Choosing Poses For Force and Stiffness Control

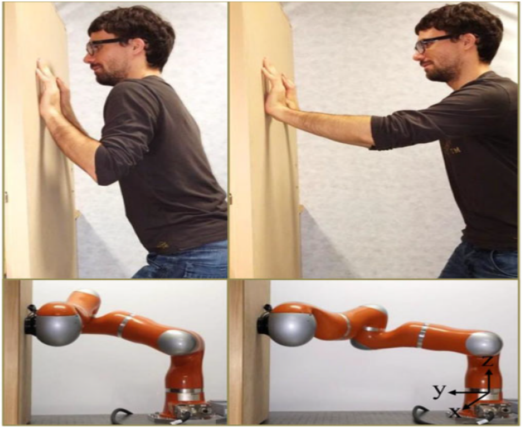
Jieming Chen, Shizeng Zhang

Institute of Control Systems
Electrical and Computer Engineering
University of Kaiserslautern

2.12.2019

Inhalt

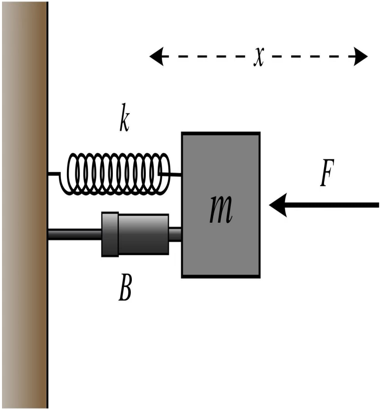
Introduction



Search Aim

For different arm configurations, the ability to maintain the stiffness are different.

Impedence Control



$$\underline{M_d}\,\ddot{\underline{\tilde{x}}} + \underline{K_d}\,\dot{\underline{\tilde{x}}} + \underline{K_p}\,\underline{\tilde{x}} = \underline{F_a}$$

$$\downarrow$$

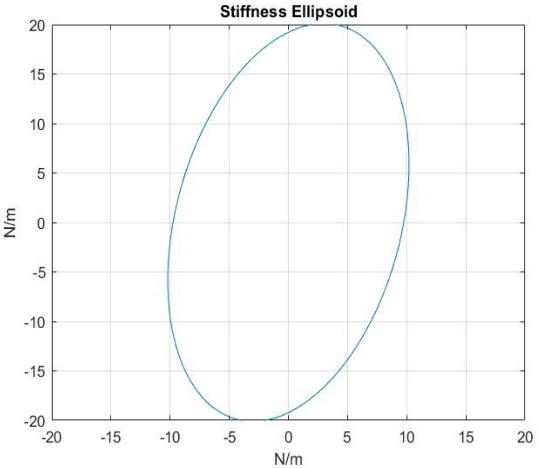
$$\underline{K_p}\,\underline{\tilde{x}} = \underline{F_a}$$

$$\downarrow$$

$$\underline{\tau} = J^T \underline{F_a}$$

τ reflects the stiffness.

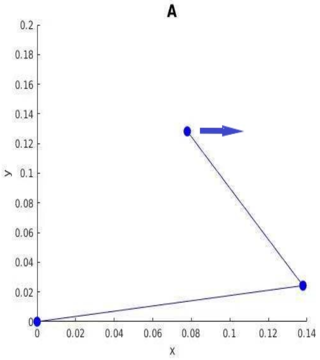
Stiffness Matrix and Stiffness Ellipsoid

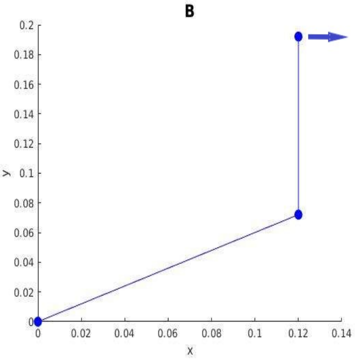


No consideration about torque limits!

