Emergence in free and correlated fermions: from impurity models to the bulk

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Summary of Work

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1. 1-channel Kondo problem: *as second author*, published in Phys. Rev. B
   Phys. Rev. B 105, 085119

2. Multi-channel Kondo problem: *as second author*, under review at Phys. Rev. B
   arXiv:2205.00790

3. Generalised Anderson impurity model: manuscript **in preparation**

4. Entanglement scaling in free fermions: manuscript **in preparation**

5. New auxiliary model approach to correlated systems: **ongoing project**

Single-channel Kondo problem

Phys. Rev. B 105, 085119

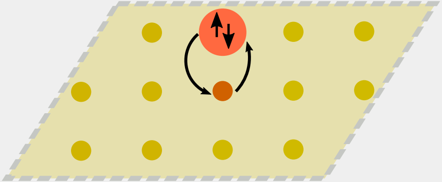
Anirban Mukherjee, *Abhirup Mukherjee*, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal

Model of impurity interacting with conduction electrons through spin-flips

1. Computation of the impurity spectral function

2. Emergence of a local Fermi liquid, and orthogonality catastrophe between local moment and singlet states

3. Calculating of thermal entropy



Kondo 1964; Wilson 1975; Andrei et al. 1983; Hewson 1993; Nozieres 1974; Anderson 1970; Tsvelick et al. 1983; Affleck et al. 1993; Goldhaber-Gordon et al. 1998; V. Borzenets et al. 2020; Sakai et al. 1989; Costi et al. 1990; Nozaki et al. 2012; Affleck 1995.

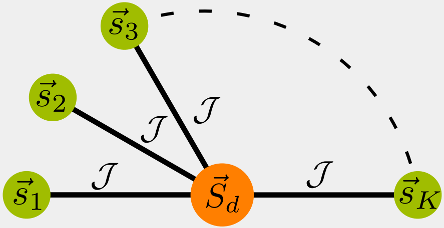
Multi-channel Kondo problem

arXiv:2205.00790

Siddhartha Patra, Abhirup Mukherjee, Anirban Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

Model of impurity interacting with multiple conduction electron channels

1. Obtaining RG fixed point Hamiltonian
2. Analytical forms for degree of compensation, magnetization and susceptibility
3. Presence of a local marginal Fermi liquid
4. Dualities of the MCK model



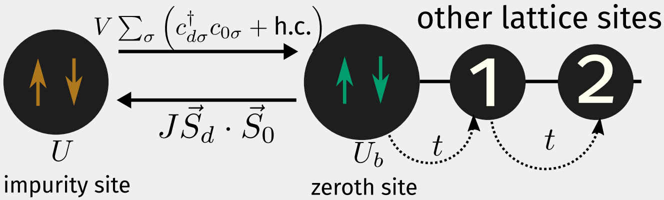
Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

Local metal-insulator transition in extended Anderson impurity model

Standard Anderson impurity model ⟶ only one stable phase (strong-coupling)

no possibility of phase transition ⟶ Introduce additional correlation

- spin-flip correlation between impurity and bath: J
- local correlation in the bath: U_b

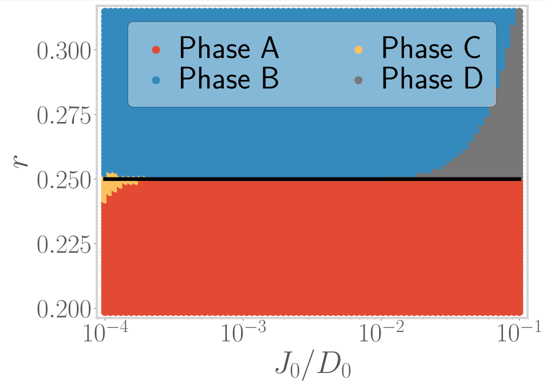


Anderson 1961; Krishna-murthy et al. 1980; Hewson 1993.

