Research Progress Report: 2023 - 2024

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Publications and Ongoing Projects

Published

- 2023 New J. Phys. 25 113011
- 2024 J. Phys. A: Math. Theor. 57 275401
- 2022 Phys. Rev. B 105, 085119
- 2023 J. Phys.: Condens. Matter 35 315601

Currently in Progress

- Development of auxiliary model-based method for interacting electronics.
- Studies of the plateau-to-plateau transition in integer quantum hall systems.

Ongoing Collaborations

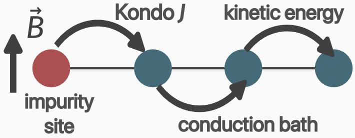
- Breakdown of Kondo screening in presence of magnetic field [DD, AM, SL]
- Quantum critical Mott MIT in a three-orbital impurity model [AK, DD, AM, NSV, SL]
- Universal features of Kondo breakdown in quantum impurity models [DD, AM, SL]
- Search for non-Fermi liquid physics in mixed-valence regime of eSIAM [AS, AM, SL]

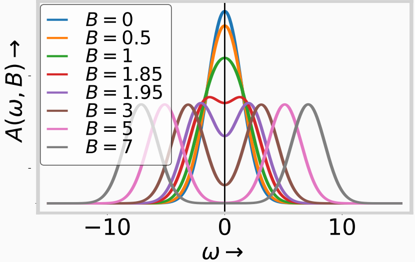
Ongoing Collaborations

Breakdown of Kondo Screening in Presence of Magnetic Field

Analysis of effect of impurity magnetic field on Kondo screening
(D Debata, A Mukherjee, S Lal) [*in preparation*]

- Models the effects of measurement on quantum system + **fermionic bath**
- Impurity undergoes localisation **transition** at large B
- Critical point displays **non-Fermi liquid** excitations.



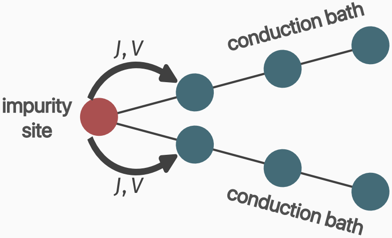


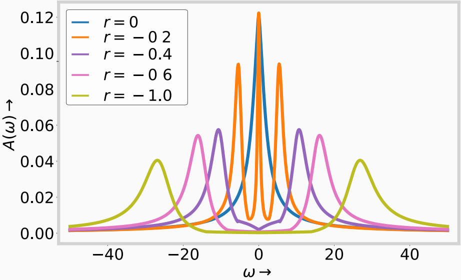
Costi 2000; Zhang et al. 2013.

Quantum critical Mott MIT in a Three-Orbital Impurity Model

Search for impurity model with a quantum critical phase intercepting a local MIT (Aashish Kumar, D Debata, A Mukherjee, N. S. Vidhyadhiraja and S Lal) [*in preparation*]

- **QC phase** obtained; gapless excitations involve composite paired objects
- Spectral function is pseudogapped at $\omega \sim 0$.
- **Self-energy** exponents characterise the non-Fermi liquid.



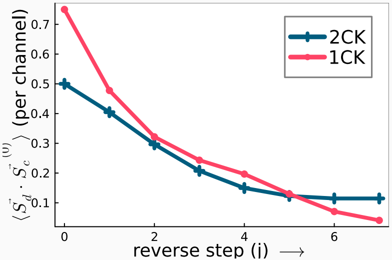


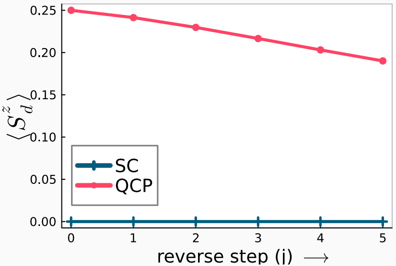
K. et al. 2023; Sen and Vidhyadhiraja 2016.

UNIVERSAL FEATURES OF KONDO BREAKDOWN IN IMPURITY MODELS

A unified framework for Kondo breakdown, in terms of entanglement measures
(D Debata, A Mukherjee, S Lal) [*in preparation*]

- Universal signatures of Kondo breakdown include **partial magnetisation** and phase shift.
- Entanglement within the **Kondo cloud** also suffers at Kondo breakdown.



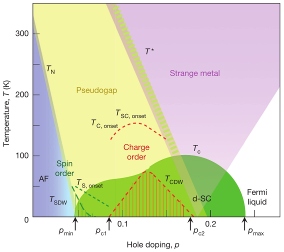


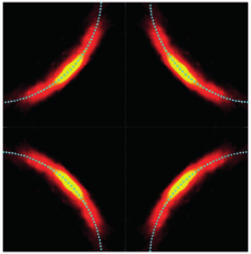
Mukherjee et al. 2023; Patra et al. 2023.

