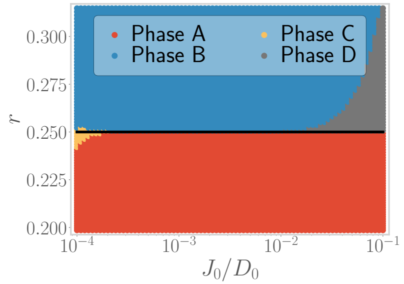


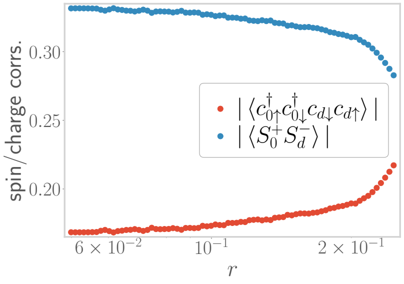
Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

Local metal-insulator transition in extended Anderson impurity model

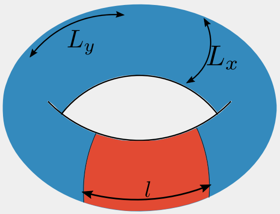


- Competition between J and U_b leads to phase transition from screened singlet phase at $|U_b| \leq 4J$ to unscreened local moment phase at $|U_b| > 4J$.
- Impurity spectral function becomes gapped beyond the critical point.
- Decoupling the impurity model leads to an effective model with the zeroth site as the correlated impurity, demonstrating the symmetry between the impurity and zeroth site.
- Geometric entanglement and mutual information track the transition by vanishing beyond the critical point.
- Subdominant pairing tendencies are observed near the quantum critical point.



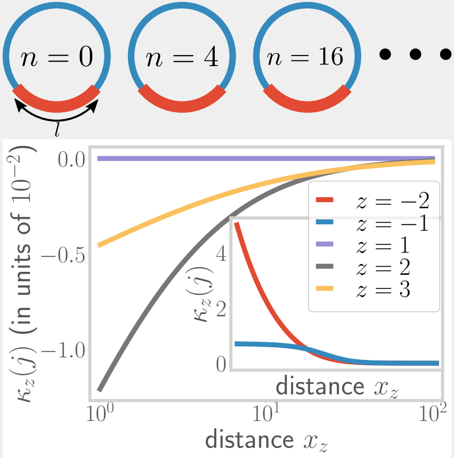


Entanglement scaling in free fermions



Entanglement scaling in free fermions

- Under coarse-graining or fine-graining in k-space, entanglement of free Dirac fermions shows hierarchical onion-like structure.
- Entanglement scaling can be used to define distances, leads to additional spatial dimension $\longrightarrow$ holography
- Emergent distances and curvature can be related to RG beta function; the sign of the curvature is topological.
- Pole structure of the entanglement tracks the Luttinger volume - invariant under the scaling transformations



Future Prospects

Future Prospects

- Better model can be obtained by taking multiple impurities and general impurity filling
- novel auxiliary model method can used for studying other models of strong-correlations as well as topologically active or flat band systems
- The URG can be applied to heavy-fermion materials towards a study of phase diagram and unconventional superconductivity, as well as Kondo insulators
- Interacting systems in a magnetic field is also a potential area of study, specifically fractional quantum hall effects

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Further Details

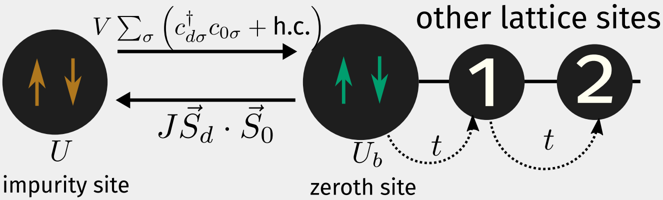
Local metal-insulator transition in extended Anderson impurity model

Extended Anderson impurity model

Standard Anderson impurity model $\longrightarrow$ only one stable phase (strong-coupling)

no possibility of phase transition $\longrightarrow$ Introduce additional correlation

- spin-flip correlation between impurity and bath: J
- local correlation in the bath: U_b

