



# Conway's Game Of LIFE

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01



What is Life?



## 01 Predates Computers

1940 mathematical concept  
John Von Neumann / Stanislaw Ulam  
Idea of Life (what does it mean)!?  
Complex Model

## 02 John Conway

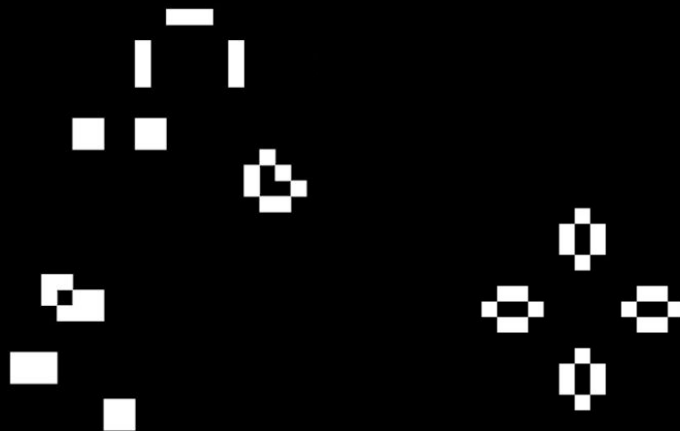
Simplified the model down,  
Published his version in *Scientific America*: 1970  
50 dollar to whoever could find a  
boundless starting configuration

## 03 Dude.... What's the Big Deal?

Popularity in the game  
exploded  
Pattern reachability /  
complex circuitry

## 04 Following Topics

How to "Play"  
Pattern Reachability  
Turing Completeness /  
limits of what's possible





# 02



How to Play / Example



# What Game? Is this even a Game?

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## 01 “0 player game”

Infinite 2 dimensional grid

Automata rules:

- Birth
- Death
- Persistence

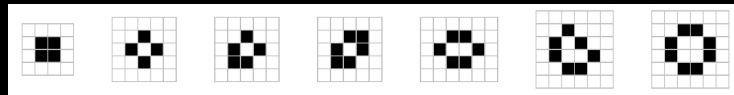


## 02 Different “Life-Forms”

Still lifes

Oscillators

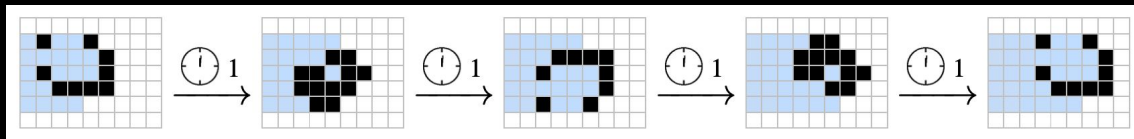
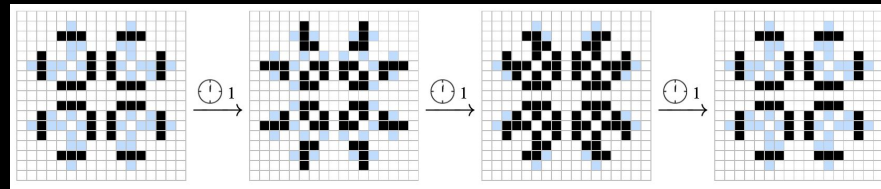
Spaceships



## 03 Example

Visual game

Ratatui





# 03

## Pattern Reachability (PREP)

# Pattern Reachability (on a Finite Grid)

## 01 Encoding GoL (4 - tuple)

d: dimensions

K / Sigma: finite alphabet of states {0,1}

W: width of neighborhood (3)

P: local mapping rule

C: Map from all cells in grid C to states in k

*Known as the configuration*

## 02 Global rule

Updates all local configurations  
simultaneously

## 03 Orbit definition

$$\text{Orbit}(X) = \{\rho^t(X) \mid t \geq 0\}$$

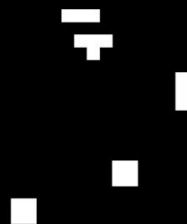
$$\text{GoL} = \langle d, k, w, p \rangle$$

$$p : \Sigma^N \rightarrow \Sigma$$

$$N = [-r, r]^d \subseteq \mathbb{Z}^d,$$

$$X : C \rightarrow \Sigma$$

$$\rho(X)(c) = p(X_c)$$



## Pattern Reachability Cont.

### 01 Formal Decision Problem: (PREP)

Does there exist a time  $t \geq 0$  such that the configuration  $P^t(X)$  contains the given target pattern  $X_0$ ?