Project I: A New Auxiliary Model Approach to Systems of Interacting Electrons

Broad Objectives

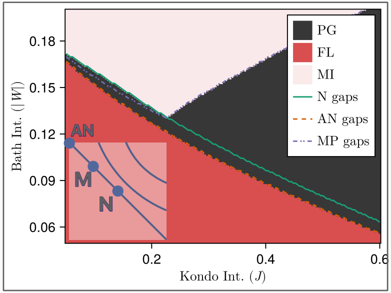
- Designing a **new method** by which to leverage quantum impurity models towards studying lattice models of interacting electrons
- Using such a method to go after the **Mott-Hubbard MIT** on the 2D square lattice
- Capturing the effects of ***k*-space anisotropy** on signatures near the transition
- Studying the **non-Fermi liquid behaviour** in the excitations near the transition





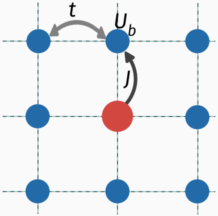
Keimer et al. 2015; Sebastian et al. 2014; Norman et al. 1998.

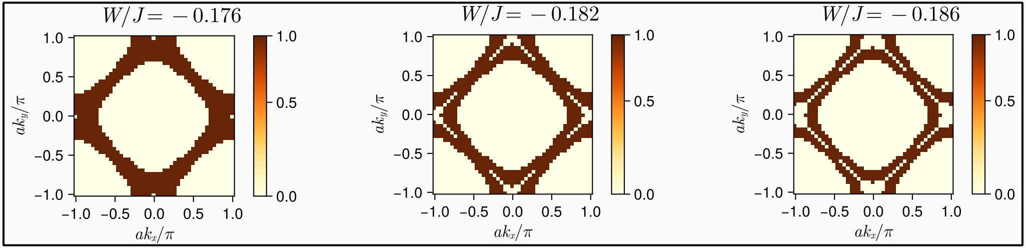
Momentum-Resolved Renormalisation Group Flows



Hamiltonian RG equations of
embedded e-SIAM

$$\Delta J^{(j)}_{\mathbf{k}_1,\mathbf{k}_2} = -\sum_{\mathbf{q}\in\mathrm{PS}} \frac{J^{(j)}_{\mathbf{k}_2,\mathbf{q}} J^{(j)}_{\mathbf{q},\mathbf{k}_1} + 4J^{(j)}_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}}}{\omega - \frac{1}{2}|\varepsilon_j| + J^{(j)}_{\mathbf{q}}/4 + W_{\mathbf{q}}/2}$$





'Periodising' the Hamiltonian and Eigenstates

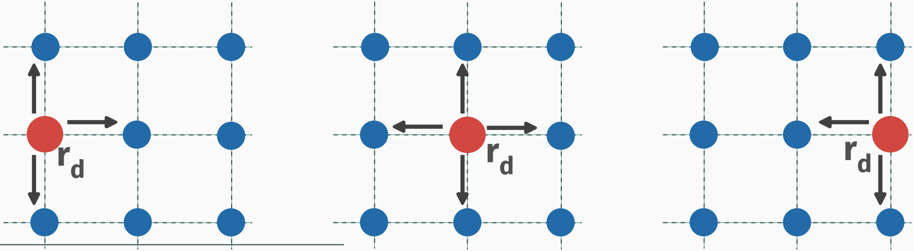
Periodising the Hamiltonian creates a **Hubbard-Heisenberg** model:

$$H_{\text{tiled}} = \sum_{\mathbf{r}} T^{\dagger}(\mathbf{r}-\mathbf{r}_d) H_{\text{aux}}(\mathbf{r}_d) T(\mathbf{r}-\mathbf{r}_d)$$

Wavefunctions can be related using a many-body **Bloch's theorem** :

$$|\Psi_{\text{gs}}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}_d} e^{i\mathbf{k}\cdot\mathbf{r}_d} \, |\psi_{\text{gs}}(\mathbf{r}_d)\rangle$$

$$H_{\text{tiled}} = -\frac{\tilde{t}}{\sqrt{Z}} \sum_{\langle \mathbf{r}_i,\mathbf{r}_j\rangle;\sigma} \left(c^{\dagger}_{\mathbf{r}_i,\sigma} c_{\mathbf{r}_j,\sigma} + \text{h.c.}\right) + \frac{\tilde{J}}{Z} \sum_{\langle \mathbf{r}_i,\mathbf{r}_j\rangle} \mathbf{S}_{\mathbf{r}_i} \cdot \mathbf{S}_{\mathbf{r}_j} - \frac{\tilde{U}}{2} \sum_{\mathbf{r}} \left(\hat{n}_{\mathbf{r},\uparrow} - \hat{n}_{\mathbf{r},\downarrow}\right)^2$$

