

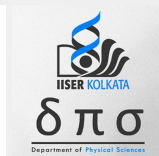
DESTRUCTION OF THE KONDO CLOUD IN THE GENERALISED SIAM: UNITARY RG PERSPECTIVE

arXiv:2111.10580v2[cond-mat.str-el]

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FEBRUARY 17, 2022

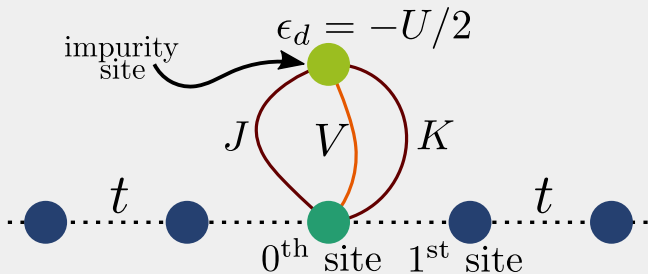


THE GENERALISED SIAM MODEL

THE MODEL

$$H = \sum_{k\sigma} \epsilon_k \tau_{k\sigma} + V \sum_{k\sigma} \left(c_{d\sigma}^\dagger c_{k\sigma} + \text{h.c.} \right) - \frac{1}{2} U (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + J \vec{S}_d \cdot \vec{s} + K \vec{C}_d \cdot \vec{C}$$

supplement 1-particle
hybridisation with **spin-
exchange** and **charge
isospin-exchange**



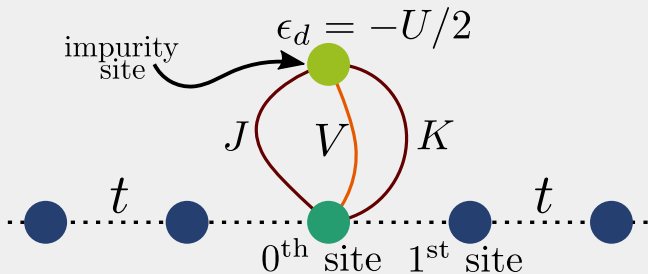
THE MODEL

$$H = \sum_{k\sigma} \epsilon_k \tau_{k\sigma} + V \sum_{k\sigma} (c_{d\sigma}^\dagger c_{k\sigma} + \text{h.c.}) - \frac{1}{2} U (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + J \vec{S}_d \cdot \vec{s} + K \vec{C}_d \cdot \vec{C}$$

$$C_d^z = \frac{1}{2} (\hat{n}_d - 1)$$

$$C_d^+ = c_{d\uparrow}^\dagger c_{d\downarrow}^\dagger$$

$$C_d^- = c_{d\downarrow} c_{d\uparrow}$$



URG OF GENERALISED SIAM

$U > 0$ ($J > 0, K < 0$): FLOW TOWARDS STRONG-COUPLING

$J \rightarrow \text{AFM}, \quad K \rightarrow \text{FM}$

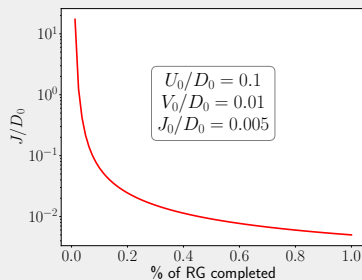
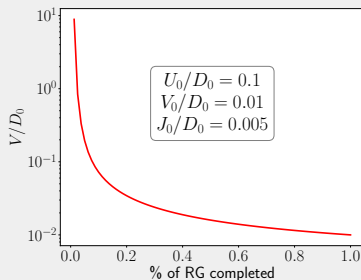
$$d_0 = \omega - \frac{D}{2} - \frac{U}{2} + \frac{K}{4}, \quad d_1 = \omega - \frac{D}{2} + \frac{U}{2} + \frac{J}{4}, \quad d_2 = \omega - \frac{D}{2} + \frac{J}{4}, \quad d_3 = \omega - \frac{D}{2} + \frac{K}{4}$$

$$\Delta V = \frac{3n_j V J}{8} \left(\frac{1}{|d_2|} + \frac{1}{|d_1|} \right) > 0$$

$$\Delta J = \frac{n_j J^2}{|d_2|} > 0$$

$$\Delta K = \frac{n_j K^2}{|d_3|} > 0$$

(K is irrelevant)



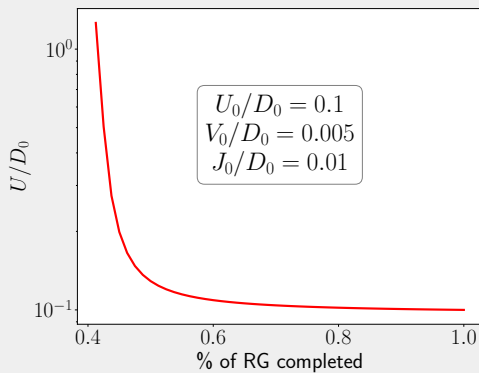
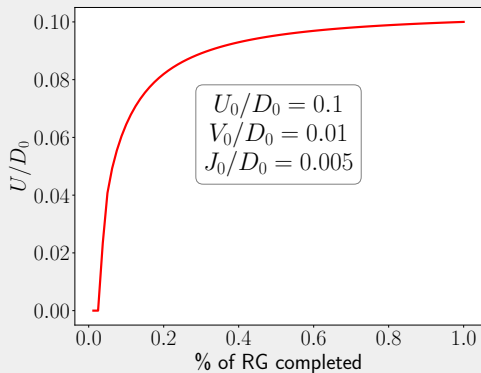
$U > 0$ ($J > 0, K < 0$): FLOW TOWARDS STRONG-COUPLING

