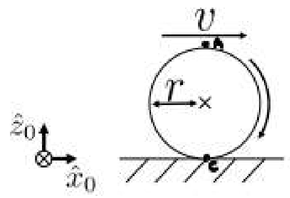
Assignment 4

Q1

1. A cylinder rolls without slipping in the $\hat{x}_0$ direction. The cylinder has a radius of r and a constant forward speed of v. What is the spatial acceleration of this cylinder expressed in $\{o\}$, ${}^{o}\mathcal{A}$ and expressed in $\{C\}$, ${}^{C}\mathcal{A}$, where frame $\{C\}$ has the same orientation as frame $\{o\}$ and its origin is at the contact point C.



The spatial velocity of the cylinder is ν_{body}.

The coordinate of ν_{body} in {o} is ${}^{o}\nu_{body}$

$$
{}^{o}\nu_{body} = \begin{bmatrix} {}^{o}\omega_{body} \\ {}^{o}v_{qbody} - {}^{o}\omega_{body} \times \overrightarrow{OR} \end{bmatrix} = \begin{bmatrix} 0 \\ v/r \\ 0 \\ 0 \\ 0 \\ v^2 t/r \end{bmatrix}
$$

the spatial accel is

$$
{}^{o}A_{body} = \frac{d}{dt}{}^{o}\nu_{body} + {}^{o}\nu \times {}^{o}\nu_{body} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ v^2/r \end{bmatrix}
$$

The adjoint matrix of ${}^{c}T_{o}$ is

$$
{}^{c}X_{o} = \begin{bmatrix} {}^{c}R_{o} & 0 \\ [{}^{c}p_{o}]{}^{c}R_{o} & {}^{c}R_{o} \end{bmatrix}
$$

In [ ]:

```
from sympy import *
r = symbols('r')
t = symbols('t')
v = symbols('v')
a_o = Matrix([0,0,0,0,0,v**2/r])
Rac = eye(3,3)
p_skew = Matrix([[0,0,0],
                 [0,0,v*t],
                 [0,-v*t,0]])
```

```
Xac = zeros(6,6)
Xac[0:3,0:3] = Rac
Xac[3:6,3:6] = Rac
Xac[3:6,0:3] = p_skew*Rac
a_c = Xac*a_o
pprint(a_c)
```

```
⎡0 ⎤
⎢  ⎥
⎢0 ⎥
⎢  ⎥
⎢0 ⎥
⎢  ⎥
⎢0 ⎥
⎢  ⎥
⎢0 ⎥
⎢  ⎥
⎢ 2⎥
⎢v ⎥
⎢──⎥
⎣r ⎦
```

Q2

$$
\begin{aligned}
\frac{d}{dt}[{}^{o}X_A^*] &= \frac{d}{dt}\begin{bmatrix} R & [p]R \\ 0 & R \end{bmatrix} \\
&= \begin{bmatrix} [w]R & \frac{d}{dt}[p]R + [p][w][R] \\ 0 & [w][R] \end{bmatrix} \\
&= \begin{bmatrix} [w]R & [v + w \times p]R + [p][w][R] \\ 0 & [w][R] \end{bmatrix} \\
&= \begin{bmatrix} [w]R & [v]R + [w][p]R - [p][w]R + [p][w][R] \\ 0 & [w][R] \end{bmatrix} \\
&= \begin{bmatrix} [w]R & [v]R + [w][p]R \\ 0 & [w][R] \end{bmatrix} \\
&= \begin{bmatrix} [w] & [v] \\ 0 & [w] \end{bmatrix} \begin{bmatrix} R & [p]R \\ 0 & R \end{bmatrix}
\end{aligned}
$$

Q3

$$
\begin{aligned}
\phi_q &= \sum_i \overrightarrow{qp_i} \times m_i v_i \\
&= \sum_i \overrightarrow{qo} \times m_i v_i + \overrightarrow{op_i} \times m_i v_i \\
&= \overrightarrow{qo} \times \sum_i m_i v_i + \sum_i \overrightarrow{opi} \times m_i v_i \\
&= \phi_o + \overrightarrow{qo} \times L
\end{aligned}
$$

Q4