Relationship between Arm Configurations and Boundries(1)

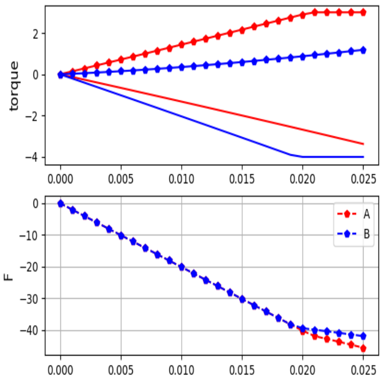
Two Different Arm Geometries





Search Aim: Find the relationship between the e-e displacement and the torque of each joint.

Relationship between Arm Configurations and Boundries(2)



For different configurations, the ability to maintain stiffness are different although they are under the same torque limit.

Calculate Method

$$F_{des} = K\delta x$$

$$\downarrow$$

$$\tau_{des} = J^T F_{des}$$

$$\downarrow$$

$$|\ \tau_{des}\ | \leq |\ \tau_{limt}\ |$$

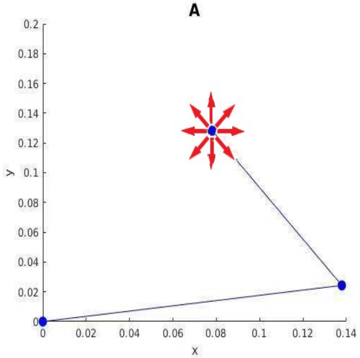
$$\downarrow$$

$$|\ \tau_{real}\ |$$

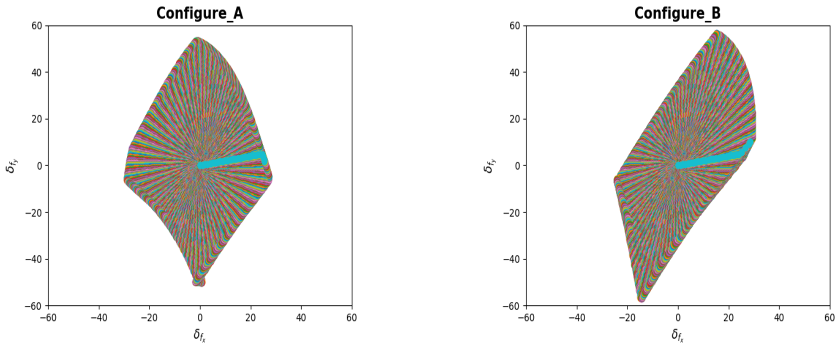
$$\downarrow$$

$$F_{real} = (J^T)^{-1}\tau_{real}$$

Apply in all Directions



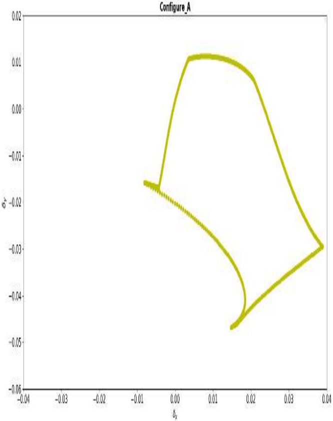
locus of $\| F \|$

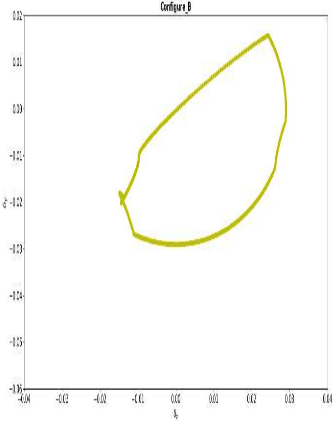


Comparison with Stiffness Matrix

Stiffness Matrix cannot always be realized because of the torque limits.