$J \rightarrow \text{AFM}, \quad K \rightarrow \text{FM}$

$$4V^2 n_j \left(\frac{1}{d_1} - \frac{1}{d_0} \right) - n_j \frac{J^2}{d_2}$$

$V > J$

$V < J$

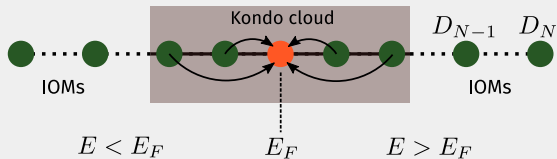


$U > 0$ FIXED POINT HAMILTONIAN

$$H^* = \sum_{k < k^*, \sigma} \epsilon_k \hat{n}_{k\sigma} + \frac{U^*}{2} (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + J^* \vec{S}_d \cdot \vec{S}_<$$

$$+ V^* \sum_{k < k^*, \sigma} (c_{d\sigma}^\dagger c_{k\sigma} + \text{h.c.})$$

$$\vec{S}_< = \frac{1}{2} \sum_{k, k' < k^*} c_{k\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{k',\beta}$$



ZERO-BANDWIDTH LIMIT OF FIXED POINT HAMILTONIAN

ZERO-BANDWIDTH LIMIT OF FIXED POINT HAMILTONIAN

Route to the zero-bandwidth model

At strong-coupling fixed point,

- kinetic energy acts as a perturbation
- **compress the bandwidth to just the Fermi surface**

$$H_{\text{zero bw}}^* = (\epsilon_F - \mu) \hat{n}_{k_F} + \frac{U^*}{2} (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + V^* \sum_{\sigma} \left(c_{d\sigma}^{\dagger} c_{0\sigma} + \text{h.c.} \right) + J \vec{S}_d \cdot \vec{S}_0$$

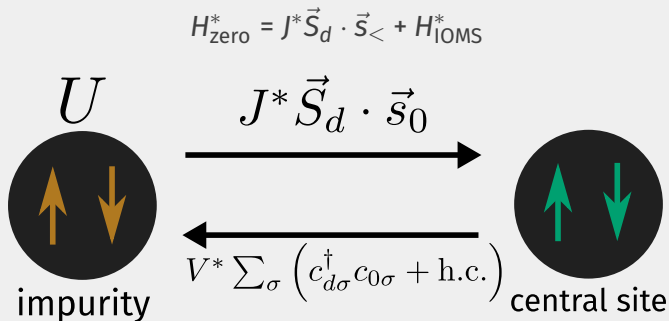
(center of motion)

- Setting $\mu = \epsilon_F$ gives a **two-site model**

$$H_{\text{zero}}^* = \frac{U^*}{2} (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + V^* \sum_{\sigma} \left(c_{d\sigma}^{\dagger} c_{0\sigma} + \text{h.c.} \right) + J \vec{S}_d \cdot \vec{S}_0$$

ZERO-BANDWIDTH LIMIT OF FIXED POINT HAMILTONIAN

Effective two-site problem



$$|\Psi\rangle_{\text{gs}} = \frac{c_s}{\sqrt{2}} (|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle) + \frac{\sqrt{1 - c_s^2}}{\sqrt{2}} (|2, 0\rangle + |0, 2\rangle), \quad c_s \rightarrow 1 \text{ as } D \rightarrow \infty$$

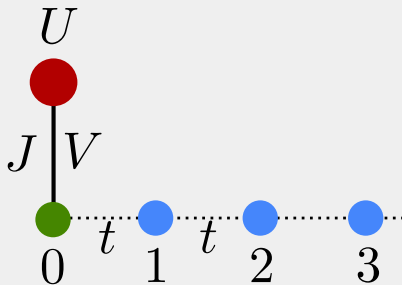
LOCAL FERMI LIQUID EXCITATIONS

LOCAL FERMI LIQUID EXCITATIONS

Effective Hamiltonian in singlet subspace

We treat the dispersion as a **real-space nearest neighbour hopping**.

$$\begin{aligned} H^* = & -\frac{U}{2} (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + J^* \vec{S}_d \cdot \vec{S}_o \\ & + V \sum_{\sigma} (c_{d\sigma}^{\dagger} c_{o\sigma} + \text{h.c.}) \\ & - t \sum_{i\sigma} (c_{i\sigma}^{\dagger} c_{i+1,\sigma} + \text{h.c.}) \end{aligned}$$



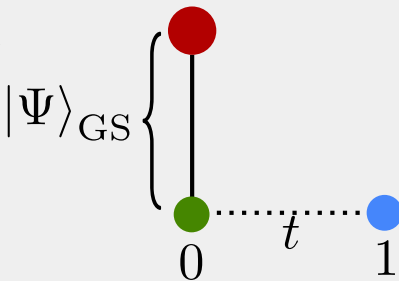
LOCAL FERMI LIQUID EXCITATIONS

Effective Hamiltonian in singlet subspace

Initially consider **just the first site**. Treat **hopping as perturbation**:

$$|\Psi\rangle_{GS}^* = c_s |SS\rangle + \sqrt{1 - c_s^2} |CT, 0\rangle$$

$$V = -t \sum_{\sigma} (c_{0\sigma}^{\dagger} c_{1,\sigma} + \text{h.c.})$$



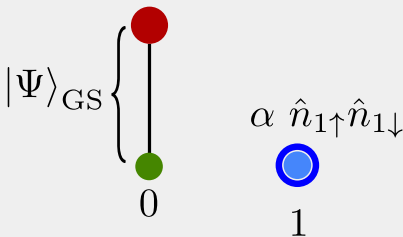
Effective Hamiltonian in singlet subspace

