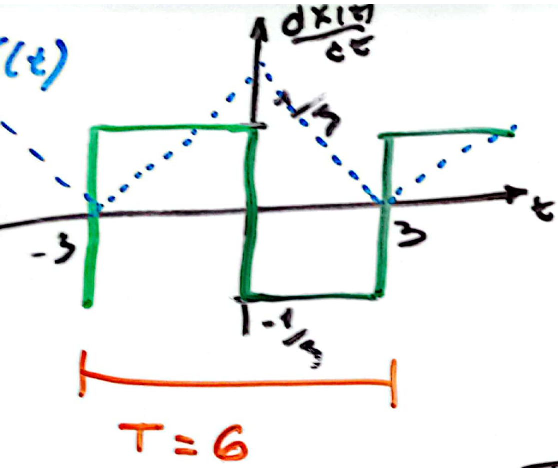
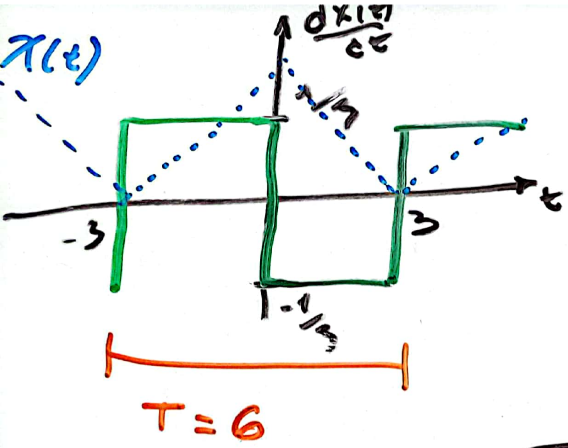


$$\frac{dx(t)}{d\tau} = \underbrace{\frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}}_{T}$$

$$T\left(\frac{dx(t)}{dt}\right) = \frac{2\,\mathrm{sen}(3\omega)}{3\omega} - \frac{4}{3\omega j}$$

$$T\left(x(t)\right) = \frac{2\,\mathrm{sen}(3\omega)}{3j\omega^2} + \frac{4}{3\omega^2}$$

$$X(j\omega) = \frac{2\,\mathrm{sen}(3\omega)}{3j\omega^2} + \frac{4}{3\omega^2}$$



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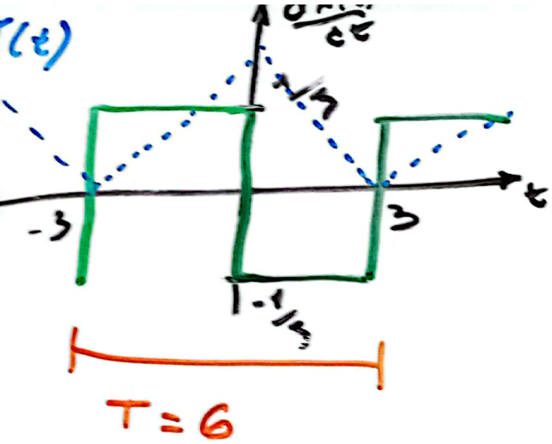
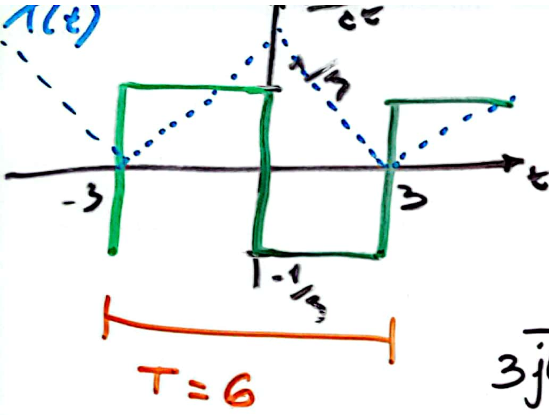


$$\frac{dx(t)}{d\tau} = \underbrace{\frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}}_{T}$$

$$T\left(\frac{dx(t)}{dt}\right) = \frac{2\,\mathrm{sen}(3w)}{3w} - \frac{4}{3wj}$$

