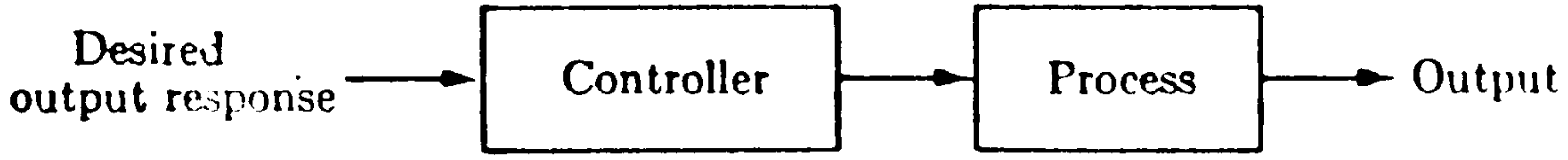
Closed loop systems

Process

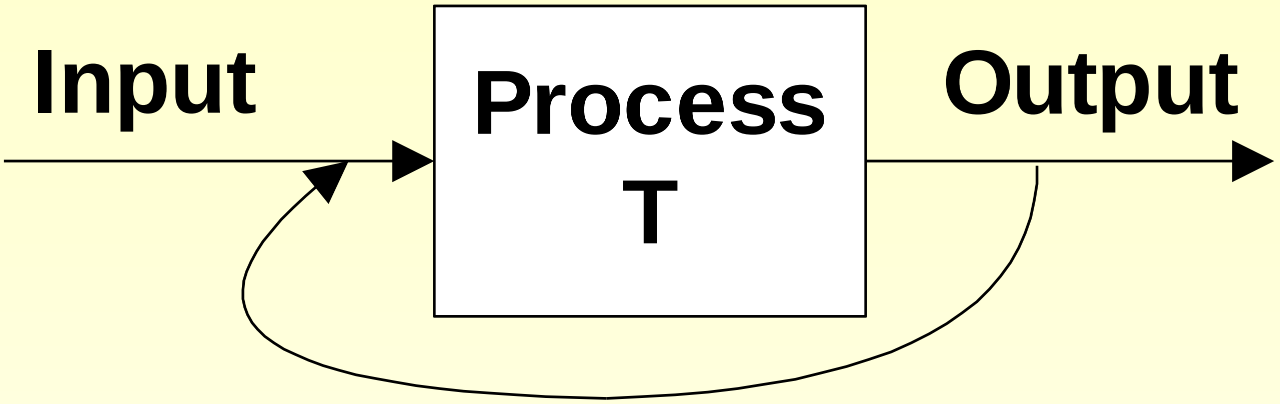
Process to be controlled.

Open loop control system



Open-loop control system.

Closed system



Negative feedback: stable levels

E.g., thermoregulation

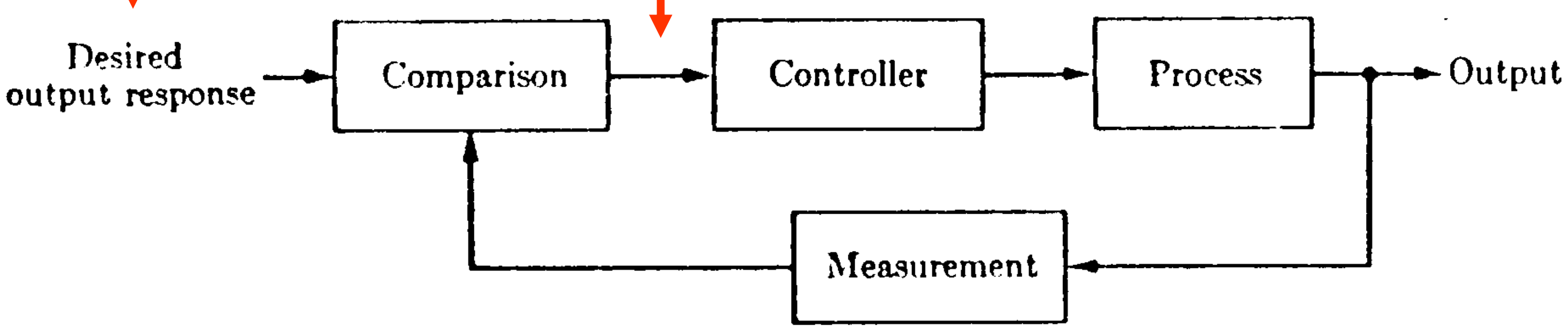
Positive feedback: on-off

E.g. action potential firing, cell division, cell death, blood coagulation, micturition

Closed loop feedback system

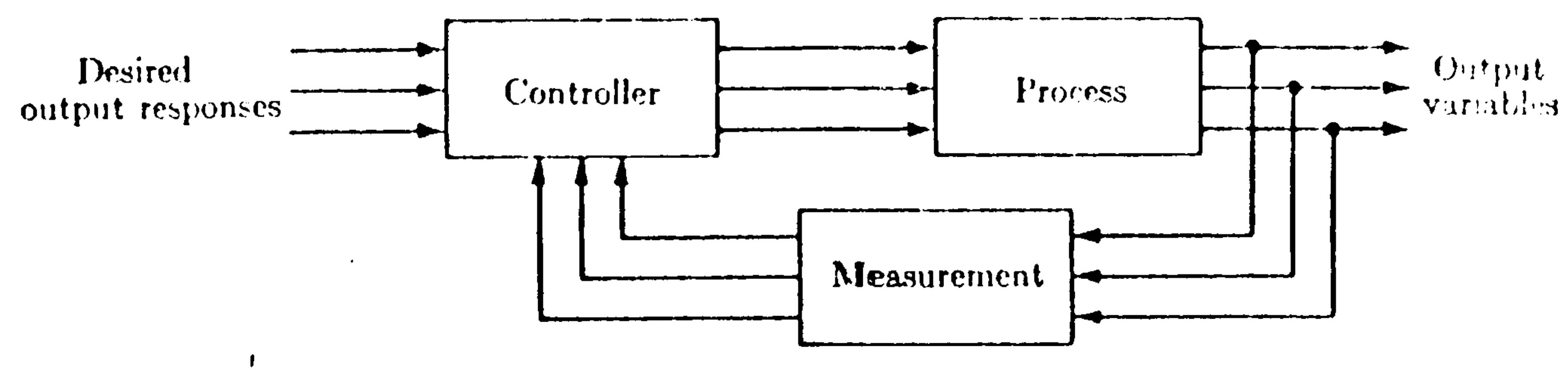
Reference signal
Command Signal
Input Signal

Difference:
Error Signal



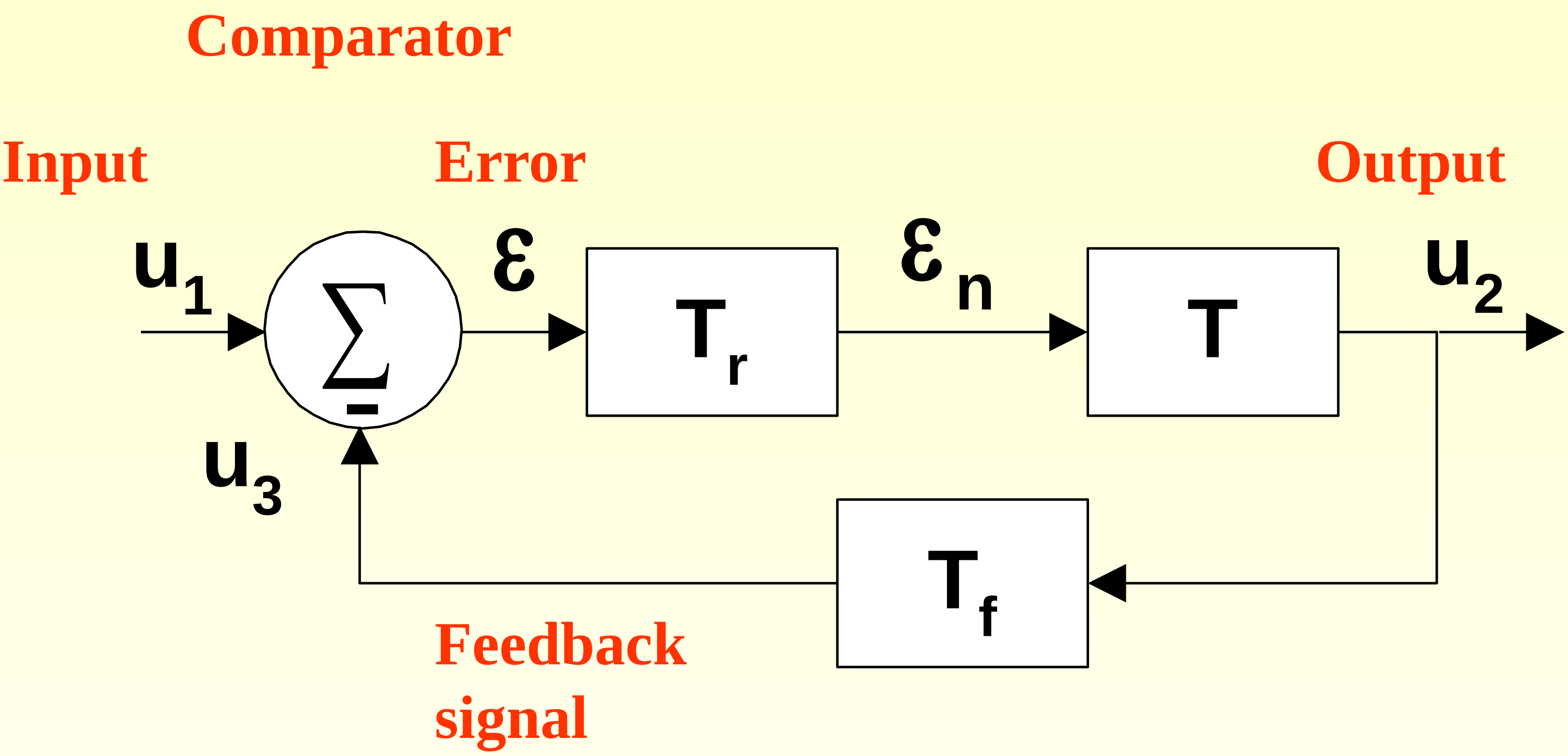
Closed-loop feedback control system.

