The Mordell-Faltings theorem

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2019 年 4 月 4 日

Some basics of algebraic number theory

Proposition 1.4

> **quotation.** Let $\{\beta_1, \cdots, \beta_n\}$ be the dual basis of $\{\alpha_1, \cdots, \alpha_n\}$ with respect to $(\ ,\ )_{\mathrm{Tr}_{K/\mathbb{Q}}}$. Then, for any $x \in O_K$, we have $x = (x, \alpha_1)_{\mathrm{Tr}_{K/\mathbb{Q}}}\beta_1 + \cdots + (x, \alpha_n)_{\mathrm{Tr}_{K/\mathbb{Q}}}\beta_n$.

Proof. hogehoge □

Lemma 1.16

> **quotation.** Because $(O_K)_P$ is a principal ideal domain, $(O_{K'})_P$ is a free $(O_K)_P$-module of rank $[K' : K]$.

Proof. hogehoge □

Lemma 1.16

> **quotation.** Thus
>
> $$\begin{aligned}\dim_{O_K/P} O_{K'}/PO_{K'} &= \dim_{O_K/P}(O_{K'})_P/P(O_{K'})_P \\ &= \dim_{O_K/P}((O_K)_P/P(O_K)_P) \otimes_{(O_K)_P} (O_{K'})_P\end{aligned}$$

Proof. hogehoge □

Adjacent to Lemma 1.17

> **quotation.** We take a integral basis $\{\omega_1, \cdots, \omega_n\}$ of O_K, we denote by $\{\beta_1, \cdots, \beta_n\}$ the dual basis with respect to $(\ ,\ )_{\mathrm{Tr}_{K/\mathbb{Q}}}$. Then we have $\mathcal{M} =$

$\mathbb{Z}\beta_1 + \cdots + \mathbb{Z}\beta_n.$

Proof. hogehoge $\square$