$$T\left(x(t)\right) = \frac{2\,\mathrm{sen}(3w)}{3jw^2} + \frac{4}{3w^2}$$

$$X(jw) = \frac{2\,\mathrm{sen}(3w)}{3jw^2} + \frac{4}{3w^2}$$



$$\frac{dx(t)}{d\tau} = \underbrace{\frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}}_{T}$$

$$\frac{1}{3j\omega}\left\{\left(\underbrace{2e^{j3\omega} - 4 + 2e^{-j3\omega}}_{2(e^{j3\omega}-e^{-j3\omega})-4}\right) + \left(\underbrace{e^{-j3\omega} - e^{j3\omega}}_{-2j\,sen(3\omega)}\right)\right\}$$

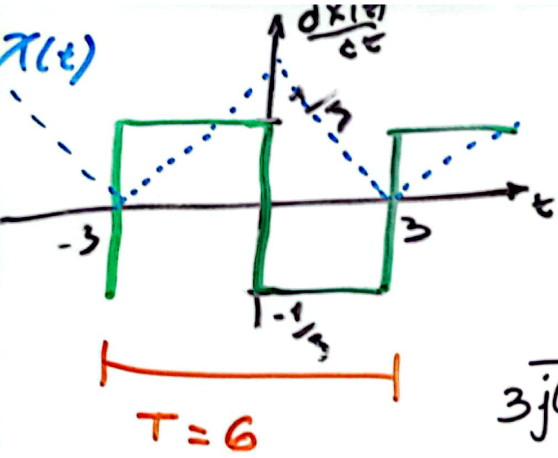
$$e^{j3\omega} - e^{-j3\omega} = \cancel{\cos(3\omega)} + j\,sen(3\omega) - \cancel{\cos(3\omega)} + j\,sen(3\omega)$$

$$2j\,sen(3\omega)$$

$$e^{-j3\omega} - e^{j3\omega} = -2j\,sen(3\omega)$$

$$\frac{1}{3\omega j}\left\{\underbrace{\left(4j\,sen(3\omega) - 4\right) - 2j\,sen(3\omega)}_{2j\,sen(3\omega)-4}\right\}$$

$$\frac{2j\,sen(3\omega) - 4}{3\omega j}$$



$$\frac{dx(t)}{d\tau} = \frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}$$

T

$$\frac{1}{3j\omega}\left(2e^{j3\omega} - 4 + 2e^{-j3\omega}\right) + \left(e^{-j3\omega} - e^{j3\omega}\right)$$

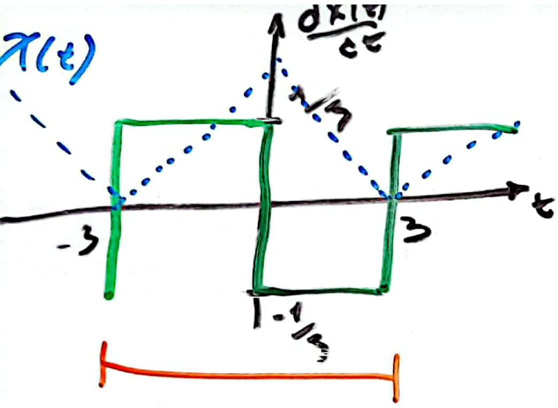
$2(e^{j3\omega} - e^{-j3\omega}) - 4$ $\qquad -2j\mathrm{sen}(3\omega)$

$$e^{j3\omega} - e^{-j3\omega} = \cos(\cancel{3\omega}) + j\mathrm{sen}(3\omega) - \cancel{\cos}(3\omega) + j\mathrm{sen}(3\omega)$$

$$2j\mathrm{sen}(3\omega)$$

$$e^{-j3\omega} - e^{j3\omega} = -2j\mathrm{sen}(3\omega)$$

$$\frac{1}{3\omega j}\left(4j\mathrm{sen}(3\omega) - 4\right) - 2j\mathrm{sen}(3\omega)$$