### 02 Certificate: 2 - Tuple

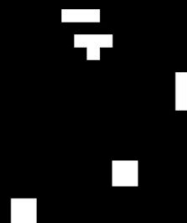
$t$ : nondeterministically guessed generation /  
computational steps needed  
 $X_0$ : Target pattern configuration

$$Z = \langle t, X_0 \rangle$$

### 03 Computation Time

Total compute time for  $t$  steps is:  $O(t * n) = O(n * t)$

For each cell  $c \in C_0$  check if  $\rho^t(X)(c) = X_0(c)$ .





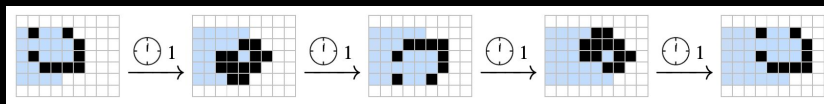
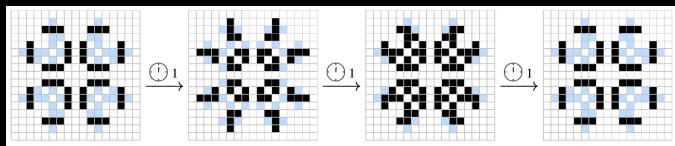
# Pattern Reachability Cont.

## 01 Oscillators

$$\mathbf{PREP}(\langle t, X_{0,p=0} \rangle) \wedge \mathbf{PREP}(\langle t+1, X_{0,p=1} \rangle) \wedge \cdots \wedge \mathbf{PREP}(\langle t+n, X_{0,p=p_n} \rangle)$$

## 02 Spaceships

$$\mathbf{PREP}(\langle t, X_{0,p=0,v=(1,-1)} \rangle) \wedge \mathbf{PREP}(\langle t+1, X_{0,p=1,v=(1,1)} \rangle) \wedge \cdots \wedge \mathbf{PREP}(\langle t+n, X_{0,p=n,v=(1,\pm 1)} \rangle)$$





# 04

Wait? What? Computation?

# More Constructions

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## 01 Gliders

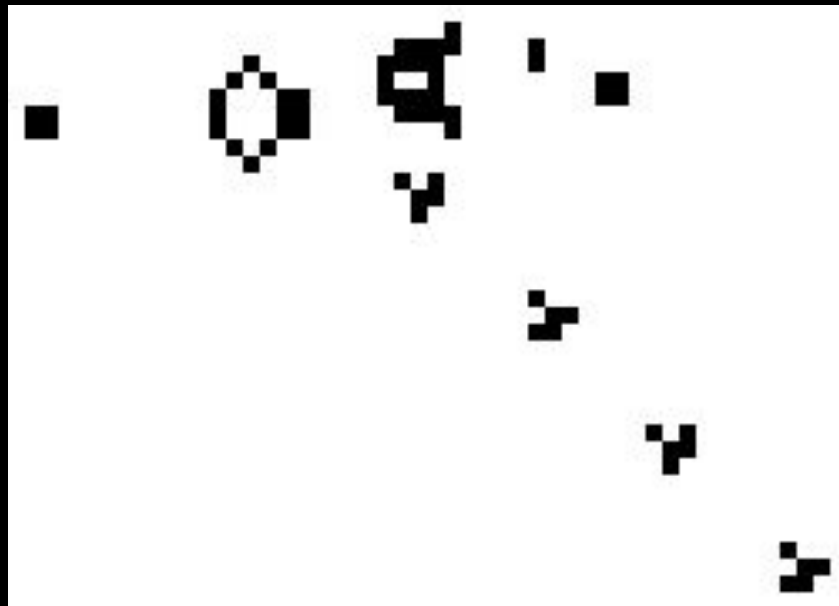
Spaceships that self-translate vertically **and** horizontally by one cell over  $n$  generations, where  $n$  is called the **period**