RG equations reveal critical point where J, V **become irrelevant**

1. orange phase: J is relevant: strong-coupling
2. blue phase: J is irrelevant: local moment
3. yellow phase: spin+charge liquid
4. gray phase: all couplings irrelevant

$r = -U_b/J$



Presence of a phase transition

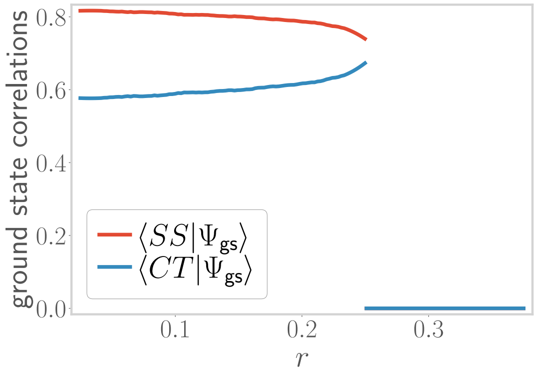
singlet ⟶ spin+charge liquid ⟶ local moment

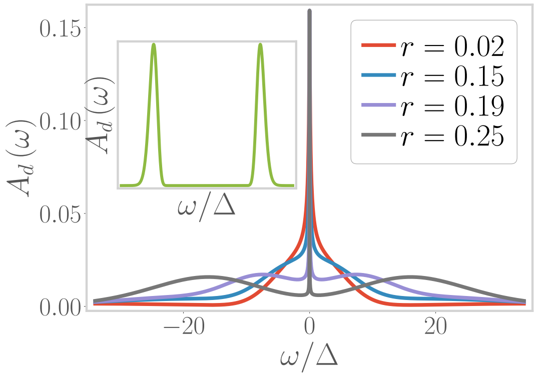
impurity spectral function gaps out

$r = -U_b/J$

$|SS\rangle = |\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle$

$|CT\rangle = |2, 0\rangle + |0, 2\rangle$



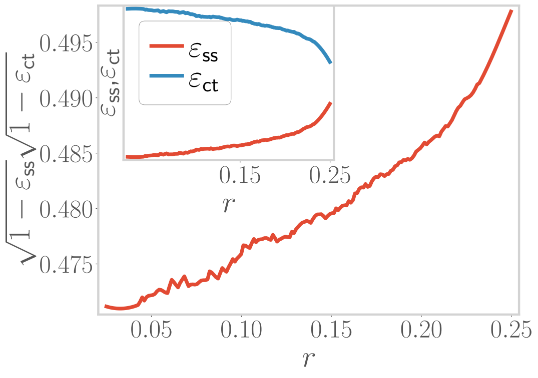


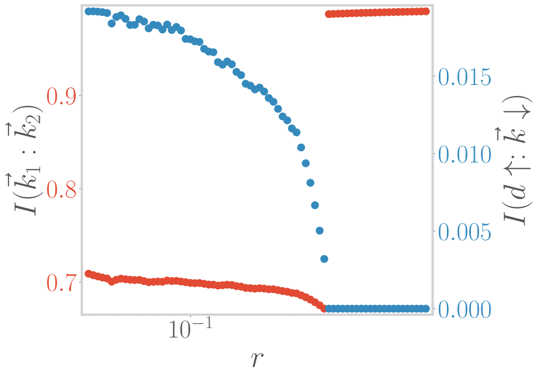
Entanglement as a probe for the transition

Geometric entanglement: $\varepsilon(\psi_1, \psi_2) = 1 - |\langle \psi_1 | \psi_2 \rangle|^2$

$\longrightarrow \sqrt{1-\varepsilon_{SS}}\sqrt{1-\varepsilon_{CT}}$ is maximised, then vanishes

Mutual information between impurity and cloud vanishes





Entanglement scaling in free fermions

Free Dirac fermions on torus: $k_x^n = \frac{2\pi}{L_x} n, \quad n \in \mathbb{Z}; \quad$ define **sparsity** $= \Delta n = 1$

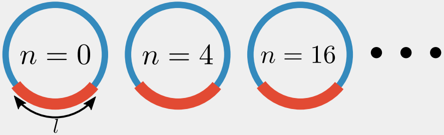
Simplest choice: the entire set

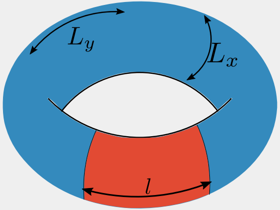
sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \ldots, -1, 0, 1, \ldots, N-2, N-1, N\}$

Coarser choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \ldots, -2, 0, 2, \ldots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \ldots, -4, 0, 4, \ldots, N-8, N-4, N\}$





Subsystem entanglement entropy: Entanglement hierarchy

$$S_{A_z(j)} = f_z(j)c\alpha L_x - c\log\left|2\sin\left(\pi f_z(j)\phi\right)\right|$$

$$i < j,\quad S_{i\cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$

- presents a **hierarchy** of entanglement $\longrightarrow$ EE distributed across RG steps
  RG transformation $\longrightarrow$ reveals entanglement

- distribution of entanglement also present in **multipartite** entanglement

Calabrese et al. 2004; Casini et al. 2005; Arias et al. 2015; Chen et al. 2017; Murciano et al. 2020.