Upto **fourth order**, effective Hamiltonian is

$$H_{\text{eff}}^* = \text{constant} + \alpha \mathcal{P}_{\text{charge}}$$

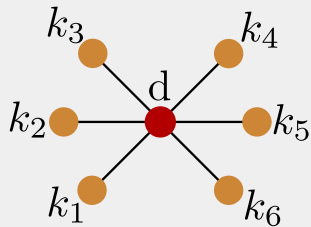
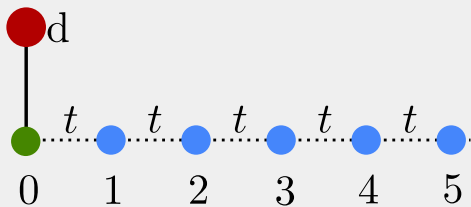
$\mathcal{P}_{\text{charge}} \longrightarrow$ projector onto $\hat{n}_1 \neq 1$

- For $U \ll V \ll J$, we get $0 < \alpha \ll 1$
- a **very weak local FL** on 1st site



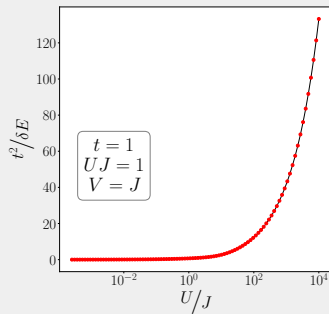
SIGNATURES OF BREAKDOWN OF SCREENING – JOURNEY TOWARDS LOCAL MOMENT PHASE

- We will work with a Hilbert space of $(6+1=)$ **7 sites**
- **Recreate RG flow** by tuning the parameters U, V, J
- **Observe various measures** of entanglement and correlation along this variation

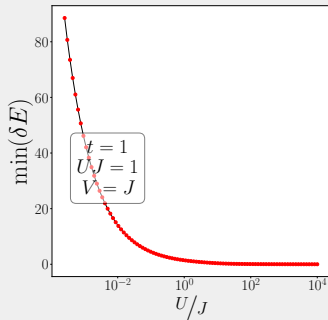


BREAKDOWN OF RENORMALISED PERTURBATION THEORY

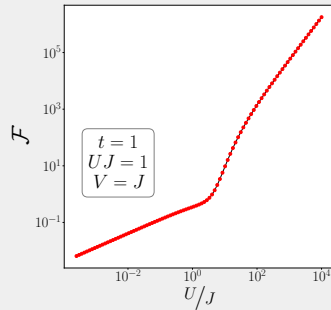
Perturbation parameter, zero mode gap and local FL strength



closing of gap,



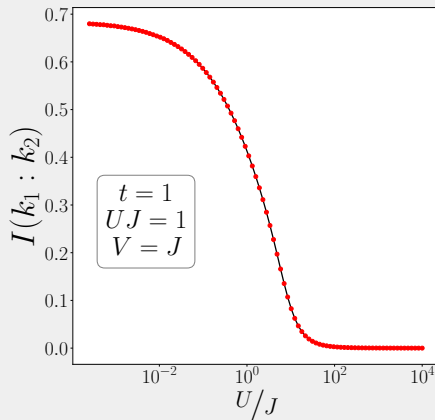
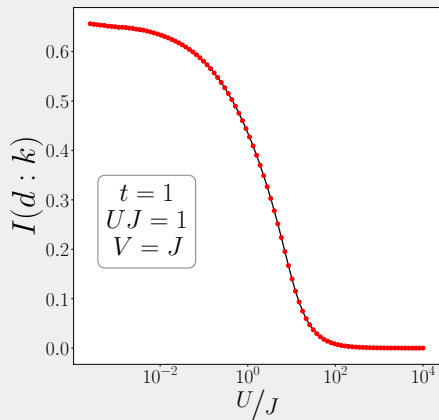
breakdown of p. theory,



extremely **correlated** LFL

DESTRUCTION OF KONDO CLOUD

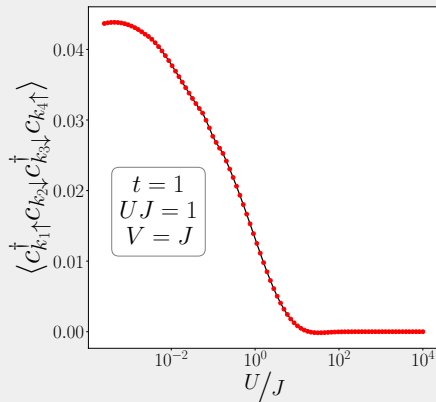
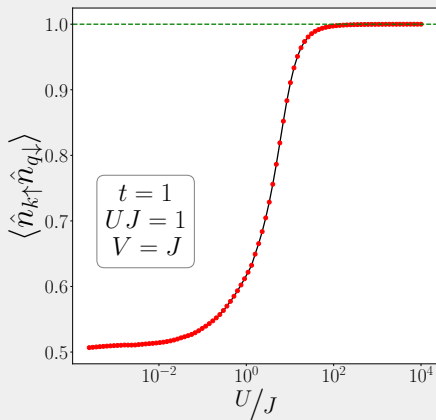
Mutual information within the Kondo cloud



