Kondo Effect & Its Breakdown: Interplay of fluctuations in zero dimension

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A huge thanks to all my collaborators!

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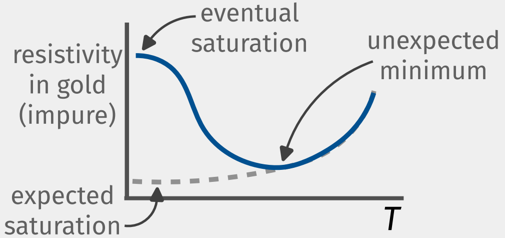
Introducing the Kondo effect

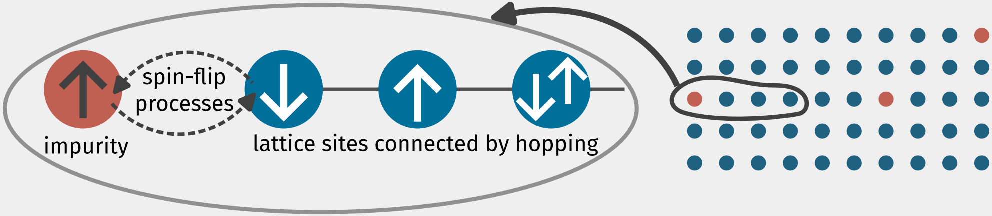
Where it all began

What is the Kondo effect?

- metal resistivity is **monotonic**: $\rho \sim T^n$
- dilute alloys show anomalous **minimum**
- resistivity eventually becomes **constant**



Can be explained using the **Kondo model**: $H_{\text{Kondo}} = KE_{\text{bath}} + J\vec{S}_{\text{imp}} \cdot \vec{S}_{\text{bath}}$



de Haas et al. 1934; Sarachik et al. 1964; Heeger 1969.

How to explain the resistance minimum & eventual saturation?

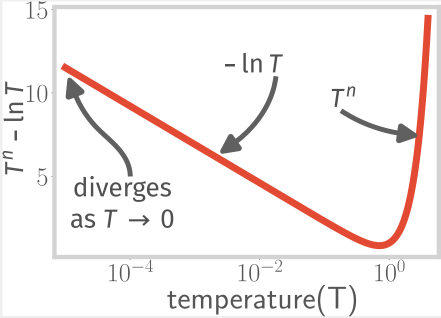
Second order perturbation theory in J gives:

$$\rho \sim T^n - \ln T$$

(Jun Kondo)

Explains the **non-monotonic** behaviour!

However, solution **diverges** at $T \rightarrow 0$!



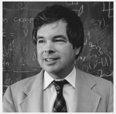
Kondo 1964.

How to explain the resistance minimum & eventual saturation?

Breakdown of perturbation theory indicates a **change in ground state**!

Obtaining T = 0 ground state requires more **powerful methods**

Numerical RG



(K. G. Wilson)

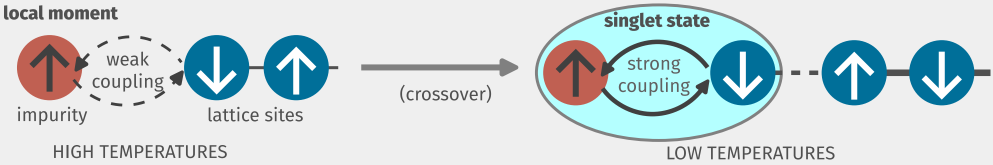
Bethe ansatz

(Natan Andrei)

Conf. field theory

(Ian Affleck)

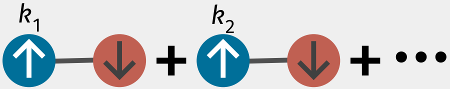
- impurity becomes **strongly coupled** at low temperatures
- local moment crosses over into **nonmagnetic** singlet

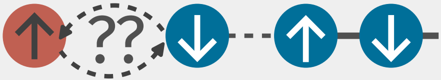


Wilson 1975; Andrei 1980; Wiegmann 1981; Affleck 1995.

Some important questions

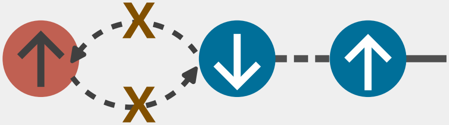
1. How do we describe the dynamics of the electrons that screen the impurity (the so-called **Kondo cloud**)?