## 02 Guns

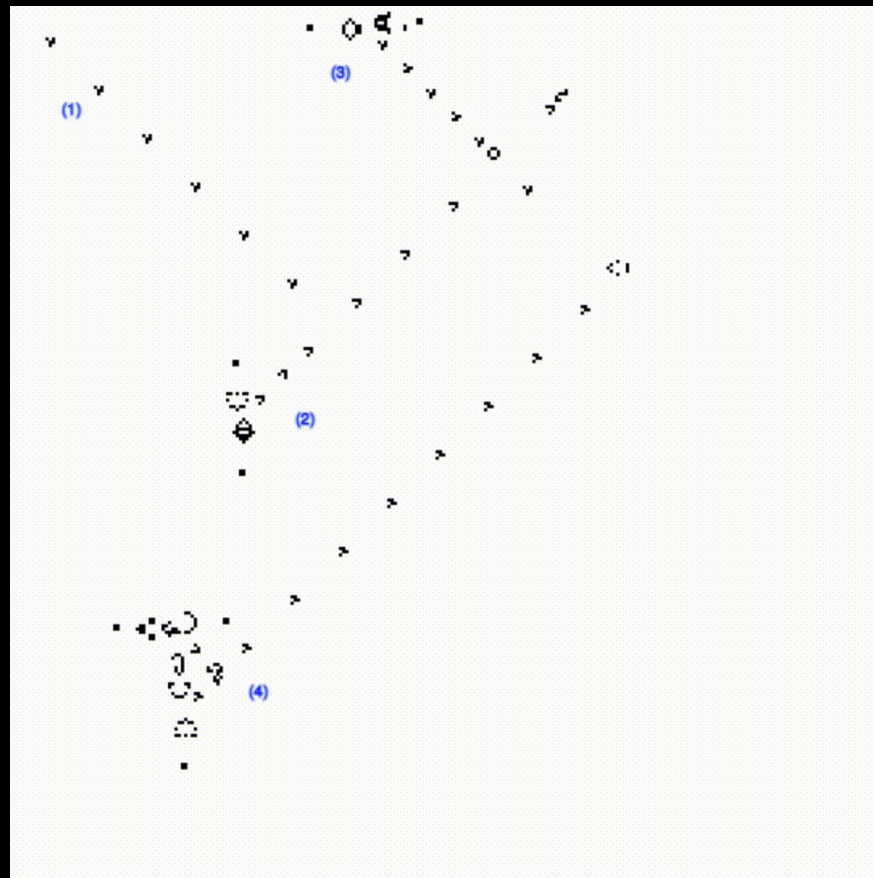
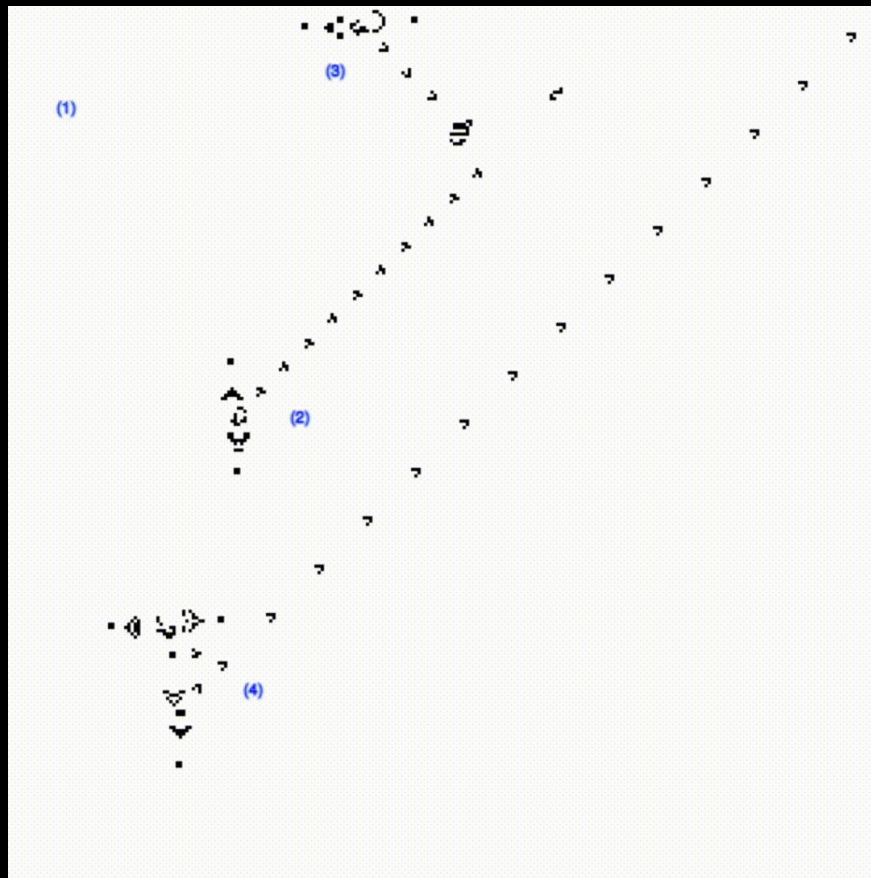
An **oscillator** that **periodically** emits **spaceships**