Mutual information = distance

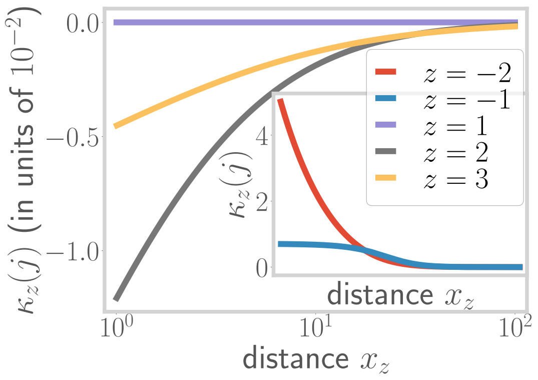
Mutual information: $I^2(A:B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j)/\Delta x_z(j), \quad v' = \Delta v_z(j)/\Delta x_z(j)$$

Curvature as well: $$\kappa_z(j) = \frac{v_z'(j)}{\left[1 + v_z(j)^2\right]^{\frac{3}{2}}}$$



Van Raamsdonk 2010; Lee et al. 2016; Mukherjee et al. 2022; Lee 2010; Lee 2014; Qi 2013; Lee et al. 2016; Mukherjee et al. 2020a; Mukherjee et al. 2020b; Ryu et al. 2006b; Ryu et al. 2006a; Nozaki et al. 2012.

RG evolution = emergent distance

- Distances and curvature can be related to an RG **beta function**
- Amounts to an **explicit demonstration** of the holographic principle
- Sign of curvature is **topological**, can be written in terms of winding numbers

Maldacena 1999; Ryu et al. 2006a; Holzhey et al. 1994.

Topological nature of geometry-independent term

$$S_{A_z(j)} = f_z(j) c \alpha L_x - \underbrace{c \log \left| 2 \sin\left(\pi f_z(j) \phi\right) \right|}_{=Q(\phi),\text{geometry-independent term}}$$

- $Q(\phi)$ is periodic in the flux ϕ, $\phi = 1$ transports a charge across Fermi surface
- pole structure of $\left(\sin\frac{\pi}{4} - |\sin\left(\pi f_z(j)\right)\phi|\right)^{-1}$ counts number of states $\longrightarrow$ tracks Luttinger volume
- Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers

Luttinger et al. 1960; Luttinger 1960; Oshikawa 2000; Seki et al. 2017; Mukherjee et al. 2020a; Heath et al. 2020.

Future Prospects

A novel auxiliary model approach

- Using local impurity models to create bulk lattice models (Bloch's theorem)

$$H_{\text{bulk}} = \sum_i H_{\text{local}}(i), \quad \Psi_{\text{bulk}}(\vec{k}) \sim \sum_i e^{i\vec{k}\cdot\vec{r}_i}\Psi_{\text{local}}(i)$$

- Relates bulk correlation functions to those of the auxiliary model
- phase transition in the extended AIM $\longrightarrow$ phase transition inthe bulk model, **metal-insulator transition** in Hubbard-Heisenberg model

A novel auxiliary model approach

- Should be useful for studying other models of strong-correlations
  - periodic Anderson/Kondo models
  - Heisenberg models
- Another potential application: topologically active systems:
  - Fractional quantum hall systems

A novel auxiliary model approach

- Method can be made more powerful by using multiple impurities
- Allows entangled liquid-like insulating phases
- Might also provide k–space resolution
  - partial gapping of Fermi surface?
  - pseudogap phases
- Extend the formalism towards higher order Greens functions
  - two-particle Greens functions, doublon-holon correlations
  - can provide more info on the MIT