Closed loop feedback system with multiple inputs and outputs



Multivariable control system.

Representation of a control system



Types of Regulators

- **Continuous**
- **Proportional (P)** $\varepsilon_n = k\varepsilon$
- **Differentiating (D)** $\varepsilon_n = k\dfrac{d\varepsilon}{dt}$
- **Integrating (I)** $\varepsilon_n = k\int \varepsilon dt$
- **Combinations (PD, PI, PID)**

Types of Regulators

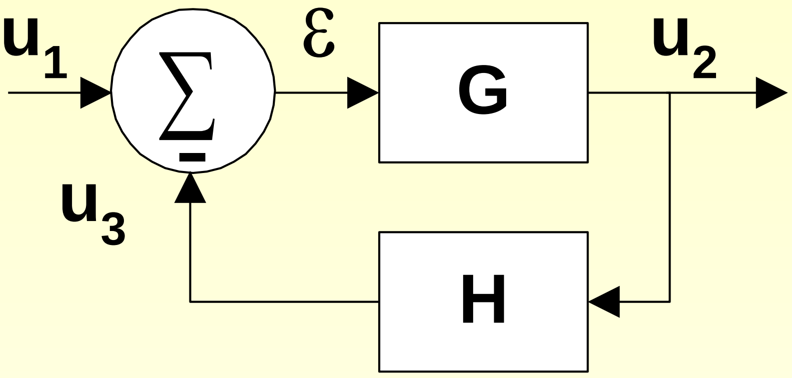
- **Discontinuous**

- **Two or more states:** **Can have only a finite number of different values dependent on the value of the error. E.g. thermostat.**

- **Constant velocity:** **$d\varepsilon_n/dt$ can only have two different values: 0 or 1**

Properties of control systems

$$u_2 = G\varepsilon$$



$$\varepsilon = u_1 - u_3$$

$$u_3 = \mathrm{H}\, u_2$$

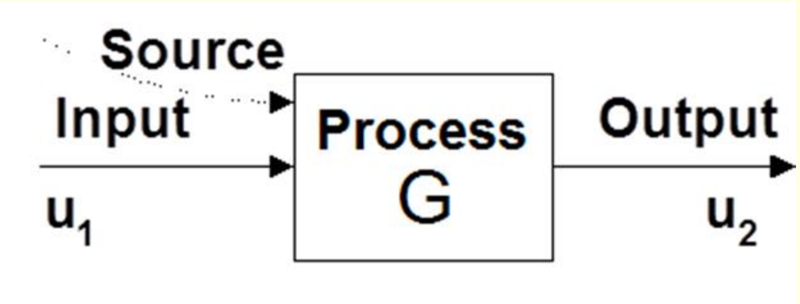
$$\varepsilon = \frac{1}{1+GH} u_1$$

$$u_2 = \frac{G}{1+GH} u_1$$

The gain of a closed system is smaller than that of the open system

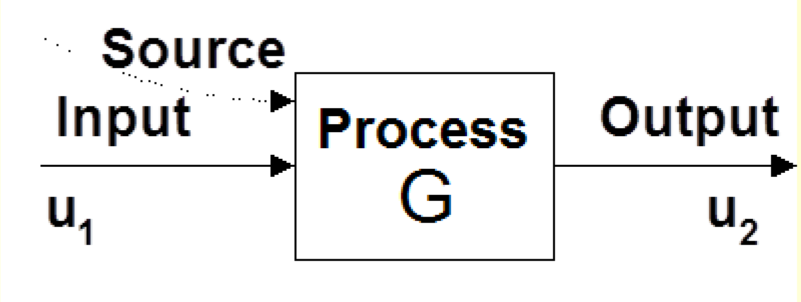
Sensitivity analysis
Definition of sensitivity

$$S_x = \frac{\frac{\delta u_2}{u_2}}{\frac{\delta x}{x}}$$



Sensitivity of open system to G

$$u_2 = Gu_1 \rightarrow \ \delta u_2 = \delta G u_1$$



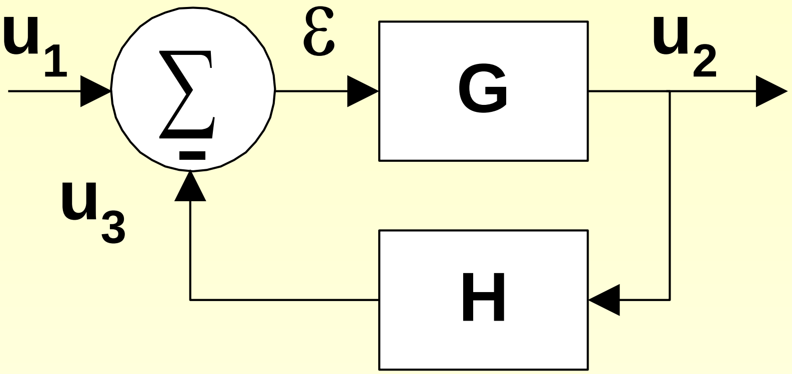
$$\frac{\delta u_2}{u_2} = \frac{\delta G u_1}{u_2} = \frac{\delta G u_1}{G u_1} = \frac{\delta G}{G}$$

$$S_G = \frac{\frac{\delta u_2}{u_2}}{\frac{\delta G}{G}} = 1$$

High sensitivity to variations in the process G

Sensitivity of closed system to G

$$u_2 = \frac{G}{1+GH} u_1$$



$$\delta u_2 = \frac{\delta G(1+GH) - GH\delta G}{(1+GH)^2} u_1$$

$$\delta u_2 = \frac{\delta G}{(1+GH)^2} u_1$$

$$\frac{\delta u_2}{u_2} = \frac{1}{1+GH} \frac{\delta G}{G}$$

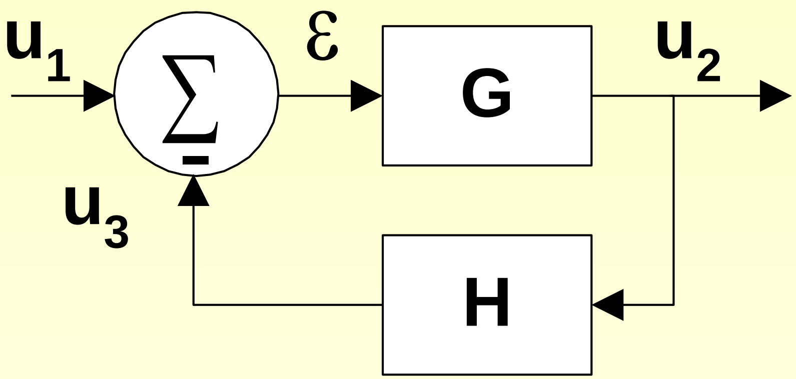
$$S_G = \frac{1}{1+GH}$$

for GH >> 1 $\Rightarrow$ $S_G \cong 1/GH$

Low sensitivity to variations in the process G

Sensitivity of closed system to H

$$u_2 = \frac{G}{1+GH} u_1$$



$$\delta u_2 = \frac{-G^2 \delta H}{(1+GH)^2} u_1$$

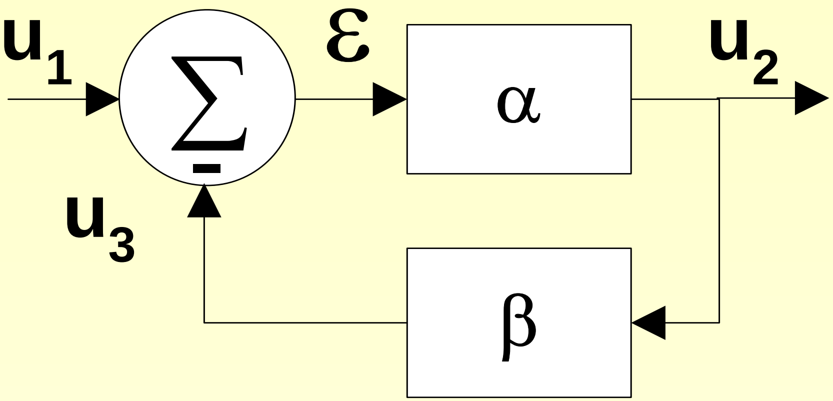
$$\frac{\delta u_2}{u_2} = \frac{-G}{1+GH} \delta H = \frac{-GH}{1+GH} \frac{\delta H}{H}$$

$$S_H = \frac{-GH}{1+GH}$$

for GH >> 1 $\Rightarrow$ $S_H \cong -1$

High sensitivity to variations in H, but this process is typically a transmission line and can be made simple to reduce errors

Proportional feedback.
Zero order process



$$u_2 = \frac{\alpha}{1+\alpha\beta} u_1$$

$$S_\alpha = \frac{1}{1+\alpha\beta}$$

For β = 1: Follower system.

