Control theory

INTRODUCTION

I DESCRIPTION OF OPEN LOOP SYSTEMS

II SYSTEM ANALYSIS

A Time domain analysis

B Frequency domain analysis

III ANALYSIS OF CLOSED SYSTEMS

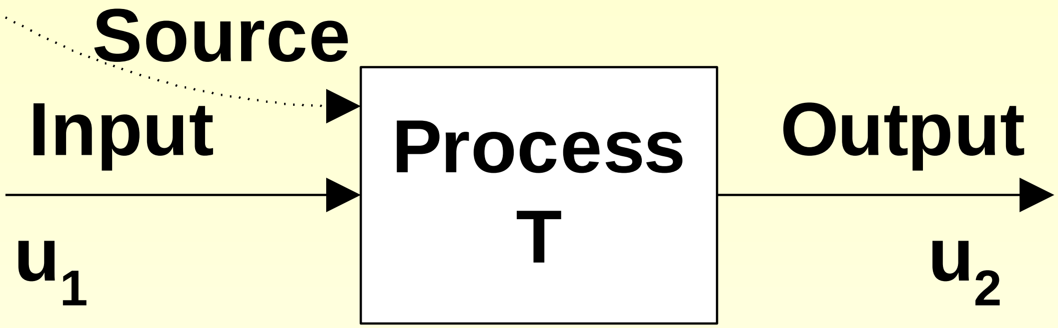
IV STABILITY OF CLOSED SYSTEMS

V PRACTICAL METHODS

VI DYNAMICS OF COMPLEX SYSTEMS

System description

System diagram



Example:

Source: potential energy of water

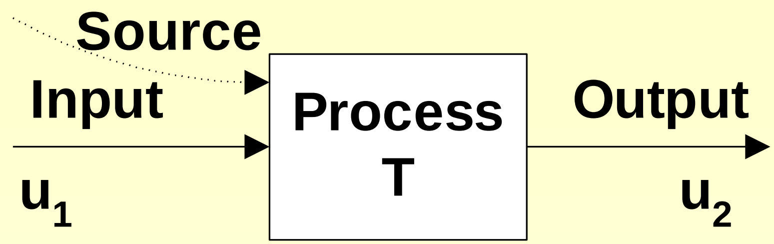
Process: change of flow by valve

Input: position of valve

Output: water flow in tube

The transfer function of a process

$$u_2 = T u_1 \quad \Longrightarrow \quad T = \frac{u_2}{u_1}$$

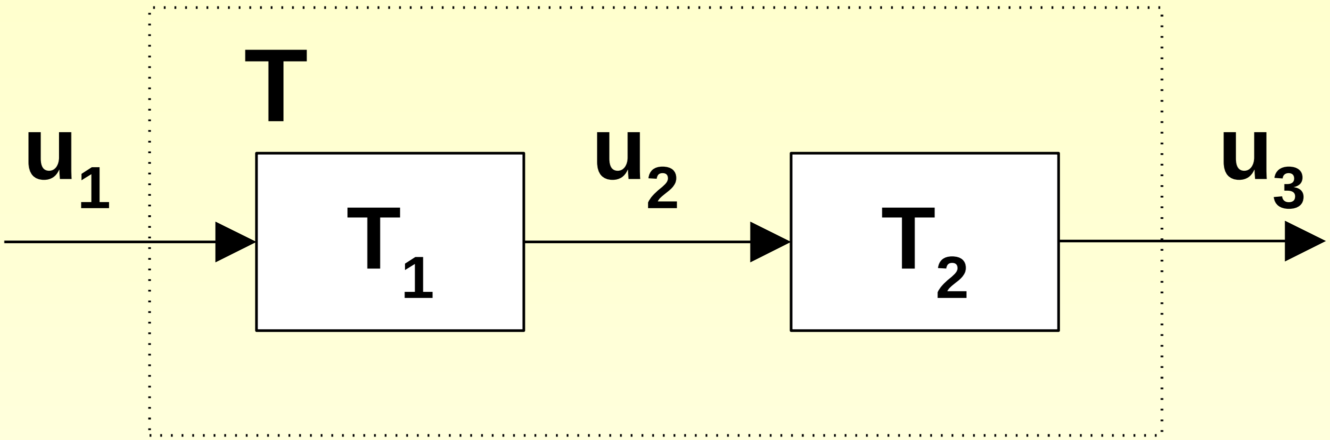


For linear time-independent systems:

$$a_n \frac{d^n u_2}{dt^n} + a_{n-1} \frac{d^{n-1} u_2}{dt^{n-1}} + \cdots + a_0 u_2 = u_1$$

$$T = \frac{u_2}{u_1} = \frac{1}{a_n D^n + a_{n-1} D^{n-1} + \cdots + a_0}$$

Systems in series



$$u_2 = T_1 u_1$$

$$u_3 = T_2 u_2$$

$$u_3 = T u_1$$

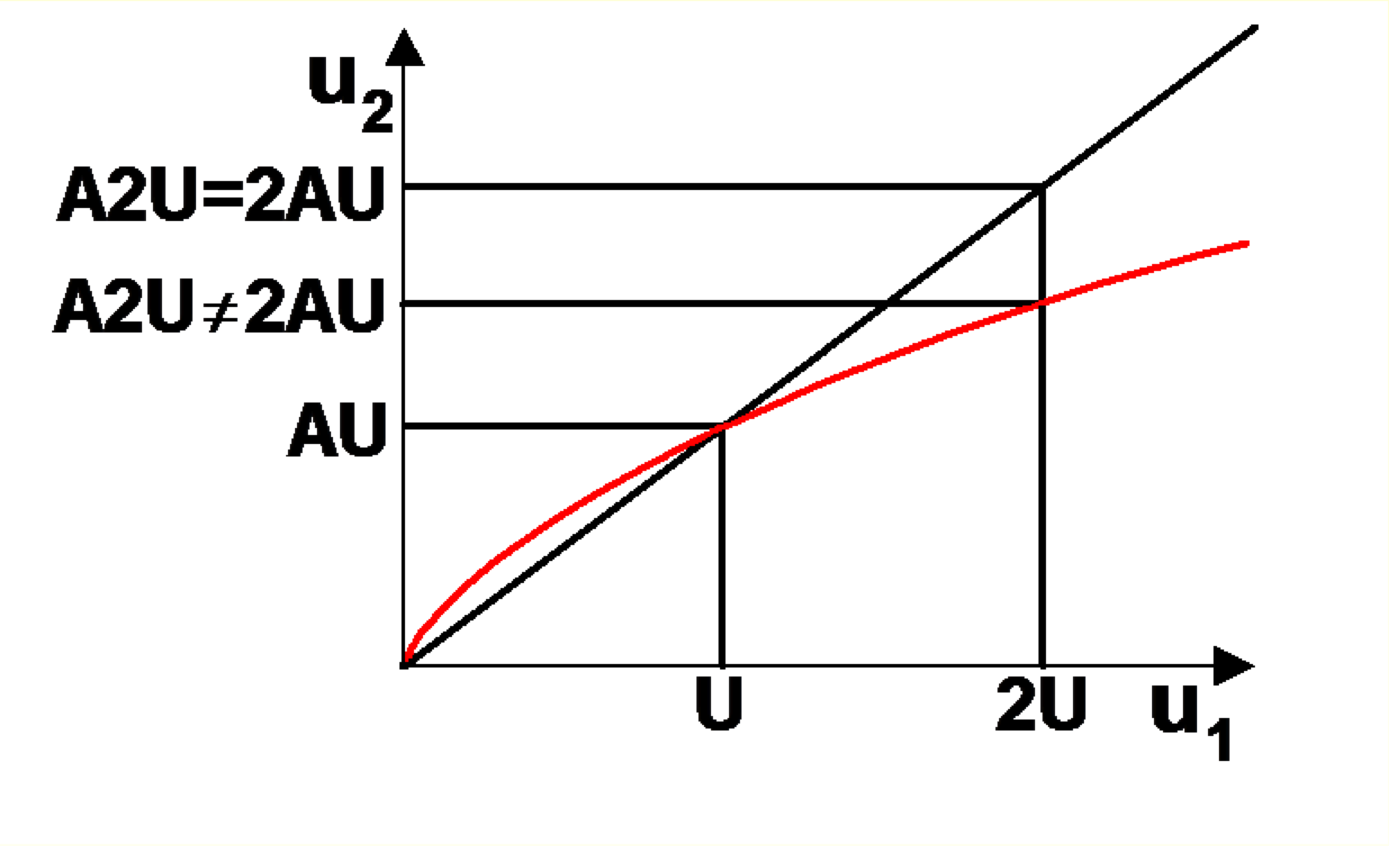
$$u_3 = T u_1 = T_2 u_2 = T_2 (T_1 u_1) = T_2 T_1 u_1$$

$$T = T_2 T_1$$

$$T_2 T_1 \neq T_1 T_2$$

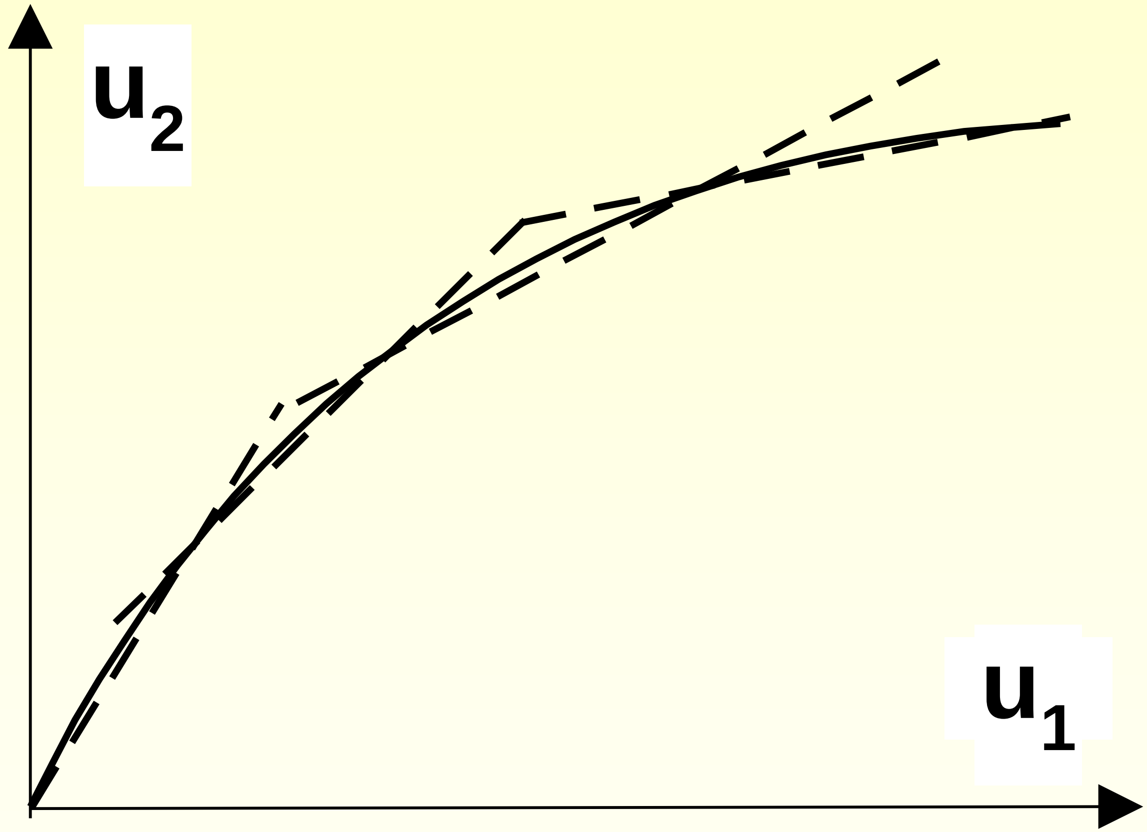
Linear and nonlinear process

$T = T(u_1)$



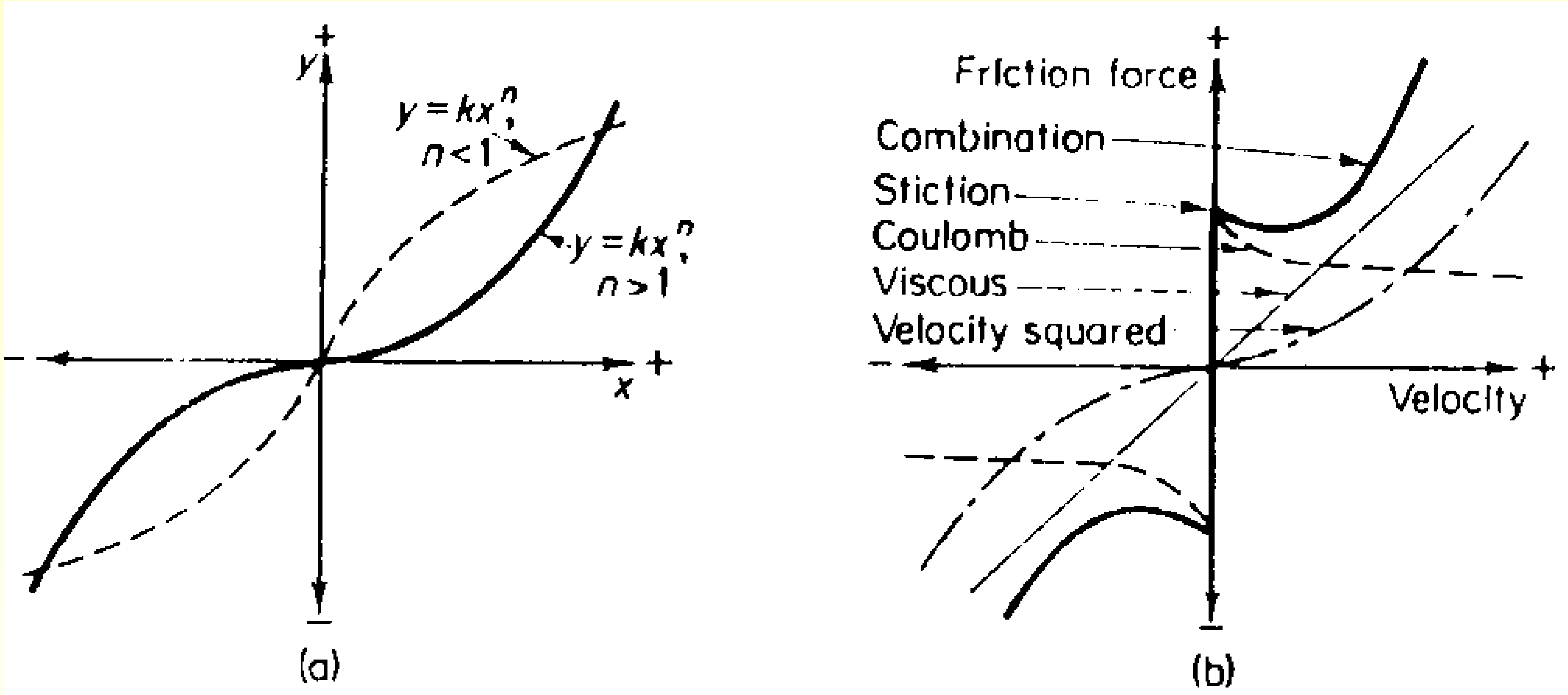
Linear approximation of nonlinear processes