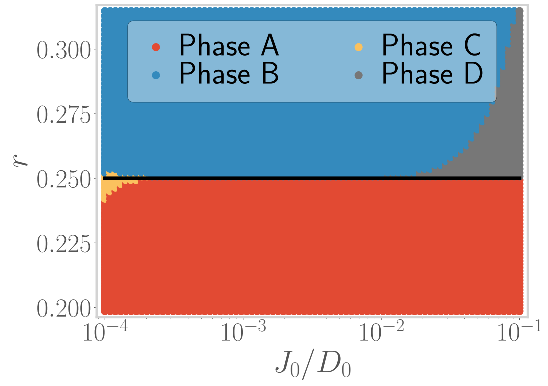


Anderson 1961; Krishna-murthy et al. 1980; Hewson 1993.

RG equations reveal critical point where J, V **become irrelevant**

1. orange phase: J is relevant: strong-coupling
2. blue phase: J is irrelevant: local moment
3. yellow phase: spin+charge liquid
4. gray phase: all couplings irrelevant

$r = -U_b/J$



Presence of a phase transition

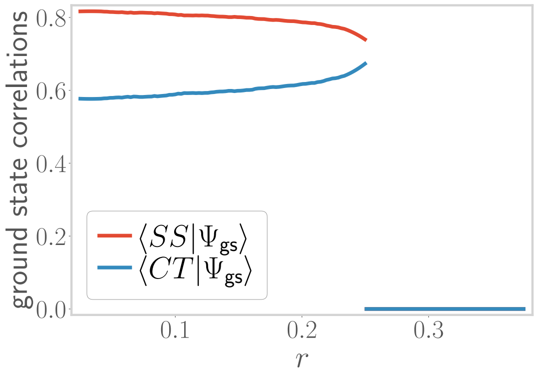
singlet ⟶ spin+charge liquid ⟶ local moment

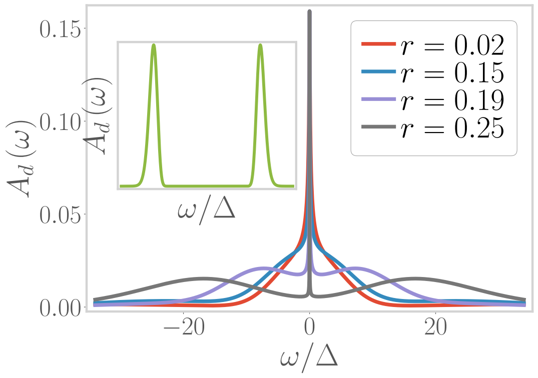
impurity spectral function gaps out

$r = -U_b/J$

$|SS\rangle = |\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle$

$|CT\rangle = |2,0\rangle + |0,2\rangle$





- Decoupling the impurity site leads to an Anderson impurity model

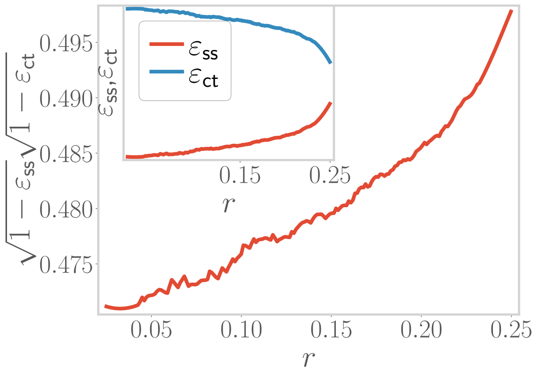
$$H_{0+\mathrm{rest}} = \underbrace{-\left(U_0 + U_b\right)\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2}_{\text{new correlated impurity}} \underbrace{-\,t \sum_{\substack{j\in\text{n.n. of } 0,\\ \sigma}} \left(c^\dagger_{0\sigma} c_{j\sigma} + \text{h.c.}\right)}_{\text{hopping between new impurity \& new bath}} \underbrace{-\,t \sum_{\langle i,j\rangle} \left(c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.}\right)}_{\text{K.E. of new bath}}$$