$u_2 = u_1/(1 + 1/\alpha)$

$\alpha \rightarrow \infty \Rightarrow u_2 \rightarrow u_1$

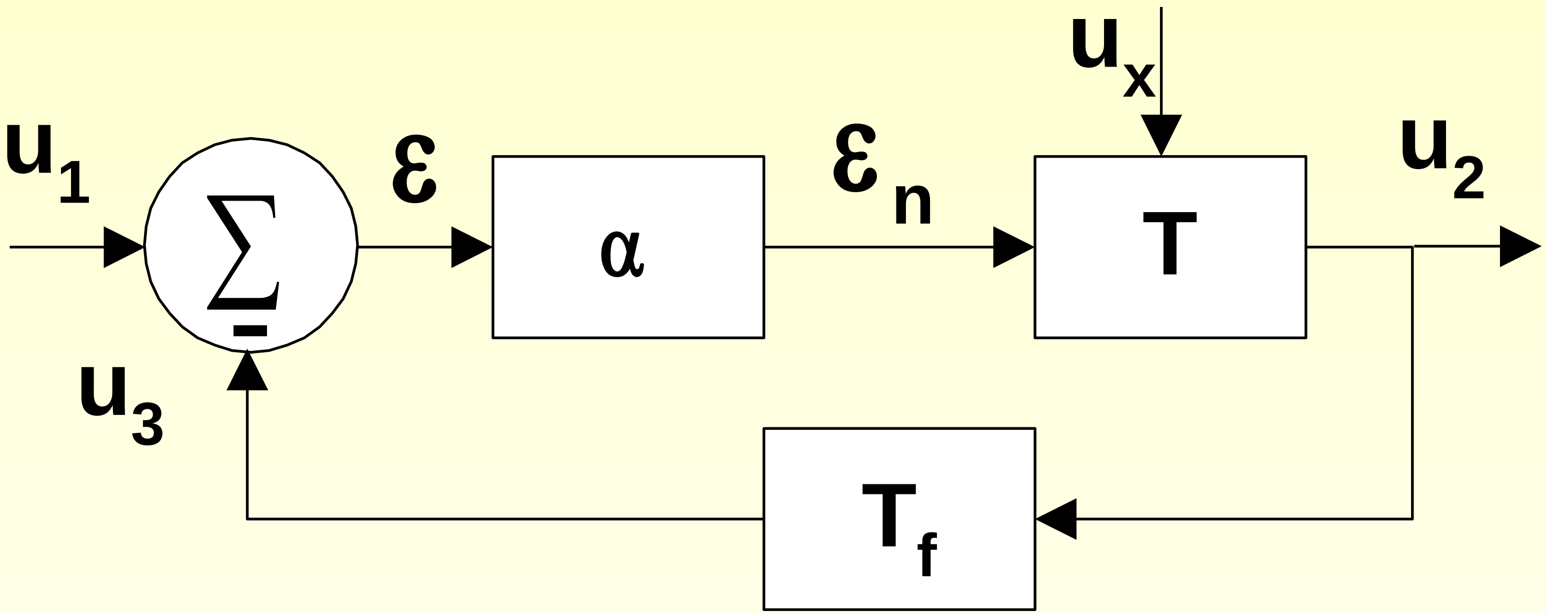
Steady state error:

$\Delta = (u_2 - u_1)/ u_1 = -1/(1 + \alpha)$

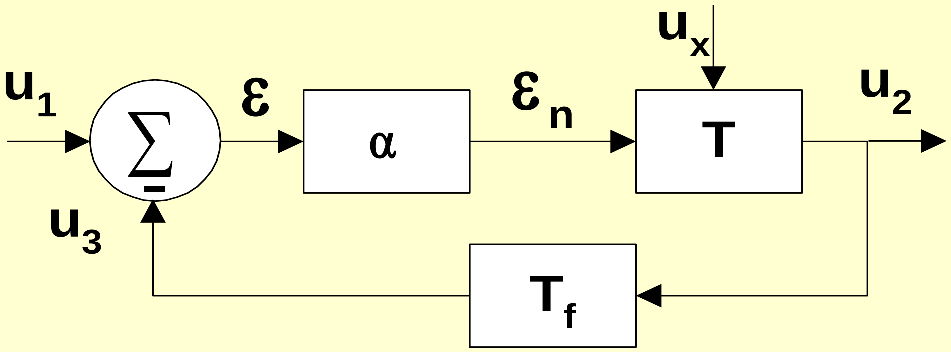
Approximation:

$\Delta \cong -1/\alpha$ for $\alpha >> 1$

First order process
(with disturbance)



First order process (with disturbance)



Open system (1)

$$\tau\frac{du_2}{dt} + u_2 = u_1 + u_x$$

Open system with amplification (2)

$$\tau\frac{du_2}{dt} + u_2 = \alpha u_1 + u_x$$

Closed system (3)

$$\tau\frac{du_2}{dt} + u_2 = \varepsilon_n + u_x$$

$$\varepsilon_n = \alpha\varepsilon \qquad \varepsilon = u_1 - u_3$$

$$\frac{\tau}{1+\alpha}\frac{du_2}{dt} + u_2 = \frac{\alpha}{1+\alpha}u_1 + \frac{1}{1+\alpha}u_x$$

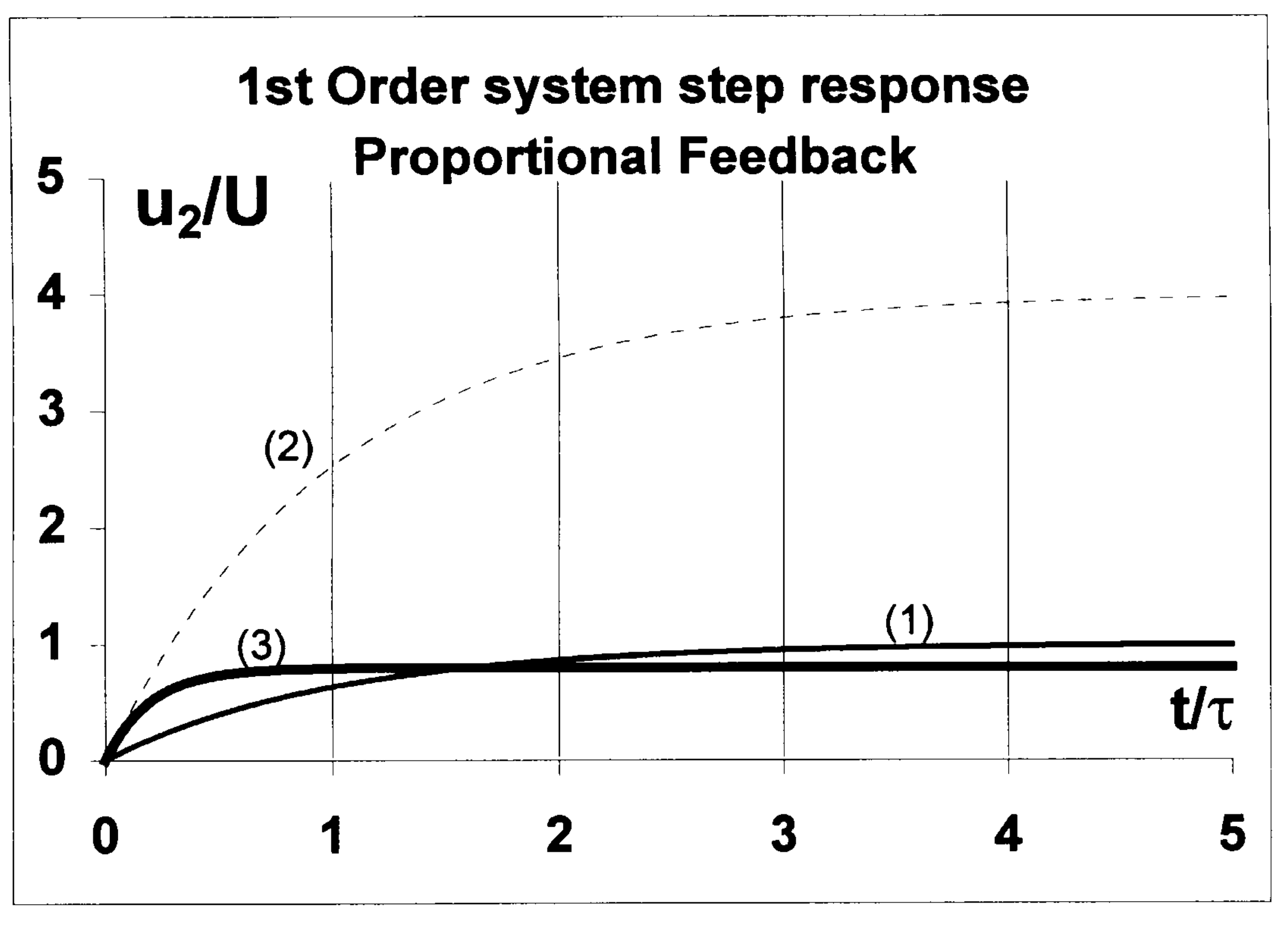
First order process (with disturbance)