■ loss of spin-flip scattering and **disappearance of Kondo cloud**

DESTRUCTION OF KONDO CLOUD

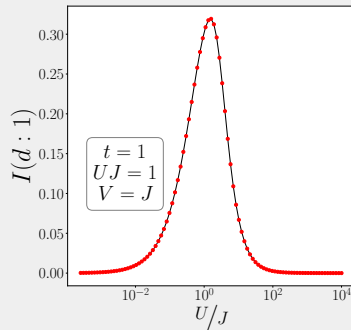
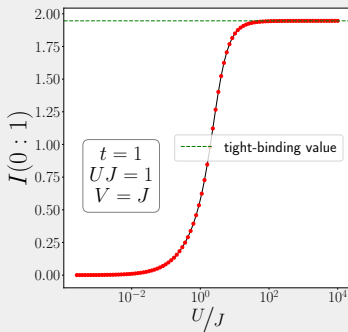
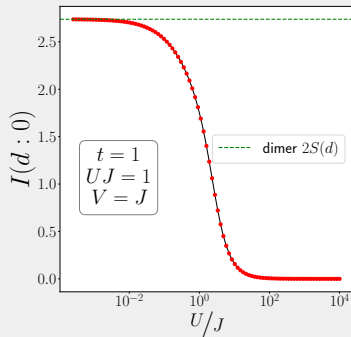
Many-particle correlations in k -space



■ loss of entanglement within the K cloud, **breakdown of screening**

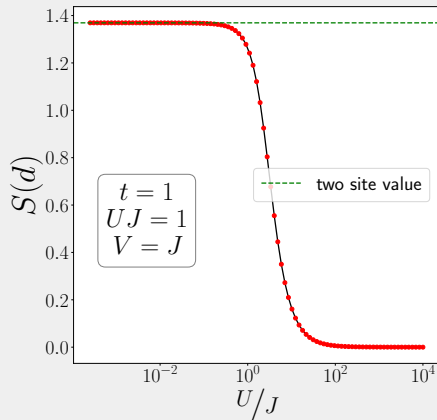
DECOUPLING OF IMPURITY SITE FROM LATTICE

Mutual information in real space



■ d and o disentangle, o gets entangled with the lattice

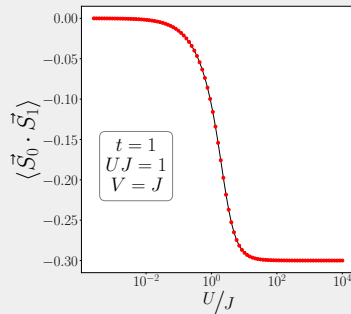
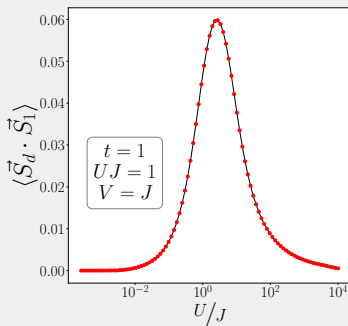
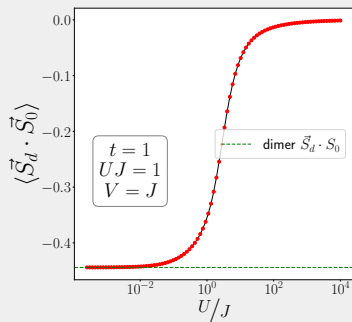
Impurity entanglement entropy



- impurity site **disentangles from the lattice**

DECOUPLING OF IMPURITY SITE FROM LATTICE

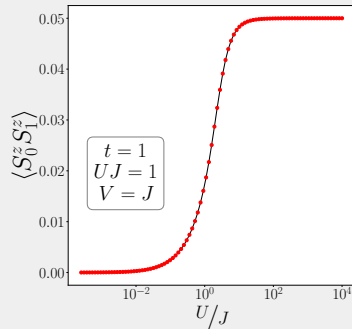
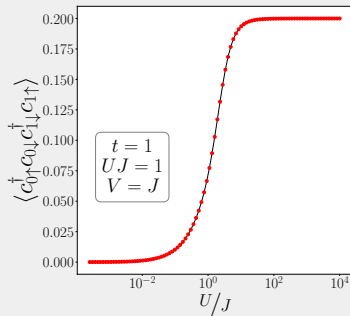
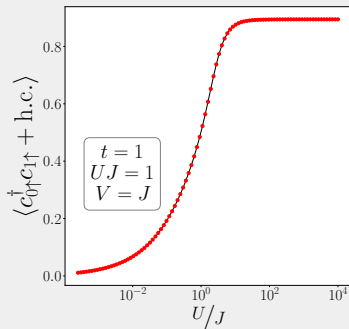
Real space spin-spin correlations



- impurity **spin compensation vanishes** (loss of screening)
- Spin correlation between 0 and 1 increases

DECOUPLING OF IMPURITY SITE FROM LATTICE

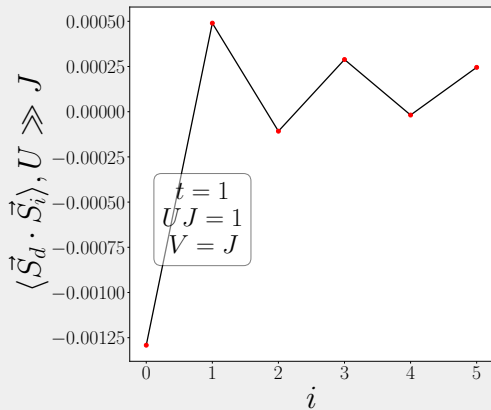
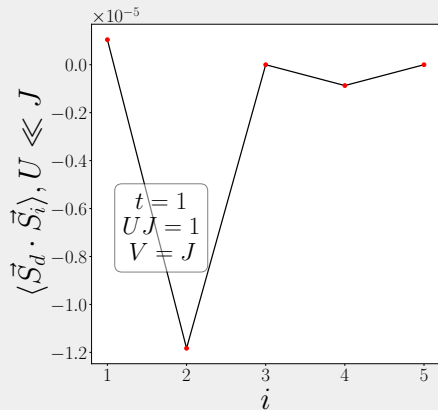
Real space diagonal and off-diagonal correlations



- **Correlations between 0 and 1 increase**
- Result of tight-binding hopping **breaking the singlet**

