ALGORITHMS FOR STABLE CONTROL OF MOBILE ROBOTS WITH OBSTACLE AVOIDANCE

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To the memory of Professor Alfredo Desages for his invaluable contribution to the advancement of the Automatic Control in Argentina.

Keywords: mobile robots; robot control; obstacle avoidance; stability analysis.

Abstract

In this paper mobile robot control laws, including obstacle avoidance based on distance sensorial information are proposed. The mobile robot is assumed to evolve in a semi-structured environment. The control systems are based on the use of the extended impedance concept, in which the relationship between fictitious forces and motion error is regulated. The fictitious forces are generated from the information provided by sensors on the distance from the obstacle to the robot. The control algorithms also prevent from the potential problem of control command saturation. The paper includes the stability analysis of the developed control systems, using positive definite potential functions.

1. Introduction

Mobile robots are mechanical devices capable of moving in some environment with a certain degree of autonomy. The environment can be classified as structured when it is well known and the motion can be planned in advance, or as partially structured when there are uncertainties which imply some on-line planning of the motions.

During the movement in partially structured environments, an obstacle can suddenly appear on the robot trajectory. Then, a sensorial system should detect the obstacle, measure its distance and orientation for calculating a control action to change the robot trajectory, thus avoiding the obstacle.

In this article it is used the concept of generalized impedance which relates fictitious forces to vehicle motion. Fictitious forces are calculated as a function of the measured distances. A similar concept for a generalized spring effect in robot manipulators is presented in (Sagués *et al.*, 1990). An application of the impedance concept to avoid obstacles with robot manipulators has been presented in (Mut *et al.*, 1992).

The control architecture presented in this paper combines two feedback loops: a motion control loop (Secchi *et al.*, 1998) and a second external impedance control loop (Hogan, 1985). This last loop provides a modification on target position when an obstacle appears on the trajectory of the mobile robot (Secchi *et al.*, 1994).

Main contributions of this paper are the design of stable motion control laws that include the actuators saturation problem; the design of a motion control structure for obstacle avoidance and its corresponding stability analysis; and the performance test of the control algorithms through simulation.

The paper is organized as follows. After this introductory section, section 2 describes the kinematic equations of an experimental robot; section 3 presents the control problem formulation; section 4 defines the fictitious force for distance feedback; section 5 presents the proposed control algorithms including their stability analysis; section 6 describes the simulation results; and finally, section 7 contains the main conclusions of the work.

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2. Kinematics Equations

Consider the unicycle-like robot positioned at a non-zero distance from a goal frame <g>. Its motion towards <g> is governed by the combined action of both the angular velocity ω and the linear velocity vector $\boldsymbol{u}$, which is always directed as one of the axes of the frame <a> attached to the robot, as depicted in figure 1.

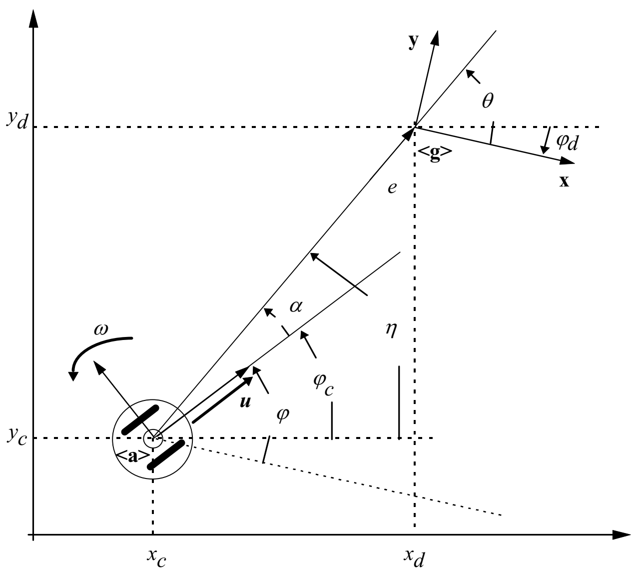

Figure 1. Position and orientation of the vehicle.