$$\tau \frac{du_2}{dt} + u_2 = u_1 + u_x \qquad \frac{\tau}{1+\alpha}\frac{du_2}{dt} + u_2 = \frac{\alpha}{1+\alpha}u_1 + \frac{1}{1+\alpha}u_x$$

$$\tau \frac{du_2}{dt} + u_2 = \alpha u_1 + u_x$$

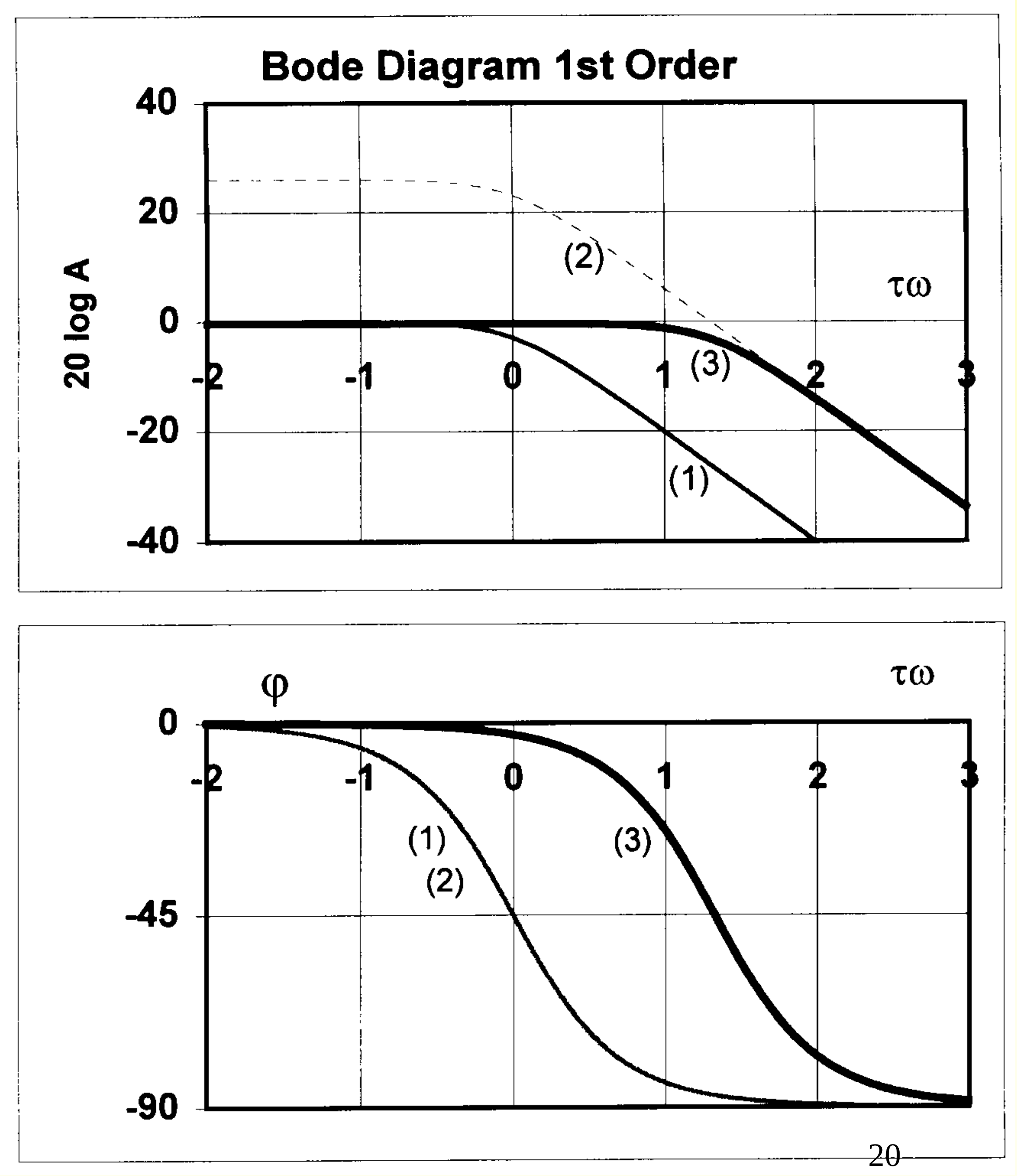
	System 1	System 2	System 3
Time constant τ'	τ	τ	$\tau/(1+\alpha)$
Gain α'	1	α	$\alpha/(1+\alpha)$
Gain of disturbance α'_x	1	1	$1/(1+\alpha)$

First order process

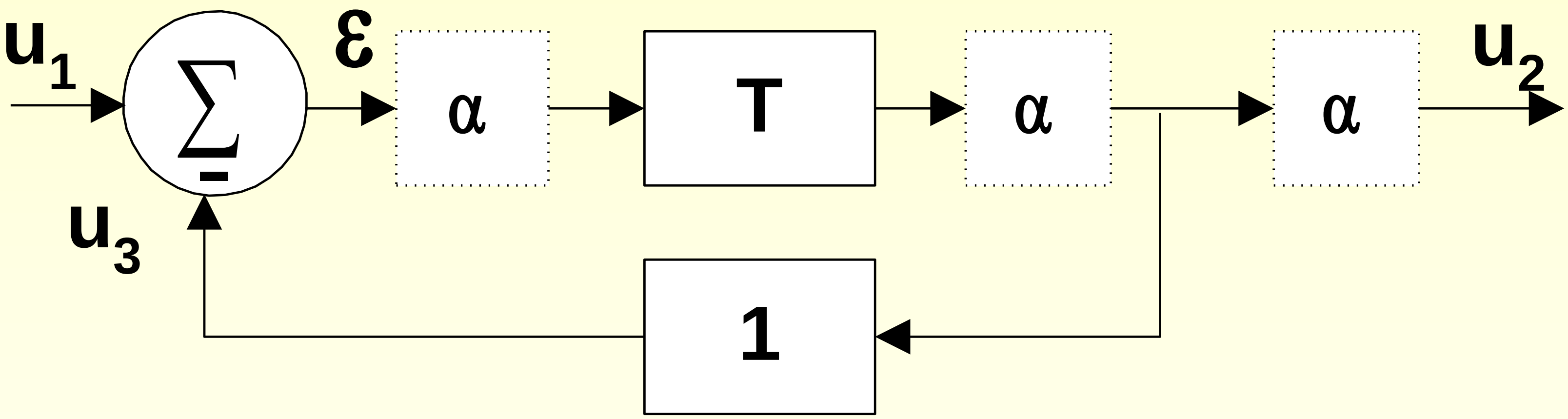


1 Open **2 Open with extra gain** **3 With unit feedback**

Bode diagram first order system



Extra amplification between comparator and regulator to compensate for the reduced gain



Second order process

Open system:

$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=u_1$$

Closed system (with extra gain):

$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=\alpha\varepsilon$$

$\varepsilon = u_1 - u_2$ (we assume unity gain feedback):

$$\frac{1}{(1+\alpha)(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{(1+\alpha)\omega_n}\frac{du_2}{dt}+u_2=\frac{\alpha}{1+\alpha}u_1$$