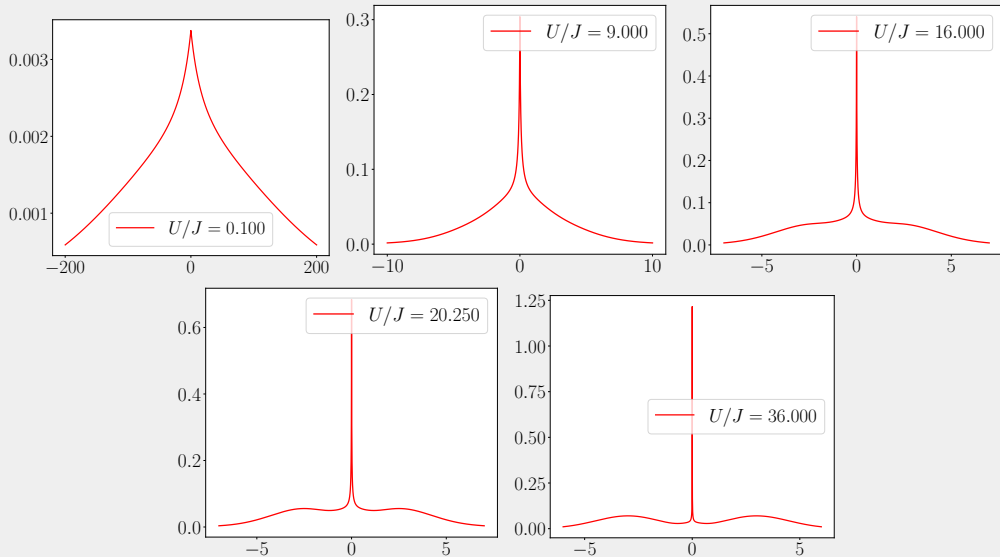
DECOUPLING OF IMPURITY SITE FROM LATTICE

Variation of real-space correlations with distance



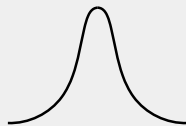
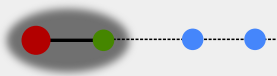
- Correlations **fall off with distance**
- Even sites are AFM in correlation, odd sites are FM

VARIATION OF SPECTRAL FUNCTION

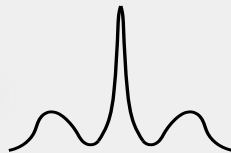


WHAT'S HAPPENING?

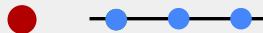
$J, V \gg U, t$: weak LFL



$U \gg J, V \sim t$: highly correlated metal



$U > 0; J, V = 0$: insulator



DISCUSSIONS & FURTHER WORK

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- Rewinding the RG flow shows the **decoupling** of the impurity site.

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- When used as an auxiliary model, this a **metal-insulator transition**.

DISCUSSIONS & FURTHER WORK

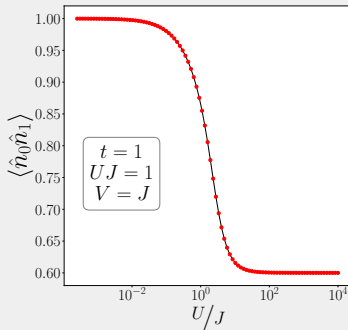
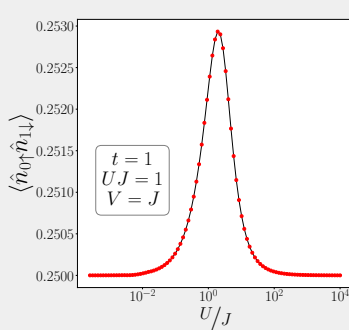
- Rewinding the RG flow shows the **decoupling** of the impurity site.
- When used as an auxiliary model, this a **metal-insulator transition**.
- **Stabilising the insulating phase under RG still remains to be done.**

DISCUSSIONS & FURTHER WORK

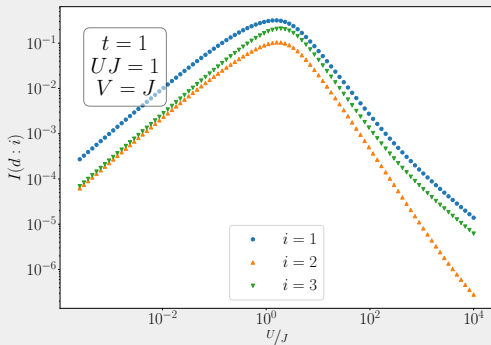
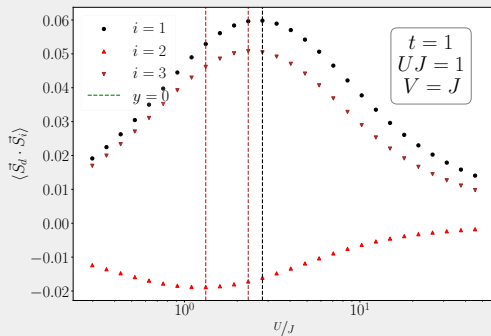
- Rewinding the RG flow shows the **decoupling** of the impurity site.
- When used as an auxiliary model, this a **metal-insulator transition**.
- Stabilising the insulating phase under RG **still remains to be done**.
- For this, we will insert a **Hubbard term on the zeroth site**, and check the RG flows.