Then, the usual set of kinematic equations, which involves Cartesian position (x,y) of the vehicle and its orientation angle φ, are

$$\begin{cases} \dot{x} = u \cdot cos\varphi \\ \dot{y} = u \cdot sin\varphi \\ \dot{\varphi} = \omega \end{cases} \tag{1}$$

where u is the magnitude of $\boldsymbol{u}$, and x, y and φ are all measured with respect to the target frame <g> origin and x-axis orientation.

Now, by representing the vehicle position in polar coordinates, and considering the error vector $\boldsymbol{e}$ with orientation θ respecting to the x-axis of frame <g>, as well as by letting $\alpha=\theta-\varphi$ be the angle measured between the main vehicle axis and the distance vector $\boldsymbol{e}$, the above kinematic equations can be rewritten (Aicardi *et al.*, 1995)

$$\begin{cases} \dot{e} = -u \cdot \cos\alpha \\ \dot{\alpha} = -\omega + u \cdot \dfrac{\operatorname{sen}\alpha}{e} \\ \dot{\theta} = u \cdot \dfrac{\operatorname{sen}\alpha}{e} \end{cases} \tag{2}$$

3. Problem Formulation

Let us consider the kinematic model of the mobile robot given by equation (2). The main characteristics of the control problem are:

1. The objective to be reached by the mobile robot itself (the target frame <g>). The problem of reaching the target frame can be formulated in two different ways: the first one is in terms of a desired motion trajectory and the second one is in terms of the target position (in this second situation we can additionally consider a desired final orientation $\theta=0$). In this paper, only the second situation is addressed, without considering the final orientation of the mobile robot when attaining the target frame.

2. The dynamic relationship (mechanical impedance) between the position error and the interaction force $\boldsymbol{F(t)}$ acting on the mobile robot. In this paper, $\boldsymbol{F(t)}$ is a fictitious force generated from the distance information coming from the exteroceptive sensors (ultrasonic sensors).

Then, the problem of motion control corresponds to the design of a controller that drives the mobile robot (the unicycle-like vehicle) to the point of coordinates $e=0$ and $\alpha=0$ (and additionally consider $\theta=0$) starting from any non zero distance from the target frame <g>. The problem of impedance control, in addition, corresponds to the design of a controller that, after detecting obstacles in the robot working environment, momentarily modifies the target position in order to avoid these obstacles.

4. Sensorial Distance Feedback

The regulation of the mechanical impedance needs some feedback of the interaction force between the robot and the environment. Interaction forces imply a physical contact with the environment, which, in the case of mobile robots, generally represents a collision. In order to avoid obstacles, it is necessary to interact with the obstacles without collision. Thus, the interaction force $\boldsymbol{F(t)}$ is represented by a fictitious force generated as a function of the robot - obstacle distance, as shown in Figure 2.

The trajectory change associated to the obstacle avoidance is performed by using the impedance concept, for which the mechanical interaction has been substituted by a distance and non-contact interaction taking into account the distance from the robot to the detected obstacle (Mut *et al.*, 1992).

The magnitude of force $\boldsymbol{F(t)}$ is computed as (Borenstein and Koren, 1991)

$$F(t) = a - b \cdot d(t) \tag{3}$$

where

a, b	are positive constants, such that $a - b \cdot dmax = 0$;
dmax	Is the maximum robot-obstacle distance measured by the sensorial system; and
d(t)	Is the robot-obstacle distance ($0 < d(t) < dmax$).

