Siegfried Bosch「Algebraic Geometry and Commutative Algebra」

https://seasawher.github.io/kitamado/

@seasawher

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6.8 Cor.7

> **quotation.** Furthermore, for any $f \in A$ such that $D(f) \subset U$, the canonical map $A \to B$ induces an isomorphism $A_f \to B_f$ via localization and, hence , using 4.3/2, a commutative diagram
>
> $$\begin{tikzcd} M \otimes_A B \arrow[r] \arrow[d] & \mathcal{F}(U) \arrow[d] \\ M \otimes_A B_f \arrow[r, "\sigma_f"] & M \otimes_A A_f \end{tikzcd}$$
>
> where σ_f is an isomorphism.

Proof. First, we show $A_f \to B_f$ is a isomorphism. The following is the proof.

$$\begin{aligned} A_f &= \mathcal{O}_X(D(f)) \\ &= \mathcal{O}_X|_U(D(f)) \\ &= \mathcal{O}_U(D(f)) \\ &= B_f \end{aligned}$$

The second part remains to be solved. □

7.1 The projective n-space

> **quotation.** Also note that the ring of global sections of the structure sheaf on $X = \mathbb{P}^n_R$ is given by the intersection
>
> $$\mathcal{O}_X(X) = \bigcap_{i=0}^{n} A_i = \bigcap_{i=0}^{n} R\left[\frac{t_0}{t_i}, \cdots, \frac{t_n}{t_i}\right] = R.$$

Proof. Consider a more general case. Let $X = \bigcup_i X_i$ be a scheme got by glueing X_i. And $\mathcal{O}_X(X_i) = A_i$ is contained by same ring B. Then, we get $\mathcal{O}_X(X) = \bigcap_i A_i$. Why?

Let $\mathcal{B}$ be a open basis of X, $\mathcal{B} = \{U \subset X \mid \exists i\, U \subset_{\text{open}} X_i\}$. For any $U \in \mathcal{B}$, restrinction maps $\bigcap_i \mathcal{O}_X(X_i) \to \mathcal{O}_X(U)$ induce a map $\bigcap_i \mathcal{O}_X(X_i) \to \varprojlim \mathcal{O}_X(U)$ by universality of the limit.

And we get the inverse map $\varprojlim \mathcal{O}_X(U) \to \bigcap_i \mathcal{O}_X(X_i)$ by gluing. Considering the definition of limit, we can use gluing axiom of structure sheaf $\mathcal{O}_X$. □