OTHER MEASURES OF CORRELATION IN GEN. SIAM

REAL SPACE CORRELATIONS



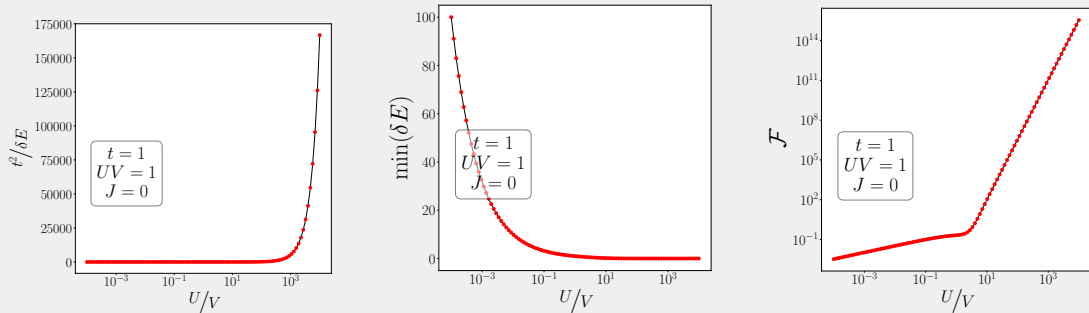
REAL SPACE CORRELATIONS AS FUNCTIONS OF DISTANCE



MEASURES OF CORRELATION IN PURE SIAM

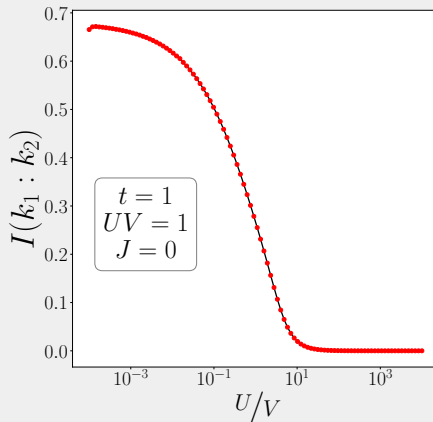
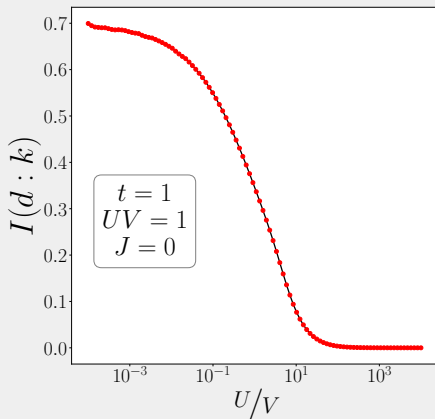
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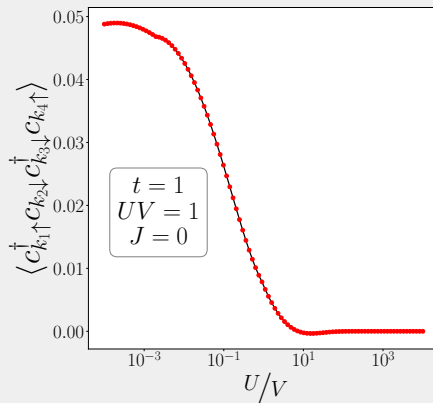
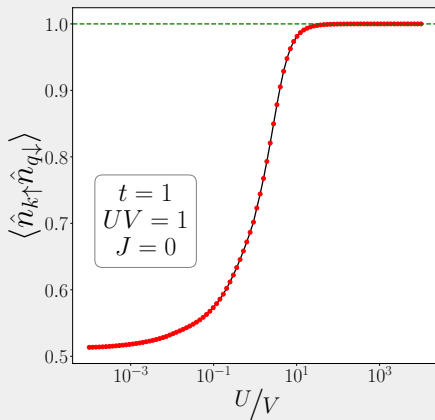
DESTRUCTION OF KONDO CLOUD

Mutual information within the Kondo cloud



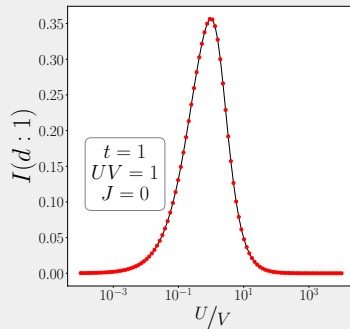
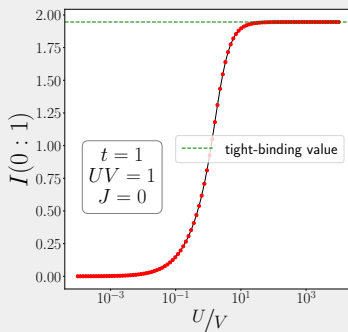
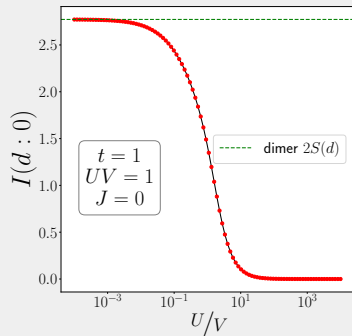
DESTRUCTION OF KONDO CLOUD

Many-particle correlations in k -space



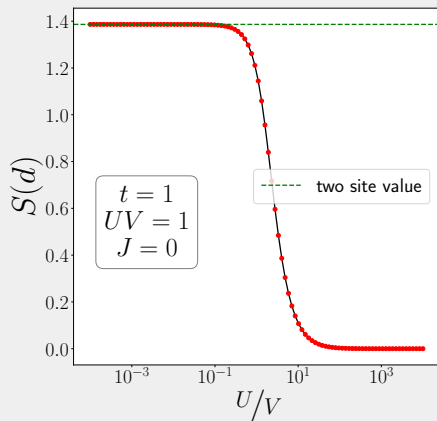
DECOUPLING OF IMPURITY SITE FROM LATTICE