$T = T(u_1)$



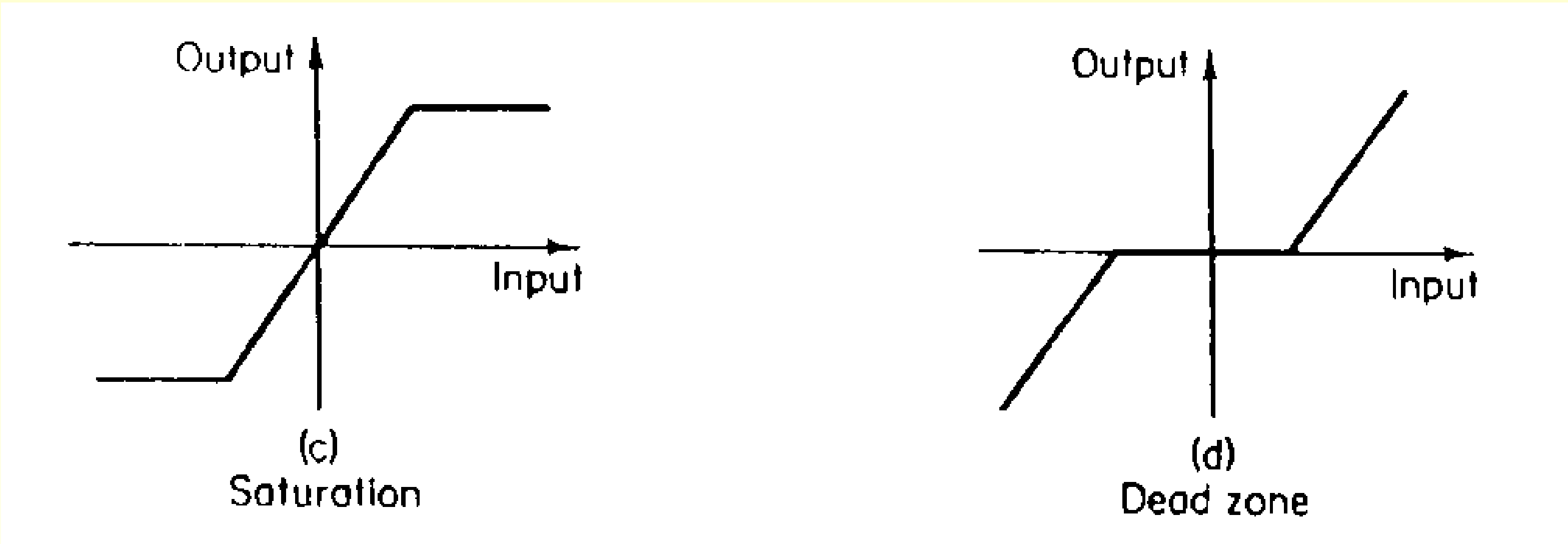
Common non-linear relations

$T = T(u_1)$



Common non-linear relations

$$T = T(u_1)$$



(c) Saturation

(d) Dead zone

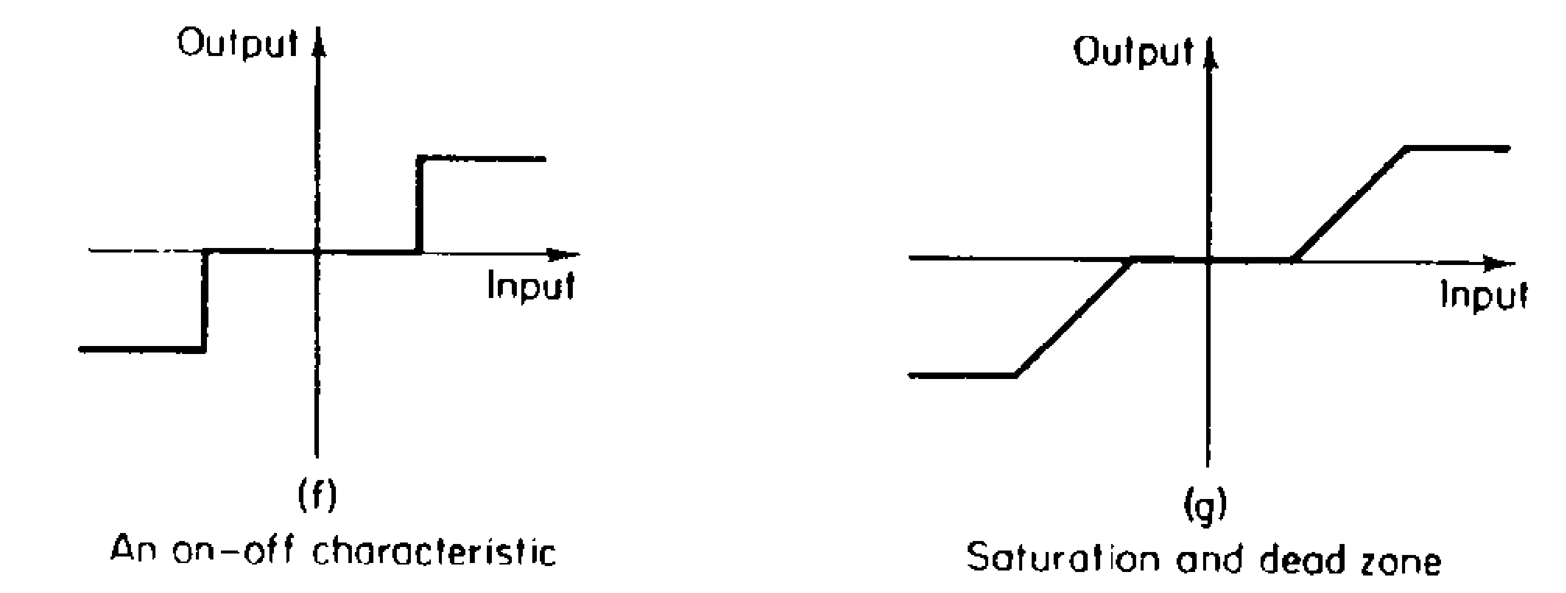
Common non-linear relations

$$T = T(u_1)$$

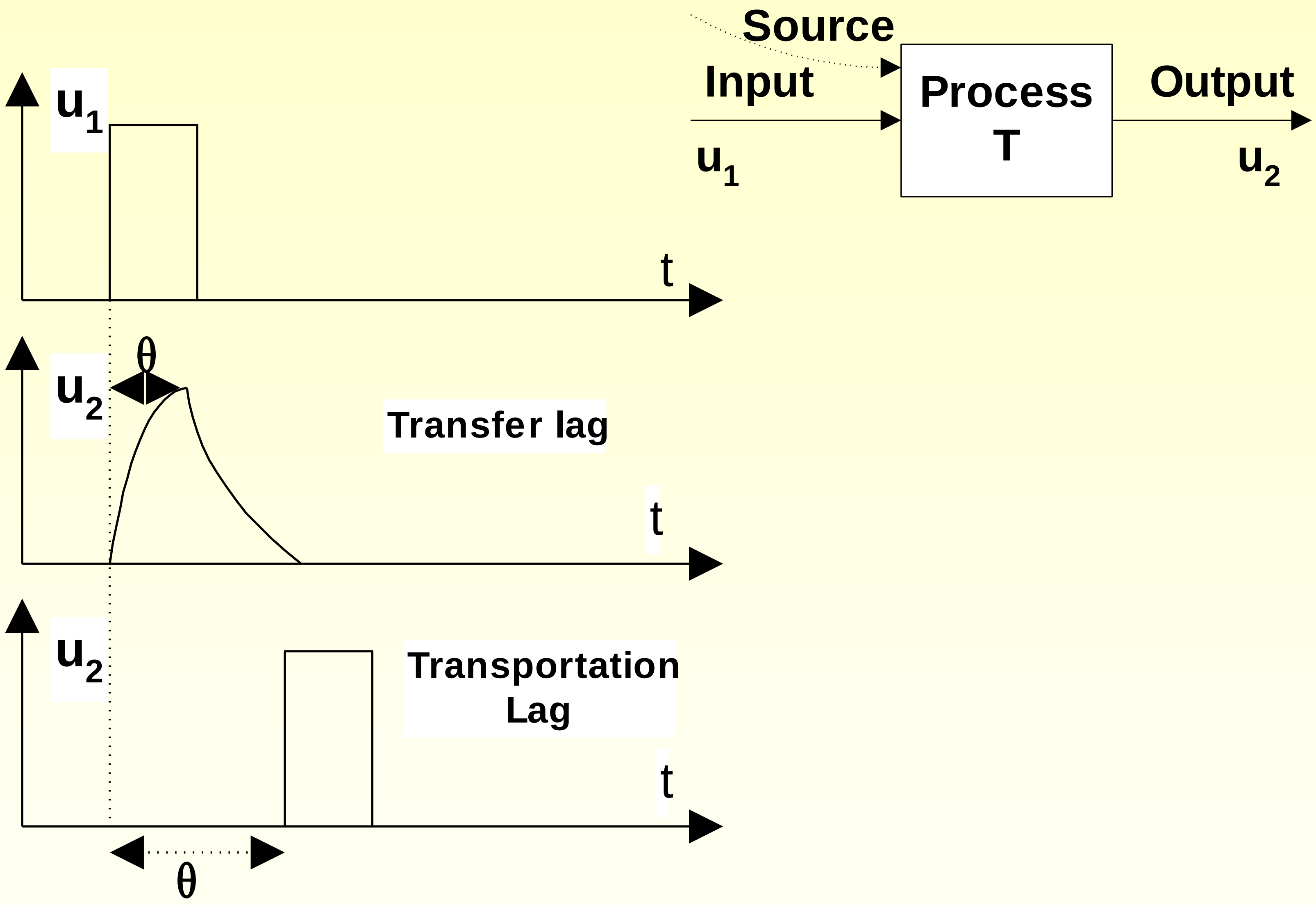
(e)
Two forms of hysteresis

Common non-linear relations

$$T = T(u_1)$$

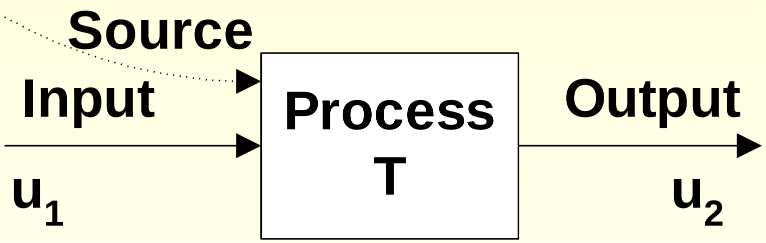


Time lags



Source
Input
Process
T
Output
u_1
u_2
u_1
t
u_2
θ
Transfer lag
t
u_2
Transportation
Lag
t
θ

System analysis



T = ?

Time domain analysis

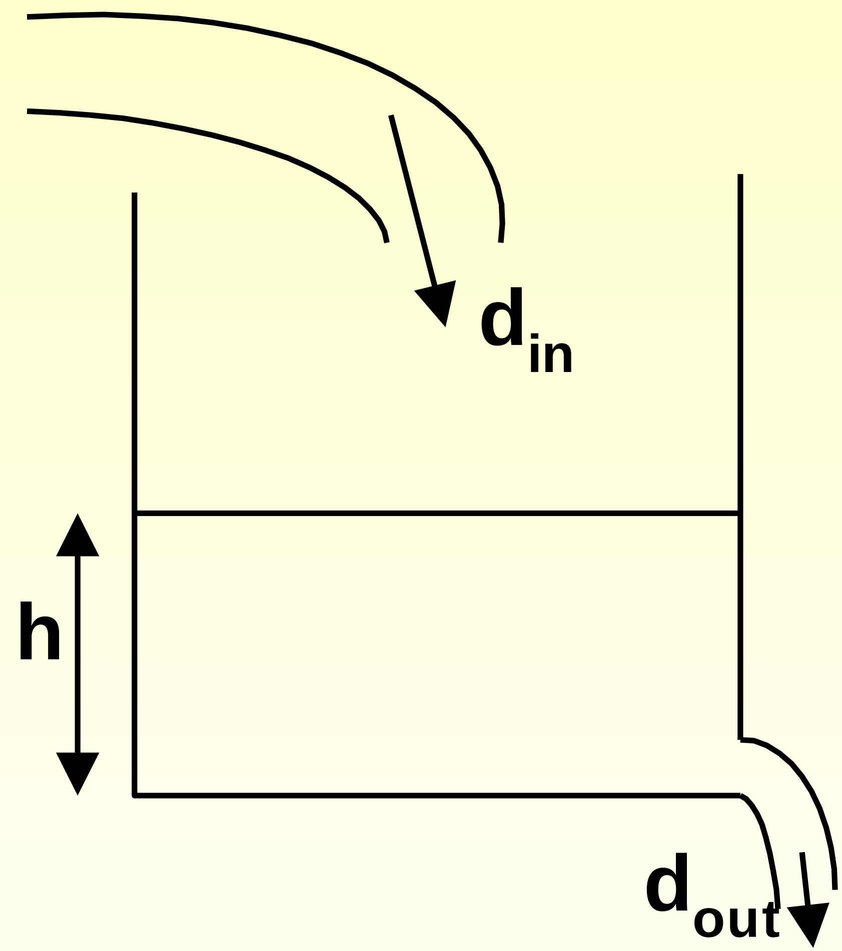
Filling of a vessel with an outflow

$$d_{out} \approx bh$$

$$\frac{dh}{dt} = a(d_{in} - d_{out}) = a(d_{in} - bh)$$

$$\frac{dh}{dt} + abh = ad_{in}$$

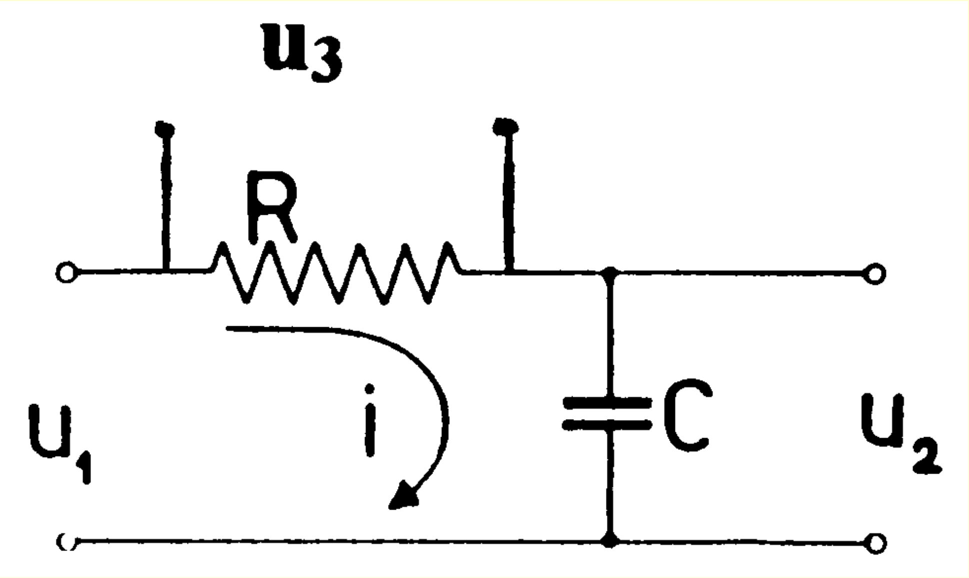
$$\frac{1}{ab}\frac{dh}{dt} + h = \frac{1}{b}d_{in}$$



Solution: steady state (dh/dt =0)

$h = d_{in}/b$ Thus, since $d_{out} = b.h$, then $d_{out} = d_{in}$

Resistance and capacitance in series



$$u_1 = u_2 + u_3$$

$$u_3 = iR$$

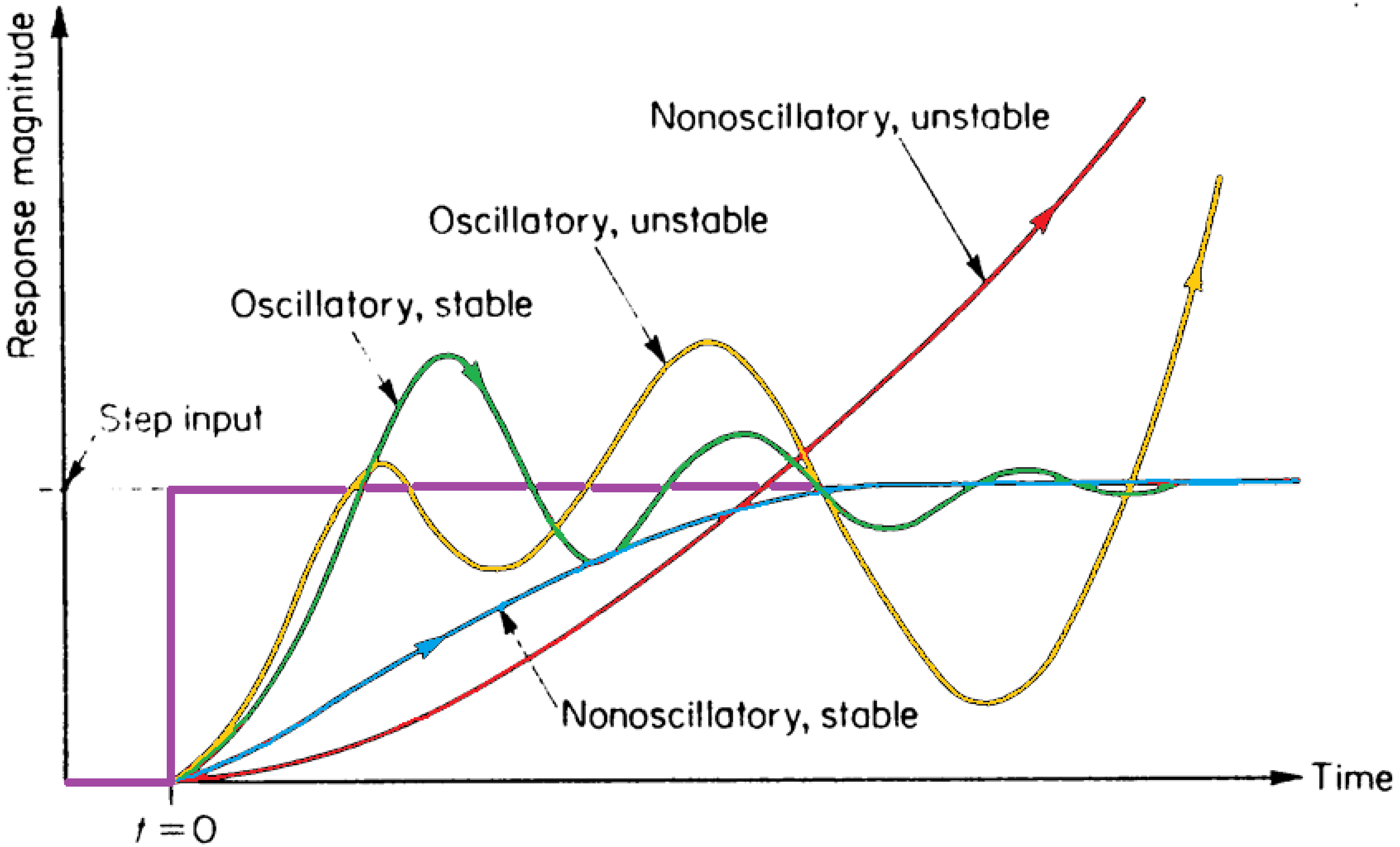
$$q = Cu_2$$

$$i = \frac{dq}{dt} = C\frac{du_2}{dt}$$

$$u_1 = u_2 + iR = u_2 + RC\frac{du_2}{dt}$$

$$RC\frac{du_2}{dt} + u_2 = u_1$$

Possible responses to a step input



Some possible responses of a system to a step input.