SFR(Stiffness Feasibility Region)(1)





$$x_{real} = F_{real}/K \tag{1}$$

SFR(Stiffness Feasibility Region)(2)

Algorithm 1 GET SFR

Ensure: The maximum range for maintaining stiffness, x_y_array;

$step\_length \leftarrow 0.001$;
for direction in direction list **do**
    Initialize the robot arm;
    while counter $<$ iter_num **do**
        $distance \leftarrow counter * step\_length$;
        $f_{ext} \leftarrow forceByDistance(distance)$;
        Update joint variables;
        Update e-e position;
        Update Jacobian Matrix;
        if joint torques reach torque limitations **then**
            Break;
        end if
    end while
    distance is stored in x_y_array ;
end for

SFR(Stiffness Feasibility Regions)(3)

Algorithm 2 GET SFR

function FORCEBYDISTANCE(*distance*)
    Calculate torque of gravity(τ_g);
    $f\_ext \leftarrow K_c \cdot distance$;
    $\tau \leftarrow jacobian^T f_{ext} + \tau_g$;
    τ then is clipped by τ limitations;
    $f_{res} \leftarrow (jacobian^T)^{-1}\tau - \tau_g$;
    Return f_{res};
end function

SFR Characteristics

SFR provides the most accurate representation of the imposed performance limits of a Cartesian stiffness controller, its calculation is computationally expensive, thus not suitable for real-time applications.

SFP(1)

stiffness feasibility polytope

Definition of SFP :

$$\delta \underline{x} \mid \|\hat{\tau}\|_\infty \leq 1$$

Deduction :

$$W_\tau = diag[\frac{1}{\tau_{lim_1}} \frac{1}{\tau_{lim_2}} \cdots \frac{1}{\tau_{lim_n}}]$$

$$\downarrow$$

$$\hat{\tau} = W_\tau \tau$$

$$\downarrow$$

$$\mid \|\hat{\tau}\|_\infty \leq 1$$

$$\downarrow$$

$$\left\| W_\tau J(q)^T K \delta x \right\|_\infty \leq 1$$

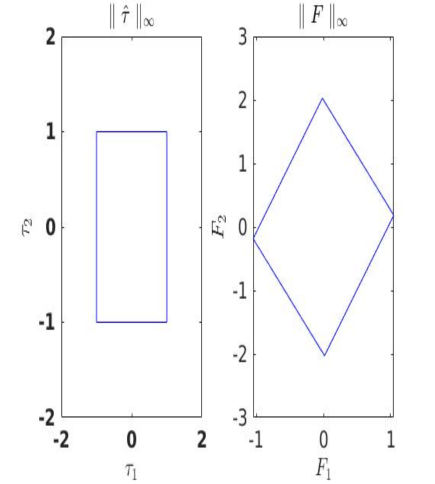


Abbildung: Two link Arm Example

SFP(2)

stiffness feasibility polytope

$$\left\| W_\tau J(q)^T K \delta x \right\|_\infty \leq 1$$

$$W_\tau J(q)^T K = UDV^T$$

$$\downarrow$$

$$\left\| W_\tau J(q)^T K \delta x \right\|_\infty = \lambda_{max} \left\| \delta x \right\|_\infty \leq 1$$

The direction of $\|\delta x\|_\infty$ *is decided by the column of u matrix.*

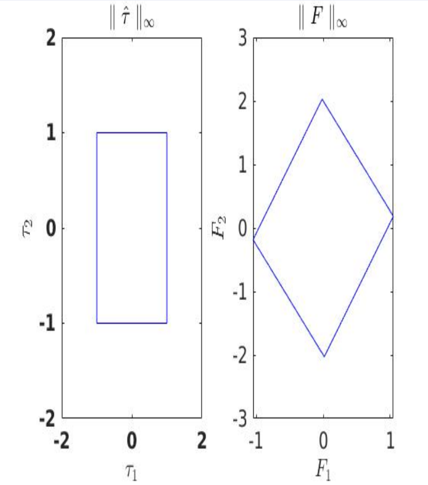


Abbildung: Two link Arm Example

SFE(1)

stiffness feasibility ellipsoid

Definition of SFE :

$\delta \underline{x} \mid \|\hat{\tau}\|_2 \leq 1$

Deduction :

$$W_\tau = diag[\frac{1}{\tau_{lim_1}} \frac{1}{\tau_{lim_2}} \cdots \frac{1}{\tau_{lim_n}}]$$