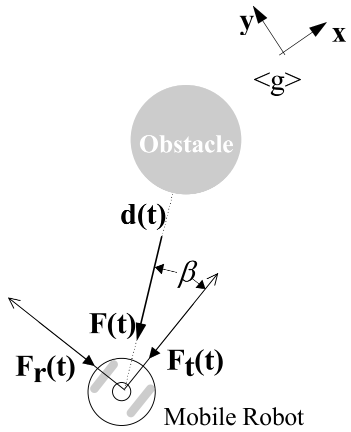

Figure 2. Action of the fictitious force **F**(t) on the mobile robot.

Figure 3 represents the block diagram of the proposed control system, where, in Cartesian coordinates,

$\mathbf{x_d}$	is the desired (x_d, y_d, φ_d) position vector ;
$\mathbf{x_r}$	is the modified (x_r, y_r, φ_r) position vector;
ψ	is the rotation angle; and
$\tilde{x}$	is the position error $\mathbf{x_r}$-$\mathbf{x_c}$.

5. Control Algorithms

One typical problem when implementing a controller is that of the practical range of control actions. If not considered in the theoretical design, possible saturation of actuators will occur and, in such a case, the design performance of the control system can not be guaranteed to be attained. In this section controller saturation is taken into account without a considerable additional calculation effort. Out of the three variables e, α and θ, the former is considered critical in terms of saturation because it directly affects the linear velocity u. Thus, in the theoretical development of the controllers, u will be guaranteed to be bounded within prescribed limits.

5.1 Motion control I: Positioning without prescribing orientation

Let the unicycle-like vehicle be initially positioned at any non zero distance from the target frame <g> and let the state variables be e and α, assumed as directly measurable for any $e>0$. Let us consider the Lyapunov candidate function

$$V(e,\alpha) = V_1 + V_2 = \frac{1}{2}\cdot\lambda\cdot e^2 + \frac{1}{2}\cdot\alpha^2 \ \ \textbf{with} \ \lambda > 0 \tag{4}$$

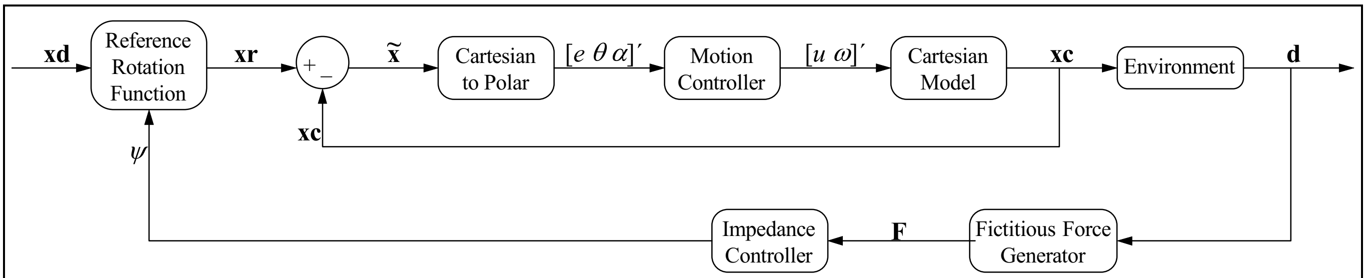

Figure 3. The block diagram of the proposed control system.

Its time derivative $\dot{V}$ along the trajectory described in (2) is given by

$$\begin{aligned} \dot{V} &= \lambda \cdot e \cdot \dot{e} + \alpha \cdot \dot{\alpha} \\ \dot{V} &= \lambda \cdot e \cdot (-u \cdot cos\alpha) + \alpha \cdot (-\omega + u \cdot \frac{sin\alpha}{e}) \\ \dot{V} &= \dot{V}_1 + \dot{V}_2 \end{aligned} \tag{5}$$

The first term in (5), corresponding to $\dot{V}_1$, can be non-positive by letting the linear velocity u have the smooth form

$$u = \gamma \cdot tanh\, e \cdot cos\alpha \quad \text{with } \lambda > 0 \tag{6}$$