2^{nd} order process with unit feedback Properties

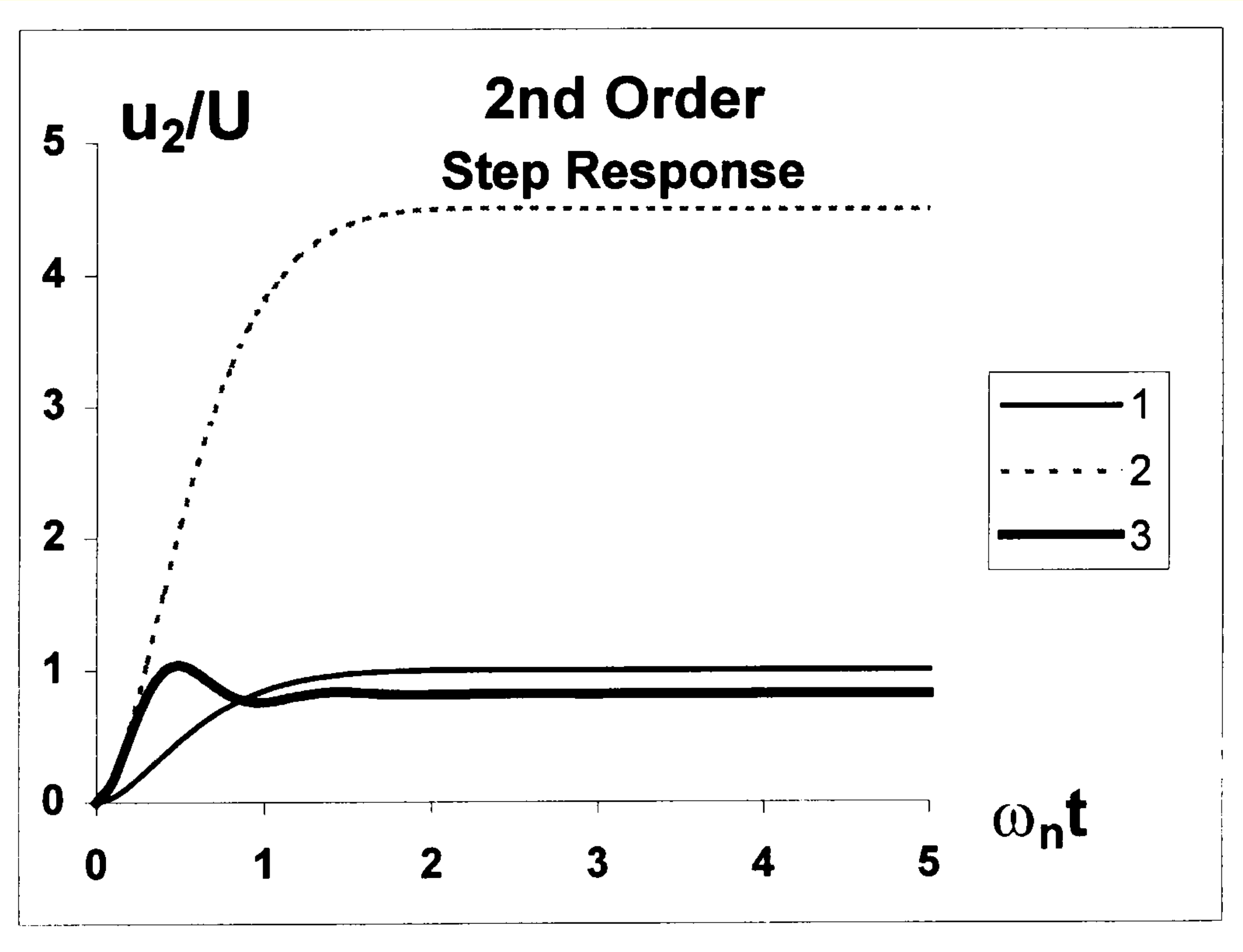
$$\omega'_n = \omega_n . \sqrt{(1+\alpha)}$$

$$\xi' = \xi / \sqrt{(1+\alpha)}$$

$$\alpha' = \alpha / (1+\alpha)$$

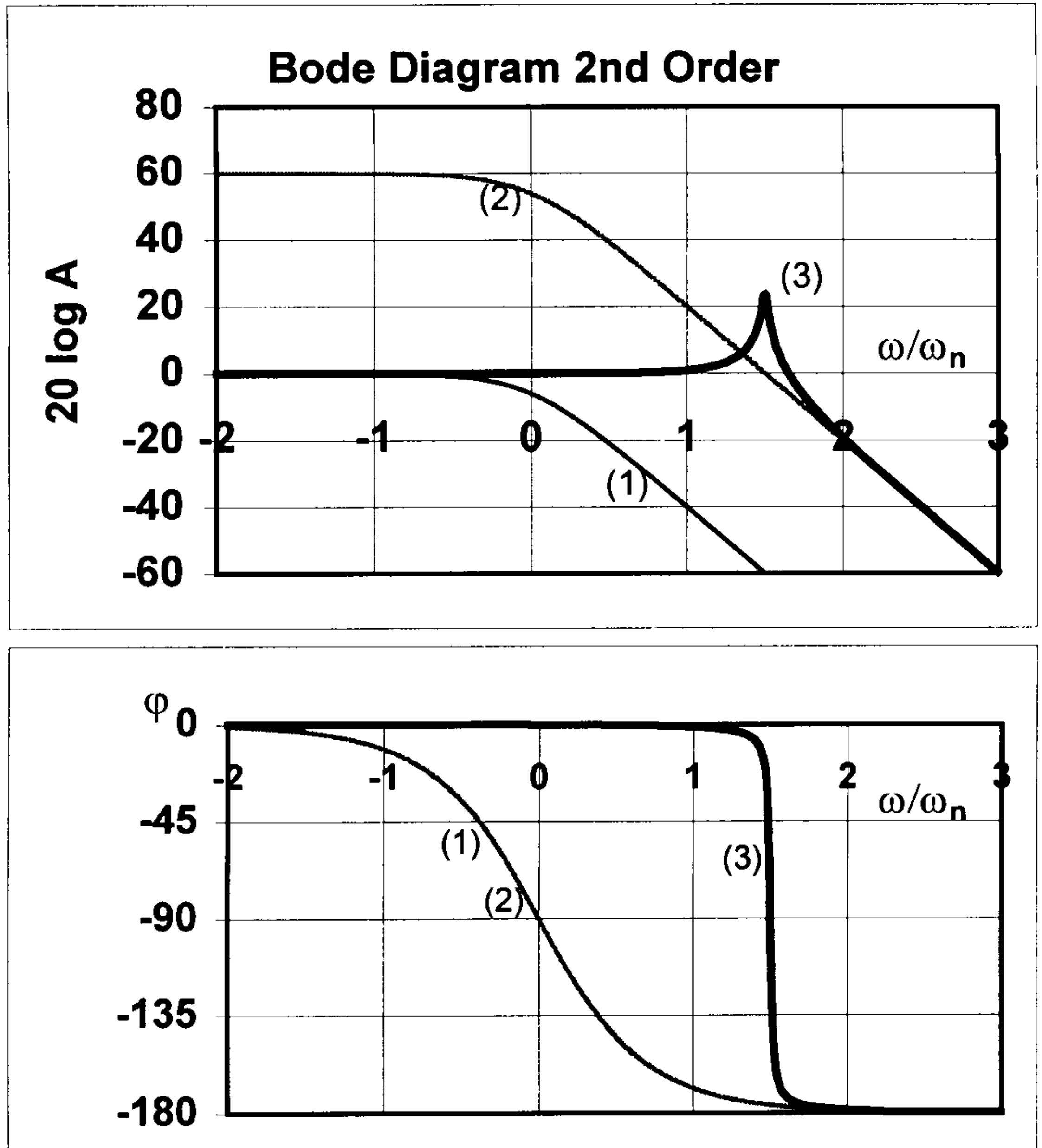
When α is high the damping factor decreases and there is risk of undamped oscillations