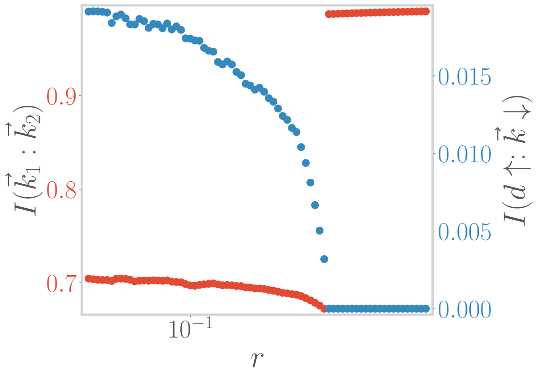
- correlated, dominant spin-flip processes lead to repulsive $U_{\text{eff}} = U_0 + U_b \sim J^*/8$
- J symmetrises the two sites, leading to similar spectral functions $\longrightarrow$ essence of self-consistency

Geometric entanglement: $\varepsilon(\psi_1, \psi_2) = 1 - |\langle\psi_1|\psi_2\rangle|^2$

$\longrightarrow \sqrt{1 - \varepsilon_{SS}}\sqrt{1 - \varepsilon_{CT}}$ is maximised, then vanishes

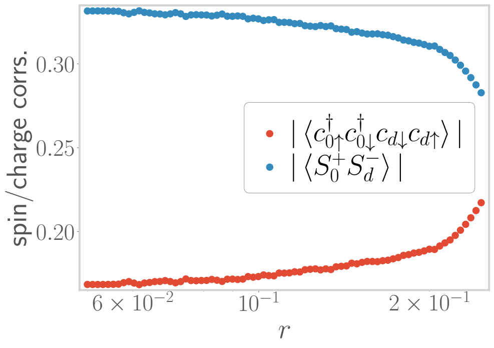
Mutual information between impurity and cloud vanishes

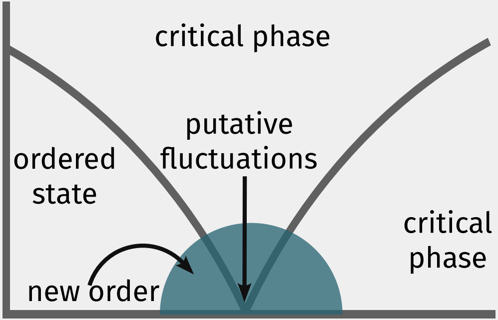




Presence of subdominant pair fluctuations

- **pairing tendencies** observed near the quantum critical point
- might lead to **superconductivity** with doping
- seen in cuprates, heavy-fermions materials, pnictides, etc





Entanglement scaling in free fermions

Free Dirac fermions on torus: $k_x^n = \frac{2\pi}{L_x} n, \quad n \in \mathbb{Z};$ define **sparsity** = $\Delta n = 1$

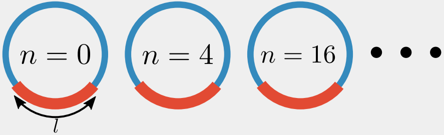
Simplest choice: the entire set

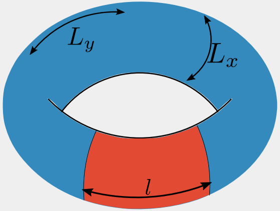
sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \ldots, -1, 0, 1, \ldots, N-2, N-1, N\}$

Coarser choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \ldots, -2, 0, 2, \ldots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \ldots, -4, 0, 4, \ldots, N-8, N-4, N\}$





Subsystem entanglement entropy: Entanglement hierarchy

$$S_{A_z(j)} = f_z(j) c \alpha L_x - c \log \left| 2 \sin\left(\pi f_z(j) \phi\right) \right|$$

$$i < j, \quad S_{i \cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$

- presents a **hierarchy** of entanglement $\longrightarrow$ EE distributed across RG steps
  RG transformation $\longrightarrow$ reveals entanglement

- distribution of entanglement also present in **multipartite** entanglement

Calabrese et al. 2004; Casini et al. 2005; Arias et al. 2015; Chen et al. 2017; Murciano et al. 2020.