$\downarrow$

$\hat{\tau} = W_\tau \tau$

$\downarrow$

$\mid \|\hat{\tau}\|_2 \leq 1$

$\downarrow$

$\hat{\tau}^T \hat{\tau} \leq 1$

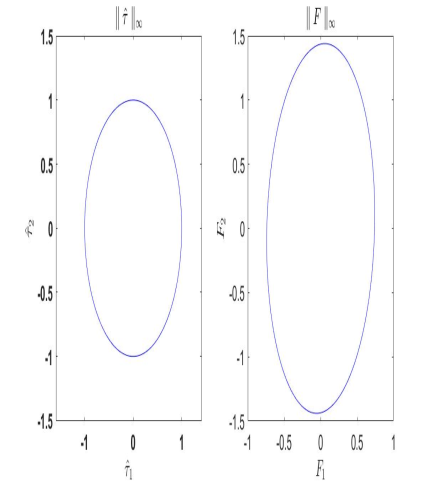


Abbildung: Two link Arm Example

SFE(2)

stiffness feasibility ellipsoid

$\hat{\tau}^T\hat{\tau} \leq 1$

$\delta x^T K^T J(q) W_\tau^T W_\tau J(q)^T K \delta x \leq 1$

$\delta x^T K^T J(q) W_\tau^T W_\tau J(q)^T K \delta x < \lambda_{max} \|\delta x\|_2$

The direction of $\|\delta x\|_2$ *is decided by the column of u matrix.*

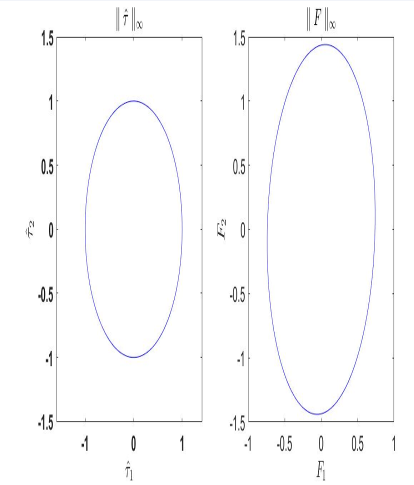
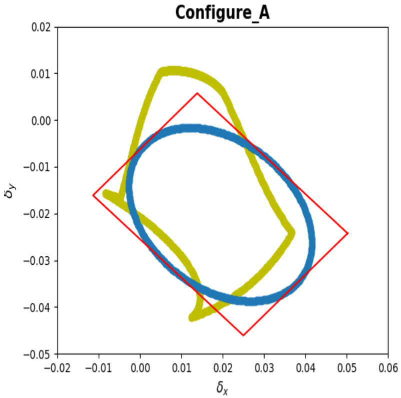
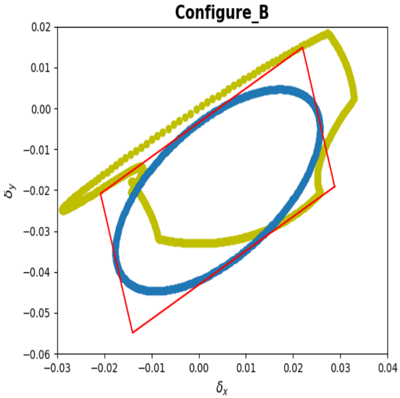


Abbildung: Two link Arm Example

SFR,SFP,SFE Comparison

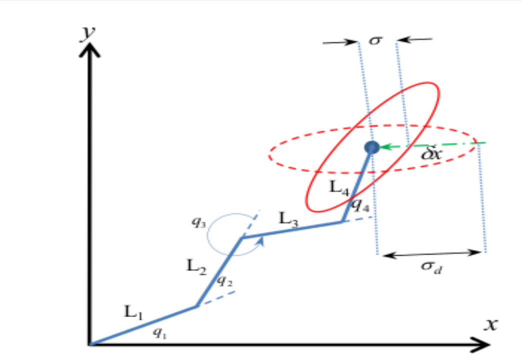
The yellow plots SFR and red plots SFP and blue plot SFE.