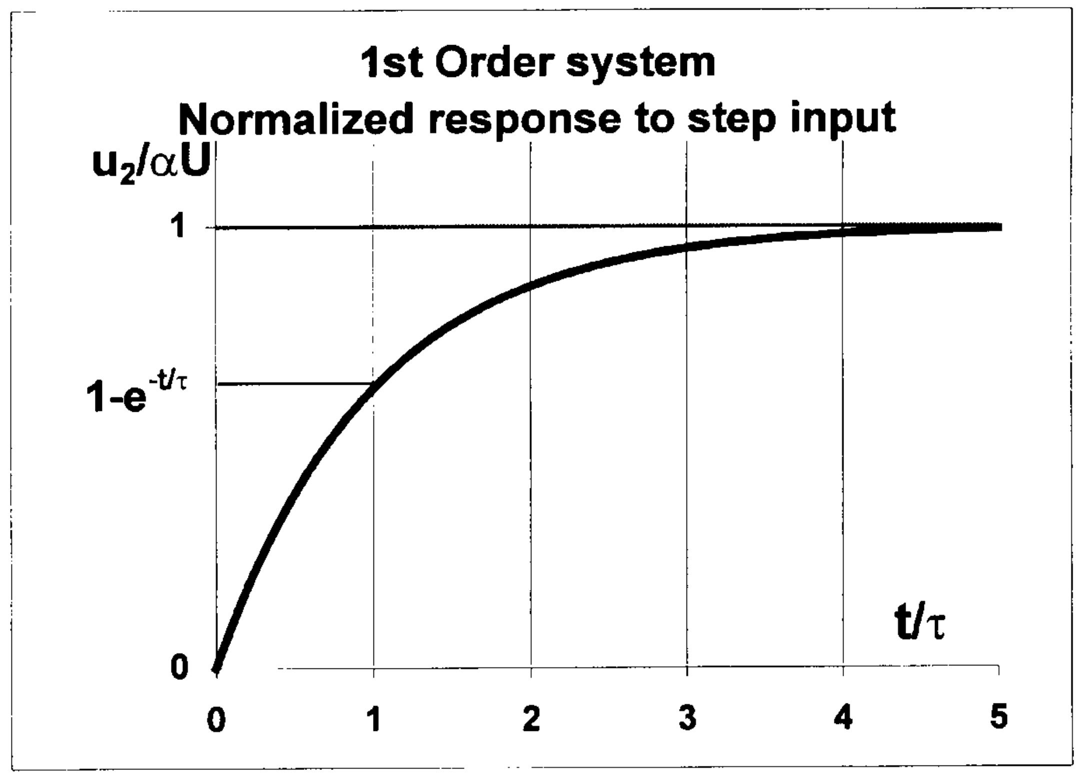
Normalized step response first order system

$$\tau \frac{du_2}{dt} + u_2 = \alpha U$$

Substitutions:

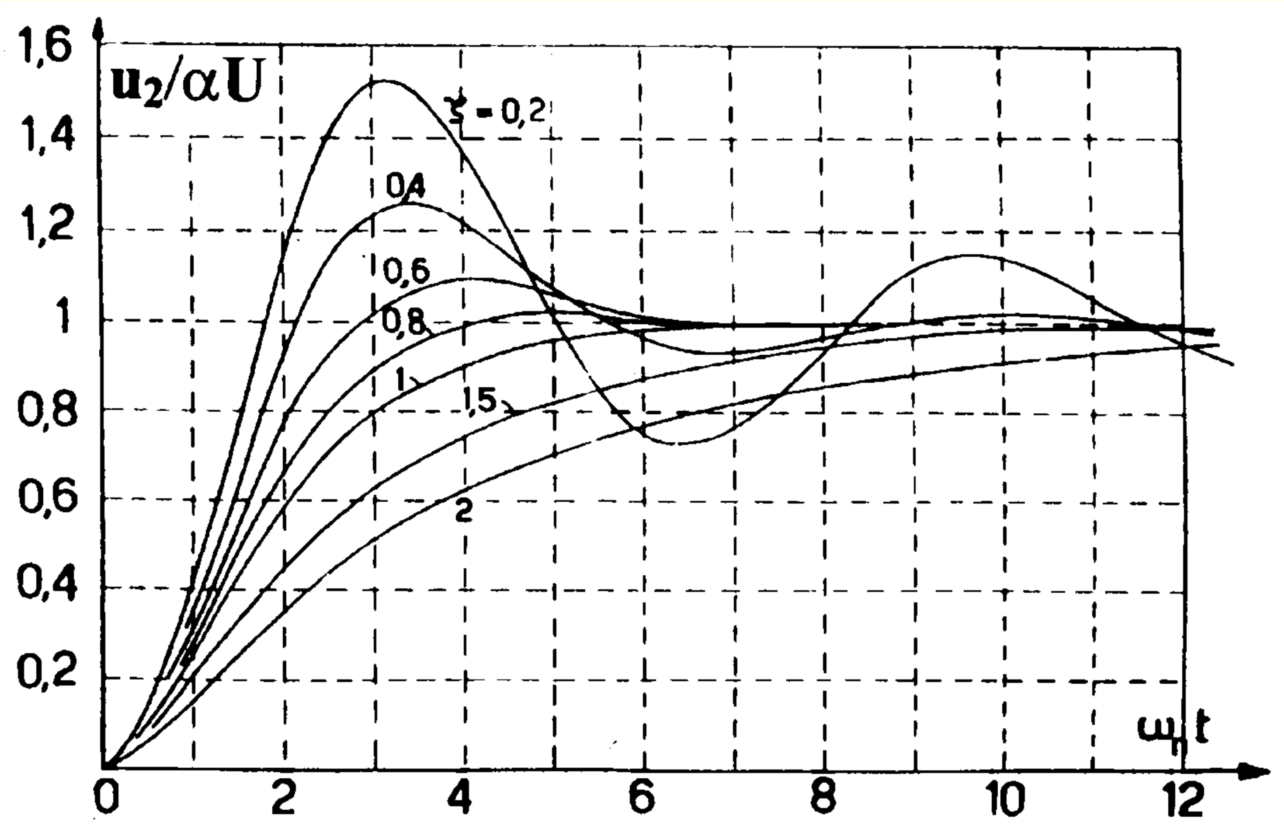
$$V = \frac{u_2}{\alpha U}; \quad \mathrm{T} = \frac{t}{\tau}$$

$$\frac{dV}{dT} + V = 1$$



Response of second order system to step input

$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=\alpha u_1 \qquad \text{(with } u_1 = U \text{ for } t \geq 0\text{)}$$



Response of second order system to step input
Differential equation and parameters

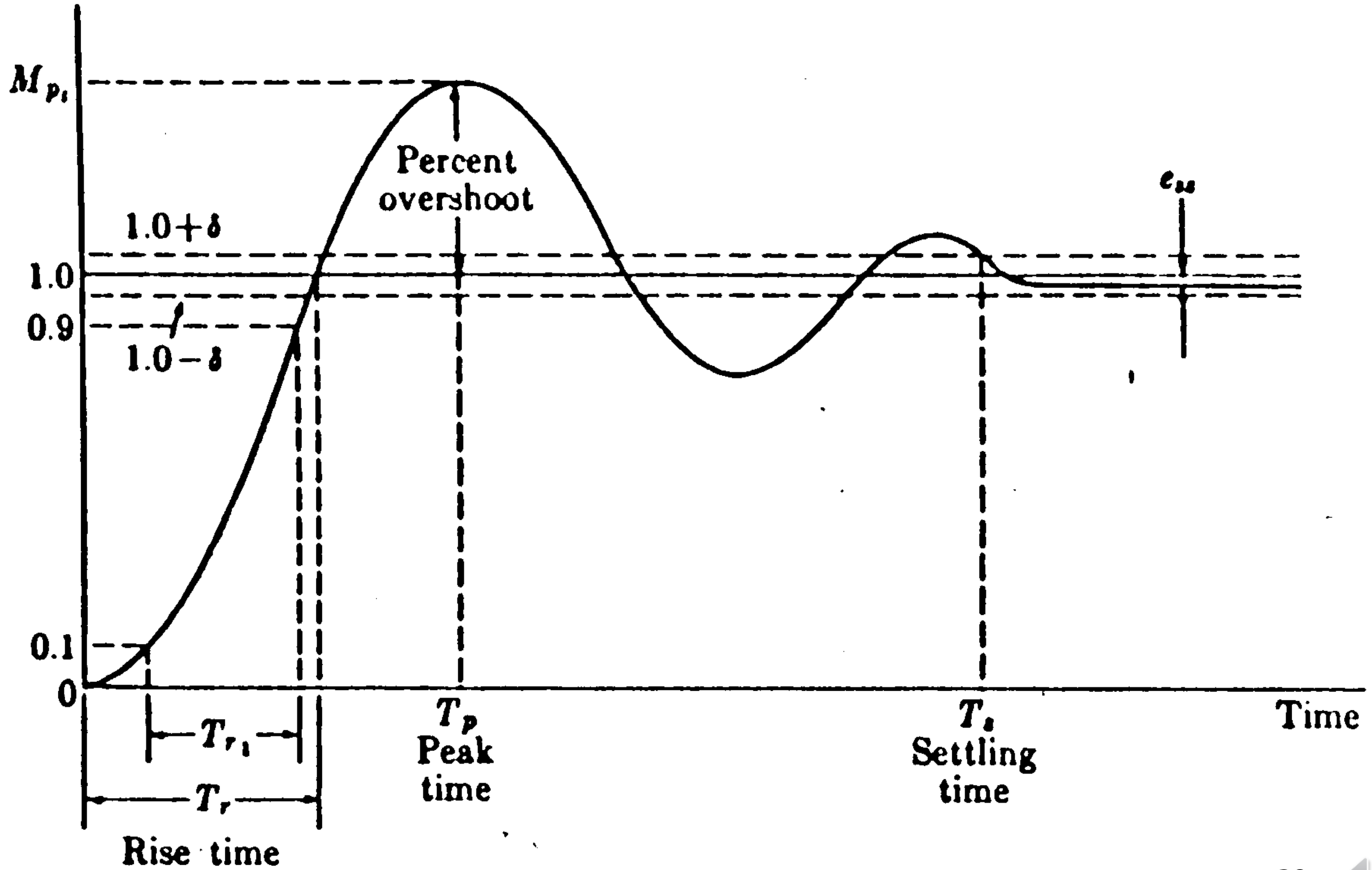
$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=\alpha U \qquad (t \geq 0)$$

α : gain

ω_n: natural frequency (eigen frequency)

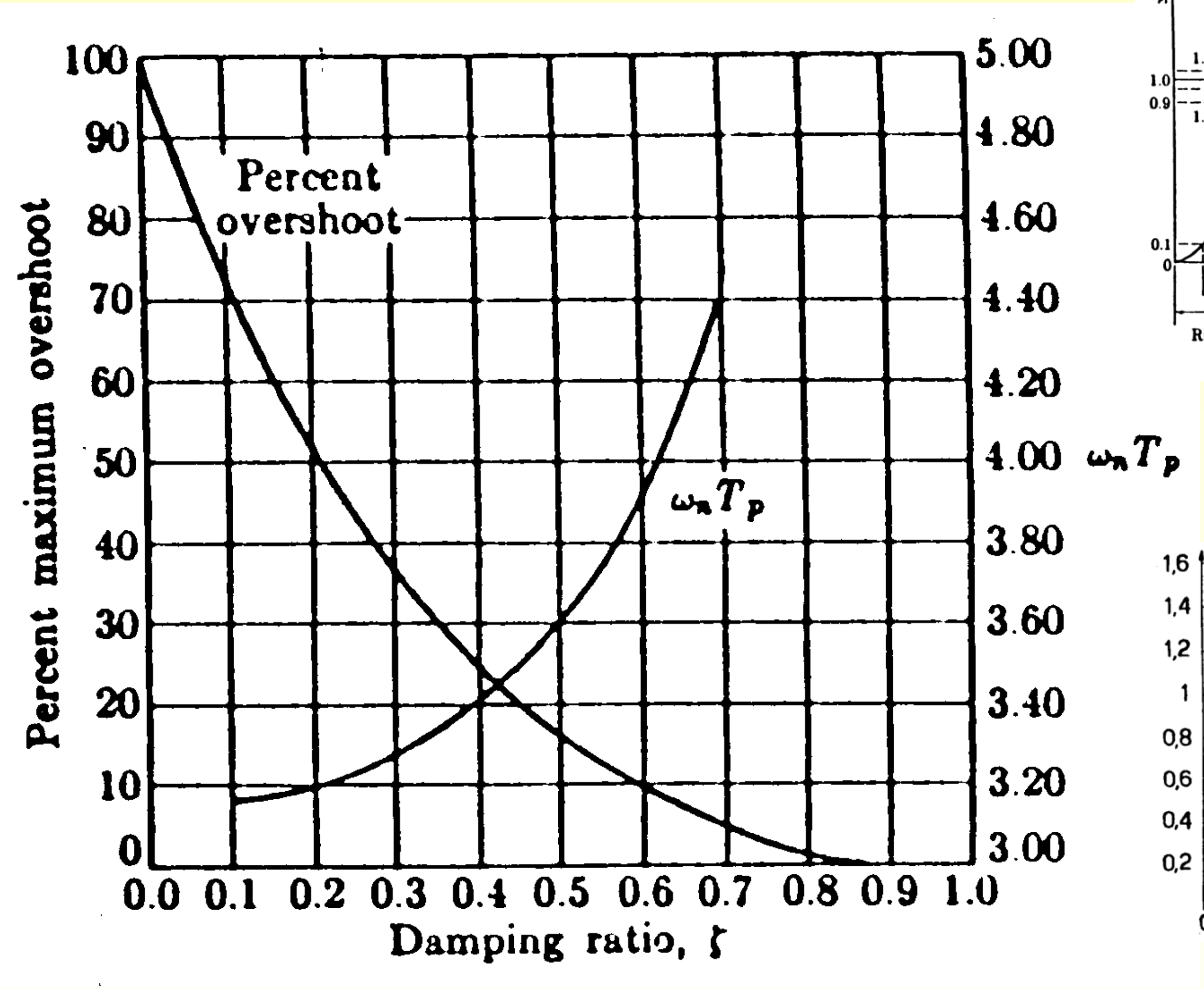
ξ : damping factor

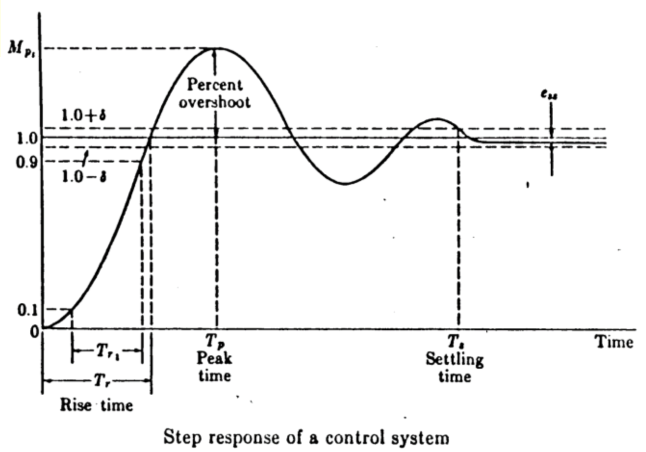
Response of second order system to a step input