2. What kind of physics can **disturb the Kondo screening** effect and distort the singlet state?

3. What is the simplest impurity model that completely destroys the Kondo effect and leads to a **phase transition**?



The single-channel Kondo problem: Anatomy of the Kondo cloud

PHYSICAL REVIEW B
covering condensed matter and materials physics

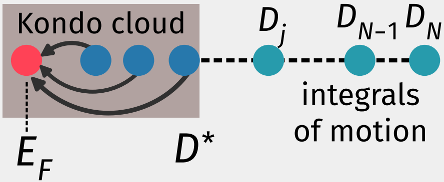
Unveiling the Kondo cloud: Unitary renormalization-group study of the Kondo model

Anirban Mukherjee, Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal

Phys. Rev. B **105**, 085119 – Published 14 February 2022

Unitary RG approach to impurity models

- Integrate out **high energy fluctuations** to reach strong-coupling low-energy theory
- Leads to **singlet ground state** and decoupled high-energy k–states
- Decoupling is carried out through **unitary transformations**

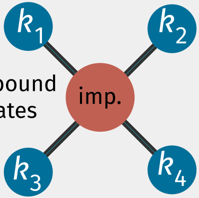


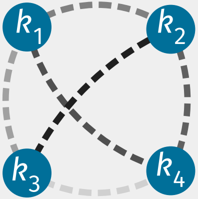
Mukherjee et al. 2020a; Mukherjee et al. 2020b.

Effective Hamiltonian for the Kondo cloud

In order to obtain a theory for the Kondo cloud,
we **trace out impurity** from fixed point Hamiltonian.

impurity bound to k- states





removing impurity generates interactions among k-states

$$KE + J^* \vec{S}_d \cdot \sum_{kk'} \vec{s}_{kk'} \longrightarrow KE + \sum_{k_1,k_2} \frac{2\varepsilon(k_1)\varepsilon(k_2)}{J^*} \hat{n}_{k_1} \hat{n}_{k_2} + \sum_{\{k_i\}} V_{1234}\, c^\dagger_{k_4\uparrow} c^\dagger_{k_3\downarrow} c_{k_2\downarrow} c^\dagger_{k_1\uparrow}$$

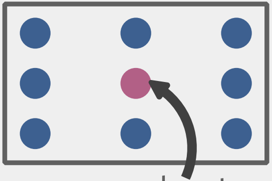
- all-to-all interactions between momentum states, **large entanglement**
- 2-particle interaction terms **not** present in Fermi liquid, are **responsible for screening**

Mukherjee et al. 2022.

Quantifying entanglement within the Kondo cloud

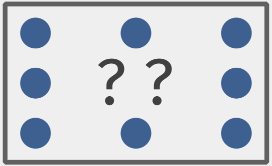
In order to demonstrate formation of Kondo cloud, we study the **variation of entanglement** and correlations under RG transformations.

- Entanglement entropy $S(A) \implies$ quantifies how much **information is gained** about the rest of the system by measuring A



measure subsystem A

$$S(A) = \text{Trace}\left(\rho_A \ln \rho_A\right)$$



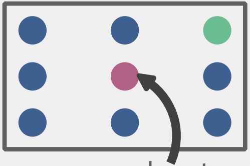
gain information about rest

Quantifying entanglement within the Kondo cloud

In order to demonstrate formation of Kondo cloud, we study the **variation of entanglement** and correlations under RG transformations.

- Entanglement entropy $S(A) \Longrightarrow$ quantifies how much **information is gained** about the rest of the system by measuring A

- Mutual information $I_2(A:B) \Longrightarrow$ quantifies how much **information about subsystem A** is gained by measuring B