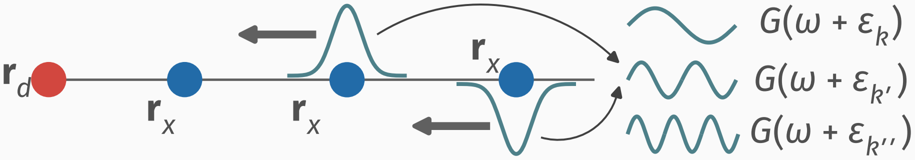


Stoyanova 2006.

Periodising the Greens Functions

Greens function = sum of 1-particle *k*-**space** Greens functions starting from **all sites** in impurity model.

$$\tilde{G}(\mathbf{r};\tilde{\omega}) = \frac{1}{N}\sum_{\mathbf{k},\mathbf{r}_x}\Big[e^{i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_p\left(\mathbf{r}_x;\omega+\varepsilon_{\mathbf{k}}\right) + e^{-i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_h\left(\mathbf{r}_x;\omega-\varepsilon_{\mathbf{k}}\right)\Big]$$



Subsequently allows periodising spectral functions and self-energies

$$\tilde{A}(\mathbf{K};\omega) = -\frac{1}{\pi}\mathrm{Im}\left[\tilde{G}(\mathbf{K};\tilde{\omega})\right]$$

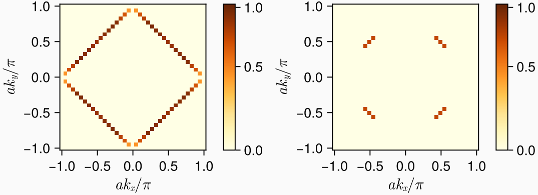
$$\tilde{\Sigma}(\mathbf{K};\omega) = \left(\tilde{G}^{(0)}(\mathbf{K};\tilde{\omega})\right)^{-1} - \left(\tilde{G}(\mathbf{K};\tilde{\omega})\right)^{-1}$$

Kotliar et al. 2001; Verret et al. 2022.

Periodising Correlation Functions and Entanglement Measures

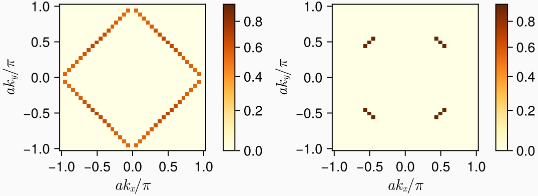
k-space spin-spin correlation

$$\tilde{S}_{\text{flip}}(\mathbf{K}_1, \mathbf{K}_2) = \frac{1}{2}\left[\sqrt{\langle S^+(\mathbf{d})\, S^-(\mathbf{K}_2)\rangle \langle S^-(\mathbf{d})\, S^+(\mathbf{K}_1)\rangle} + \text{h.c.}\right]$$



k-space reduced density matrix

$$\bar{\rho}_{\mathbf{K},\sigma} = \frac{1}{2}\left[c^\dagger_{\mathbf{K},\sigma}\rho_{\text{gs}}(\mathbf{r}_c)c_{\mathbf{r}_c,\sigma} + c^\dagger_{\mathbf{r}_c,\sigma}\rho_{\text{gs}}(\mathbf{r}_c)c_{\mathbf{K},\sigma}\right] + \text{h.c.}$$



Rosenberg et al. 2022; Meixner et al. 2024.

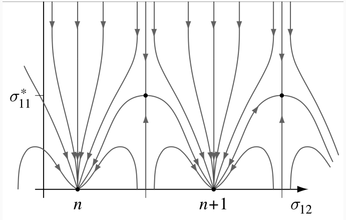
Outstanding Questions

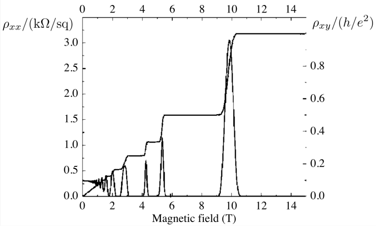
- A better understanding of the mechanism of the pseudogap **phase diagram**
- Calculation of **spectral functions** and self-energies
- Characterisation of **non-Fermi liquid** behaviour in the pseudogapped region

Project II: Search for Punctured-Chern Topology at IQHE Transitions

Broad Objectives

- Obtaining the **IQHE phase diagram** from a model of 2D lattice electrons
- Characterising the plateau-to-plateau transition **critical point** through a topological invariant
- Checking the robustness of our conclusions to the addition of **disorder**

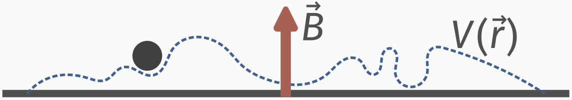




Khmelnitskii 1983; Altland and Simons 2010; Prange and Girvin 1987.

