Bayesian statistics

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Derivatives computation

In this section, we consider a Gaussian mixture model.

$$X \sim \sum_{k=1}^{K} \omega_k \mathcal{N}(\mu_k, \sigma_k)$$
$$f_X(x) = \sum_{k=1}^{K} \omega_k f_k(x)$$

The derivatives with respect to ω

$$\frac{\partial \log f}{\partial \omega_k}(x) = \frac{f_k(x)}{f(x)}$$
$$\frac{\partial^2 \log f}{\partial \omega_k \partial \omega_j}(x) = -\frac{f_k(x) f_j(x)}{f^2(x)}$$

The derivatives with respect to μ

$$\frac{\partial \log f}{\partial \mu_k}(x) = \omega_k \frac{x - \mu_k}{\sigma_k^2} \frac{f_k(x)}{f(x)}$$
$$\frac{\partial^2 \log f}{\partial \mu_k \partial \mu_j}(x) = -w_k.w_j(\frac{x - \mu_k}{\sigma_k^2})(\frac{x - \mu_j}{\sigma_j^2})\frac{f_k(x) f_j(x)}{f^2(x)}$$
$$\frac{\partial^2 \log f}{\partial \mu_k^2}(x) = \frac{\omega_k}{\sigma_k^2} \frac{f_k(x)}{f(x)}(1 - \frac{(x - \mu_k)^2}{\sigma_k^2}(1 - \omega_k \frac{f_k(x)}{f(x)})$$

The derivatives with respect to σ

$$\frac{\partial \log f}{\partial \sigma_k}(x) = \frac{\omega_k}{\sigma_k}(\frac{(\mu_k - x)^2}{\sigma_k^2} - 1)\frac{f_k(x)}{f(x)}$$
$$\frac{\partial^2 \log f}{\partial \sigma_k \partial \sigma_j}(x) = -\frac{\omega_k \omega_j}{\sigma_k \sigma_j}(\frac{(\mu_k - x)^2}{\sigma_k^2} - 1)(\frac{(\mu_j - x)^2}{\sigma_j^2} - 1)\frac{f_k(x) f_j(x)}{f^2(x)}$$
$$\frac{\partial^2 \log f}{\partial \sigma_k^2}(x) =$$

The cross-derivatives

$$\frac{\partial^2 \log f}{\partial \omega_k \partial \mu_j}(x) = \omega_j \frac{\mu_j - x}{\sigma_j^2}\frac{f_k . f_j(x)}{f^2(x)}$$
$$\frac{\partial^2 \log f}{\partial \omega_k \partial \sigma_j}(x) = \frac{\omega_j}{\sigma_j}(\frac{(\mu_j - x)^2}{\sigma_j^2} - 1)\frac{f_k . f_j(x)}{f^2(x)}$$
$$\frac{\partial^2 \log f}{\partial \omega_k \partial \mu_k}(x) = f_k(x)(\frac{x - \omega_k}{\sigma_k^2})\frac{w_k f_k(x) - f(x)}{f^2(x)}$$
$$\frac{\partial^2 \log f}{\partial \omega_k \partial \sigma_k}(x) = \frac{1}{\sigma_k}(\frac{(\mu_k - x)^2}{\sigma_k^2} - 1)\frac{f(x) f_k(x) - w_k f_k^2(x)}{f^2(x)}$$
$$\frac{\partial^2 \log f}{\partial \mu_k \partial \sigma_j}(x) =$$
$$\frac{\partial^2 \log f}{\partial \mu_k \partial \sigma_k}(x) =$$