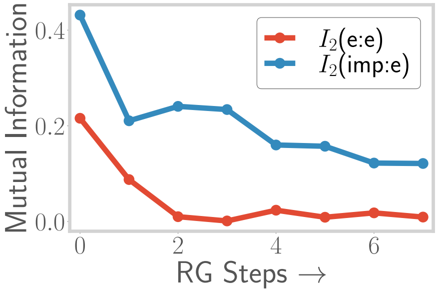
measure subsystem A

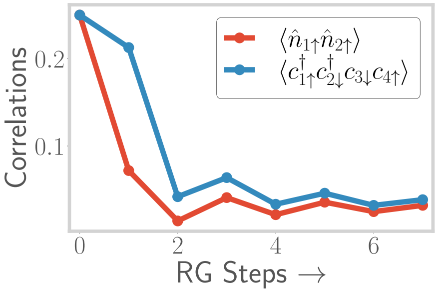
$$I_2(A:B) = S(A) + S(B) - S(A \cup B)$$

gain information about B

Quantifying entanglement within the Kondo cloud

Both entanglement and k-space correlations **increase** as RG proceeds from UV to IR.





- The former shows the formation of the **Kondo singlet**
- The latter shows the growth of two-particle correlations in the **Kondo cloud**

Mukherjee et al. 2022.

Distorting the Kondo singlet: The multi-channel Kondo Problem

IOPscience

Journal of Physics: Condensed Matter

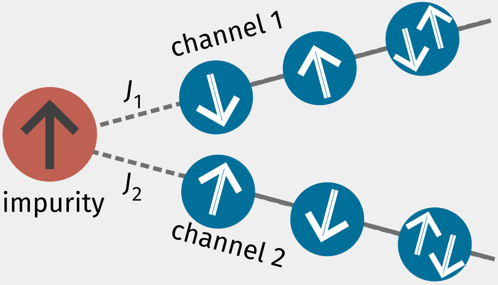
Frustration shapes multi-channel Kondo physics: a star graph perspective

Siddhartha Patra[1], Abhirup Mukherjee[1], Anirban Mukherjee[1], N S Vidhyadhiraja[2], A Taraphder[3] and Siddhartha Lal[4,1]

What is the multichannel Kondo problem?

Single impurity interacting with **multiple channels** in the bath

$$H_{\text{Kondo}} = KE_{\text{bath}} + \sum_{l} J_l \vec{S}_{\text{imp}} \cdot \vec{s}^{(l)}$$



Known to display divergent impurity susceptibility as $T \to 0$, indicating **incomplete screening**

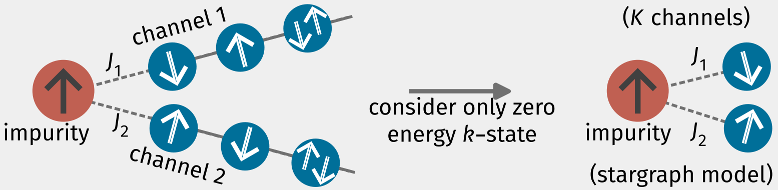
Other anomalous features: orthogonality catastrophe, **non-Fermi liquid** excitations, diverging specific heat

Nozières, Ph. et al. 1980; Affleck et al. 1993; Emery et al. 1992; Andrei et al. 1984.

Can a simpler model make this more intuitive?

At $T = 0$ and in the ground state, consider **only** lowest energy state in bath.

$$KE_{\text{bath}} + \sum_{l=1}^{K} J_l \vec{S}_{\text{imp}} \cdot \vec{s}^{(l)} \Longrightarrow \sum_{l=1}^{K} J_l \vec{S}_{\text{imp}} \cdot \vec{s}_0^{\,(l)}$$



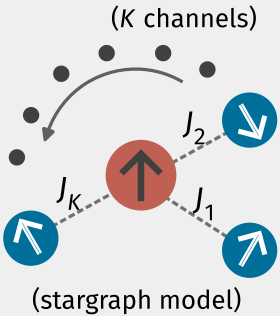
Patra et al. 2023.

Can a simpler model make this more intuitive?

Model is (analytically) solvable : $\{|S^z_{\text{tot}}\rangle\}$

Ground state is **degenerate** for $K > 1$

- Degeneracy for $K > 1$ shows orthogonality catastrophe
- $S^z_{\text{tot}} \neq 0$ in ground states shows incomplete screening



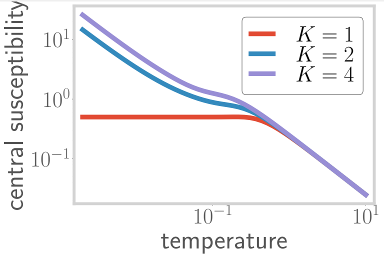
Patra et al. 2023.