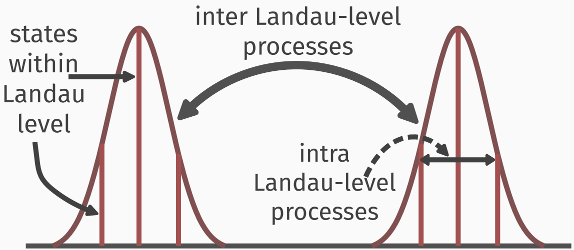
Non-interacting electrons, magnetic field, one-particle potential

$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 + V(\mathbf{r})$$



In the absence of $V(\mathbf{r})$, produces decoupled **Landau levels** with large degeneracy.

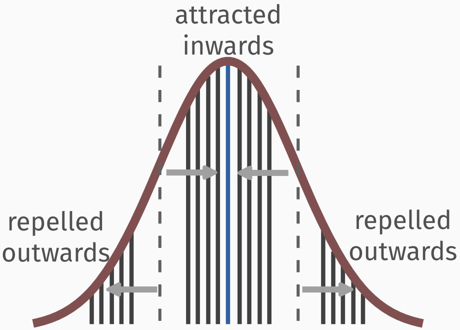
$V(\mathbf{r})$ leads to **scattering** among these states.

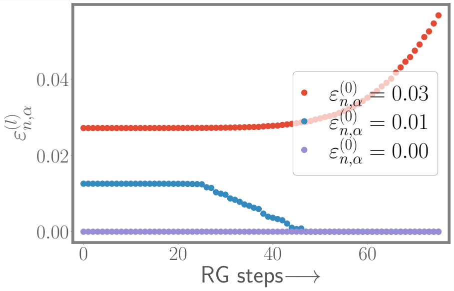


URG Analysis of Intra Landau-level Processes

$$H_n^* = \sum_{\varepsilon_{n,\alpha}\sim 0} \varepsilon_{n,\alpha} c^\dagger_{n,\alpha} c_{n,\alpha} + \sum_{|\varepsilon^*_{n,\alpha}|>\Delta^*} \varepsilon^*_{n,\alpha} c^\dagger_{n,\alpha} c_{n,\alpha} + \sum_{\varepsilon_{n,\alpha_1},\varepsilon_{n,\alpha_2}\sim 0} L^*_{\alpha_1,\alpha_2}(n)\left(c^\dagger_{n,\alpha_1} c_{n,\alpha_2} + \text{h.c.}\right)$$

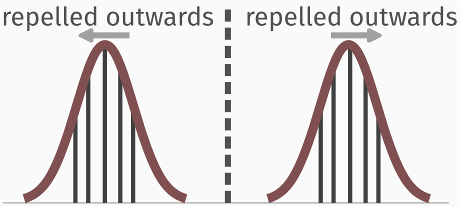
States within a window are **attracted** towards central state, with relevant forward scattering among them.





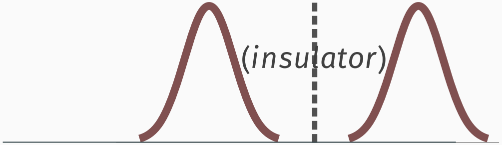
Landau levels are repelled away from chemical potential (**stability**).

LLs below chemical potential are decoupled and filled.



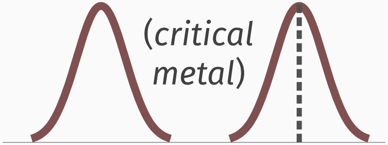
Insulating phase

No longitudinal transport.
Central state allows transverse transport.



Critical point

Marginal scattering processes at Fermi level. Lead to longitudinal resistivity.



Cain, Römer, and Raikh 2003; Cain and Römer 2005.

Outstanding Questions

- Description of the metal obtained at the **critical point** (self-energy, etc.).
- **Topological invariant** to characterise the critical point (Chern number, etc.)

Future Plans

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- Finish the embedded eSIAM project and the IQHE projects.
- Study heavy-fermion physics using auxiliary mode approach (simple extension of the embedded eSIAM project).

Thank You!