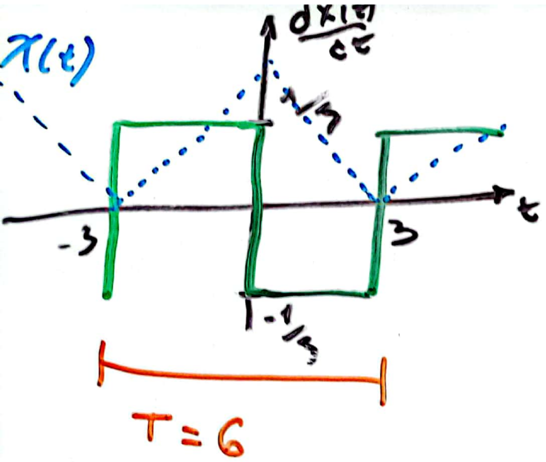
T=6

$$\frac{dx(t)}{d\tau} = \underbrace{\frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}}_{T}$$

$$\frac{2}{3}\left(\int_{-3}^{0} e^{-j\omega t}\,dt - \int_{0}^{3} e^{-j\omega t}\,dt\right) - \frac{1}{3}\int_{-3}^{3} e^{-j\omega t}\,dt$$

$$\frac{2}{3}\left\{ -\frac{e^{-j\omega t}}{j\omega}\Bigg|_{-3}^{0} + \frac{e^{-j\omega t}}{j\omega}\Bigg|_{0}^{3} \right\} + \frac{1}{3}\left(\frac{e^{-j\omega t}}{j\omega}\Bigg|_{-3}^{3} \right)$$

$$\frac{2}{3}\left\{ \frac{e^{j3\omega}}{j\omega} - \frac{1}{j\omega} + \frac{e^{-j3\omega}}{j\omega} - \frac{1}{j\omega} \right\} + \frac{1}{3}\left(\frac{e^{-j3\omega}}{j\omega} - \frac{e^{j3\omega}}{j\omega} \right)$$

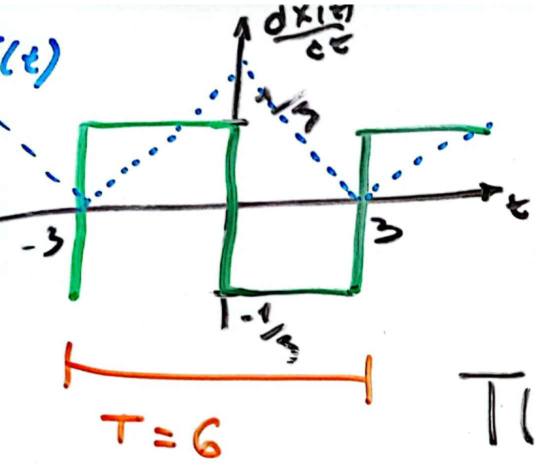


$$\frac{dx(t)}{d\tau} = \underbrace{\frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}}_{\text{Ειυη } T}$$

$$\frac{2}{3}\int_{-\frac{T}{2}}^{\cancel{\frac{T}{2}}\,0} \cancel{u(\tau-3)}^{\,1}\, e^{-j\omega t}\, d\tau - \int_{\cancel{-\frac{T}{2}}\,0}^{\frac{T}{2}} \cancel{u(t)}_{\,1}\, e^{-j\omega t}\, d\tau - \dots$$

$$\frac{1}{3}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j\omega t}\, d\tau$$

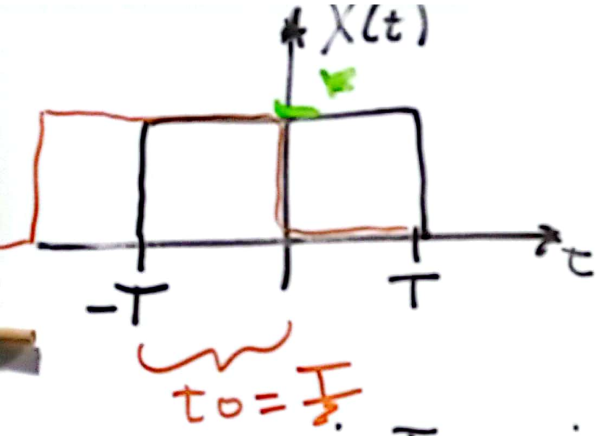
$$\frac{1}{3}\left(-\frac{e^{-j\omega t}}{j\omega}\Bigg|_{-\frac{T}{2}}^{\frac{T}{2}}\right)$$