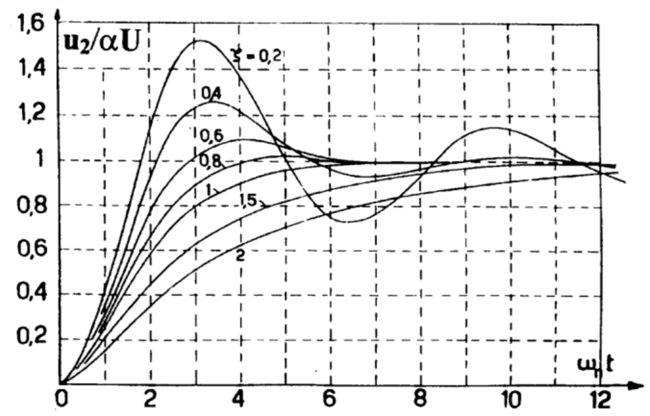


Step response of a control system

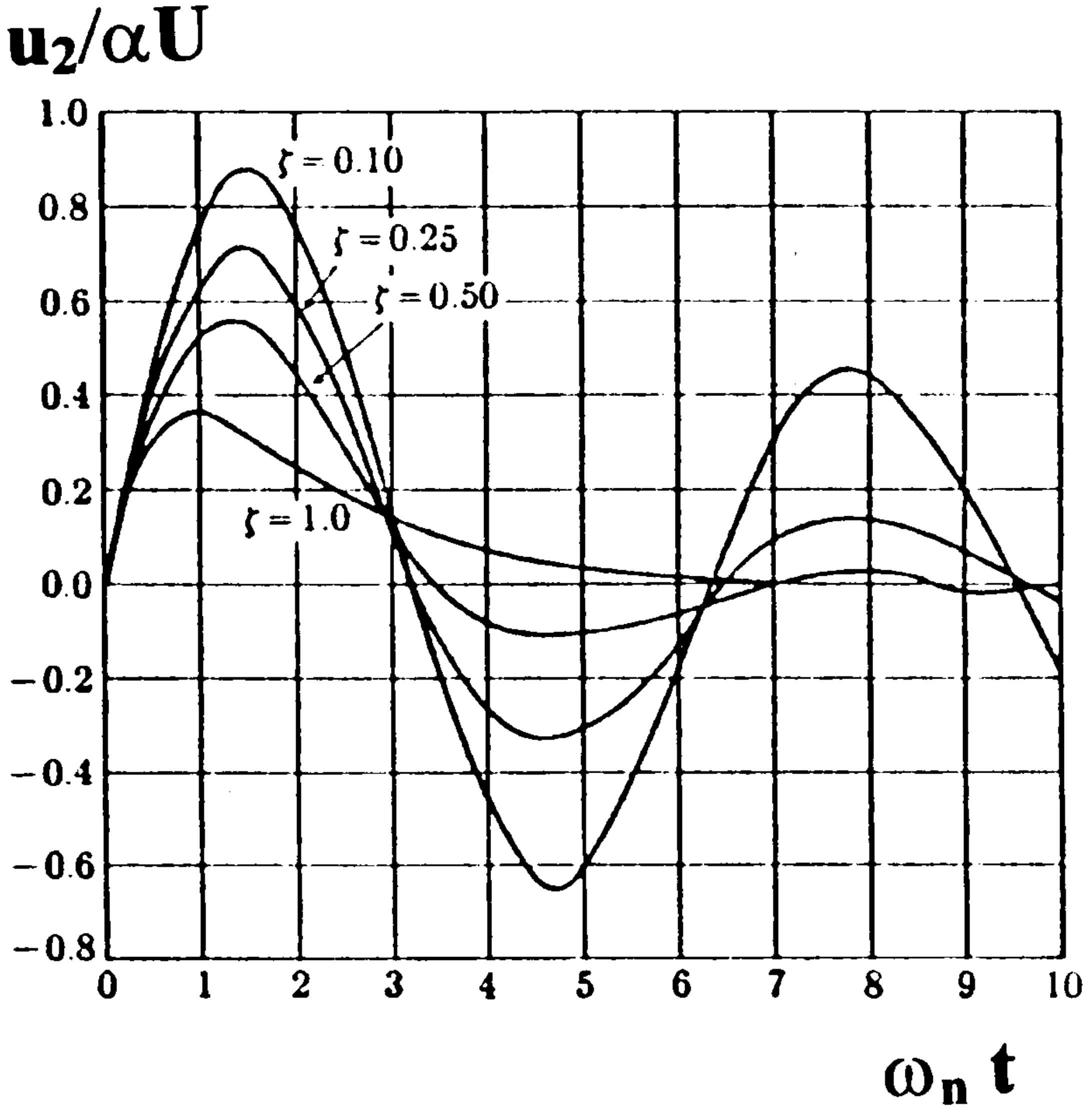
Overshoot of second order system as function of damping factor







Response of second order system to an impulse input



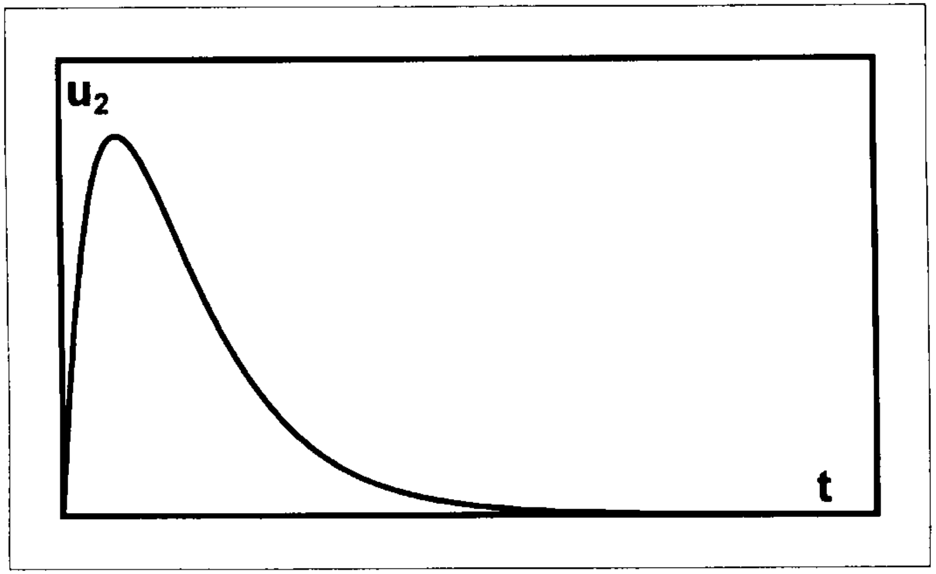
Response of a second-order system for an impulse function input.

Response of system of order -1 to a step input

First order: $$\tau \frac{du_2}{dt} + u_2 = \alpha u_1$$

Order -1: $$\int u_2 dt = u_1 \quad u_2 = \frac{du_1}{dt}$$

Speedometer: Real response to a step input

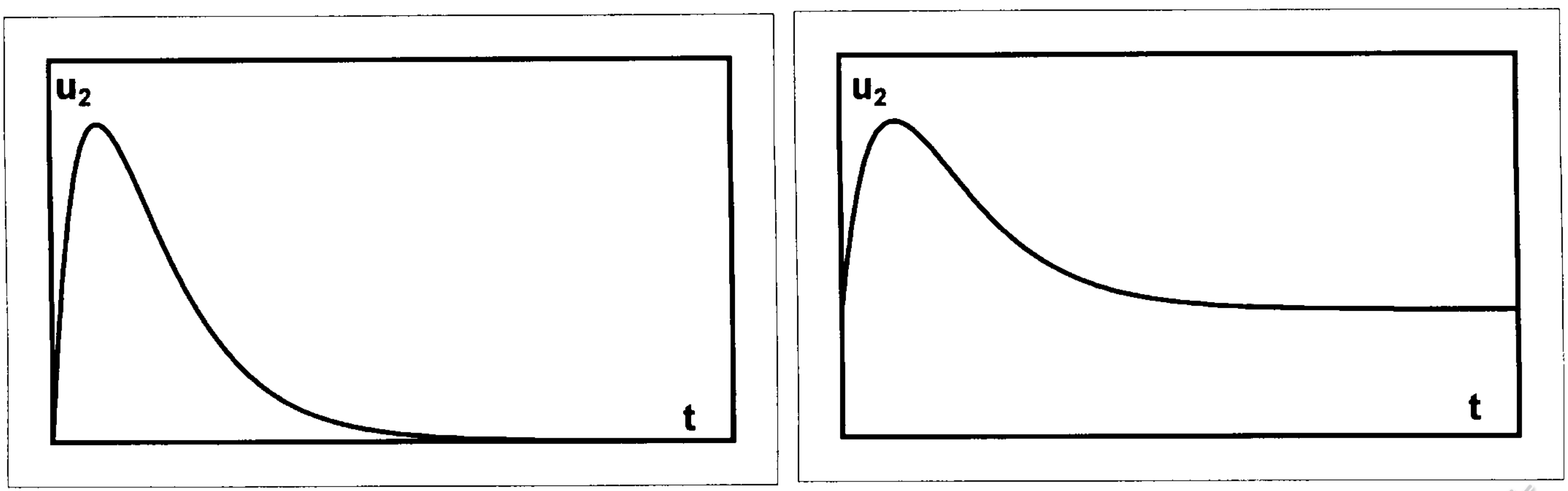


Response of system of order -1 to a step input

First order: $\tau \dfrac{du_2}{dt} + u_2 = \alpha u_1$

Order -1: $\int u_2 dt = u_1 \quad u_2 = \dfrac{du_1}{dt} \qquad u_2 = a\dfrac{du_1}{dt}$

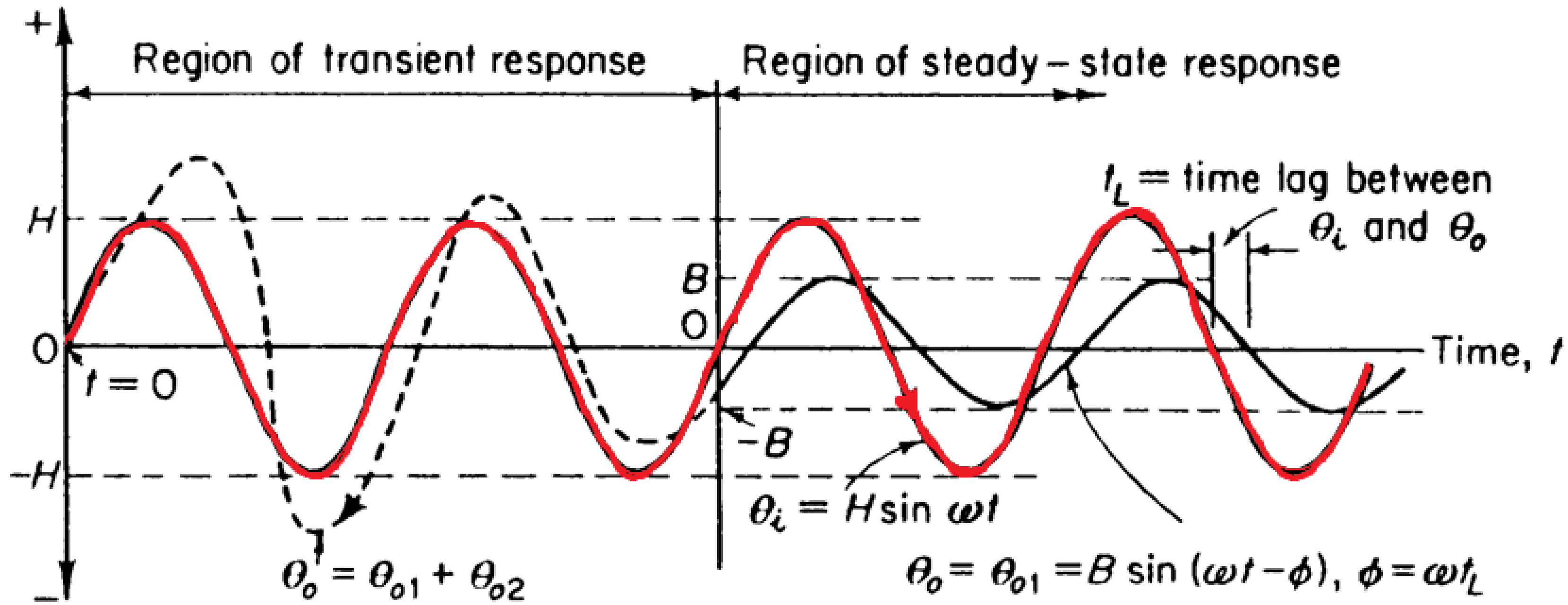
Speedometer: Real response to a step input



Frequency domain analysis

Typical system response to sinus input

Red trace: input Black trace: output



Typical system response to sinusoidal input: θ_{o2} is transient term of response; θ_{o1} is steady-state term of response.

Frequency domain analysis of first order system

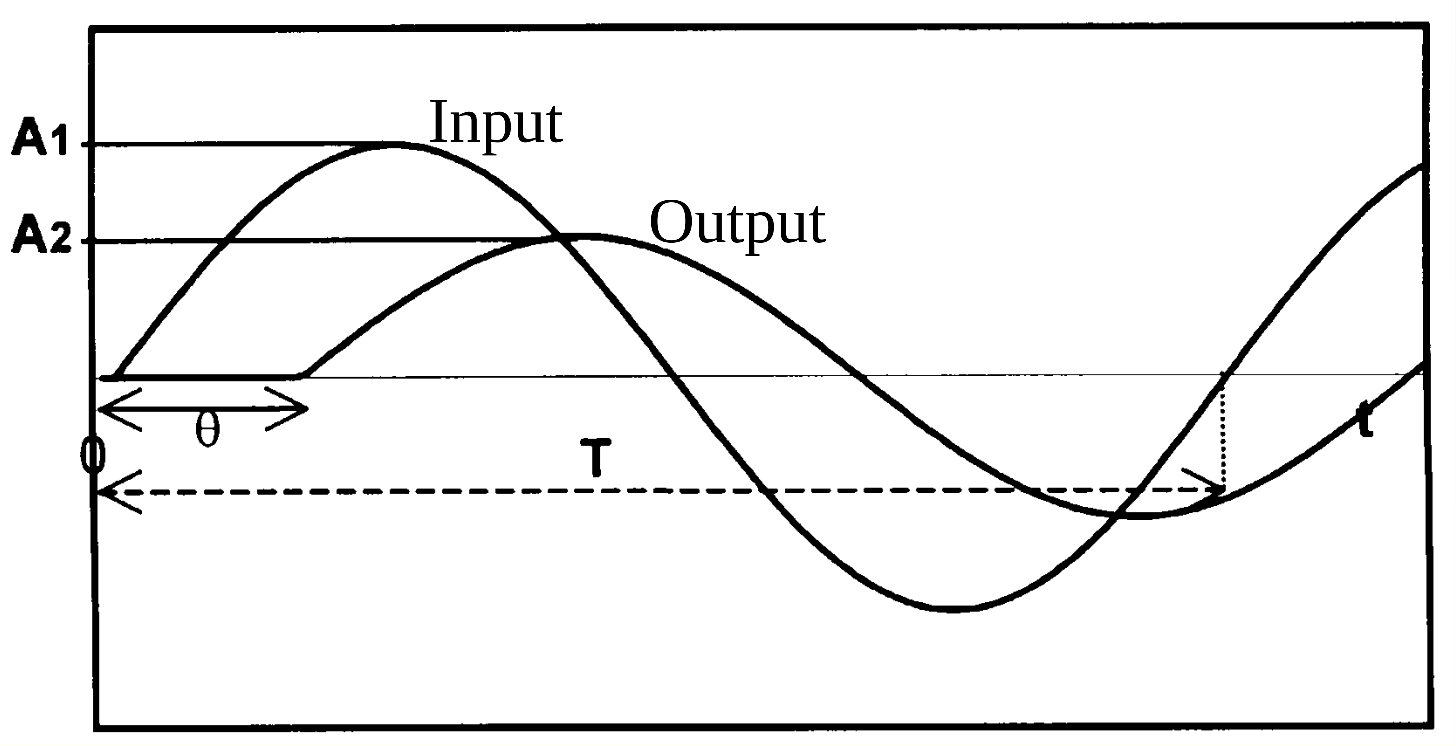
$$\tau \frac{du_2}{dt} + u_2 = \alpha u_1 \qquad \tau \frac{du_2}{dt} + u_2 = \alpha A_1 \sin(\omega t)$$

Solution:

- Transient terms (a number of exponential functions)
- ***Steady state term of the form*** $A_2 \sin(\omega t + \varphi)$

Steady-state system response to sinus input

$$\tau \frac{du_2}{dt} + u_2 = \alpha A_1 \sin(\omega t)$$



Frequency domain analysis first order system

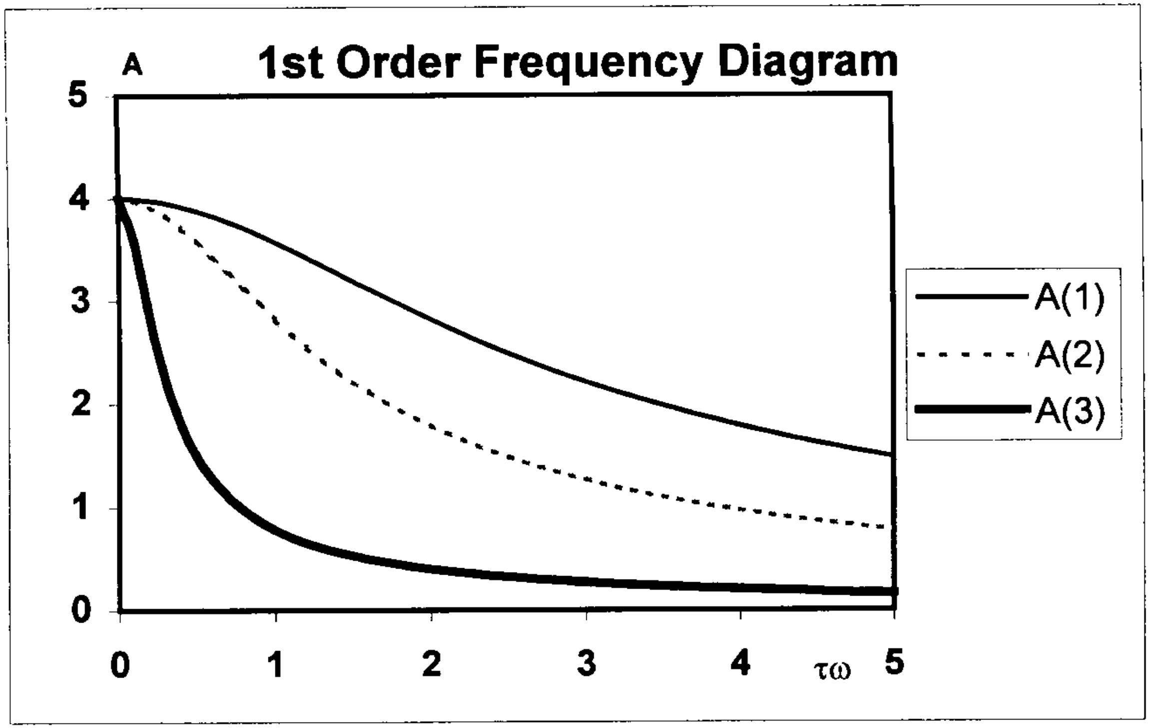
$$\tau \frac{du_2}{dt} + u_2 = \alpha A_1 \sin(\omega t)$$

