Emergence in correlated fermions: From impurity models to the bulk

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RPC Presentation 2022-2023

Emergent Phenomena in Quantum Matter Group
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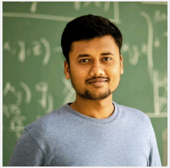
Siddhartha Lal

Anirban Mukherjee



Siddhartha Patra

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A huge thanks to all my collaborators!

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List of publications, preprints and ongoing projects

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  2022 **Phys. Rev. B** 105, 085119.

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- ✓ Frustration shapes multi-channel Kondo physics: a star graph perspective.

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- We are currently finishing the development of a new auxiliary model-based approach to studying correlated systems in the bulk.

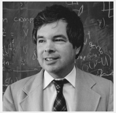
- We have begun work on a project that aims to provide new insights on the plateau-to-plateau transition in integer quantum hall systems.

How to explain the resistance minimum & eventual saturation?

Breakdown of perturbation theory indicates a **change in ground state**!

Obtaining T = 0 ground state requires more **powerful methods**

Numerical RG



(K. G. Wilson)

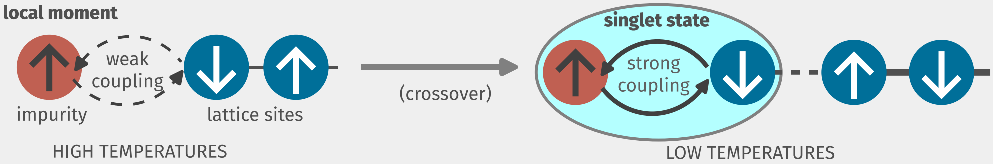
Bethe ansatz

(Natan Andrei)

Conf. field theory

(Ian Affleck)

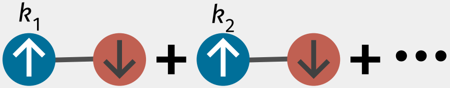
- impurity becomes **strongly coupled** at low temperatures
- local moment crosses over into **nonmagnetic** singlet

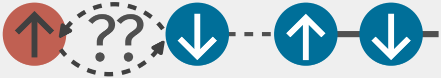


Wilson 1975; Andrei 1980; Wiegmann 1981; Affleck 1995.

Some important questions

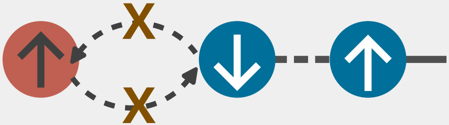
How do we describe the dynamics of the electrons that screen the impurity (the so-called **Kondo cloud**)?
[Mukherjee et al 2023 Phys. Rev. B 105, 085119]





What kind of physics can **disturb the Kondo screening** effect and distort the singlet state?
[Patra et al 2023 J. Phys.: Condens. Matter 35 315601]

What is the simplest impurity model that completely destroys the Kondo effect and leads to a **phase transition**?
[Mukherjee et al 2023 arXiv:2302.02328]



The single-channel Kondo problem: Anatomy of the Kondo cloud

PHYSICAL REVIEW B
covering condensed matter and materials physics

Unveiling the Kondo cloud: Unitary renormalization-group study of the Kondo model

Anirban Mukherjee, Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal

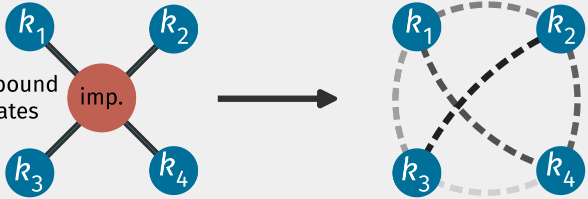
Phys. Rev. B **105**, 085119 – Published 14 February 2022

Effective Hamiltonian for the Kondo cloud

We first applied the **unitary RG** to obtain a low energy fixed point theory.

To obtain a theory for the Kondo cloud, we **trace out impurity** from fixed point Hamiltonian.

impurity bound to k– states



removing impurity generates interactions among k–states

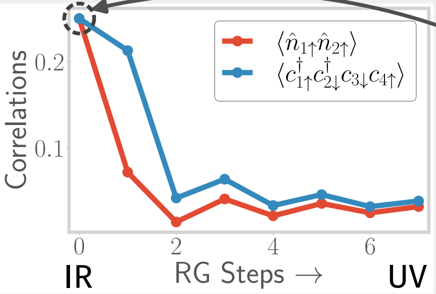
$$KE + J^*\vec{S}_d \cdot \sum_{kk'} \vec{s}_{kk'} \longrightarrow KE + \sum_{k_1,k_2} \frac{2\varepsilon(k_1)\varepsilon(k_2)}{J^*}\hat{n}_{k_1}\hat{n}_{k_2} + \sum_{\{k_i\}} V_{1234}\, c^\dagger_{k_4\uparrow} c^\dagger_{k_3\downarrow} c_{k_2\downarrow} c^\dagger_{k_1\uparrow}$$