$$\frac{dx(t)}{d\tau} = \underbrace{\frac{2}{3}u(\tau+3) - \frac{2}{3}u(\tau) - \frac{1}{3}}_{\text{Ειυη } T}$$

$$T\left(\frac{dx(t)}{dt}\right) = Ta_k = \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{dx(t)}{d\tau} e^{-j\omega t} dt$$

$$T(x(t)) = X(j\omega) = \frac{1}{j\omega}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{dx(t)}{d\tau} e^{-j\omega t} dt$$



$$T a_K = \int_{-T}^{T} x(t) e^{-j\omega t} dt \qquad \omega = \frac{2\pi K}{T}$$

$$= K \int_{-T}^{T} e^{-j\omega t} dt = -\frac{e^{-j\omega t}}{j\omega}\Big|_{-T}^{T}$$

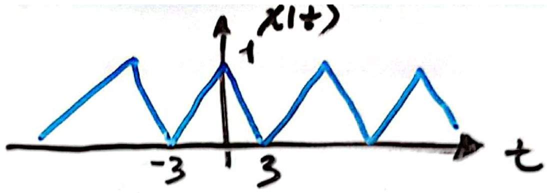
$$-\frac{e^{-j\omega T}}{j\omega} + \frac{e^{j\omega T}}{j\omega} = \frac{1}{j\omega}\left(e^{j\omega T} - e^{-j\omega T}\right)$$

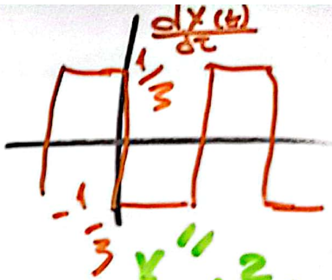
$$= \frac{1}{j\omega}\left(\cancel{\cos(\omega T)} + j\operatorname{sen}(\omega T) - \cancel{\cos(\omega T)} + j\operatorname{sen}(\omega T)\right)$$

$$X(j\omega) = \frac{2}{\omega}\operatorname{sen}(\omega T) K$$

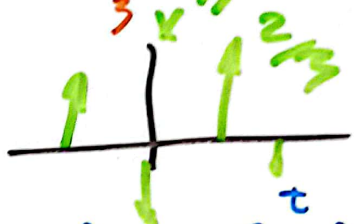
$$X(j\omega) = e^{-j\omega t_o} K \frac{2}{\omega}\operatorname{sen}(\omega T)$$

Sea $x(t)$





a) Calcular $X(j\omega)$



$$X(j\omega) = Na_k = \sum x(t)\, e^{-j\overbrace{\frac{2\pi}{N}}^{\omega}kt}$$

$$x(t) = \begin{cases} 1 + \frac{t}{3}, & -3 \geq t \leq 0 \\ 1 - \frac{t}{3}, & 0 < t \leq 3 \end{cases}$$

$$\frac{dx(t)}{dt} = \begin{cases} \frac{1}{3}, & (-3, 0) \\ -\frac{1}{3}, & (0, 3) \end{cases}$$

