

EMERGENCE IN FREE AND CORRELATED FERMIONS: FROM IMPURITY MODELS TO THE BULK

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SUMMARY OF WORK

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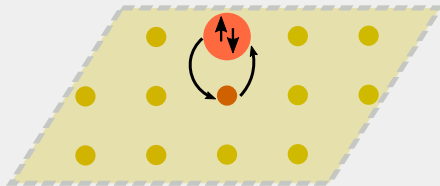
1. 1-channel Kondo problem: *as second author*, **published** in Phys. Rev. B
2. Multi-channel Kondo problem: *as second author*, **under review** at Phys. Rev. B
3. Generalised Anderson impurity model: manuscript **in preparation**
4. Entanglement scaling in free fermions: manuscript **in preparation**
5. New auxiliary model approach to correlated systems: **ongoing project**

SINGLE-CHANNEL KONDO PROBLEM

SINGLE-CHANNEL KONDO PROBLEM

Model of impurity interacting with conduction electrons through spin-flips

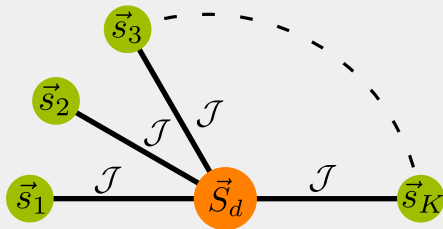
1. Computation of the impurity spectral function
2. Emergence of a local Fermi liquid, and orthogonality catastrophe between local moment and singlet states
3. Calculating of thermal entropy



MULTI-CHANNEL KONDO PROBLEM

Model of impurity interacting with multiple conduction electron channels

1. Obtaining RG fixed point Hamiltonian
2. Analytical forms for degree of compensation, magnetization and susceptibility
3. Presence of a local marginal Fermi liquid
4. Dualities of the MCK model



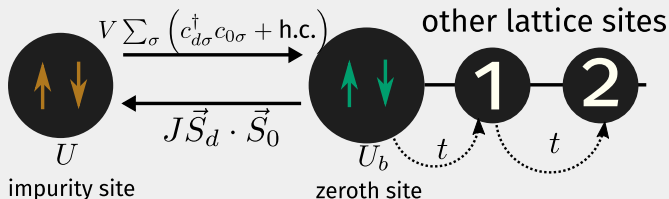
LOCAL METAL-INSULATOR TRANSITION IN EXTENDED ANDERSON IMPURITY MODEL

EXTENDED ANDERSON IMPURITY MODEL

Standard Anderson impurity model \rightarrow only one stable phase (strong-coupling)

no possibility of phase transition \rightarrow Introduce additional correlation

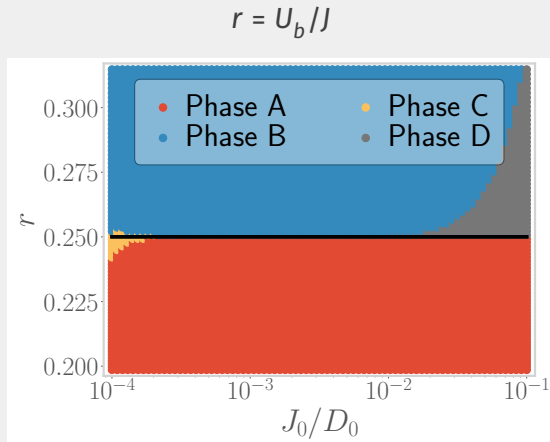
- spin-flip correlation between impurity and bath: J
- local correlation in the bath: U_b