Can a simpler model make this more intuitive?

Magnetisation of central node goes as

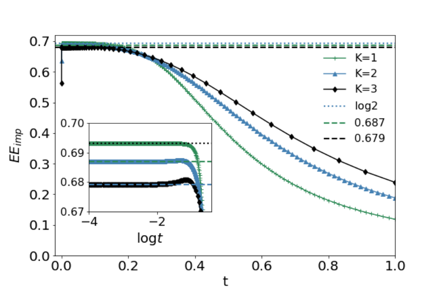
$$m = \frac{1 - K}{2(1 + K)}$$

- Susceptibility shows $T = 0$ **divergence** for $K > 1$
- Magnetisation can be linked to **scattering phase shifts** and hence to RG equation



Patra et al. 2023.

Can a simpler model make this more intuitive?



Upon including excitations above the stargraph

- entanglement entropy of central node shows **discontinuity** $\implies$ orthogonality catastrophe
- effective Hamiltonian for these excitations shows **non-Fermi liquid** physics

Patra et al. 2023.

How to destroy the Kondo cloud: Effect of local interactions in the bath

arXiv > cond-mat > arXiv:2302.02328

Condensed Matter > Strongly Correlated Electrons

[Submitted on 5 Feb 2023]

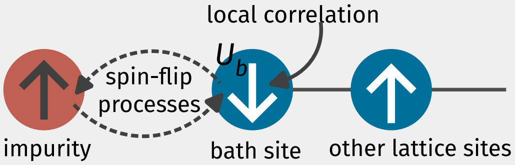
Kondo frustration via charge fluctuations: a route to Mott localisation

Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

WHAT IS THE NEW PHYSICS INGREDIENT?

Add **local correlation** on bath (zeroth) site coupled to impurity

$$KE_{\text{bath}} + J\vec{S}_{\text{imp}} \cdot \vec{S}_{\text{bath}} - U_b\left(\vec{S}_{\text{bath}}\right)^2$$

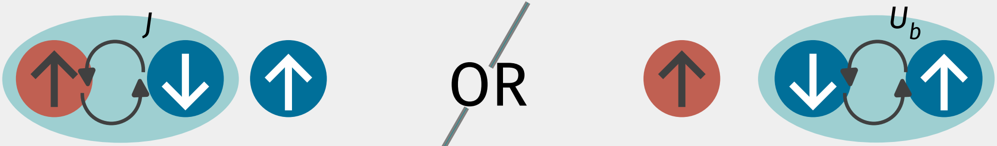


What is the new physics ingredient?

URG equations show that an **attractive** U_b competes with J:

$$\Delta J \sim J^2 + 4U_b J \implies \textbf{phase transition at } J = -4U_b$$

This happens because the zeroth site is **frustrated**.



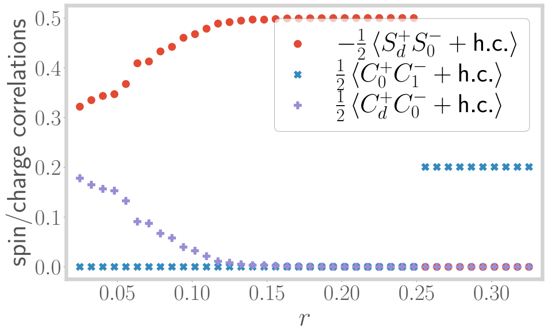
singlet state (favour J)?

decoupled local moment (favour U_b)?

Nature of the transition

Correlation functions show **discontinuity** across transition.

Impurity correlations vanish, bath correlations become non-zero.



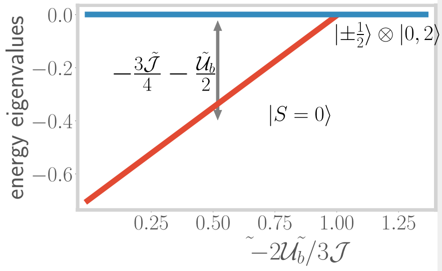
Shows that **pairing correlations** in the bath are responsible for the transition.

Nature of the transition

Manifests as a **level crossing** in the zero bandwidth problem.