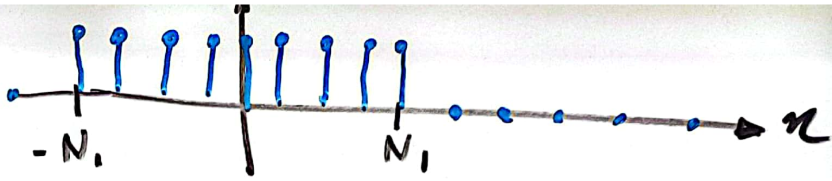
$$x(t) \xleftrightarrow{F} X(j\omega)$$

$$x'(t) \longleftrightarrow j\omega X(j\omega)$$

$$x''(t) \longleftrightarrow -\omega^2 X(j\omega)$$

$$\boxed{X(j\omega) = \frac{-x''(t)}{\omega^2}}$$

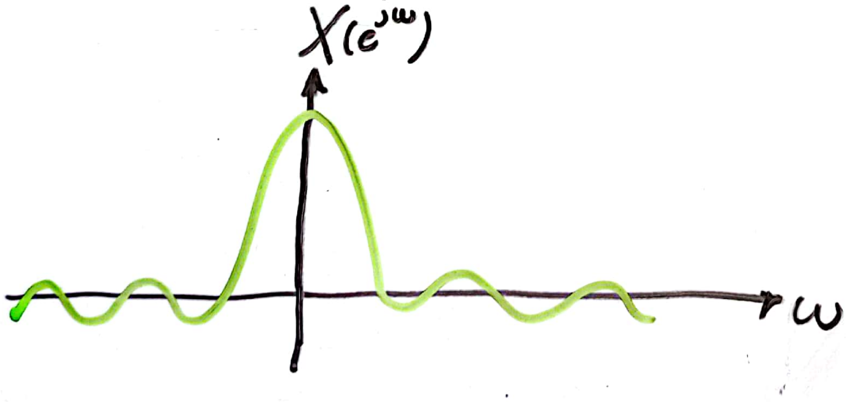
$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & |n| > N_1 \end{cases}$$



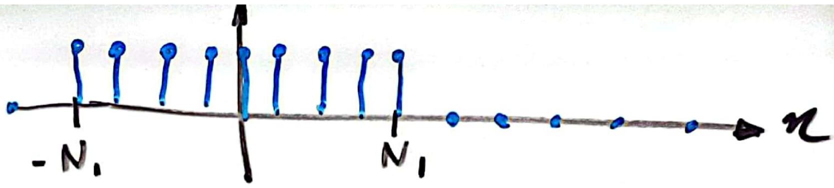
$$\boxed{X(e^{j\omega})} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = e^{j\omega N_1} \sum_{k=0}^{2N_1} e^{-jk\omega}$$

S. Geometrica $:= \frac{1-e^{n}}{1-e}$

$$e^{j\omega N_1}\left(\frac{1-e^{-j\omega(2N_1+1)}}{1-e^{-j\omega}}\right) = \boxed{\frac{\operatorname{sen}\left(\omega\left(N_1+\frac{1}{2}\right)\right)}{\operatorname{sen}\left(\frac{\omega}{2}\right)}}$$



$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & |n| > N_1 \end{cases}$$



$$\boxed{X(e^{j\omega})} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = e^{j\omega N_1} \sum_{k=0}^{2N_1} e^{-jk\omega}$$

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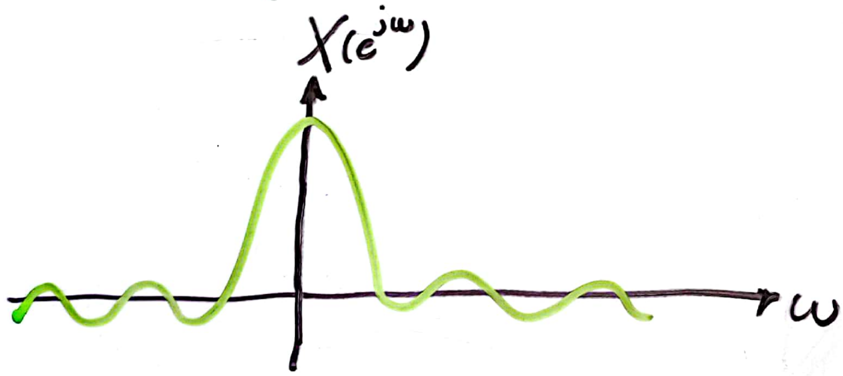


$$\hat{x}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} \, dw$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\sum_{k}^{\infty} x[k] e^{-jwk} \right) e^{jwn} \, dw$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{2\pi} x[k] \int_{-\pi}^{\pi} e^{jw(n-k)} \, dw \rightarrow \text{cuando } n=k = 1$$