RG PHASE DIAGRAM

RG equations reveal critical point where J, V **become irrelevant**

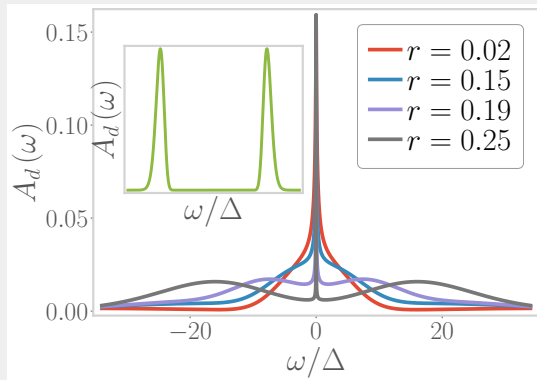
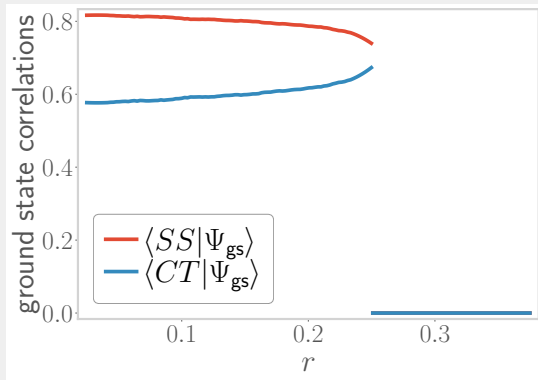
1. orange phase: J is relevant: strong-coupling
2. blue phase: J is irrelevant: local moment
3. yellow phase: spin+charge liquid
4. gray phase: all couplings irrelevant



PRESENCE OF A PHASE TRANSITION

singlet \longrightarrow spin+charge liquid \longrightarrow local moment

impurity spectral function gaps out

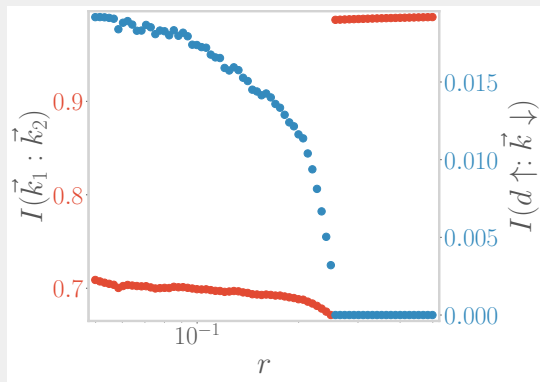
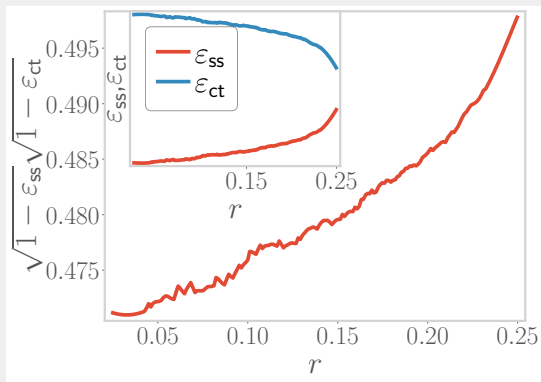


ENTANGLEMENT AS A PROBE FOR THE TRANSITION

Geometric entanglement: $\varepsilon(\psi_1, \psi_2) = 1 - |\langle \psi_1 | \psi_2 \rangle|^2$

$\rightarrow \sqrt{1 - \varepsilon_{SS}} \sqrt{1 - \varepsilon_{CT}}$ is maximised, then vanishes

Mutual information between impurity and cloud vanishes



ENTANGLEMENT SCALING IN FREE FERMIONS

Free Dirac fermions on torus: $k_x^n = \frac{2\pi}{L_x}n$, $n \in \mathbb{Z}$; define **sparsity** = $\Delta n = 1$

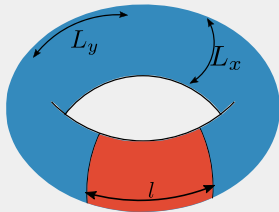
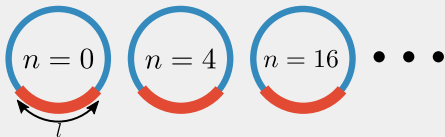
Simplest choice: the entire set

sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \dots, -1, 0, 1, \dots, N-2, N-1, N\}$