## 03 Gosper's Glider Gun

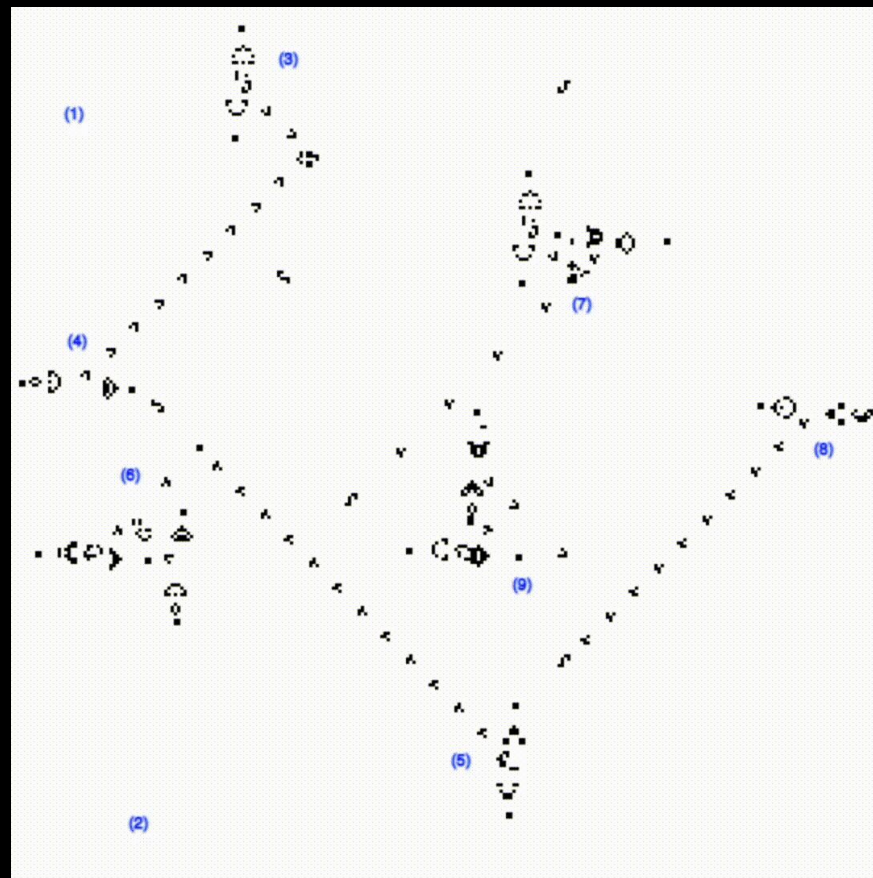
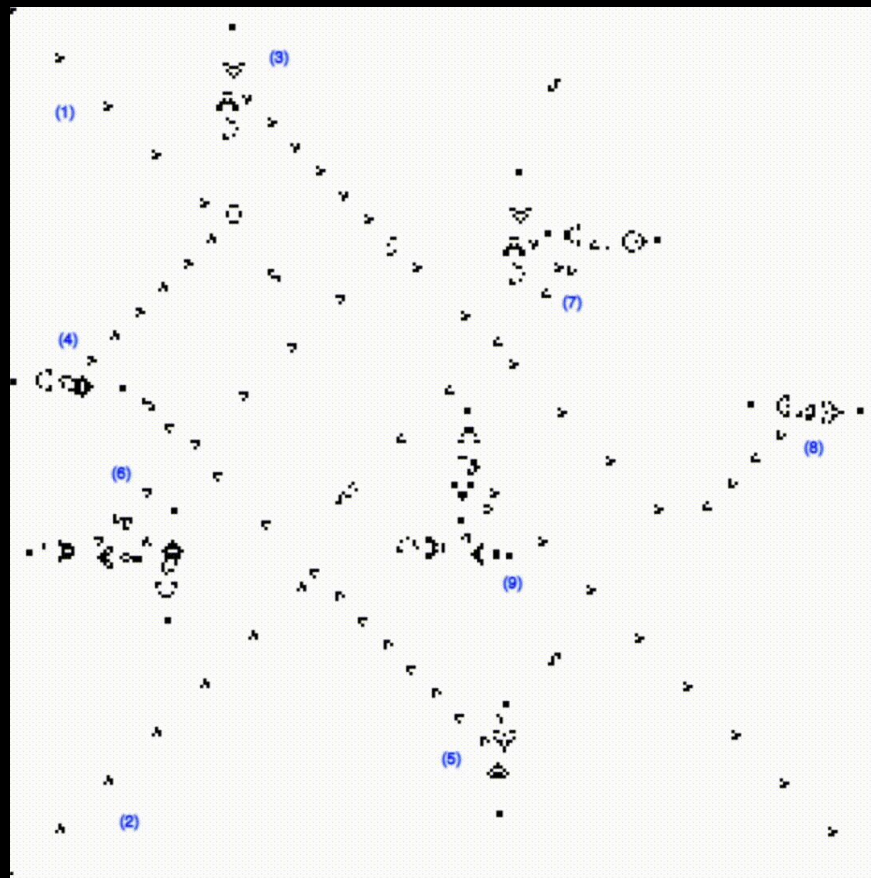
First discovered by Bill Gosper in 1970  
A gun that emits gliders in one of 4 directions (NW, NE, SW, SE)