2nd order feedback process Step response

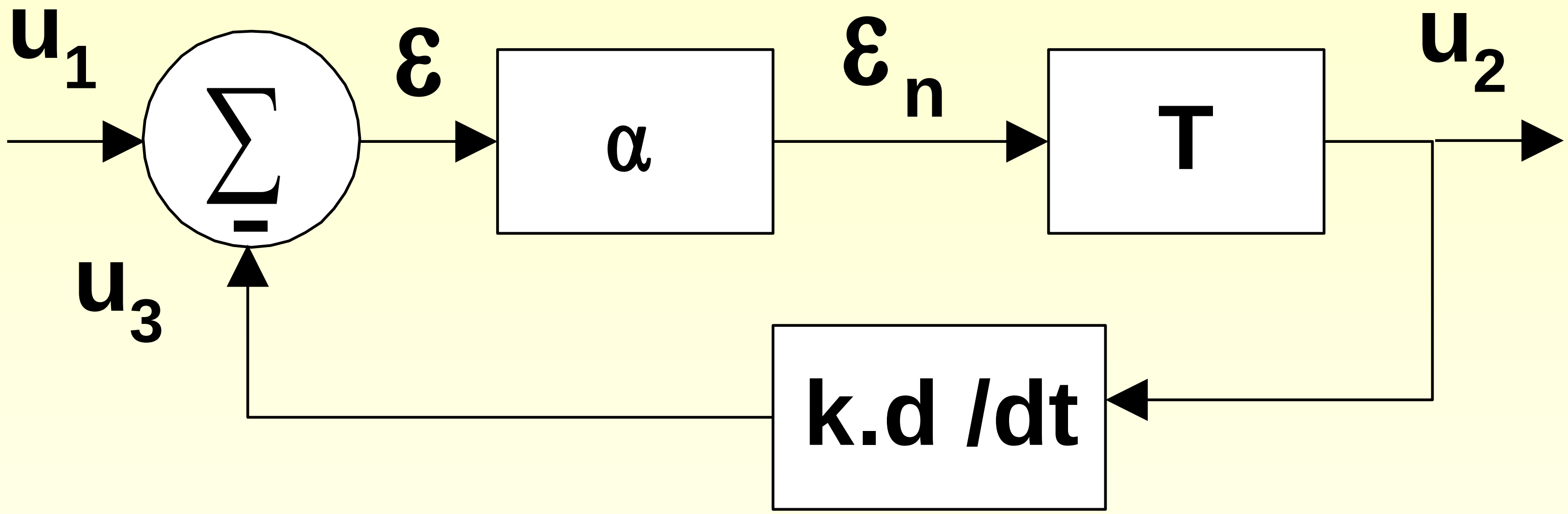


1 Open **2 Open with extra gain** **3 With unit feedback**

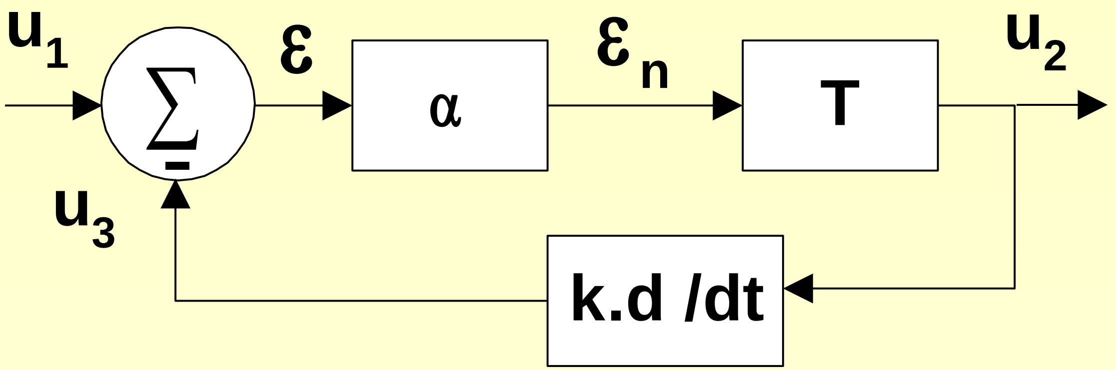
2^{nd} order feedback process. Bode diagram



Differentiating feedback



Differentiating feedback: Parameters



$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=\alpha\varepsilon$$

$$\varepsilon = u_1 - u_3 \qquad u_3 = k\frac{du_2}{dt}$$

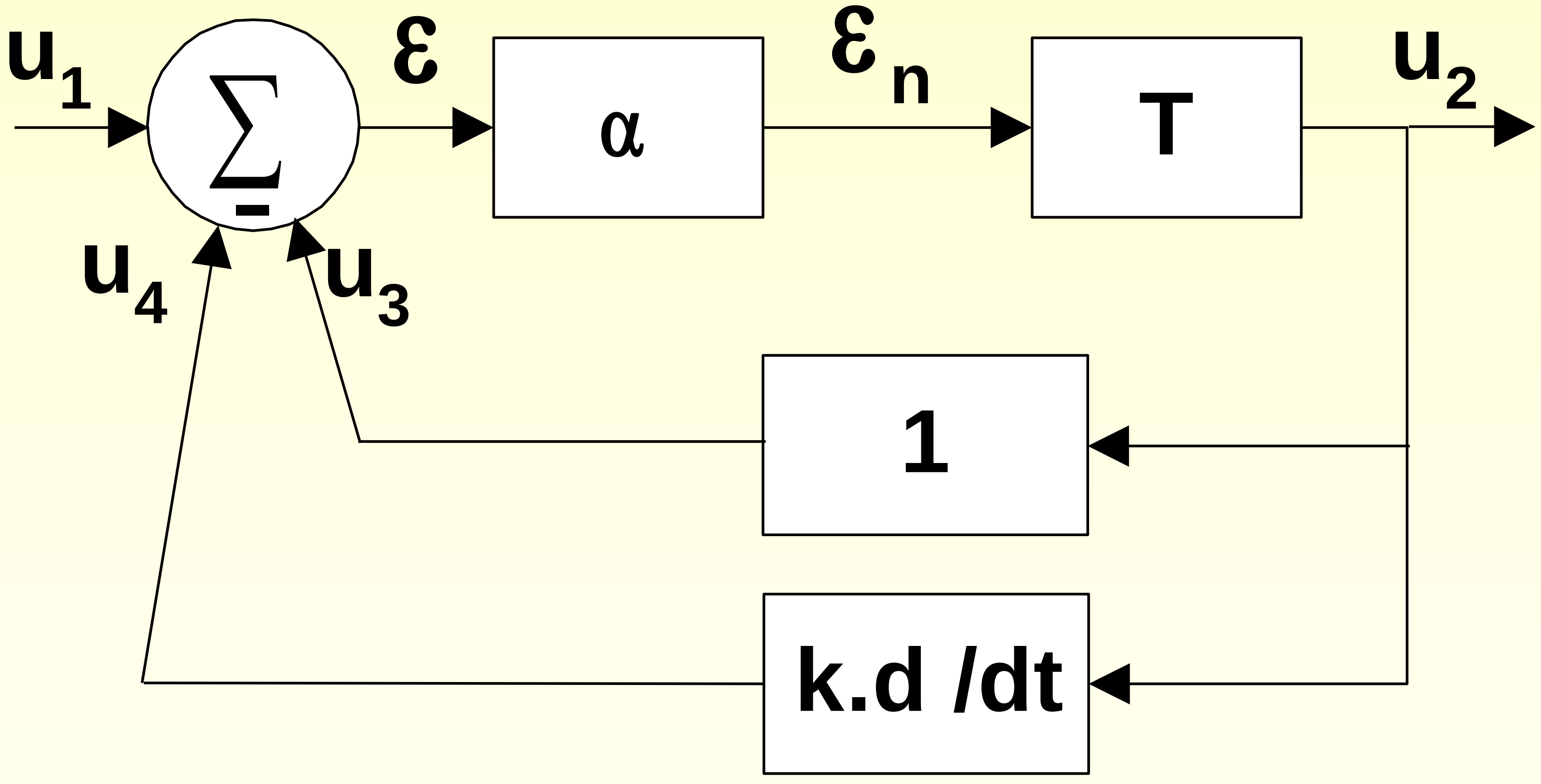
$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2(\xi+\omega_n\alpha k/2}{\omega_n}\cdot\frac{du_2}{dt}+u_2=\alpha u_1$$