Coarser choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \dots, -2, 0, 2, \dots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \dots, -4, 0, 4, \dots, N-8, N-4, N\}$



$$S_{A_z(j)} = f_z(j) c \alpha L_x - c \log |2 \sin(\pi f_z(j) \phi)|$$

$$i < j, S_{i \cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$



- presents a **hierarchy** of entanglement \rightarrow EE distributed across RG steps
RG transformation \rightarrow reveals entanglement
- distribution of entanglement also present in **multipartite** entanglement

MUTUAL INFORMATION = DISTANCE

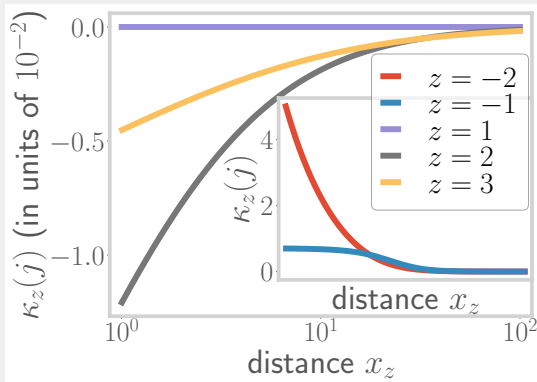
Mutual information: $I^2(A : B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j) / \Delta x_z(j), \quad v' = \Delta v_z(j) / \Delta x_z(j)$$

$$\text{Curvature as well: } \kappa_z(j) = \frac{v'_z(j)}{[1 + v_z(j)^2]^{\frac{3}{2}}}$$



- Distances and curvature can be related to an RG **beta function**
- Amounts to an **explicit demonstration** of the holographic principle
- Sign of curvature is **topological**, can be written in terms of winding numbers

$$S_{A_z(j)} = f_z(j)c\alpha L_x - \underbrace{c \log |2 \sin(\pi f_z(j)\phi)|}_{=Q(\phi), \text{ geometry-independent term}}$$

- $Q(\phi)$ is periodic in the flux ϕ , $\phi = 1$ transports a charge across Fermi surface
- pole structure of $\left(\sin \frac{\pi}{4} - |\sin(\pi f_z(j)\phi)|\right)^{-1}$ counts number of states \rightarrow tracks Luttinger volume
- Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers

FUTURE PROSPECTS

- Using local impurity models to create bulk lattice models (Bloch's theorem)

$$H_{\text{bulk}} = \sum_i H_{\text{local}}(i), \quad \psi_{\text{bulk}}(\vec{k}) \sim \sum_i e^{i\vec{k} \cdot \vec{r}_i} \psi_{\text{local}}(i)$$

- Relates bulk correlation functions to those of the auxiliary model
- phase transition in the extended AIM \longrightarrow phase transition in the bulk model, **metal-insulator transition** in Hubbard-Heisenberg model

- Should be useful for studying other models of strong-correlations
 - ▶ periodic Anderson/Kondo models
 - ▶ Heisenberg models
- Another potential application: topologically active systems:
 - ▶ Fractional quantum hall systems

- Method can be made more powerful by using multiple impurities
- Allows entangled liquid-like insulating phases
- Might also provide k -space resolution
 - ▶ partial gapping of Fermi surface?
 - ▶ pseudogap phases
- Extend the formalism towards higher order Greens functions
 - ▶ two-particle Greens functions, doublon-holon correlations
 - ▶ can provide more info on the MIT

- Materials with very high quasiparticle masses
- Outstanding questions exist about the nature of phases and phase transitions
 - ▶ microscopic justification of certain phases
 - ▶ theory for the strange metal excitations
 - ▶ microscopic justification for the origin of unconventional superconductivity
- the URG, MERG and auxiliary model methods should prove useful

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