Pose Optimization for Force and Stiffness(1)

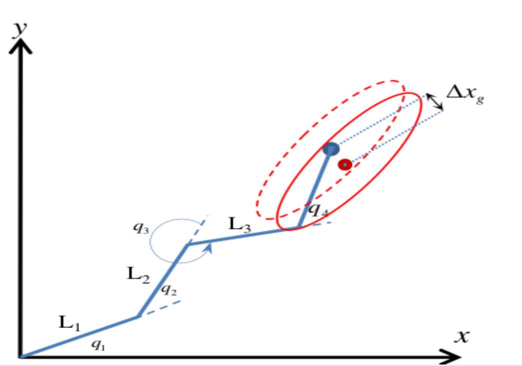
optimization objective(1): SFR Geometry



σ: the length in which the robot arm could maintain stiffness in ceartin direction. The aim is to try to enlarge this σ by changing configuration and rotating this ellipsoid.

Pose Optimization for Force and Stiffness(2)

optimization objective(2): Minimize Gravity Effect



The aim is to reduce the translation of the SFE due to gravity in the task space.

Pose Optimization for Force and Stiffness(3)

Build optimization problem

- Cost function for SFR Geometry:

$$(W_\tau J^T(q) K_c \sigma \frac{\delta x}{\|\delta x\|})^T (W_\tau J^T(q) K_c \sigma \frac{\delta x}{\|\delta x\|}) = 1$$

$$V_1 = \sigma = (\frac{\delta x}{\|\delta x\|}^T K_c J(q) W_\tau^2 J(q)^T K_c \frac{\delta x}{\|\delta x\|})^{-1/2}$$

- Cost function for minimizing gravity effect:

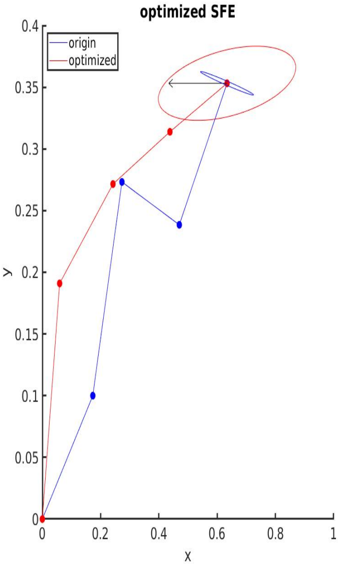
$$V_2 = \triangle x_g^T \triangle x_g$$

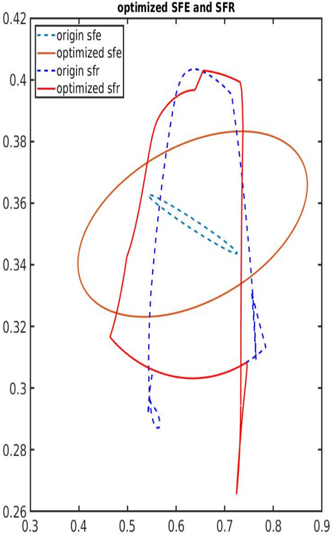
$$\triangle x_g = K_c^{-1} G_q \quad and \quad G_q = (J^T)^{-1} \tau_g$$

- Optimazation problem:

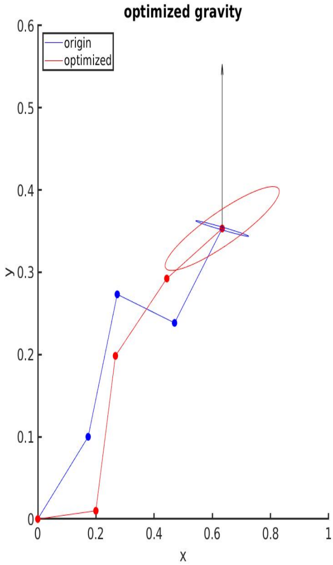
$$\begin{aligned} \min_{\underline{q}} \quad & V = weightingfactor_1 * V_1 + weightingfactor_2 * V_2, \\ \text{s.t.} \quad & forwardKinematic(\underline{q}) = \underline{X}_0, \\ & \underline{q_{min}} < \underline{q} < \underline{q_{max}}. \end{aligned}$$