Siempre se cumple

$$\hat{x}[n] = \sum_{k=-\infty}^{\infty} x[k] \, \delta[n-k] = x[n]$$

$$X(e^{j\omega}) = N a_k$$

$$\bar{X}[n] = \sum \frac{1}{N} X(e^{j\omega}) e^{j\omega n}$$

$$\omega_0 = \frac{2\pi}{N} \rightarrow \frac{1}{N} = \frac{\omega_0}{2\pi}$$

$$\bar{X}[n] = \frac{1}{2\pi} \sum X(e^{j\omega}) e^{j\omega n} \omega_0$$

Antitransformada de Fourier

$$\boxed{X[n] = \frac{1}{2\pi} \int X(e^{j\omega}) e^{j\omega n} d\omega}$$

$$X(e^{jw}) = N a_k$$

$$\bar{X}[n] = \sum \frac{1}{N} X(e^{jw}) e^{jwn}$$

$$w_0 = \frac{2\pi}{N} \rightarrow \frac{1}{N} = \frac{w_0}{2\pi}$$

$$\bar{X}[n] = \frac{1}{2\pi} \sum X(e^{jw}) e^{jwn} w_0$$

Antitransformada de Fourier

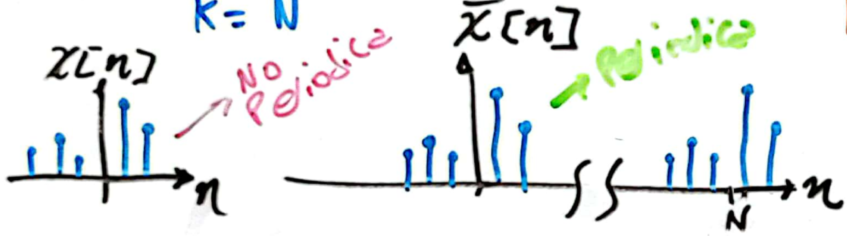
$$\boxed{X[n] = \frac{1}{2\pi} \int X(e^{jw}) e^{jwn} dw}$$

Transformada de Fourier tiempo discreto $\boxed{\omega_0 = \frac{2\pi}{N}}$

$$\tilde{x}[n] = \sum_{k=N} a_k e^{j\omega_0 k n}$$

$$a_k = \frac{1}{N} \sum_{n=}^{N} x[n] e^{-jk\omega_0 n}$$

Sea



Con $N \to \infty$

$\tilde{x}[n] = x[n]$

$x[n] = \tilde{x}[n]$ en un periodo, $\omega = k\omega_0$

transformada de Fourier

$$\boxed{X(e^{j\omega}) = N a_k = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}}$$

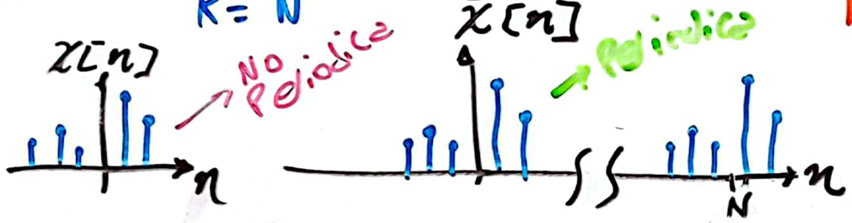
$$a_k = \frac{1}{N} X(e^{j\omega})$$

Transformada de Fourie tiempo discreto $\boxed{\omega_0 = \frac{2\pi}{N}}$

$$\tilde{x}[n] = \sum_{k=N} a_k e^{j\omega_0 k n}$$

$$a_k = \frac{1}{N} \sum_{n:}^{N} x[n] e^{-jk\omega_0 n}$$

Sea



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transformada de Fourier

$$X(e^{j\omega}) = N a_k = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Rta es f del sist   Si el sist LTI es estable

siempre $\exists H$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

un sist estable   si x(t) conv
y(t) conv

$$Y(j\omega) = H(j\omega) X(j\omega)$$

Para sist LTI inestables Transf. de Laplace