Heavy-fermion materials

- Materials with very high quasiparticle masses
- Outstanding questions exist about the nature of phases and phase transitions
  - microscopic justification of certain phases
  - theory for the strange metal excitations
  - microscopic justification for the origin of unconventional superconductivity
- the URG, MERG and auxiliary model methods should prove useful

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References I

- Affleck, Ian (1995). “Conformal field theory approach to the Kondo effect”. In: *Acta Phys.Polon.B* 26.1869.
- Affleck, Ian and Andreas W.W. Ludwig (1991). “Critical theory of overscreened Kondo fixed points”. In: *Nuclear Physics B* 360.2, pp. 641–696. ISSN: 0550-3213.
- Affleck, Ian and Andreas WW Ludwig (1993). “Exact conformal-field-theory results on the multichannel Kondo effect: Single-fermion Green’s function, self-energy, and resistivity”. In: *Physical Review B* 48.10, p. 7297.
- Anderson, P. W. (1961). “Localized Magnetic States in Metals”. In: *Phys. Rev.* 124 (1), pp. 41–53.
- Anderson, PW (1970). “A poor man’s derivation of scaling laws for the Kondo problem”. In: *Journal of Physics C: Solid State Physics* 3.12, p. 2436.
- Andrei, N, K Furuya, and J H Lowenstein (1983). “Solution of the Kondo problem”. In: *Rev. Mod. Phys.* 55, p. 331.
- Arias, Raúl E, David D Blanco, and Horacio Casini (2015). “Entanglement entropy as a witness of the Aharonov–Bohm effect in QFT”. In: *Journal of Physics A: Mathematical and Theoretical* 48.14, p. 145401.
- Bulla, R, A C Hewson, and Th Pruschke (1998). “Numerical renormalization group calculations for the self-energy of the impurity Anderson model”. In: *Journal of Physics: Condensed Matter* 10.37, pp. 8365–8380.
- Calabrese, Pasquale and John Cardy (2004). “Entanglement entropy and quantum field theory”. In: *Journal of Statistical Mechanics: Theory and Experiment* 2004.06, P06002.
- Casini, H, C D Fosco, and M Huerta (2005). “Entanglement and alpha entropies for a massive Dirac field in two dimensions”. In: *Journal of Statistical Mechanics: Theory and Experiment* 2005.07, P07007–P07007.
- Chen, Xiao et al. (2017). “Two-cylinder entanglement entropy under a twist”. In: *Journal of Statistical Mechanics: Theory and Experiment* 2017.4, p. 043104.

References II

- Costi, T.A. and A.C. Hewson (1990). “A new approach to the calculation of spectra for strongly correlated systems”. In: *Physica B: Condensed Matter* 163.1, pp. 179–181. ISSN: 0921-4526.
- Emery, V. J. and S. Kivelson (1992). “Mapping of the two-channel Kondo problem to a resonant-level model”. In: *Phys. Rev. B* 46 (17), pp. 10812–10817.
- Gan, J (1994). “On the multichannel Kondo model”. In: 6.24, pp. 4547–4568.
- Goldhaber-Gordon, D. et al. (1998). “Kondo effect in a single-electron transistor”. In: *Nature* 391.6663, pp. 156–159. ISSN: 1476-4687.
- Heath, Joshuah T and Kevin S Bedell (2020). “Necessary and sufficient conditions for the validity of Luttinger’s theorem”. In: *New Journal of Physics* 22.6, p. 063011.
- Hewson, A. C. (1993). *The Kondo Problem to Heavy Fermions*. Cambridge University Press.
- Holzhey, Christoph, Finn Larsen, and Frank Wilczek (1994). “Geometric and renormalized entropy in conformal field theory”. In: *Nuclear Physics B* 424.3, pp. 443–467. ISSN: 0550-3213.
- Kondo, Jun (1964). “Resistance minimum in dilute magnetic alloys”. In: *Progress of theoretical physics* 32.1, pp. 37–49.
- Krishna-murthy, H. R., J. W. Wilkins, and K. G. Wilson (1980). “Renormalization-group approach to the Anderson model of dilute magnetic alloys. I. Static properties for the symmetric case”. In: *Phys. Rev. B* 21 (3), pp. 1003–1043.
- Lee, Ching Hua and Xiao-Liang Qi (2016). “Exact holographic mapping in free fermion systems”. In: *Physical Review B* 93.3, p. 035112.
- Lee, Sung-Sik (2010). “Holographic description of quantum field theory”. In: *Nuclear Physics B* 832.3, pp. 567–585.
- — (2014). “Quantum renormalization group and holography”. In: *Journal of High Energy Physics* 2014.1, p. 76.