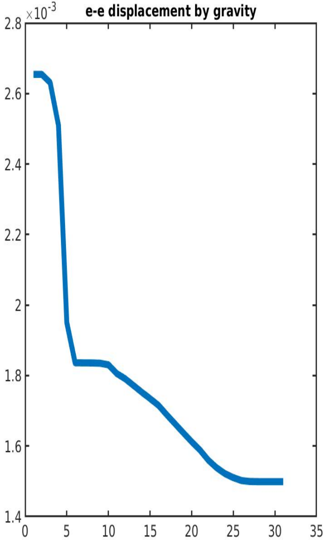
Result of Optimizing SFR geometry



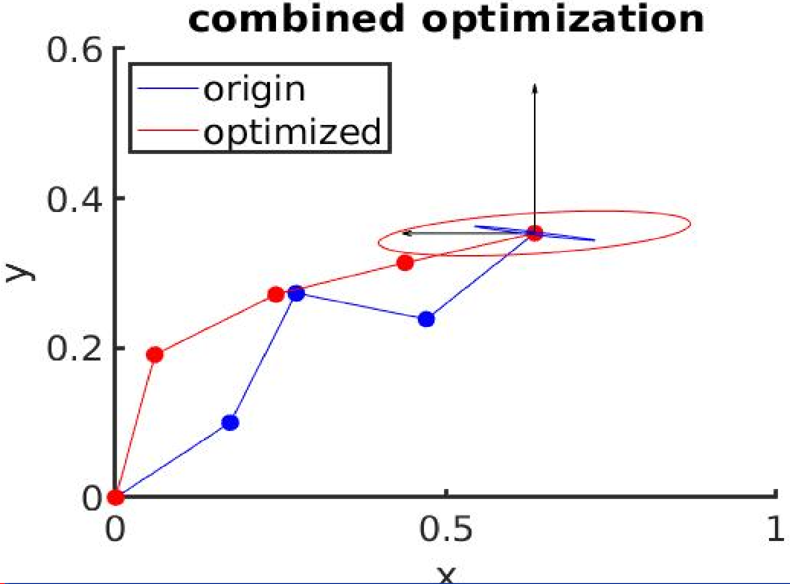


Result of minimizing the gravity effect





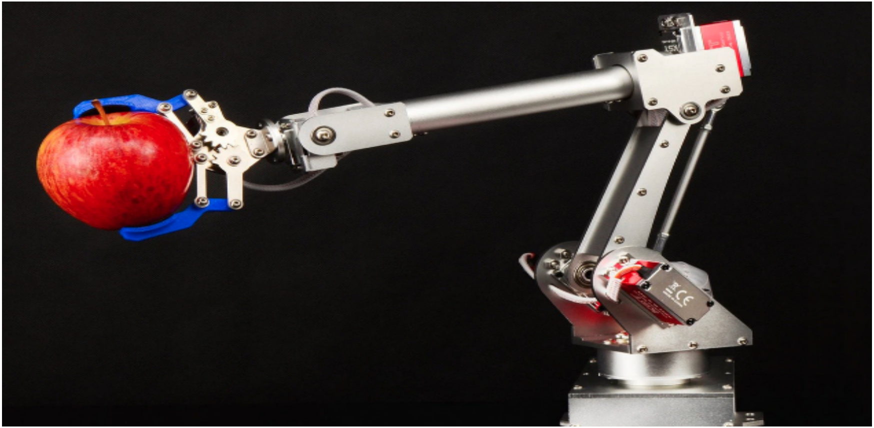
Result of Combined Optimization



Conclusion

In this paper, we research how large the area that the robot could maintain stiffness is, under the practical torque limitations and gravity effect. So that we introduce the concept of SFR and explain the role of configuration or pose of robot arm. Then to simplify calculation, SPE, SFP are also introduced. If the robot arm has redundancy, we could treat the problem that which configuration is the best for maintaining stiffness as a optimization problem. We also try minimizing the gravity effect for deviation of SFR, and combining both cost functions together.

Prospect



When the robot arm holding a stuff tracks trajectory, we could involve the maintaining stiffness ability in the cost function.