It is clear that $\gamma = |u_{max}|$. According to the velocity u in (6), $\dot{V}_2$ in (5) becomes

$$\dot{V}_2 = \alpha \cdot \left(-\omega + \gamma \cdot \frac{tanh\ e}{e} \cdot sin\alpha \cdot \cos\alpha \right) \tag{7}$$

which can also be made non-positive, by letting the angular velocity ω have the smooth form

$$\omega = k \cdot \alpha + \gamma \cdot \frac{\tanh e}{e} \cdot \sin\alpha \cdot \cos\alpha \quad \text{with } k > 0 \tag{8}$$

where $|\omega_{max}| = k \cdot \pi + \gamma \cdot 0{,}5$, and thus leading to the following expression for the time derivative of the original Lyapunov function V

$$\dot{V} = -\lambda \cdot \gamma \cdot e \cdot \tanh e \cdot \cos^2 \alpha - k \cdot \alpha^2 < 0$$

$$\dot{V}(e,\alpha) = 0 \Rightarrow \begin{cases} e(t) \\ \alpha(t) \end{cases} \to 0 \text{ when } t \to \infty$$

which results in a negative definite function. This means asymptotic convergence to zero of state variables, thus verifying the control objective.

Now it is important to analyze θ in order to know about the orientation of the mobile robot when it reaches the target position.

Remark 1: Considering $\dot{\alpha}$ in equation (2) and the equation (8) and (6) we have

$$\dot{\alpha} + k \cdot \alpha = 0$$

The solution is

$$\alpha = \alpha_0 \cdot \exp(-k \cdot t) \tag{9}$$

that is bounded for all t and tends to zero as t tends to infinite. Now, by referring to equation (8), it becomes clear that $\omega(t) \to 0$ *when* $t \to \infty$.

Remark 2: From (2) and (6)

$$\dot{\theta} = \gamma \frac{tanh\ e}{e} sin\,\alpha\ cos\,\alpha \tag{10}$$

As $\alpha(t) \to 0$ when $t \to \infty$ then $\dot{\theta}(t) \to 0$ when $t \to \infty$.

Remark 3: Considering that $\frac{\tanh e}{e} \le 1$, the time integral of (10) is bounded by

$$\begin{aligned} \theta(t) - \theta(0) &= \int_0^t \dot{\theta}(t) \cdot d\tau \le \int_0^t \frac{\gamma}{2} sin\big(2 \cdot \alpha_0 \cdot \exp(k \cdot t)\big) \cdot d\tau \\ &= -\frac{\gamma}{2} \cdot \left(2 \cdot \alpha_0 \cdot \exp(-k \cdot t) - \frac{8 \cdot \alpha_0^3 \cdot \exp(-3k \cdot t)}{18} + \frac{32 \cdot \alpha_0^5 \cdot \exp(-5k \cdot t)}{600} - \dots \right)_0^t \end{aligned} \tag{11}$$

The integral is bounded for all t. This implies that $\theta(t)$ is bounded.

From remark 2 and remark 3 we see that $\theta(t) \to$ constant when $t \to \infty$. Then, the final value of the robot orientation when approaching the target position is constant, which means that the robot does not keep rotating about its own center.

Remark 4: As $\alpha = \theta - \varphi$ and α and θ are bounded as shown in remarks 1 and 3, then φ is also bounded.

5.2 Motion control II : Positioning with prescribed orientation

Let again the unicycle-like vehicle be initially positioned at any non zero distance from the target frame <g> and let the state variables be e, θ and α, directly measurable for any $e>0$. Let us consider the Lyapunov candidate function

$$\begin{aligned} V(e,\theta,\alpha) &= V_1 + V_2 \\ V(e,\theta,\alpha) &= \frac{1}{2} \cdot \lambda \cdot e^2 + \frac{1}{2} \cdot \alpha^2 + \frac{1}{2} \cdot \kappa \cdot \theta^2 \end{aligned} \tag{12}$$

