URG Analysis of Electron in a Periodic Potential

Role of the center of mass

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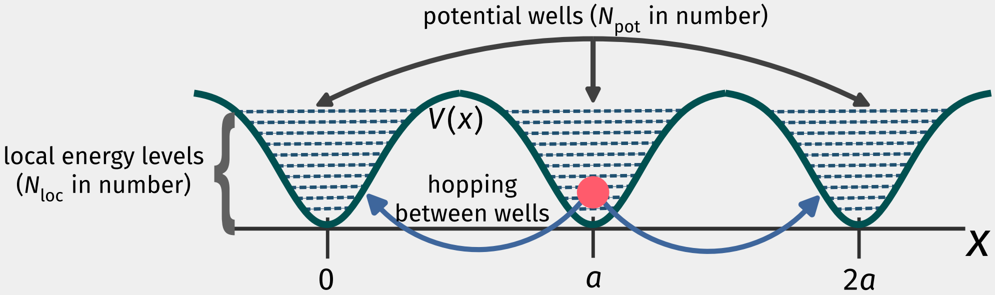
- We conclude by connecting this problem to that of the **IQHE**.

The Problem of a Particle in a Periodic Potential (PPP)

$$H = \int_{-\infty}^{\infty} dx \, c^{\dagger}(x) \left[\hat{p}^2 / 2m + V(x) \right] c(x), \quad V(x+a) = V(x)$$



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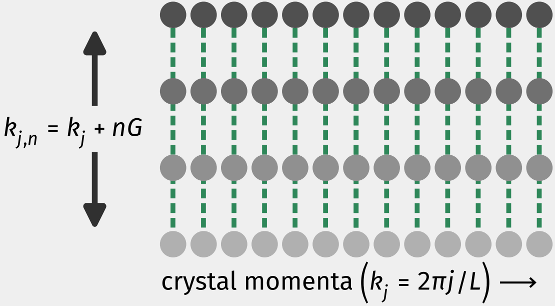
Potential only connects momentum states separated by a reciprocal lattice vector.

$$\langle k + q | V | k \rangle = \delta_{q,G} V(G)$$

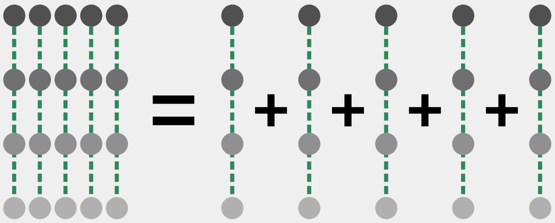
Leads to conserved **crystal momenta**: $\{k_j < G\}$



The PPP as a Particle on a Circle

The conserved crystal momenta leads to a block-diagonal form of the Hamiltonian.

$$H = \sum_k H(k), \quad H(k) \sim \left(-i\hbar \frac{\partial}{\partial x'} + \hbar k\right)^2 + V(x')$$

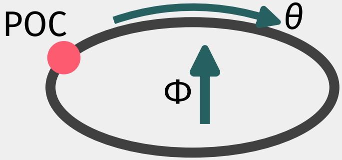


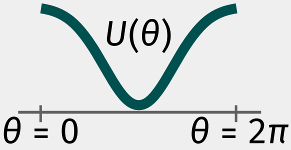
The PPP as a Particle on a Circle

Define dimensionless position and momentum.

$$H(k) = \frac{\hbar^2}{2ma^2}\left(\hat{Q} + \Phi/2\pi\right)^2 + U(\theta)$$

Hamiltonian is that of a **particle on a circle**. Flux is $\Phi = ka$.





THANK YOU.