- all-to-all interactions between momentum states, **large entanglement**
- 2-particle interaction terms **not** present in Fermi liquid, are **responsible for screening**

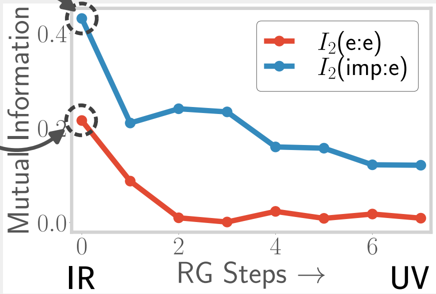
Mukherjee et al. 2022; Mukherjee et al. 2020a; Mukherjee et al. 2020b.

Quantifying entanglement within the Kondo cloud

In order to demonstrate formation of Kondo cloud, we study the **variation of entanglement** and correlations under RG transformations.



formation of singlet



- Both entanglement and k-space correlations **increase** as RG proceeds from UV to IR.
- This shows the formation of the **Kondo singlet** and the growth of two-particle correlations in the **Kondo cloud**.

Mukherjee et al. 2022.

Distorting the Kondo singlet: The multi-channel Kondo Problem

IOPscience

Journal of Physics: Condensed Matter

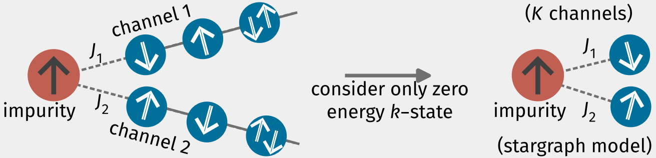
Frustration shapes multi-channel Kondo physics: a star graph perspective

Siddhartha Patra[1], Abhirup Mukherjee[1], Anirban Mukherjee[1], N S Vidhyadhiraja[2], A Taraphder[3] and Siddhartha Lal[4,1]

What is the multichannel Kondo problem?

Single impurity interacting with **multiple channels** in the bath

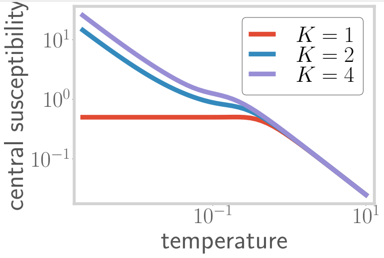
$$H_{\text{Kondo}} = KE_{\text{bath}} + \sum_l J_l \vec{S}_{\text{imp}} \cdot \vec{s}^{(l)}$$



Known to display divergent $T = 0$ impurity susceptibility (incomplete screening), and orthogonality catastrophe, **non-Fermi liquid** excitations.

Zero bandwidth limit is (analytically) solvable: $\{|S^z_{\text{tot}}\rangle\}$

- Ground state degeneracy for $K > 1$ explains **orthogonality catastrophe**
- $S^z_{\text{tot}} \neq 0$ in ground states shows incomplete screening
- Excitations shows **non-Fermi liquid** physics in the form of inter-channel scattering.



Nozières, Ph. et al. 1980; Affleck et al. 1993; Emery et al. 1992; Andrei et al. 1984.

How to destroy the Kondo cloud: Effect of local interactions in the bath

arXiv > cond-mat > arXiv:2302.02328

Condensed Matter > Strongly Correlated Electrons

[Submitted on 5 Feb 2023]

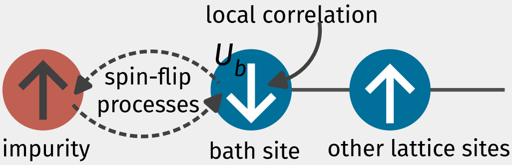
Kondo frustration via charge fluctuations: a route to Mott localisation

Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

What is the new physics ingredient?