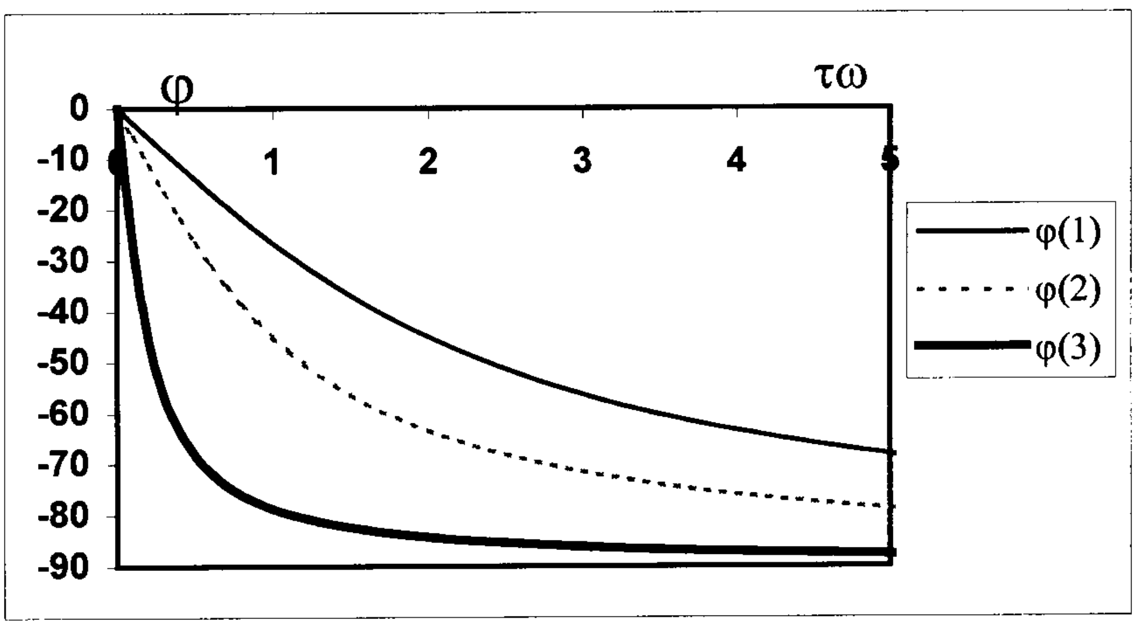
Steady-state output: $u_2 = A_2 \sin(\omega t + \varphi)$

$A = A_2/A_1$: Gain
T: Period
ν: Frequency
ω: Pulsation
φ: Phase shift
θ: Delay

$T = 1/\nu$ and $\omega = 2\pi\nu$

$\varphi = 2\pi\,\theta / T = \theta\, 2\pi\nu = \theta\,\omega$





First order frequency diagram

$\omega = 0$

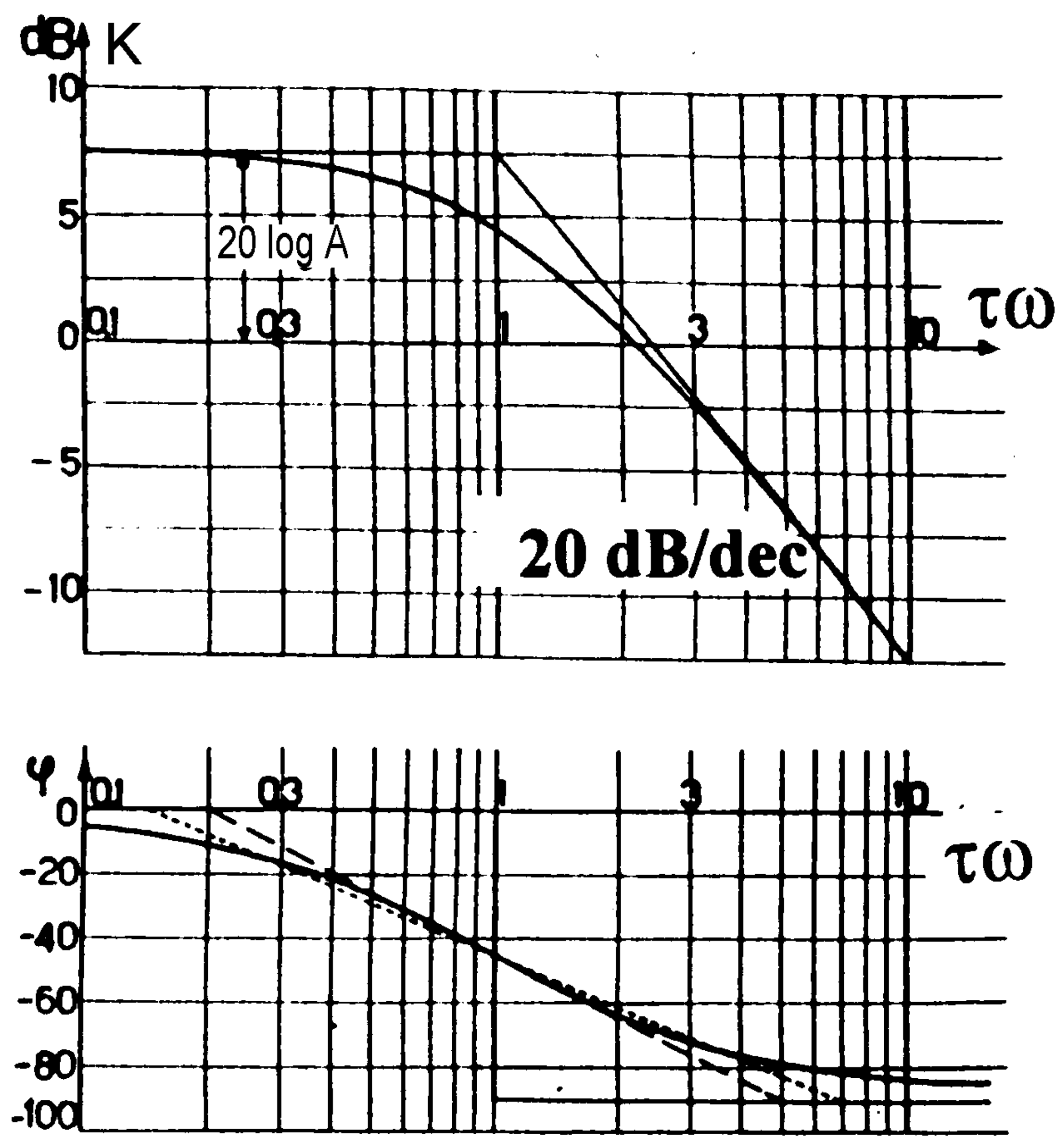
$\Rightarrow A = \alpha$

$\varphi = 0$

$\omega = \infty$

$\Rightarrow A = 0$

$\varphi = -90^{o} = -\pi/2$



Bode diagram
1st order system

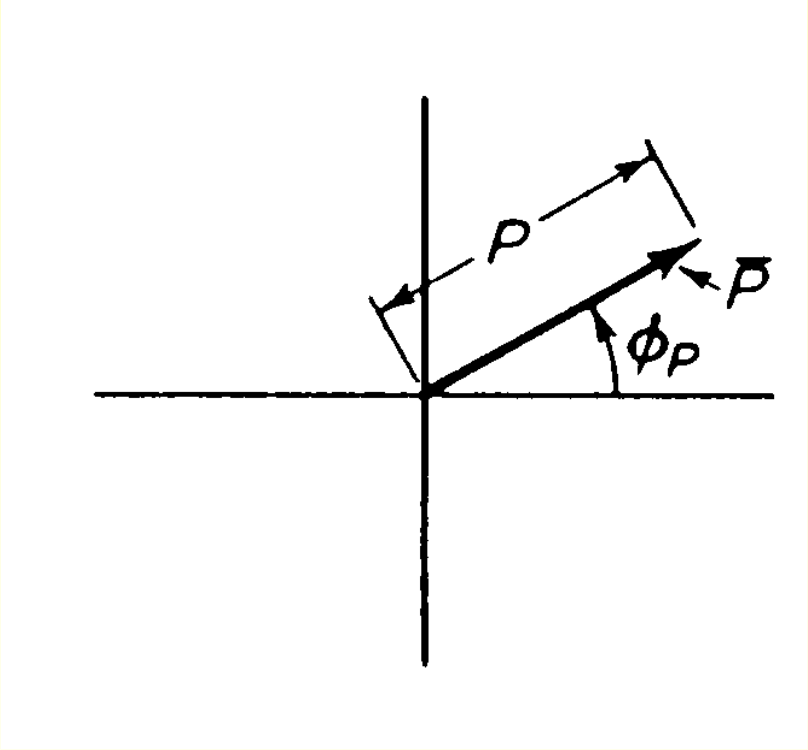
$$K = 20 \log \frac{\alpha}{\sqrt{(1 + \tau^2\omega^2)}}$$

$$\varphi = \mathrm{arctg}\,(-\,\tau\omega)$$

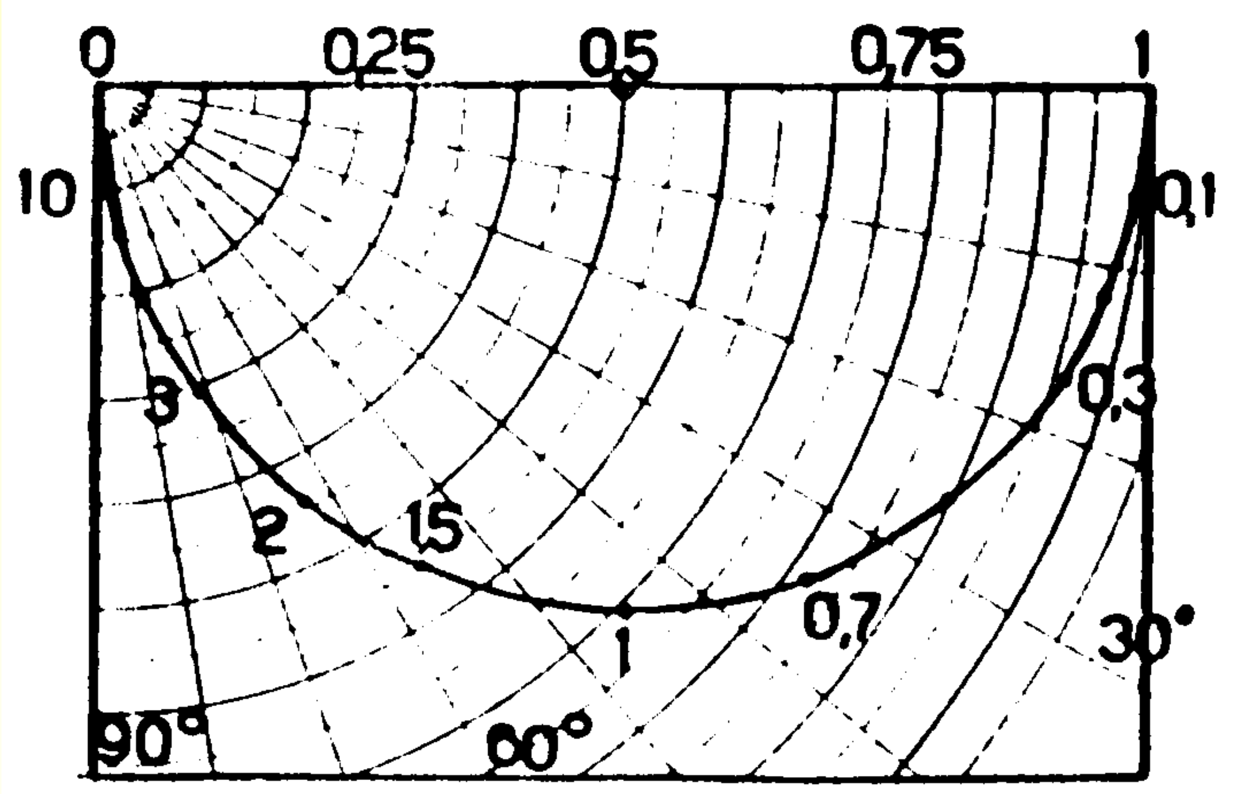
Slope = 20 dB/dec

φ_{max} = -90°

Phasor representation



Nyquist diagram first order system



$$A = \frac{\alpha}{\sqrt{(1 + tg^2\varphi)}}$$

$$A = \frac{\alpha \sqrt{\cos^2 \varphi}}{\sqrt{(\cos^{+2}\varphi + \sin^{+2}\varphi)}}$$