Mutual information in real space



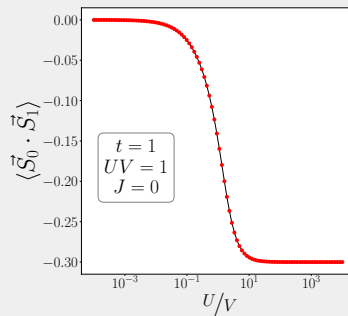
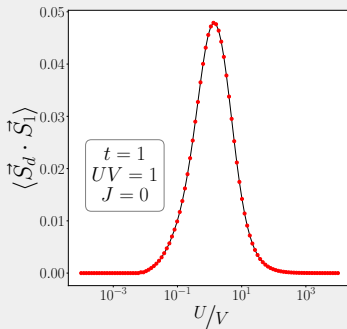
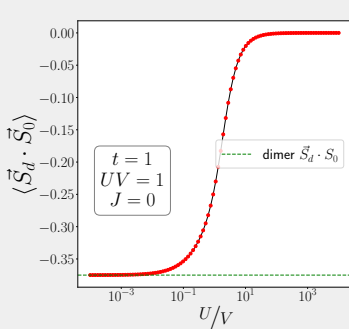
DECOUPLING OF IMPURITY SITE FROM LATTICE

Impurity entanglement entropy



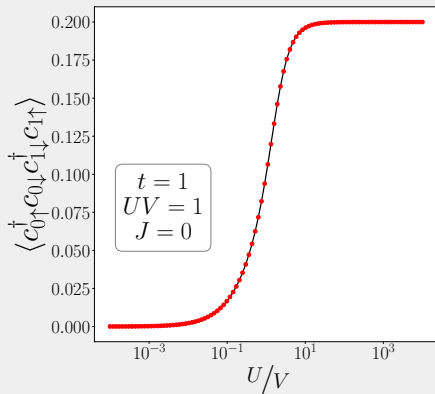
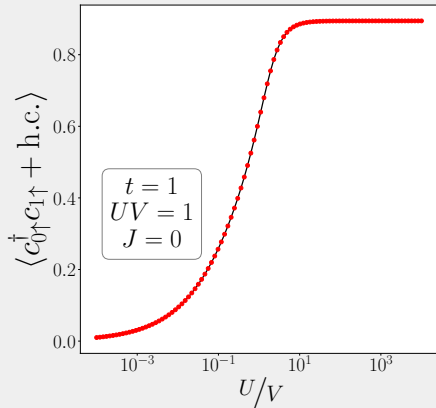
DECOUPLING OF IMPURITY SITE FROM LATTICE

Real space spin-spin correlations



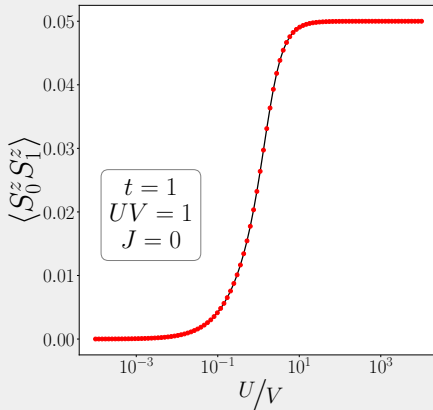
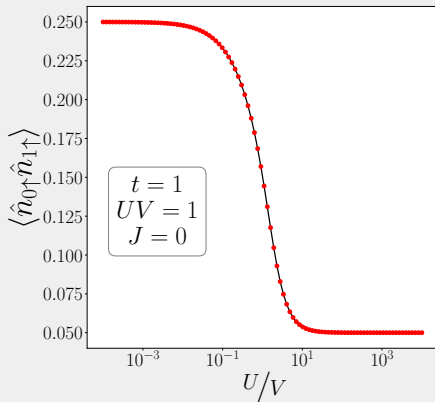
DECOUPLING OF IMPURITY SITE FROM LATTICE

Real space off-diagonal 1-particle and 2-particle correlations



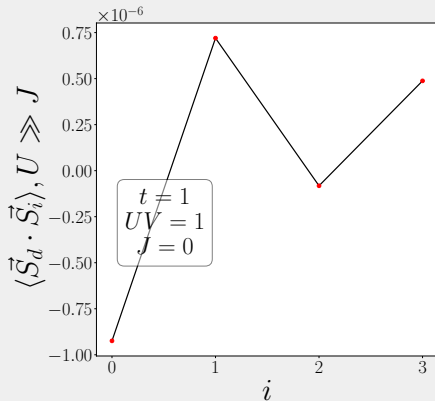
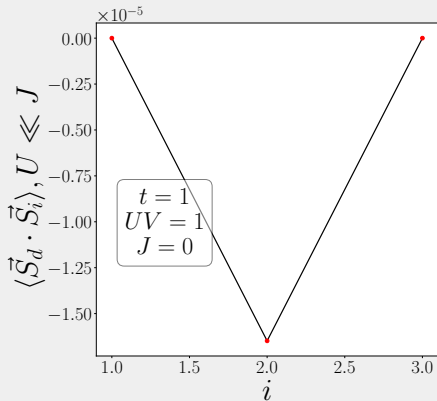
DECOUPLING OF IMPURITY SITE FROM LATTICE