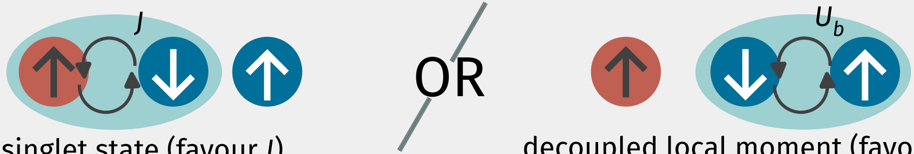
Add **local correlation** on bath (zeroth) site coupled to impurity

$$KE_{\text{bath}} + J\vec{S}_{\text{imp}} \cdot \vec{S}_{\text{bath}} - U_b \left(\vec{S}_{\text{bath}}\right)^2$$



URG equations show that an **attractive** U_b frustrates the zeroth site.

$$\Delta J \sim J^2 + 4U_b J \implies \textbf{phase transition} \text{ at } J = -4U_b$$



singlet state (favour J) — decoupled local moment (favour U_b)

Such a model sheds light on the Mott MIT in ∞-dimensions (as seen from DMFT).

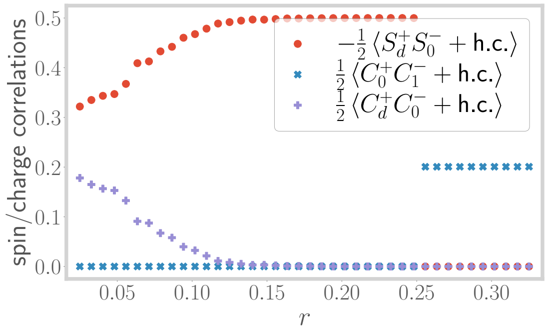
Georges et al. 1996.

Nature of the transition

Across the transition,

- impurity correlations vanish
- bath correlations become non-zero

Shows that **pairing correlations** in the bath are responsible for the transition.

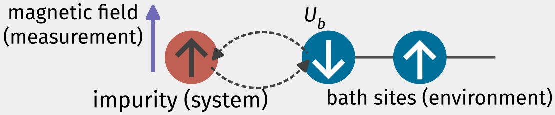


The state **precisely at the transition** is special:

- non-Fermi liquid excitations
- **fractional** impurity magnetisation and occupancy

Concluding remarks

- Our analyses often link entanglement measures with correlations, providing bridges between the worlds of condensed matter and quantum information.
- Models of Kondo breakdown can be used to study the effects of measurement on a system coupled to a bath.



- The Kondo model with attractive U_b term has applications in studying the physics of Mott transitions.


Thank you, all of you.

How to explain the resistance minimum & eventual saturation?

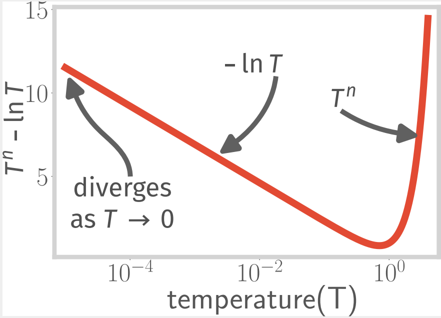
Second order perturbation theory in J gives:

$$\rho \sim T^n - \ln T$$

Explains the **non-monotonic** behaviour!

(Jun Kondo)

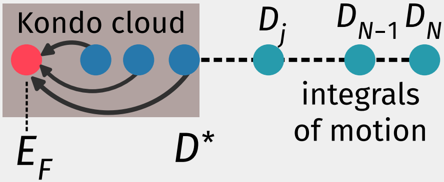
However, solution **diverges** at $T \to 0$!



Kondo 1964.

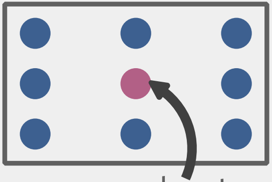
Unitary RG approach to impurity models

- Integrate out **high energy fluctuations** to reach strong-coupling low-energy theory
- Leads to **singlet ground state** and decoupled high-energy k–states
- Decoupling is carried out through **unitary transformations**



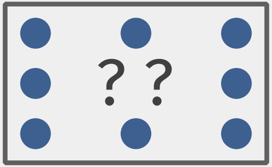
Mukherjee et al. 2020a; Mukherjee et al. 2020b.

- Entanglement entropy $S(A) \Longrightarrow$ quantifies how much **information is gained** about the rest of the system by measuring A



measure subsystem A

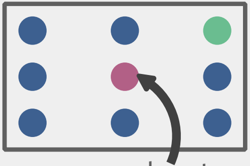
$$S(A) = \mathrm{Trace}\left(\rho_A \ln \rho_A\right)$$



gain information about rest

- Entanglement entropy $S(A) \Longrightarrow$ quantifies how much **information is gained** about the rest of the system by measuring A

- Mutual information $I_2(A : B) \Longrightarrow$ quantifies how much **information about subsystem A** is gained by measuring B



measure subsystem A

$$I_2(A : B) = S(A) + S(B) - S(A \cup B)$$

gain information about B