$$\mathbf{A = \alpha \cos \varphi}$$

$\omega = 0 \Rightarrow A = \alpha$

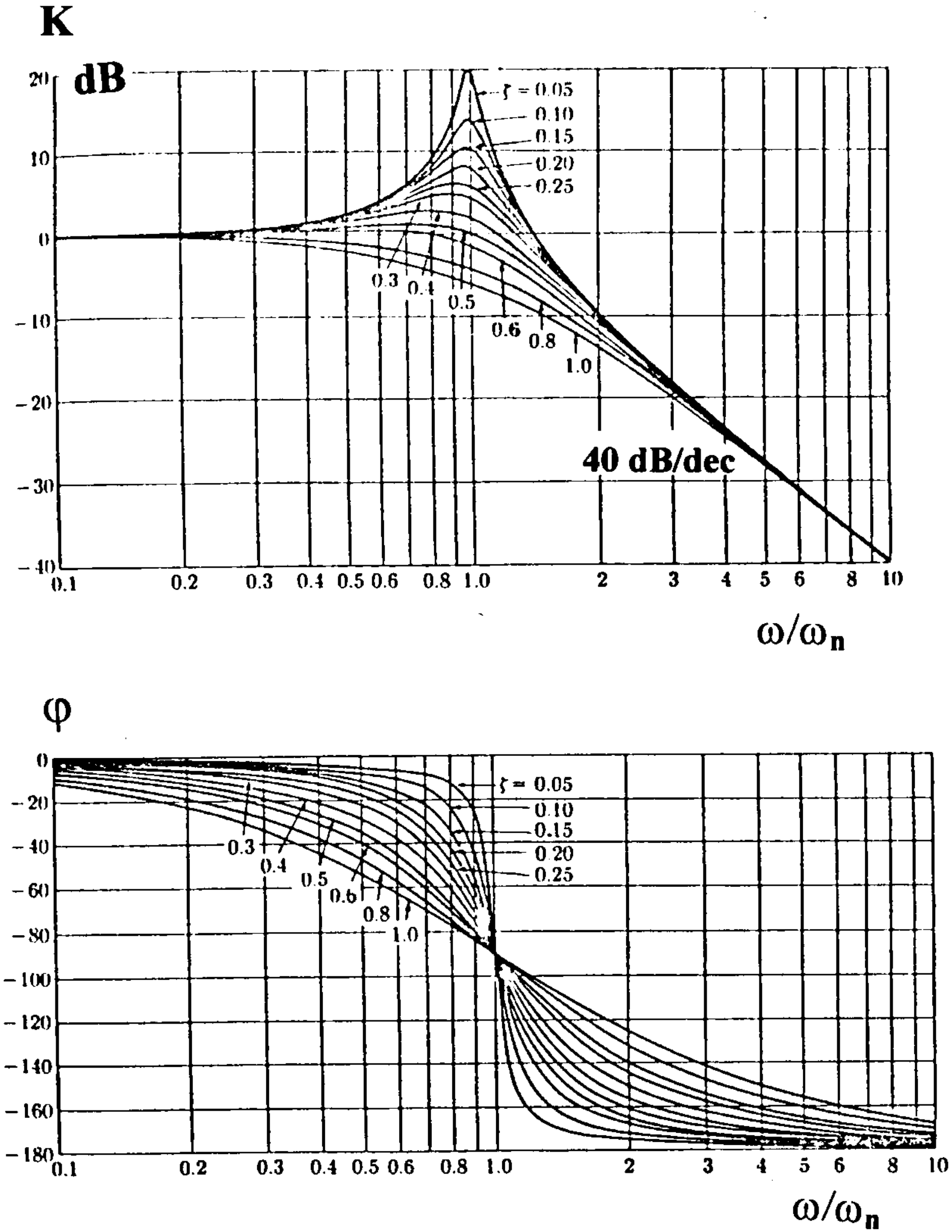
$\varphi = 0$

$\omega = \omega_b \Rightarrow A = \alpha/\sqrt{2}$

$\varphi = -\pi/4$

$\omega = \infty \Rightarrow A = 0$

$\varphi = -\pi/2$



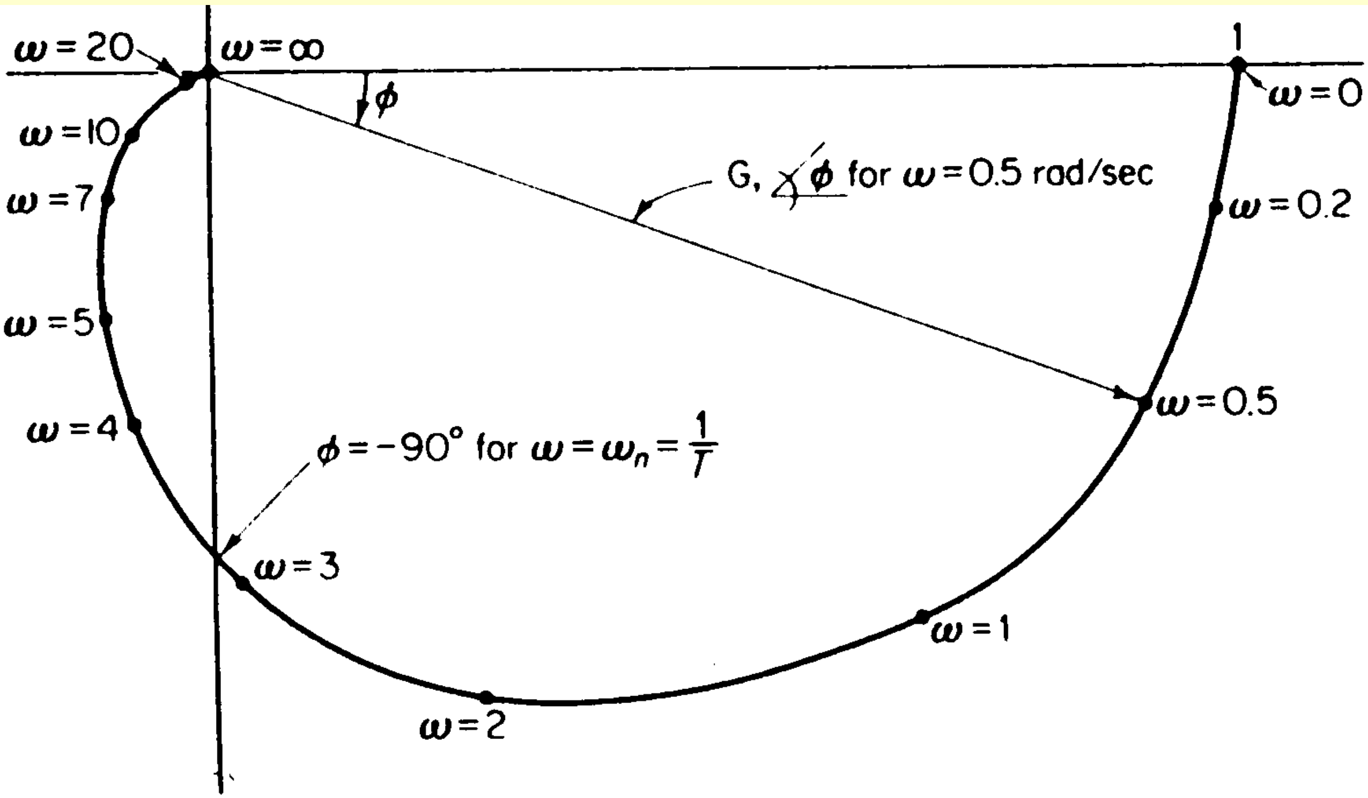
Bode diagram 2nd order system

$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=\alpha U$$

Slope = 40 dB/dec

φ_{max} = -180°

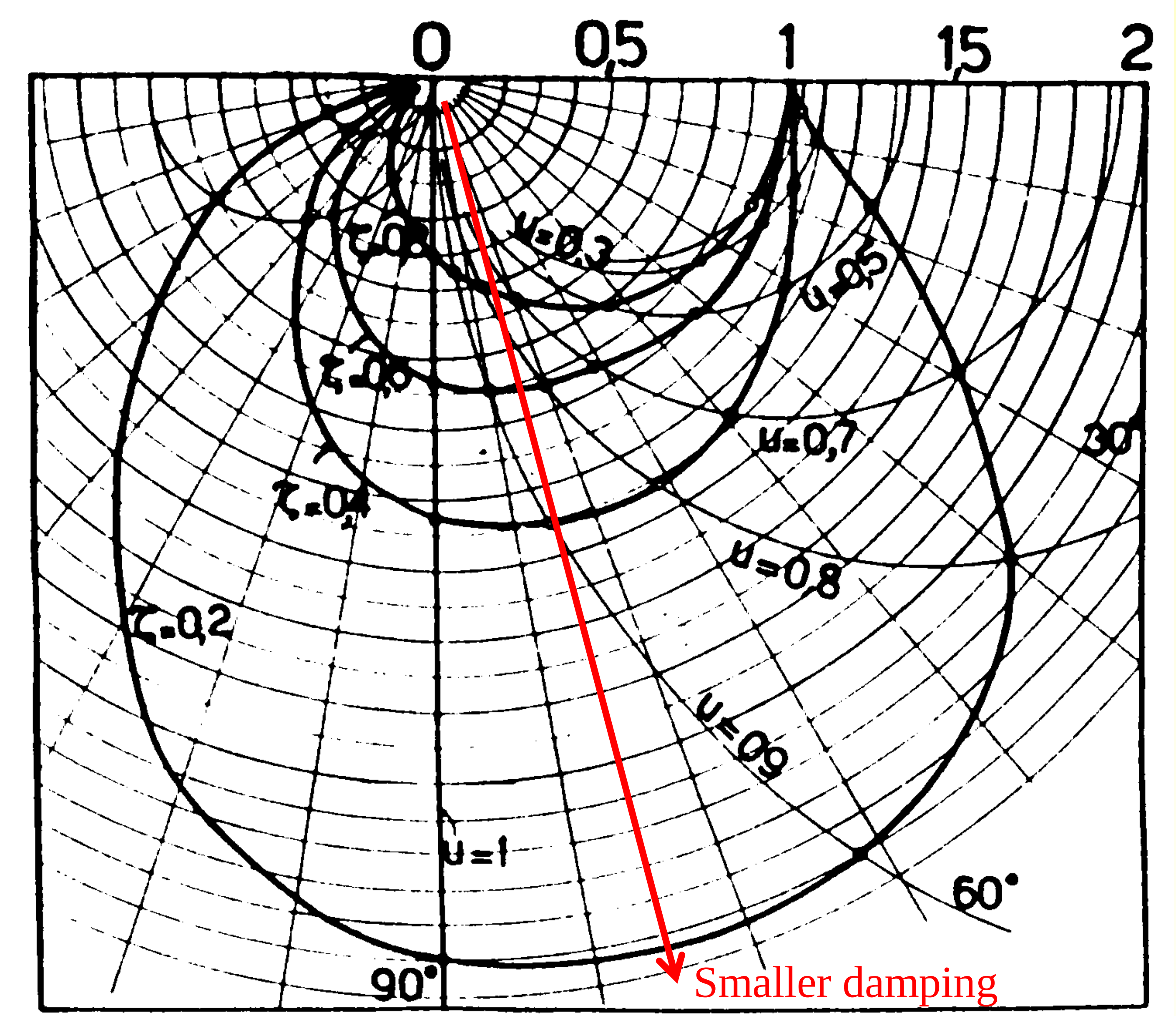
Nyquist diagram of second order system

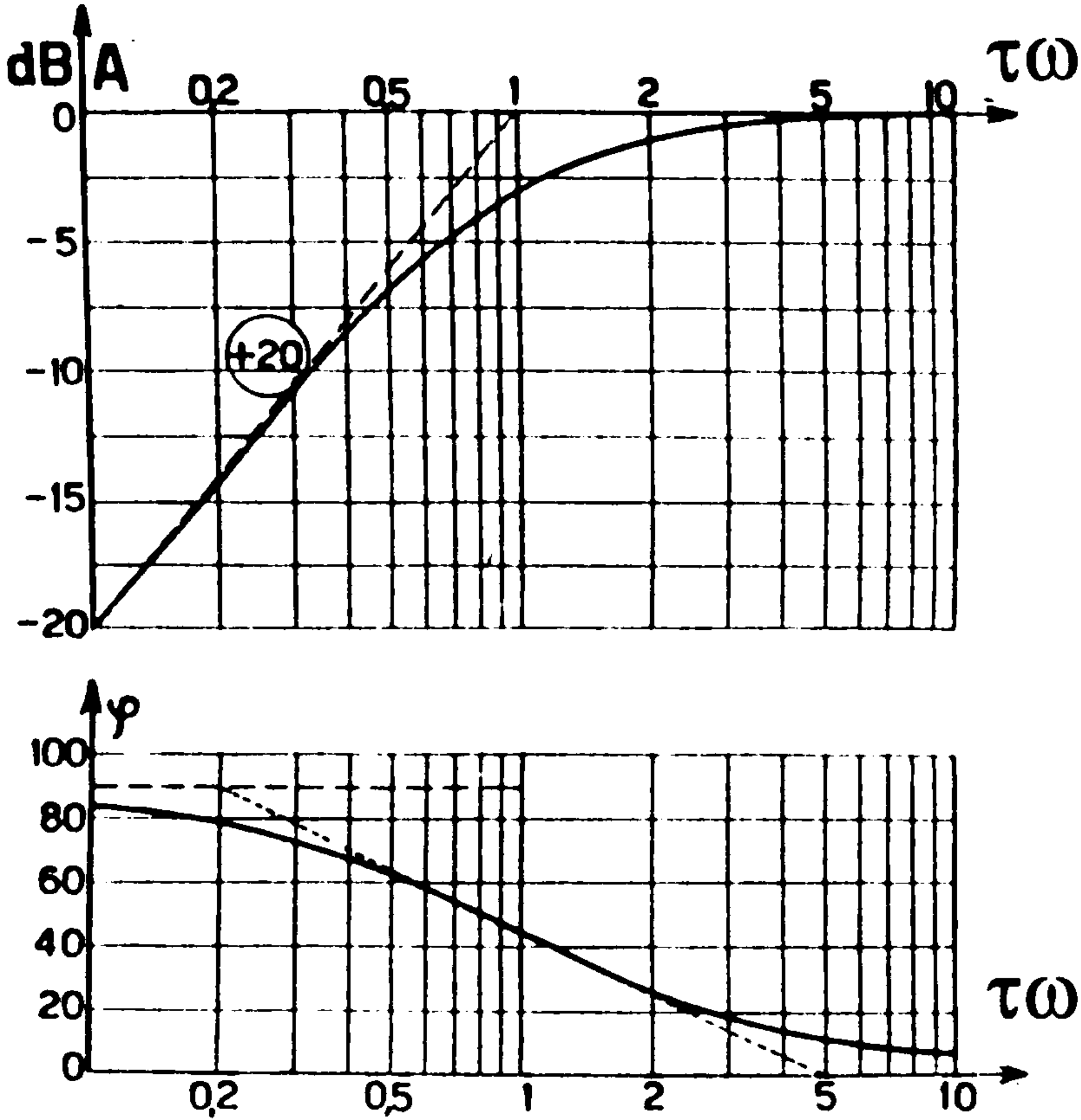


ω, rad/sec	0	0.2	0.5	1	2	3	4	5	7	10	20	100	∞
G	1	0.994	0.96	0.87	0.65	0.49	0.35	0.26	0.16	0.09	0.024	~ 0	0
ϕ, degrees	0	−8	−19.5	−38	−67	−87	−102	−113	−128	−142	−160	−176	−180