with λ, $\kappa > 0$. Its time derivative $\dot{V}$ along the trajectory described in (2) is given by

$$\dot{V} = \lambda \cdot e \cdot \dot{e} + \alpha \cdot \dot{\alpha} + \kappa \cdot \theta \cdot \dot{\theta}$$

$$\dot{V} = \underbrace{\lambda \cdot e \cdot (-u \cdot \cos\alpha)}_{\dot{V}_1} + \underbrace{\alpha \cdot \left(-\omega + u \cdot \frac{\sin\alpha}{\mathrm{e}} \right) + \kappa \cdot \theta \cdot \left(u \cdot \frac{\sin\alpha}{\mathrm{e}} \right)}_{\dot{V}_2} \tag{13}$$

The first term in (13), corresponding to $\dot{V}_1$, can be non-positive by letting the linear velocity u have the smooth form

$$u = \gamma \cdot tanh\, e \cdot cos\alpha \quad \text{with } \gamma > 0 \tag{14}$$

where $\gamma = |u_{max}|$. According to the velocity u in (14), $\dot{V}_2$ in (13) becomes

$$\dot{V}_2 = \alpha \cdot \left(-\omega + \gamma \cdot \frac{tanh\, e}{e} \cdot sin\alpha \cdot cos\alpha \right) + \kappa \cdot \gamma \cdot \theta \cdot \frac{tanh\, e}{e} \cdot sin\alpha \cdot cos\alpha \tag{15}$$

which can also be made non-positive, by letting the angular velocity ω have the smooth form

$$\omega = k \cdot \left(\alpha + r \cdot \frac{\theta^2}{\alpha} \right) + \kappa \cdot \gamma \cdot \frac{\tanh e}{e} \cdot \theta \cdot \frac{sin\alpha}{\alpha} \cdot \cos\alpha + \gamma \cdot \frac{\tanh e}{e} \cdot sin\alpha \cdot \cos\alpha \tag{16}$$

with k, $r > 0$. Where $|\omega_{max}| = k \cdot (\pi + r \cdot \pi) + \kappa \cdot \gamma \cdot \pi + \gamma \cdot 0{,}5$; and thus leading to the following expression for the time derivative of the original global Lyapunov function V

$$\dot{V} = -\lambda \cdot \gamma \cdot e \cdot \tanh e \cdot \cos^2 \alpha - k \cdot \left(\alpha^2 + r \cdot \theta^2\right) < 0$$

$$\dot{V}(e,\alpha,\theta) < 0 \Rightarrow \begin{cases} e(t) \\ \alpha(t) \\ \theta(t) \end{cases} \rightarrow 0 \text{ when } t \rightarrow \infty$$

which results in a negative definite form. This means that the state variables asymptotically converge to zero in accomplishment of the control objective.

The control action of equation (16) cannot be implemented for α=0 To avoid this problem, we propose the use of a lower bound for this variable in the first term of (16). It is now necessary to verify that the stability conditions are kept.

Adding and subtracting the term $\left(k \cdot r \cdot \frac{\theta^2}{\alpha_0}\right)$, where $\alpha_0 = \delta \cdot \mathrm{sign}(\alpha), \quad \delta > 0$, equation (16) can be rewritten as

$$\omega = \omega_0 + k \cdot r \cdot \theta^2 \left[\frac{\alpha_0 - \alpha}{\alpha_0 \cdot \alpha}\right]$$

where ω_0 is the expression of (16) with α_0 in the first term, and

$$\omega = \begin{cases} \omega_0 + k \cdot r \cdot \theta \cdot \left[\frac{\alpha_0 - \alpha}{\alpha_0 \cdot \alpha}\right] & \text{if } |\alpha| \geq \delta \\ \omega_0 & \text{if } |\alpha| < \delta \end{cases} \qquad (17)$$

From equation (17), three cases can be analyzed.

