

# HIERARCHICAL STRUCTURE AND TOPOLOGICAL CONTENT OF ENTANGLEMENT OF FREE FERMIONS

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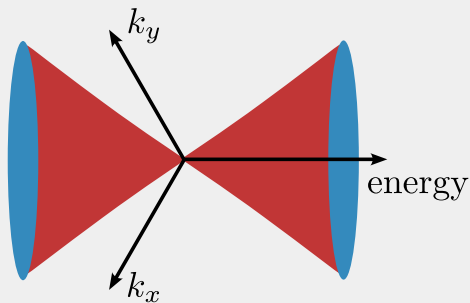
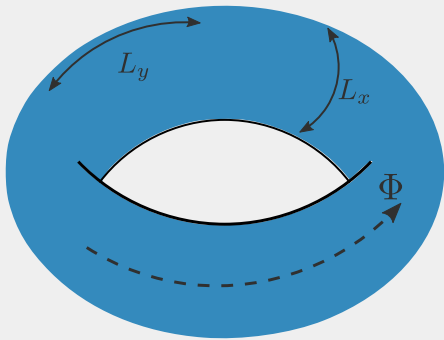
# INTRODUCTION

Massless Dirac fermions on a 2-torus

$$\mathcal{L} = i\bar{\psi}\gamma_{\mu}\partial_{\mu}\psi$$

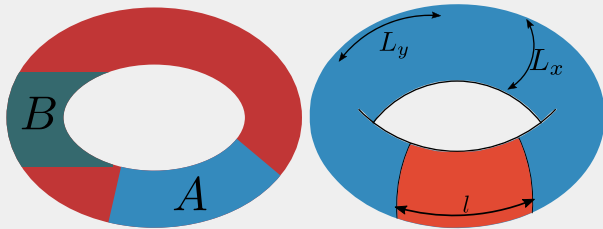
In presence of an Aharonov-Bohm flux

$$\mathcal{L} = \bar{\psi} (i\gamma_{\mu} + e\mathbf{A}_{\mu}) \partial_{\mu}\psi$$



# MEASURES OF ENTANGLEMENT

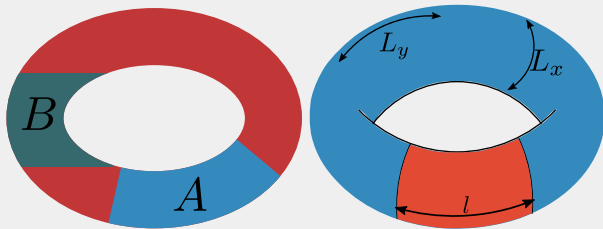
$$\rho = |\Psi\rangle \langle \Psi| \longrightarrow \text{density matrix}$$



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$\rho = |\Psi\rangle \langle \Psi| \longrightarrow$  density matrix

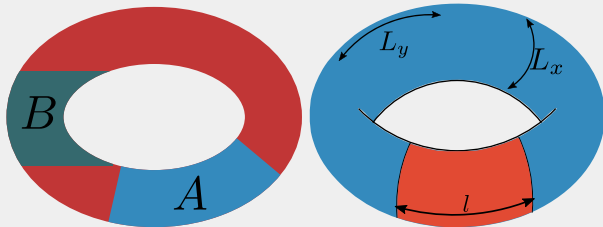
$\rho_A =$  partial trace over system A  
 $\longrightarrow$  reduced DM



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$\rho = |\Psi\rangle\langle\Psi| \longrightarrow$  **density matrix**

$\rho_A =$  partial trace over system A  
 $\longrightarrow$  **reduced DM**



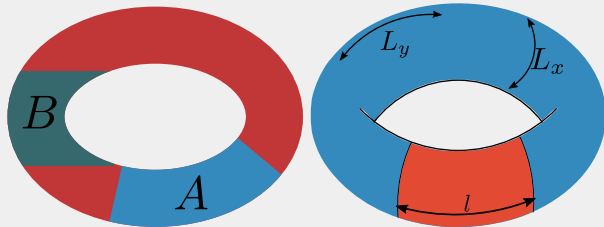
$S(A) = -\text{Tr} [\rho_A \ln \rho_A] \longrightarrow$  **entanglement entropy** of A

$\longrightarrow$  quantifies information shared between A and rest

# MEASURES OF ENTANGLEMENT

$\rho = |\Psi\rangle\langle\Psi| \longrightarrow$  **density matrix**

$\rho_A =$  partial trace over system A  
 $\longrightarrow$  **reduced DM**



$I(A : B) = S(A) + S(B) - S(A \cup B) \longrightarrow$  **mutual information** between A and B  
 $\longrightarrow$  quantifies information shared between A and B

## ENTANGLEMENT OF FREE FERMIONS

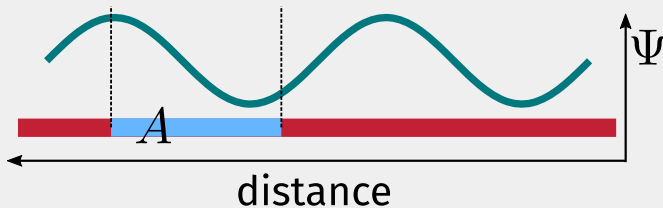
Diagonal in  $k$ -space  $\longrightarrow$  **Vanishing** entanglement in momentum space



# ENTANGLEMENT OF FREE FERMIONS

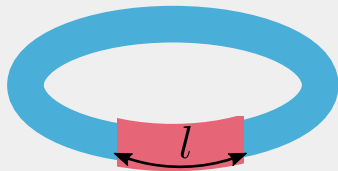
Diagonal in  $k$ -space  $\longrightarrow$  **Vanishing** entanglement in momentum space

Off-diagonal in  $r$ -space  $\longrightarrow$  **Fluctuations** exist in real space  
 $\longrightarrow$  leads to entanglement in real space



# ENTANGLEMENT OF FREE FERMIONS

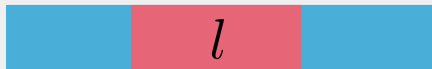
1D-ring of massless fermions:  $\frac{2}{3} \ln \left( \frac{L}{\pi a} \sin \frac{\pi l}{L} \right)$



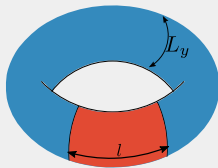
1D-line of massless fermions:  $\frac{1}{3} \ln \left( \frac{2L}{\pi a} \sin \frac{\pi l}{L} \right)$



1D-line of relativistic fermions:  $-\frac{1}{3} \ln (ma)$



2D-torus of massless fermions:  $\propto \frac{L_y}{\epsilon}$



WHAT ARE WE GOING AFTER?

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- Effect of a magnetic flux on the entanglement
- Distribution of the entanglement among subsystems of various sizes
- Emergent space generated by the transformations between these subsystems
- Curvature and related quantities of this space