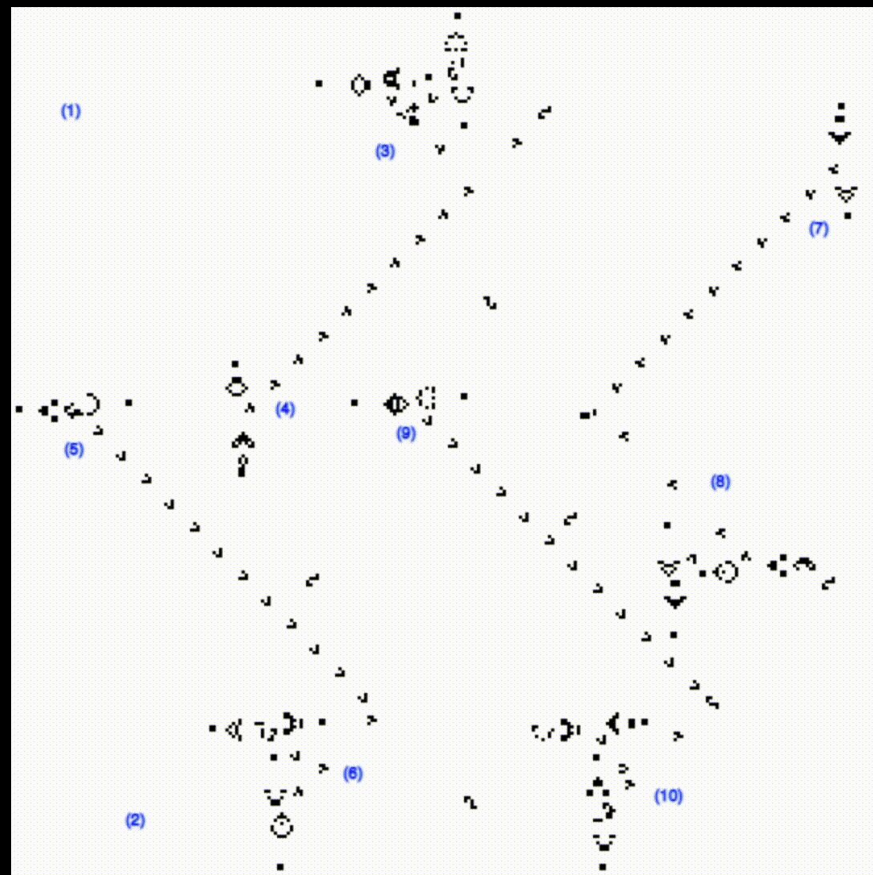
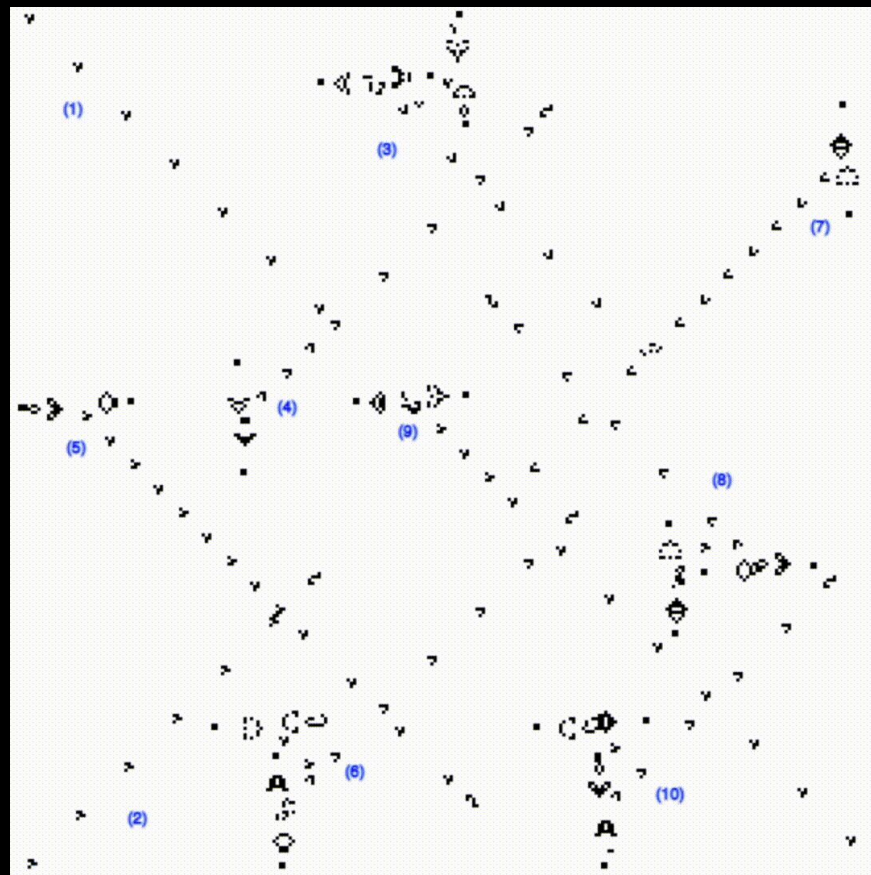
## Logical Gates: NOT