Frequency response of the system $\theta_o/\theta_i = 1/(1 + 0.7D + 0.1D^2)$

Nyquist diagram of second order system

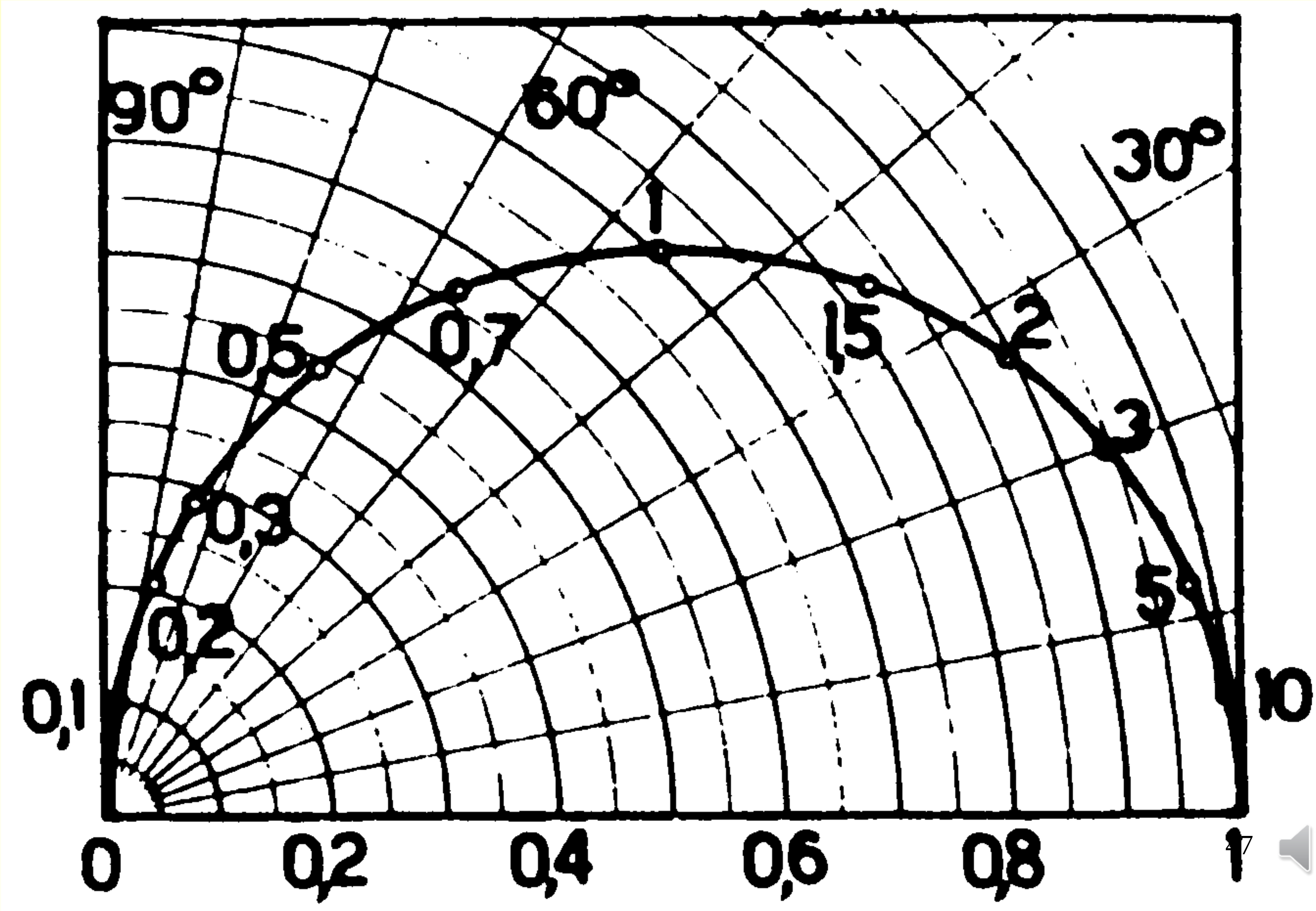




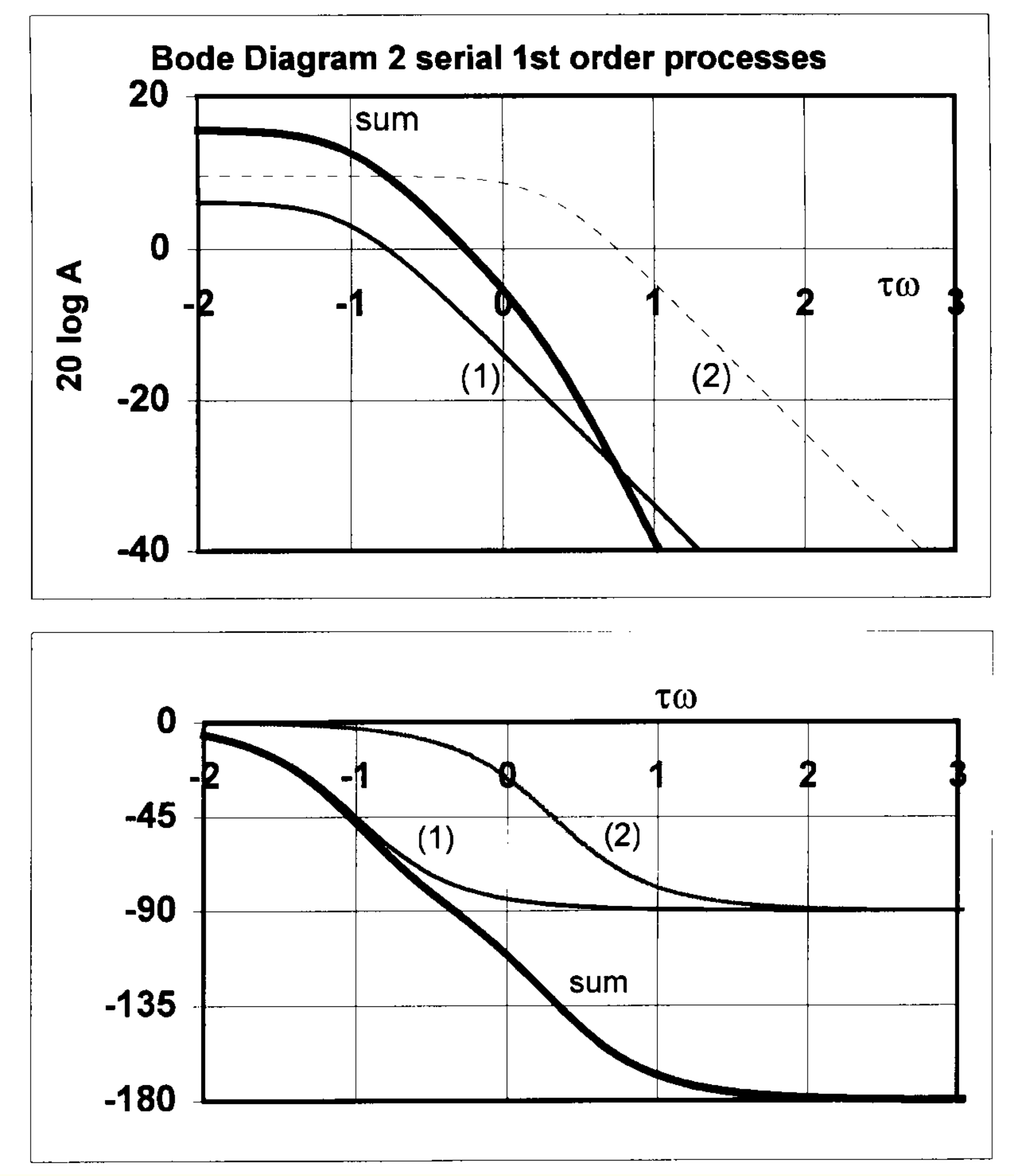
Bode diagram system order -1

$$\int u_2 dt = u_1$$

Nyquist diagram system of order -1



Bode diagram of 2 systems in series



$T = T_2\,T_1$

$A = A_{(1)} \,.\, A_{(2)}$

$\varphi = \varphi_{(1)} + \varphi_{(2)}$

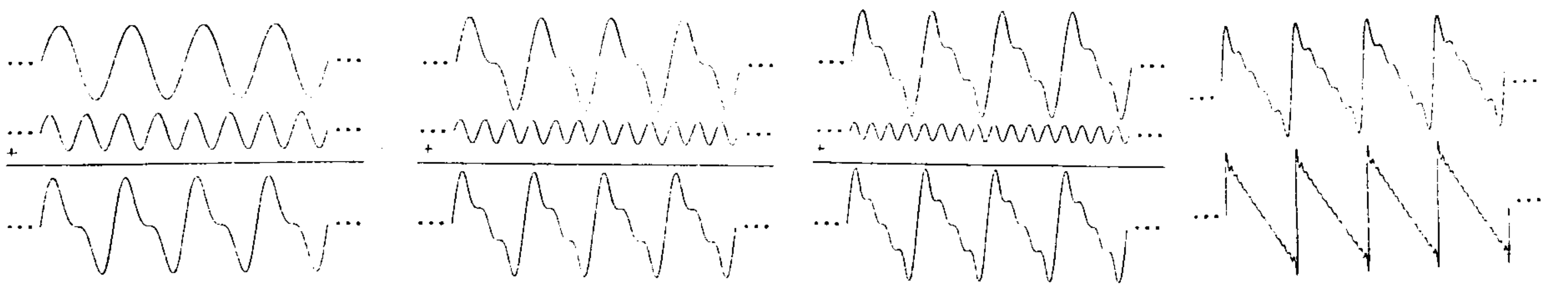
$K = 20 \log A$

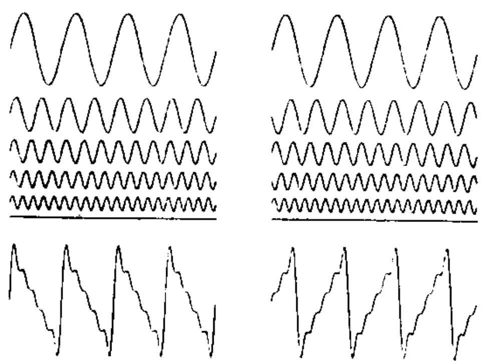
$K = 20 \log A_{(1)} \,.\, A_{(2)}$

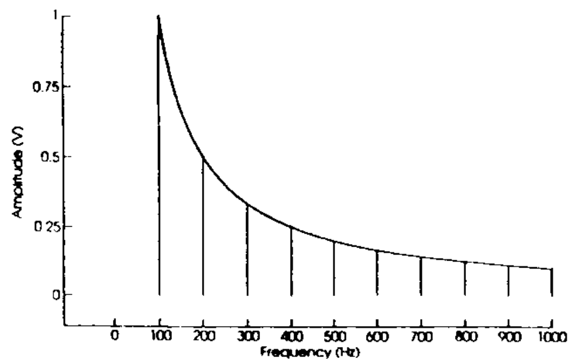
$K = 20 \log A_{(1)} + 20 \log A_{(2)}$

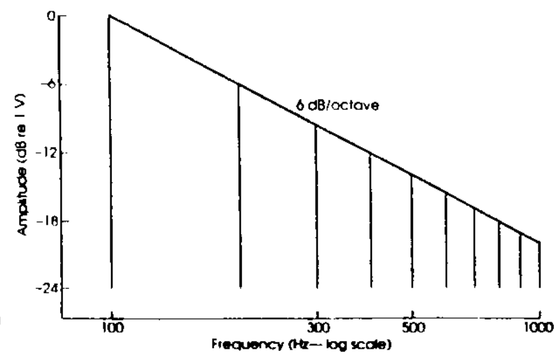
$K = K_{(1)} + K_{(2)}$

Fourier series

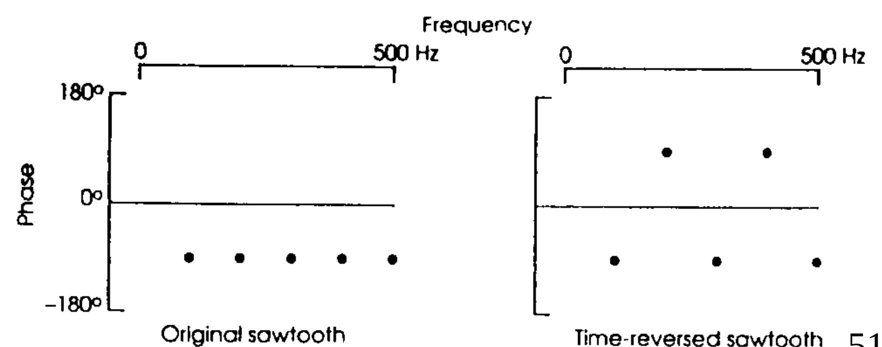




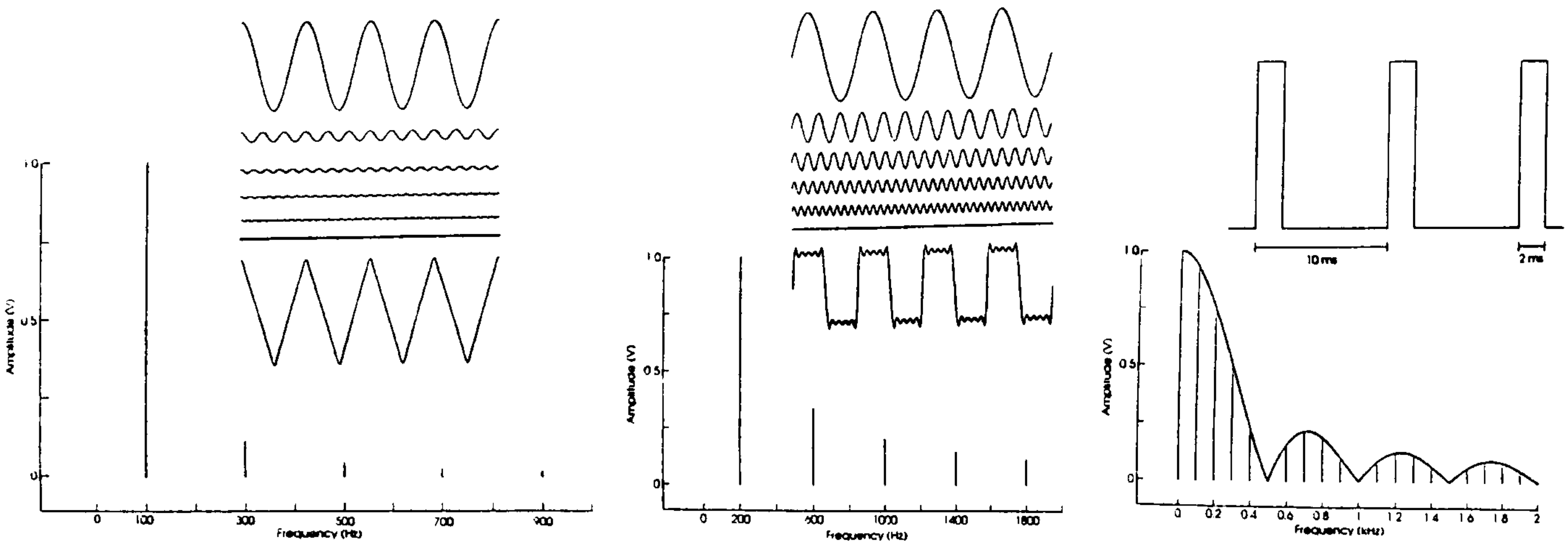




Frequency (Hz)	Amplitude (V)
100	1
200	1/2
300	1/3
400	1/4
500	1/5
600	1/6



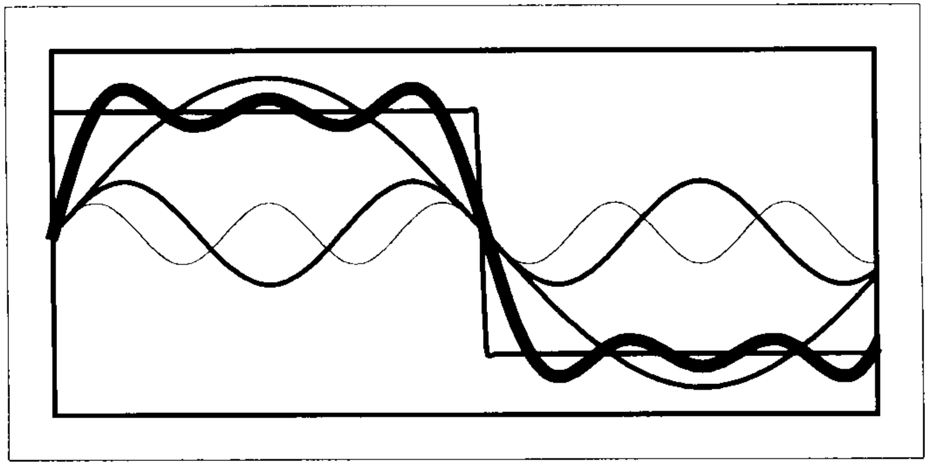
Examples of Fourier series

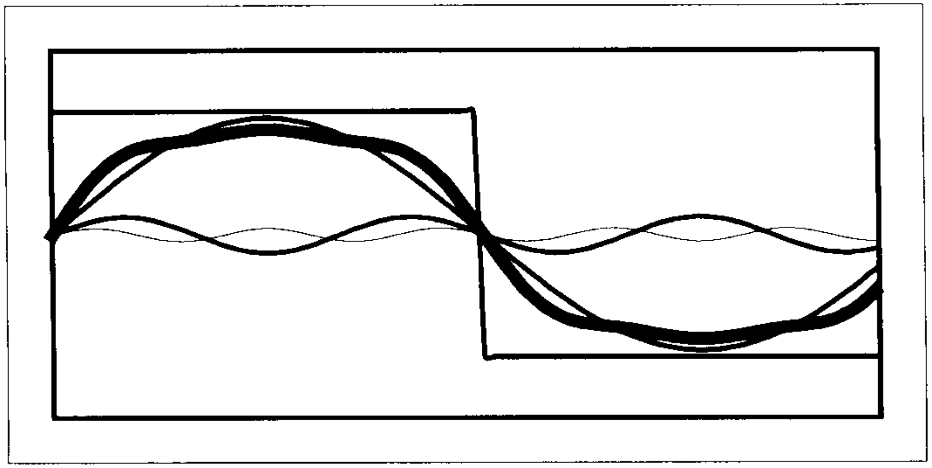


Fourier analysis of step response

Step input decomposed in sinus waves

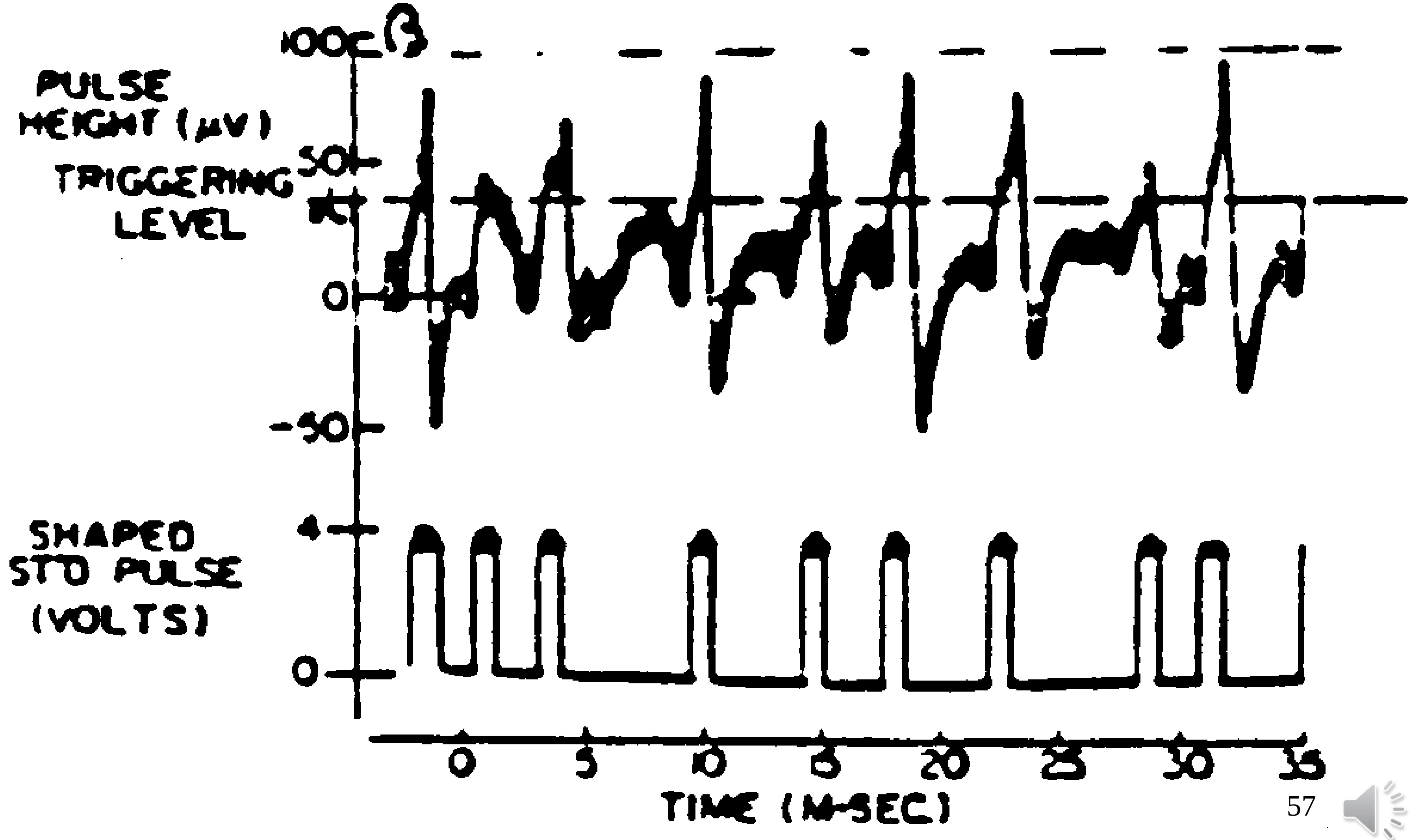
Output signal showing reduction of the amplitude for the high frequency components