$\omega'_n = \omega_n$

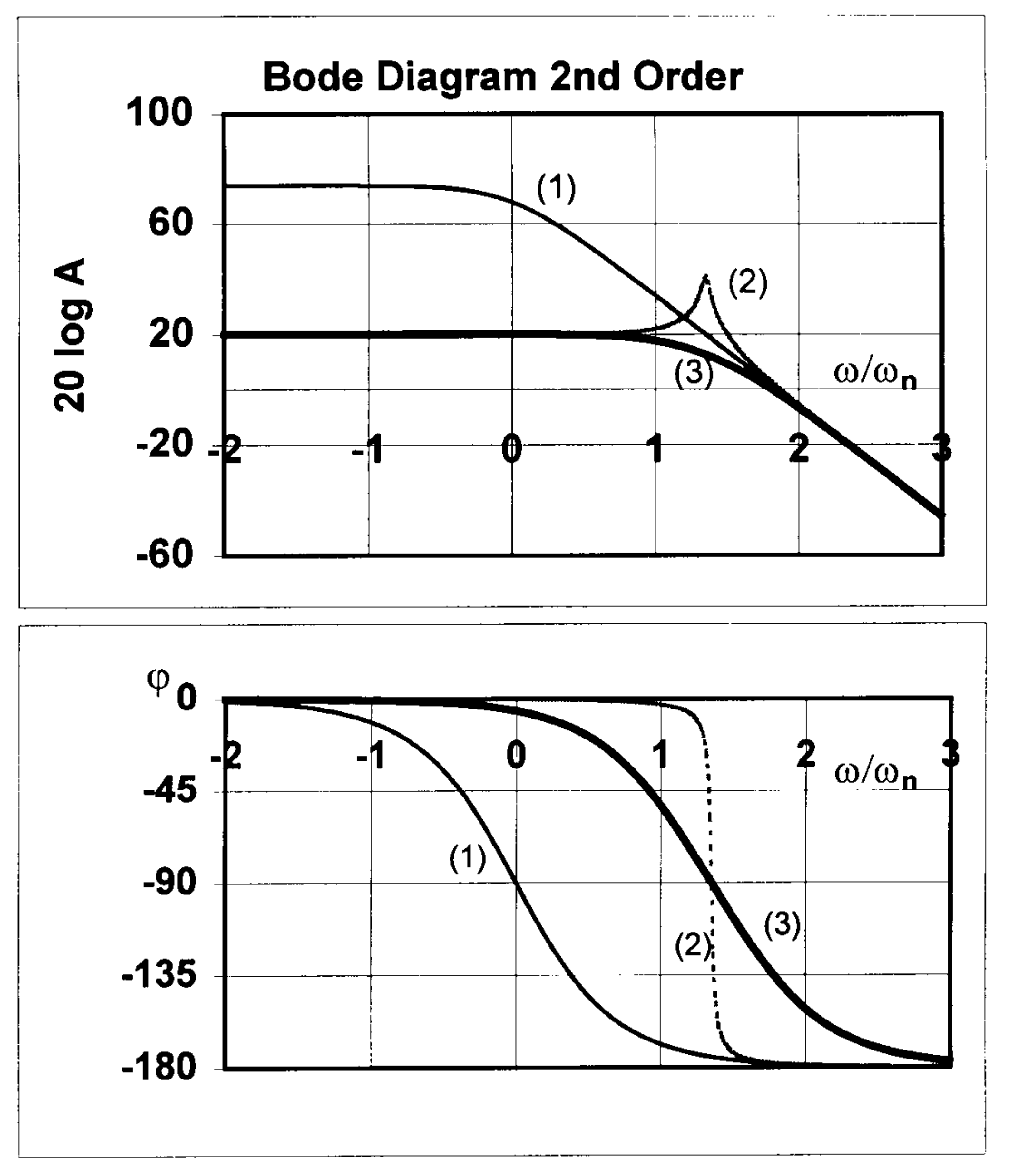
$\xi' = \xi+\omega_n\alpha k/2$

$\alpha' = \alpha$

Proportional feedback with differentiating correction



Proportional feedback with differentiating correction Bode diagram



1 Open 2 Proportional feedback 3 Proport. feedback with diff. corr.

Integrating feedback

$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2=\alpha\varepsilon$$

$$\varepsilon = u_1 - u_3 \qquad\qquad u_3 = k\int u_2 dt$$

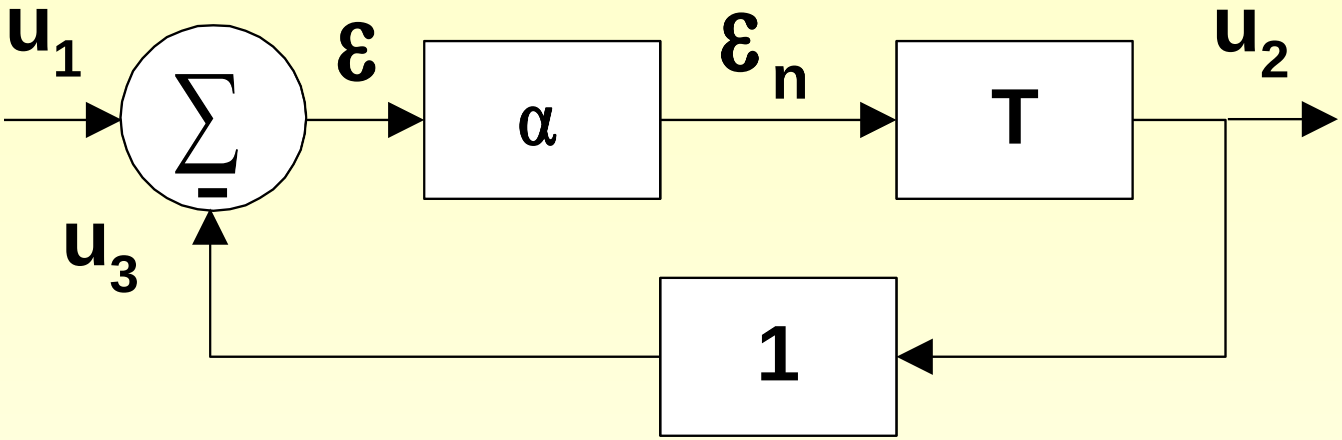
$$\frac{1}{(\omega_n)^2}\frac{d^2u_2}{dt^2}+\frac{2\xi}{\omega_n}\frac{du_2}{dt}+u_2+\alpha k\int u_2 dt=\alpha u_1$$

Differentiation of both sides of the equation:

$$\frac{1}{(\omega_n)^2}\frac{d^3u_2}{dt^3}+\frac{2\xi}{\omega_n}\frac{d^2u_2}{dt^2}+\frac{du_2}{dt}+\alpha k u_2=\alpha\frac{du_1}{dt}$$

Stability of feedback systems

Stability of feedback systems



With T a system of order > 2

There is a frequency ω_k for which $\phi_{u_2}(\omega_k) = -180^o$

$u_2 = A_2.\sin(\omega_k t - 180^o) = -A_2.\sin\omega_k t$

$u_3 = u_2$

$\varepsilon = u_1 - u_3$

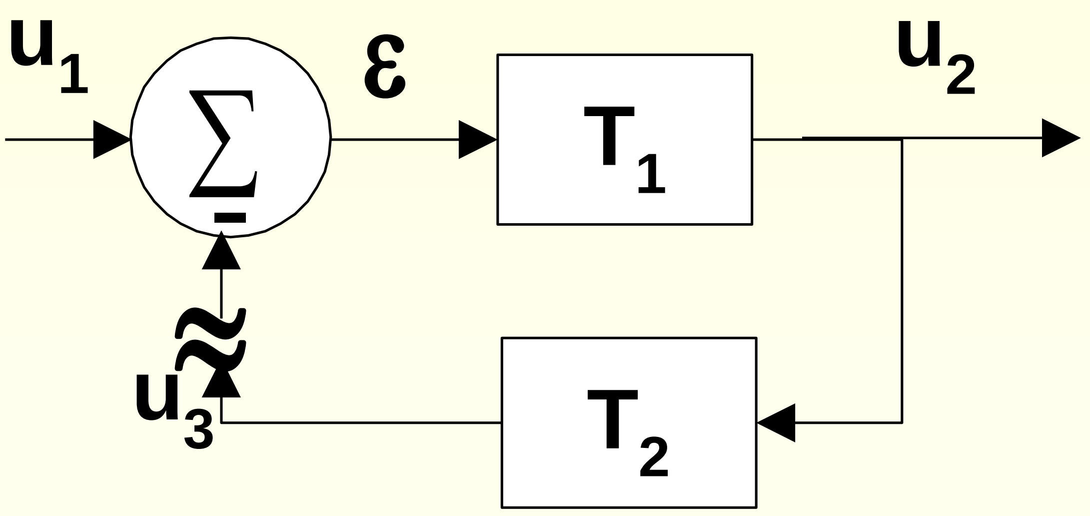
$\varepsilon = A_1.\sin\omega_k t - (-A_2.\sin\omega_k t)$

$\varepsilon = (A_1 + A_2).\sin\omega_k t > A_1.\sin\omega_k t$

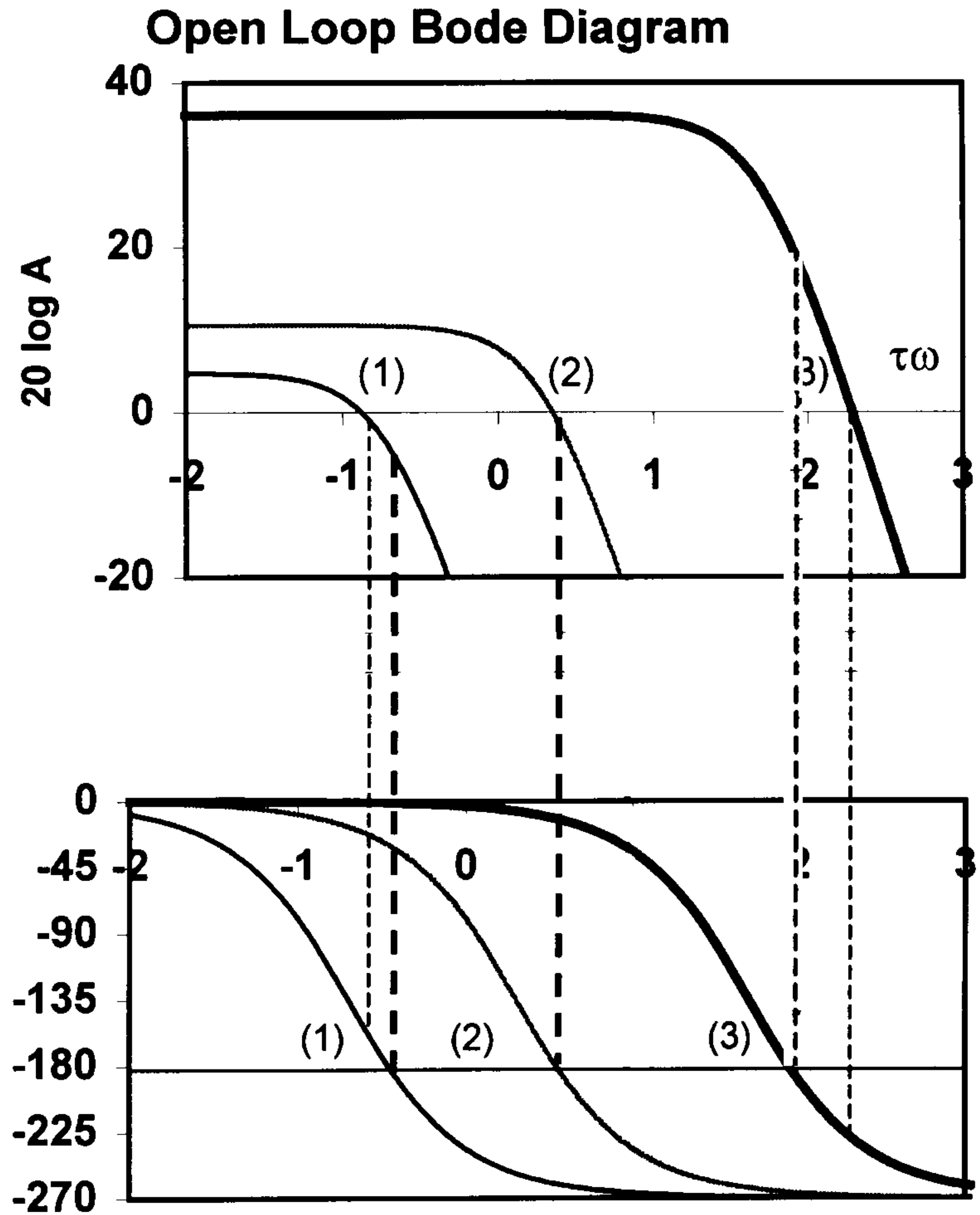
Nyquist criterion

A system with negative feedback is unstable if the open loop gain is larger than or equal to one at the frequency at which the open loop phase shift is = 180°