## Logical Gates: AND



## Logical Gates: OR

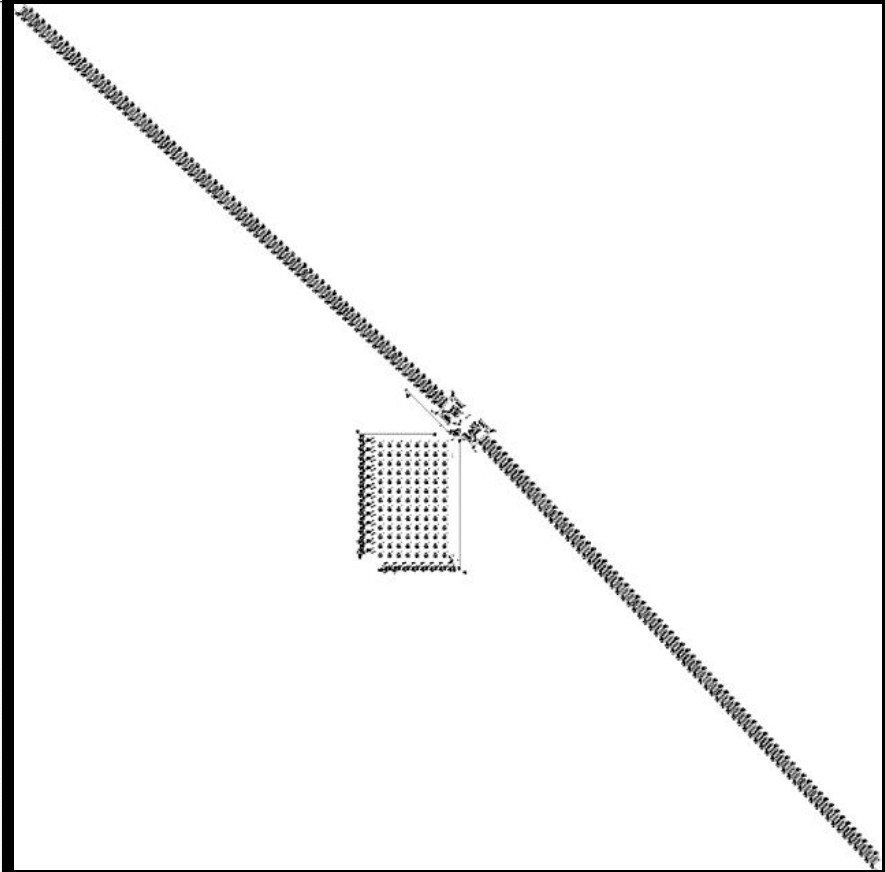
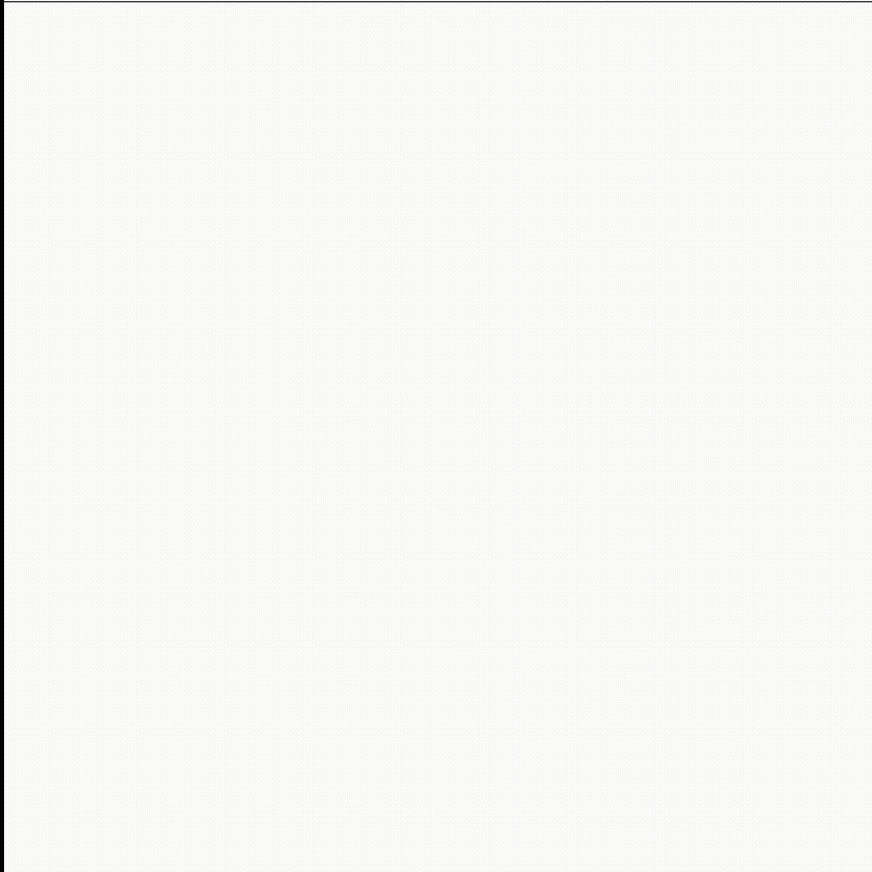


# Examples

## 4-Bit Adder (adding 5 and 6 here)

UTM

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**GoL is Turing Complete!**

The background is a solid black field filled with white pixelated elements. These elements include various star shapes, some with long, thin trails suggesting movement or light streaks. There are also several constellation-like patterns formed by small white squares and circles, connected by thin white lines. The overall effect is a digital, retro-style representation of a starry night sky.

THANK YOU