Case I: $|\alpha| \geq \delta$: Here α_0 is equal to α, then

$$\dot{V}_0 = -\lambda \cdot \gamma \cdot tanh\, e \cdot e \cdot \cos^2 \alpha - k \cdot \left(\alpha^2 + r \cdot \theta^2 \cdot \frac{\alpha}{\alpha_0}\right) = \dot{V}$$

which leads to the situation already analyzed.

Case II : $|\alpha| < \delta$ and $\alpha \neq 0$ **:** Function $\dot{V}_0$ becomes

$$\dot{V}_0 = -\lambda \cdot \gamma \cdot e \cdot tanh\, e \cdot cos^2\alpha - k \cdot \left(\alpha^2 + r \cdot \theta^2 \cdot \frac{\alpha}{\alpha_0}\right)$$

In this case $0 < \alpha/\alpha_0 \leq 1$, thus implying that $\dot{V}_0$ is negative definite and asymptotic convergence of control errors to zero is again verified.

Case III : Evolution of $\theta(t)$ when α=0 and $\dot{\alpha} = 0$. In this case

$$k \cdot r \cdot \theta^2 \cdot \frac{\alpha}{\alpha_0} = 0$$

thus it is not evident the convergence to zero of signal $\theta(t)$. We can now recall the LaSalle´s theorem for autonomous systems (Vidyasagar, 1993) by noting that:

1. The system is autonomous.
2. There exists a set $\mathbf{S}(e,\theta,\alpha) / \dot{V}_0 = 0$.
3. If $\dot{V}_0 = 0$, it means that $\alpha(t) = 0$ and $e(t) = 0$. From equations (2) in closed loop

$$\dot{\theta} = \gamma \cdot \frac{tanh\; e}{e} sin\alpha \cdot \cos\alpha$$

when $\alpha(t) = 0$, $\dot{\theta}(t) = 0$ which means $\theta(t)$ *=constant.*

4. Now we can obtain the constant value of $\theta(t)$ in the set **S**. From equations (2) in closed loop, when $\alpha(t) = 0$ and $e(t) = 0$ and consequently $\dot{\alpha}(t) = 0$

$$\dot{\alpha} = -\underbrace{\alpha}_{=0} - k \cdot \frac{\overbrace{\theta^2}^{=ctte}}{\alpha_0} - \kappa \cdot \gamma \cdot \overbrace{\theta}^{=ctte} \cdot \underbrace{\frac{tanh\, e}{e}}_{=1} \cdot \underbrace{\frac{sin\alpha}{\alpha}}_{=1} \cdot \underbrace{cos\alpha}_{=1} = 0$$

It is immediately concluded that $\theta = 0$ in **S**. Following La Salle theorem, this means that control error signals converge asymptotically to zero.

As a general conclusion and since for the three cases the error signals converge asymptotically to zero, the control objective is guaranteed for the controller with bounded ω control action.

5.3 Impedance Control

The desired impedance is defined as

$$Z = Bs + K$$

and the impedance relationship is taken as

$$x_a = Z^{-1} \cdot F_t \qquad (18)$$

where

B, K are positive constants;

F_t is the component of **F** (fictitious force) in the direction of robot movement.

Constant B represent a damping effect and K a spring effect in the interaction between the mobile robot and the obstacle.

By referring to figure 3, it is taken

$$\psi = x_a \cdot sign(Fr)$$

where F_r is the component of **F** (fictitious force) perpendicular to the direction of robot movement. Then, the transformation

$$\mathbf{x_r} = \begin{bmatrix} cos\psi & sen\psi & 0 \\ -sen\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x_d}$$

is applied, and the position error is calculated as $\tilde{\mathbf{x}} = \mathbf{x_r} - \mathbf{x_c}$