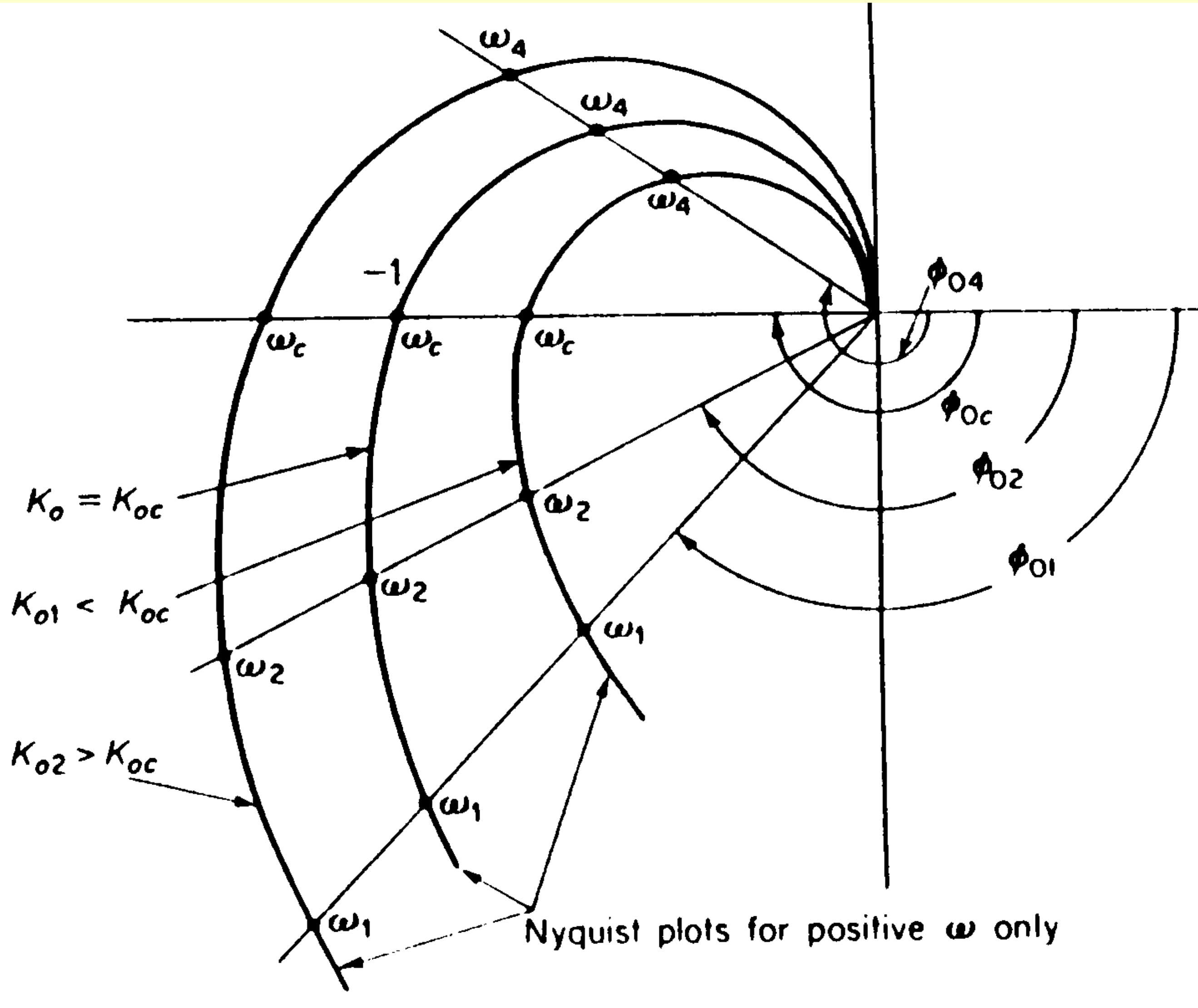
Open loop transfer function: T_1T_2



Nyquist criterion in Bode diagram

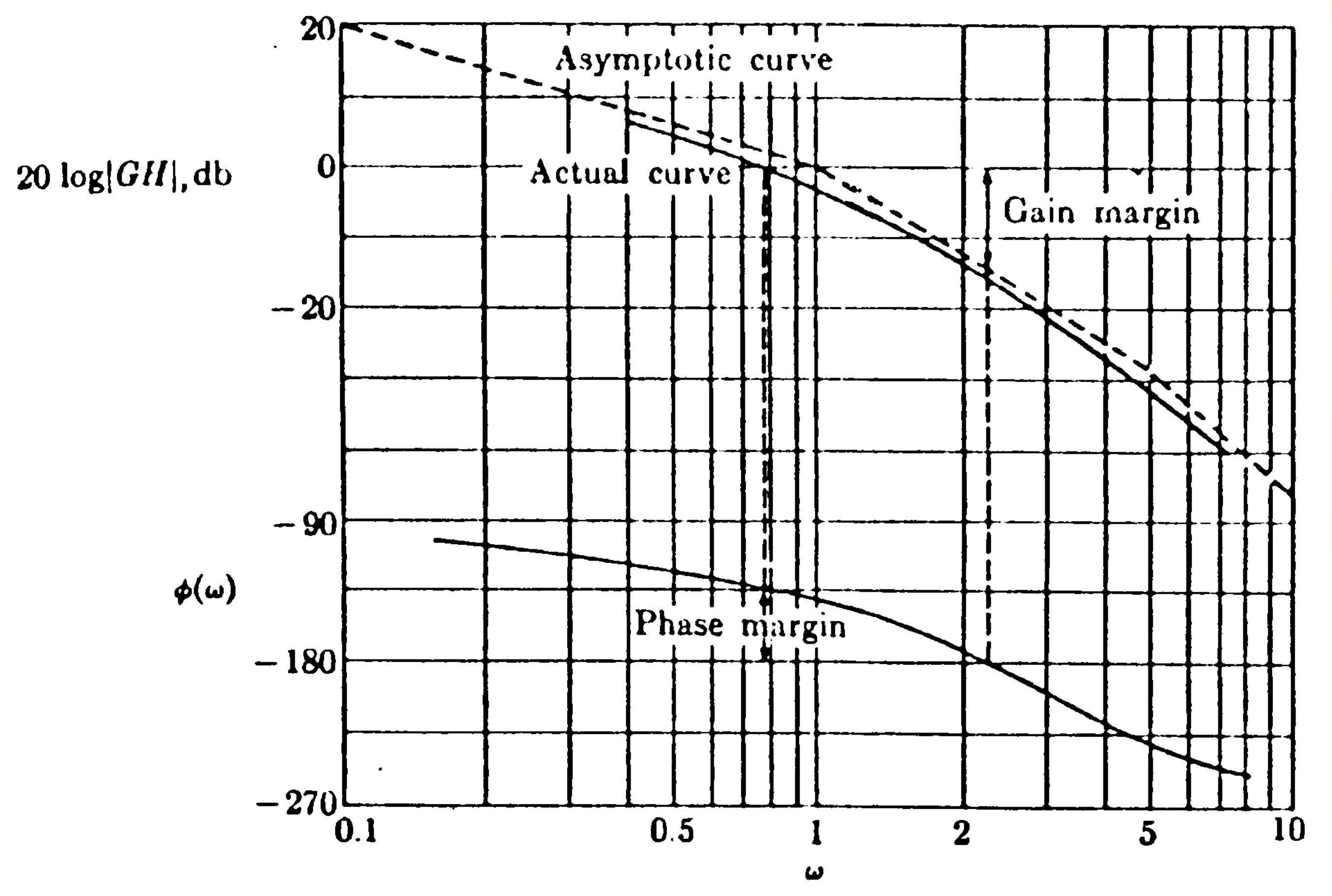


Nyquist criterion in polar plot



Effect of K_o on Nyquist plots of the form

Gain and phase margin



Gain and phase shifts that can be introduced without the system becoming unstable