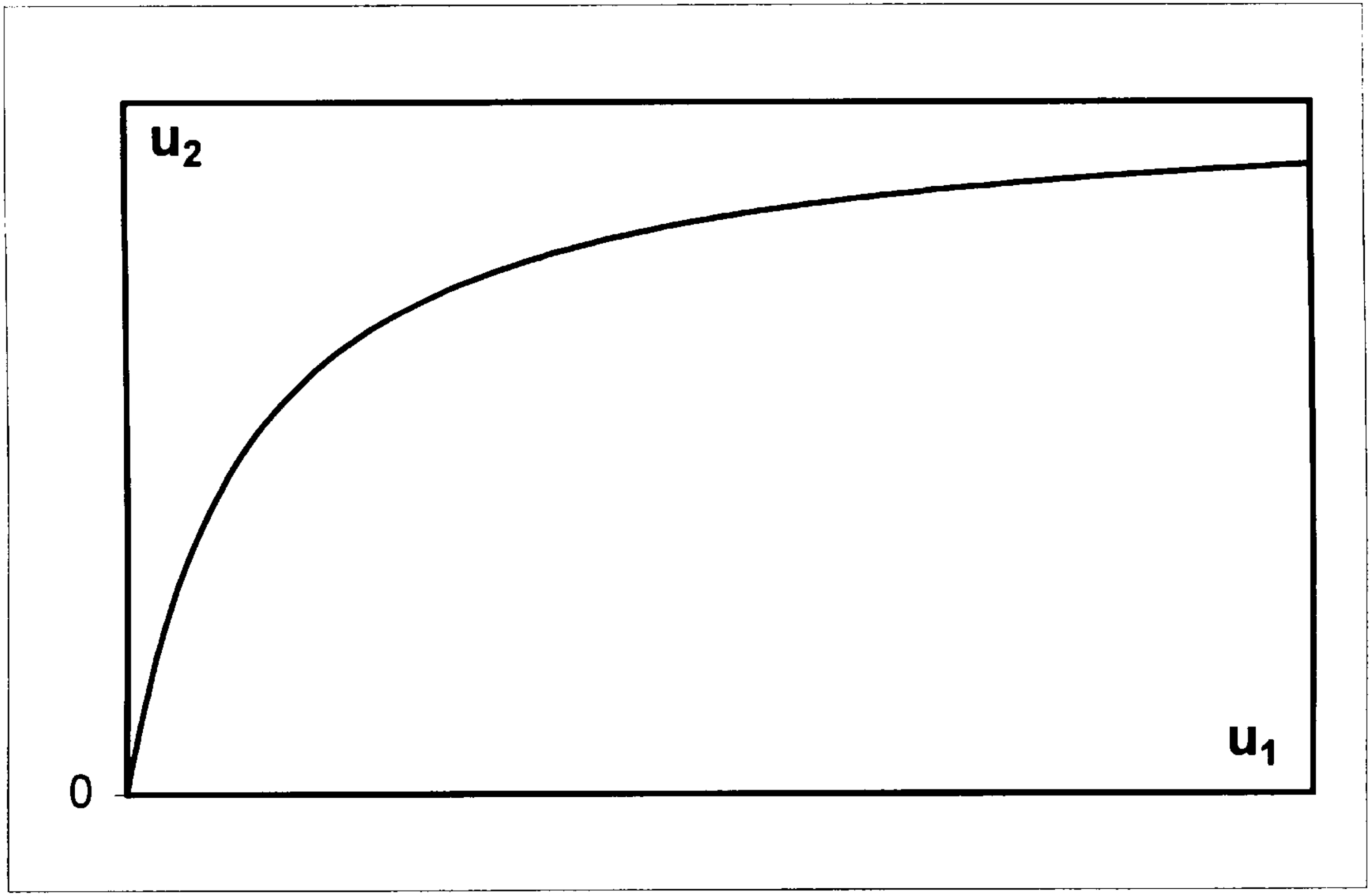


Example:
Photoreceptor ganglion
of the crayfish

Crayfish photoreceptor ganglion activity

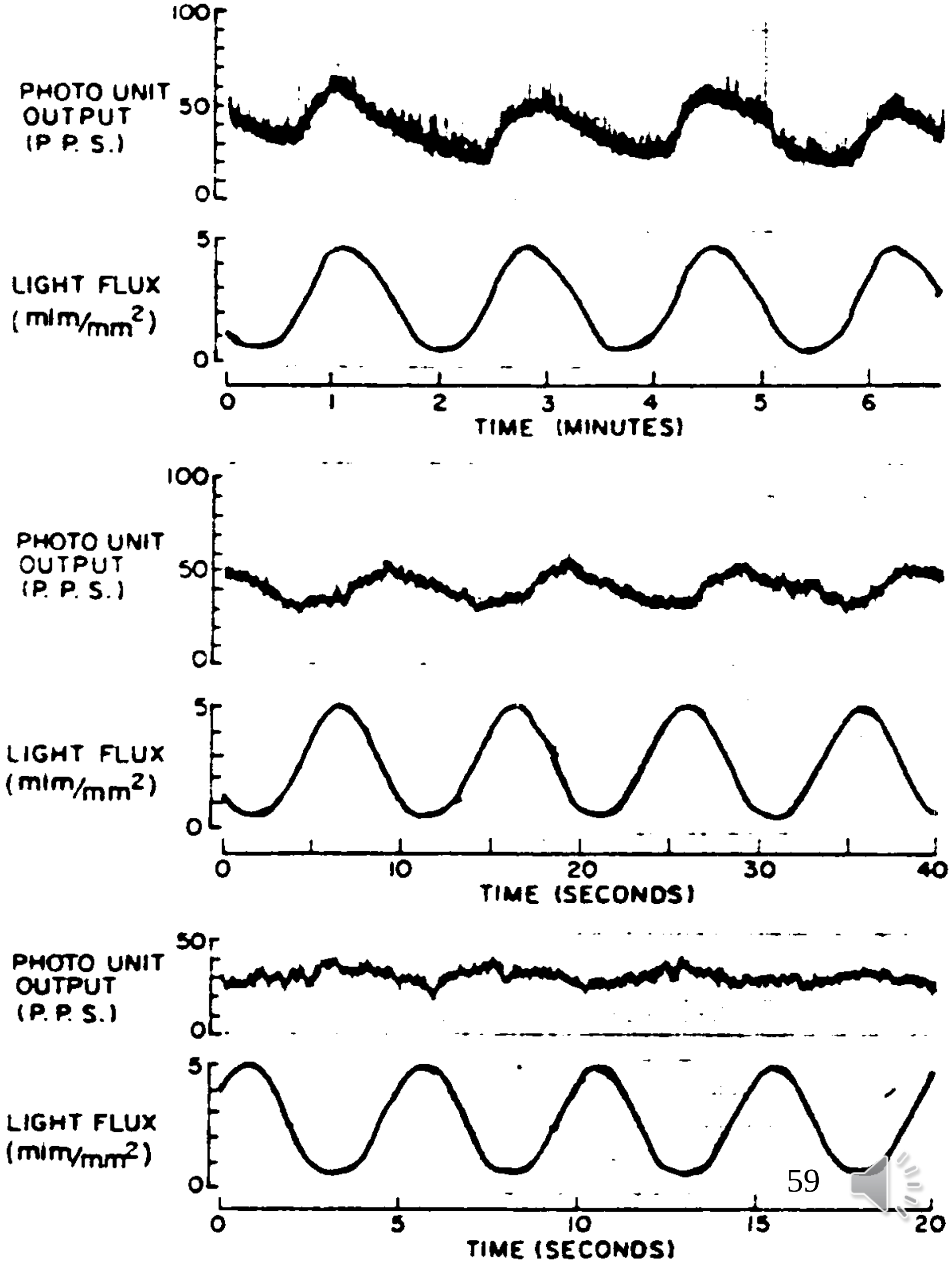


Crayfish photoreceptor ganglion
Steady-state transfer function

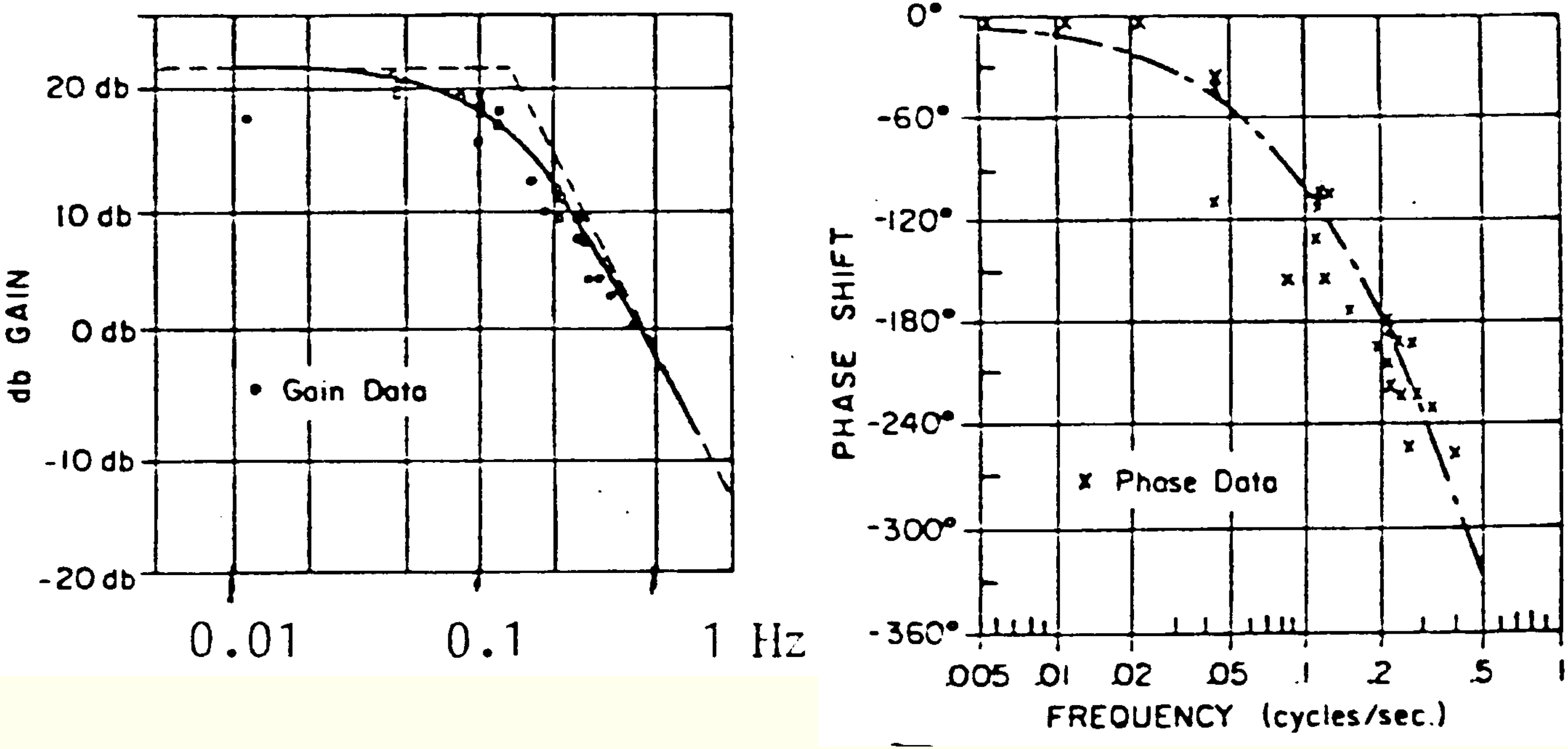


Crayfish photoreceptor ganglion

Response to sinus input



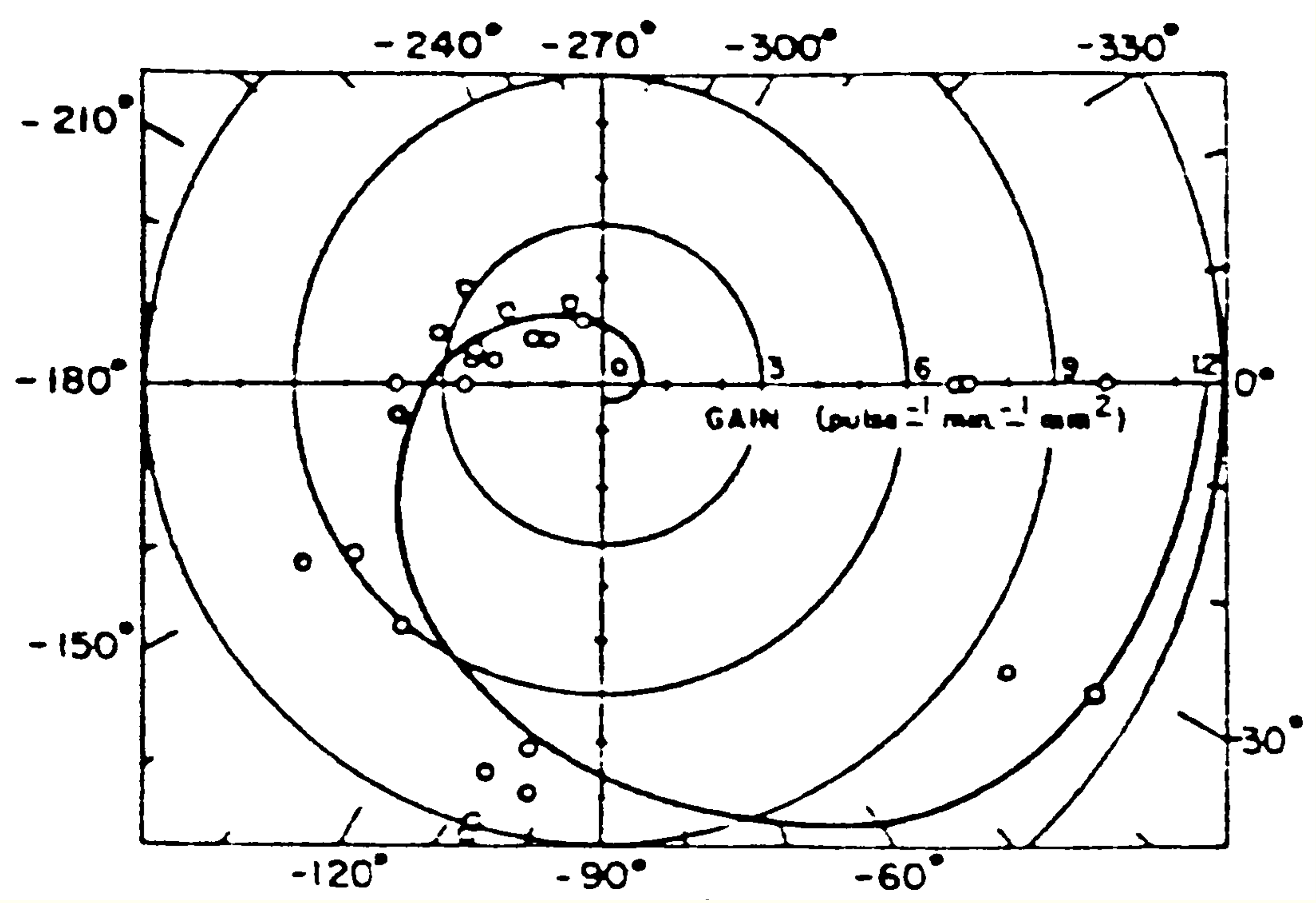
Crayfish photoreceptor ganglion
Bode diagram: example



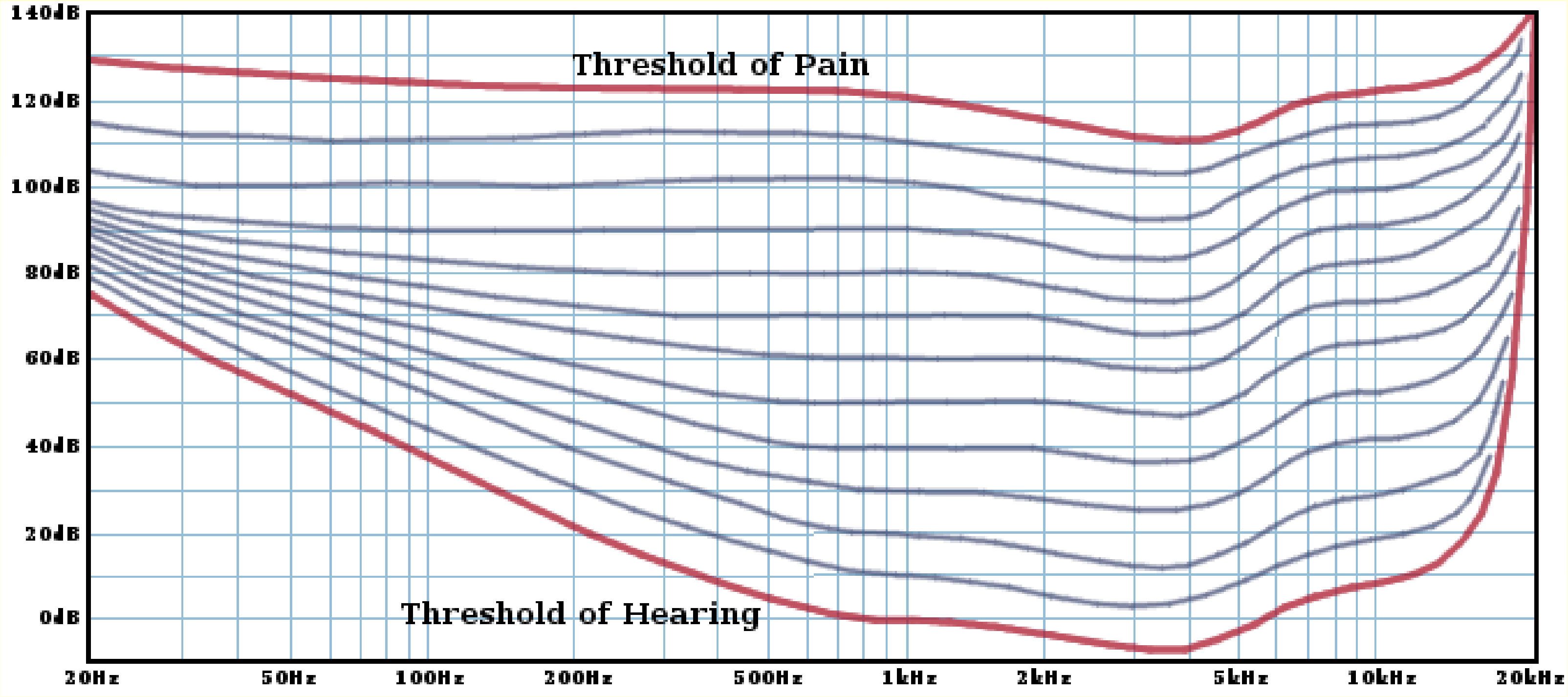
Analysis average results different expts: 20 dB/dec: First order

$K = 22$ dB, $\tau = 1.3$ s, $\theta = 0.5$ s

Crayfish photoreceptor ganglion Nyquist diagram



Human hearing frequency response



Human hearing frequency response