References III

- Luttinger, JM (1960). “Fermi surface and some simple equilibrium properties of a system of interacting fermions”. In: *Physical Review* 119.4, p. 1153.
- Luttinger, Joaquin Mazdak and John Clive Ward (1960). “Ground-state energy of a many-fermion system. II”. In: *Physical Review* 118.5, p. 1417.
- Maldacena, Juan (1999). “The large-N limit of superconformal field theories and supergravity”. In: *International journal of theoretical physics* 38.4, pp. 1113–1133.
- Mukherjee, Anirban and Siddhartha Lal (2020a). “Unitary renormalisation group for correlated electrons-I: a tensor network approach”. In: *Nuclear Physics B* 960, p. 115170.
- — (2020b). “Unitary renormalisation group for correlated electrons-II: insights on fermionic criticality”. In: *Nuclear Physics B* 960, p. 115163.
- — (2022). “Superconductivity from repulsion in the doped 2D electronic Hubbard model: an entanglement perspective”. In: *Journal of Physics: Condensed Matter* 34.27, p. 275601.
- Murciano, Sara, Paola Ruggiero, and Pasquale Calabrese (2020). “Symmetry resolved entanglement in two-dimensional systems via dimensional reduction”. In: *Journal of Statistical Mechanics: Theory and Experiment* 2020.8, p. 083102.
- Nozaki, Masahiro, Shinsei Ryu, and Tadashi Takayanagi (2012). “Holographic geometry of entanglement renormalization in quantum field theories”. In: *Journal of High Energy Physics* 2012.10, p. 193.
- Nozieres, P (1974). “A “Fermi-liquid” description of the Kondo problem at low temperatures”. In: *Journal of Low Temperature Physics* 17, p. 31.
- Nozières, Ph. and Blandin, A. (1980). “Kondo effect in real metals”. In: *J. Phys. France* 41.3, pp. 193–211.

References IV

- Oshikawa, Masaki (2000). "Topological approach to Luttinger's theorem and the Fermi surface of a Kondo lattice". In: *Physical Review Letters* 84.15, p. 3370.
- Qi, Xiao-Liang (2013). "Exact holographic mapping and emergent space-time geometry". In: *arXiv preprint arXiv:1309.6282*.
- Ryu, Shinsei and Tadashi Takayanagi (2006a). "Aspects of holographic entanglement entropy". In: *Journal of High Energy Physics* 2006.08, p. 045.
- — (2006b). "Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence". In: *Physical review letters* 96.18, p. 181602.
- Sakai, Osamu, Yukihiro Shimizu, and Tadao Kasuya (1989). "Single-Particle and Magnetic Excitation Spectra of Degenerate Anderson Model with Finite f–f Coulomb Interaction". In: *Journal of the Physical Society of Japan* 58.10, pp. 3666–3678. eprint: `https://doi.org/10.1143/JPSJ.58.3666`.
- Seki, Kazuhiro and Seiji Yunoki (2017). "Topological interpretation of the Luttinger theorem". In: *Physical Review B* 96.8, p. 085124.
- Tsvelick, A M and P B Wiegmann (1983). "Exact results in the theory of magnetic alloys". In: *Adv. in Phys.* 32, p. 453.
- Tsvelick, A. M. and P. B. Wiegmann (1985). "Exact solution of the multichannel Kondo problem, scaling, and integrability". In: *Journal of Statistical Physics* 38.1, pp. 125–147. ISSN: 1572-9613.
- V. Borzenets, Ivan et al. (2020). "Observation of the Kondo screening cloud". In: *Nature* 579.7798, pp. 210–213. ISSN: 1476-4687.
- Van Raamsdonk, Mark (2010). "Building up spacetime with quantum entanglement". In: *General Relativity and Gravitation* 42.10, pp. 2323–2329.

References V

- Wilson, Kenneth G. (1975). “The renormalization group: Critical phenomena and the Kondo problem”. In: *Rev. Mod. Phys.* 47 (4), pp. 773–840.