Real space diagonal correlations

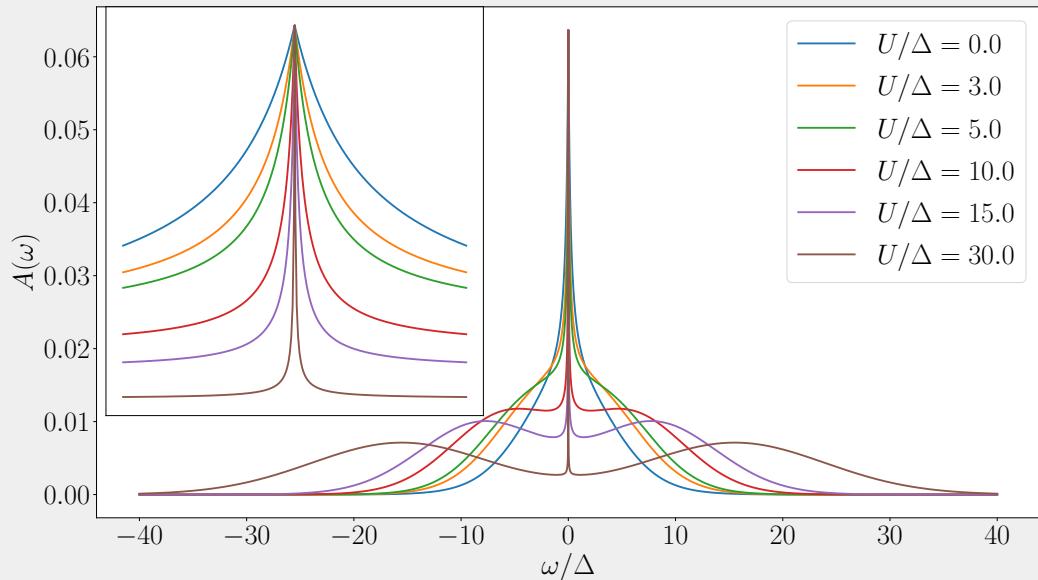


DECOUPLING OF IMPURITY SITE FROM LATTICE

Variation of real-space correlations with distance



IMPURITY SPECTRAL FUNCTION (GEN. SIAM)



Form of Kondo cloud Hamiltonian

$$V_{1234} = (\epsilon_{k_1} - \epsilon_{k_3}) \left[1 - \frac{2}{J^*} (\epsilon_{k_3} - \epsilon_{k_1} + \epsilon_{k_2} + \epsilon_{k_4}) \right]$$

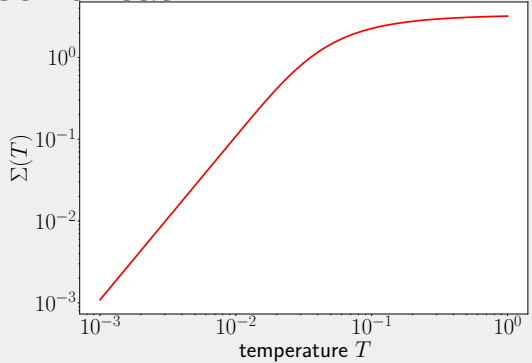
- Mixture of **Fermi liquid** and **two-particle off-diagonal scattering term**
- Fermi liquid part: **result of Ising scattering**
- 2P off-diagonal term: **Non-Fermi liquid** in character - **result of spin-flip scattering**
- NFL part **leads to screening** and formation of singlet

Impurity specific heat

- Fermi-liquid part renormalises one-particle **self-energy**

$$\bar{\epsilon}_k = \epsilon_k + \Sigma_k$$

$$\Sigma_k = \sum_{k'\sigma'} \frac{\epsilon_{k'} \epsilon_k}{J^*} \delta n_{k',\sigma'}$$



- Fermi-liquid part renormalisation
one-particle **self-energy**

Impurity specific heat

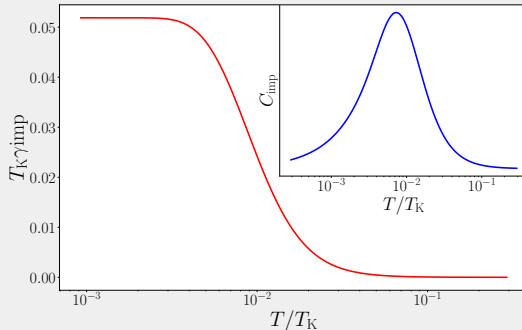
$$C_V = \gamma \times T$$

$$\bar{\epsilon}_k = \epsilon_k + \Sigma_k$$

$$\Sigma_k = \sum_{k', \sigma'} \frac{\epsilon_{k'} \epsilon_k}{J^*} \delta n_{k', \sigma'}$$

- Compute renormalisation in C_V :

$$C_{\text{imp}} = \sum_{k, \sigma} \frac{1}{T^2} \left[\frac{(\bar{\epsilon}_k)^2 e^{\beta \bar{\epsilon}_k}}{(e^{\beta \bar{\epsilon}_k} + 1)^2} - \frac{(\epsilon_k)^2 e^{\beta \epsilon_k}}{(e^{\beta \epsilon_k} + 1)^2} \right]$$



Wilson 1975; Andrei, Furuya, and Lowenstein 1983; Wiegmann 1981.

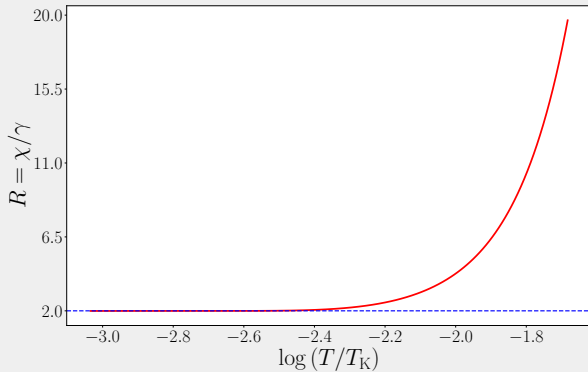
$$R = \frac{\chi}{\gamma}$$

$$\chi(T \rightarrow 0) = \frac{1}{2J^*}$$

$$\gamma(T \rightarrow 0) = \frac{1}{4J^*}$$

R saturates to 2 as $T \rightarrow 0$

Wilson ratio



Wilson 1975; Andrei, Furuya, and Lowenstein 1983; Wiegmann 1981.