The state precisely at the transition is special:

- non-Fermi liquid excitations
- **fractional** impurity magnetisation and occupancy



Concluding remarks and takeaways

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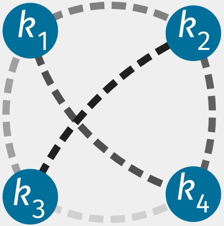
We have answered the questions posed at the beginning:

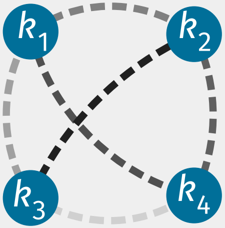
- obtained the effective theory for the Kondo cloud,

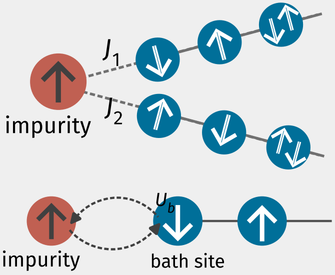


Concluding remarks

We have answered the questions posed at the beginning:

- obtained the effective theory for the Kondo cloud,
- learnt multiple ways of hampering the Kondo effect, and

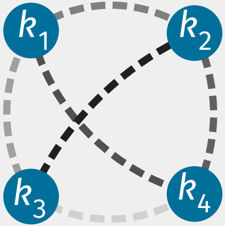


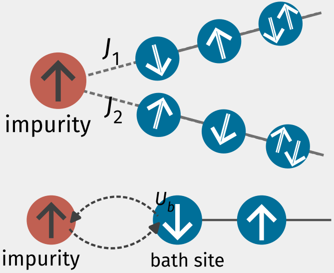


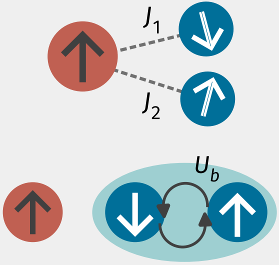
CONCLUDING REMARKS

We have answered the questions posed at the beginning:

- obtained the effective theory for the Kondo cloud,
- learnt multiple ways of hampering the Kondo effect, and
- studied the various phases arising from this.

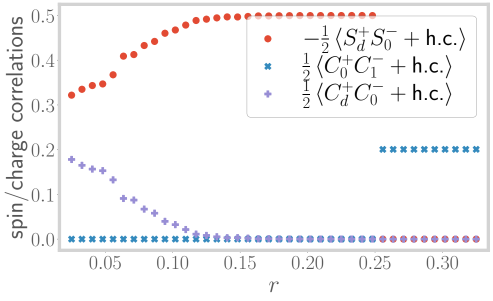


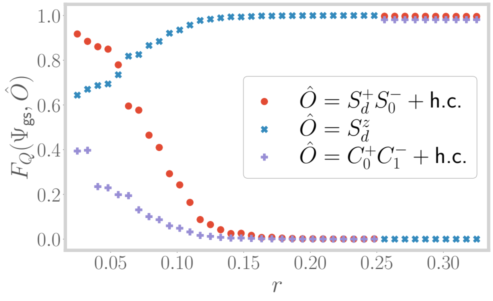




Takeaways

- Our analyses often link entanglement measures with correlations, providing bridges between the worlds of condensed matter and quantum information.





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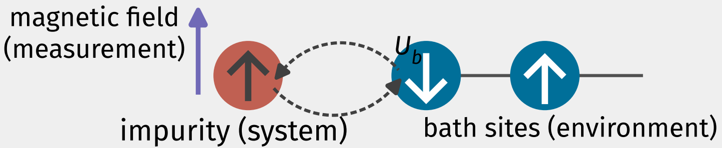
- Our analyses often link entanglement measures with correlations, providing bridges between the worlds of condensed matter and quantum information.
- Models of Kondo breakdown can be used to study the effects of measurement on a system coupled to a bath.



Takeaways

- Our analyses often link entanglement measures with correlations, providing bridges between the worlds of condensed matter and quantum information.
- Models of Kondo breakdown can be used to study the effects of measurement on a system coupled to a bath.
- The Kondo model with attractive U_b term has applications in studying the physics of Mott transitions.

That's all!

Thanks to everyone.