Mutual information = distance

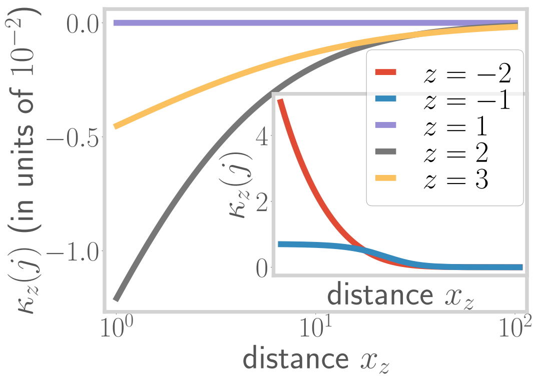
Mutual information: $I^2(A:B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j)/\Delta x_z(j), \quad v' = \Delta v_z(j)/\Delta x_z(j)$$

Curvature as well: $\kappa_z(j) = \dfrac{v_z'(j)}{\left[1 + v_z(j)^2\right]^{\frac{3}{2}}}$



Van Raamsdonk 2010; Lee et al. 2016; Mukherjee et al. 2022; Lee 2010; Lee 2014; Qi 2013; Lee et al. 2016; Mukherjee et al. 2020a; Mukherjee et al. 2020b; Ryu et al. 2006b; Ryu et al. 2006a; Nozaki et al. 2012.

RG evolution = emergent distance

- Distances and curvature can be related to an RG **beta function**
- Amounts to an **explicit demonstration** of the holographic principle
- Sign of curvature is **topological**, can be written in terms of winding numbers

Maldacena 1999; Ryu et al. 2006a; Holzhey et al. 1994.

Topological nature of geometry-independent term

$$S_{A_z(j)} = f_z(j) c \alpha L_x - \underbrace{c \log \left| 2 \sin\left(\pi f_z(j)\phi\right)\right|}_{=Q(\phi),\text{geometry-independent term}}$$

- $Q(\phi)$ is periodic in the flux ϕ, $\phi = 1$ transports a charge across Fermi surface
- pole structure of $\left(\sin\frac{\pi}{4} - |\sin\left(\pi f_z(j)\right)\phi|\right)^{-1}$ counts number of states $\longrightarrow$ tracks Luttinger volume
- Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers

Luttinger et al. 1960; Luttinger 1960; Oshikawa 2000; Seki et al. 2017; Mukherjee et al. 2020a; Heath et al. 2020.

Future Prospects

Improvements to the auxiliary model

- Better model can be obtained by using multiple impurities
- Allows entangled liquid-like insulating phases
- Might also provide k–space resolution
  - partial gapping of Fermi surface?
  - pseudogap phases
- Introducing general impurity filling
  - new phases?
  - dominant pair fluctuations?

- Using local impurity models to create bulk lattice models (Bloch's theorem)

$$H_{\text{bulk}} = \sum_i H_{\text{local}}(i), \quad \Psi_{\text{bulk}}(\vec{k}) \sim \sum_i e^{i\vec{k}\cdot\vec{r}_i}\Psi_{\text{local}}(i)$$

- Relates bulk correlation functions to those of the auxiliary model

- phase transition in the extended AIM $\longrightarrow$ phase transition inthe bulk model, **metal-insulator transition** in Hubbard-Heisenberg model

- Should be useful for studying other models of strong-correlations
  - periodic Anderson/Kondo models
  - Heisenberg models
- Another potential application: topologically active systems:
  - Fractional quantum hall systems
- Extend the formalism towards higher order Greens functions
  - two-particle Greens functions, doublon-holon correlations
  - can provide more info on the MIT

Heavy-fermion materials

- Materials with very high quasiparticle masses
- Outstanding questions exist about the nature of phases and phase transitions
  - microscopic justification of certain phases
  - theory for the strange metal excitations
  - microscopic justification for the origin of unconventional superconductivity
- the URG, MERG and auxiliary model methods should prove useful