When the fictitious force is zero, $\mathbf{x_r} = \mathbf{x_d}$, and the objective of the motion control loop is achieved, meaning that $\tilde{\mathbf{x}} \rightarrow 0$ as $t \rightarrow \infty$. The Cartesian to polar coordinate transformation is performed through

$$\begin{cases} e = \sqrt{\left(x_d - x_c\right)^2 + \left(y_d - y_c\right)^2} \\ \eta = atan3\left[\left(y_d - y_c\right),\left(x_d - x_c\right)\right] \\ \theta = \eta - \varphi_d \\ \alpha = \eta - \varphi_c \end{cases}$$

where : $\mathbf{x_c}$'=[$x_c \;\; y_c \;\; \varphi_c$] is the vector of Cartesian coordinates of the robot, $\mathbf{x_d}$'=[$x_d \;\; y_d \;\; \varphi_d$] is the vector of Cartesian coordinates of the destination position and *atan*3 is the arc tangent function that covers a 2π range angle in positive and negative directions.

6. Simulation Example

In the following simulation examples, the values u_{max}=0.3m/sec and ω_{max}=0.8rad/sec have been considered in order to select design parameters and avoid saturation of control actions. The parameters value of the impedance controller are K=3 Nt/rad and B=0.02 Nt.sec/rad.

6.1 Motion Control I : Positioning without prescribed

orientation (x_d=3.5, y_d=5)

Figure 4 shows the trajectory described by the mobile robot, for a case in which an obstacle appears on its original trajectory towards the target.

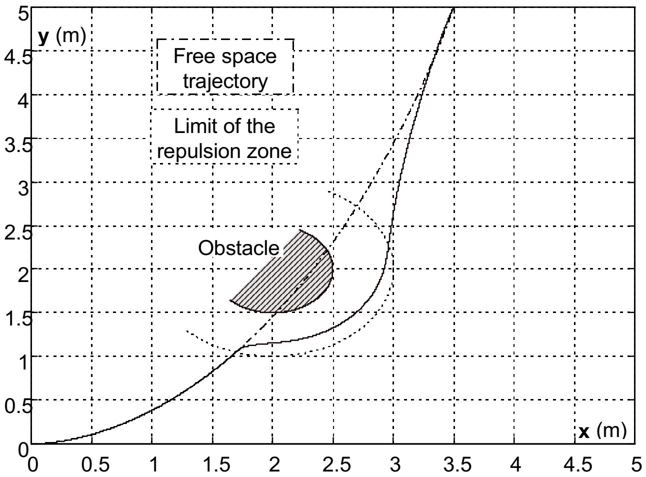

Figure 4. Trajectory described by the mobile robot to avoid an obstacle on its path.

Figure 5 shows the corresponding linear and angular velocities along that trajectory. For this example, the impedance control loop is active when the mobile robot finds an obstacle at less than 0.5 m.

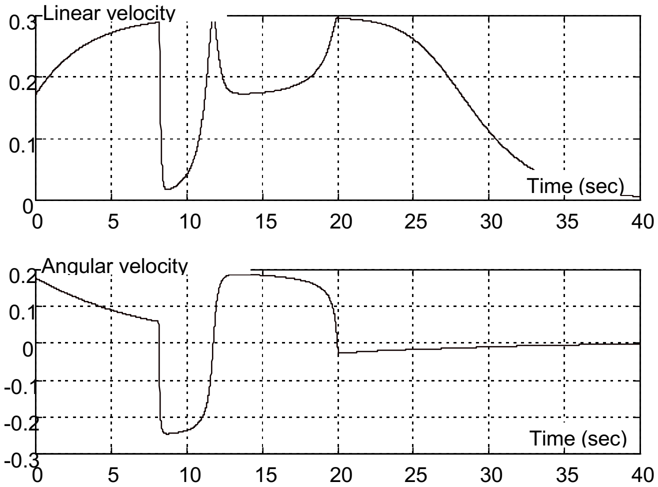

Figure 5. Linear and angular velocities of the mobile robot when avoiding obstacles.

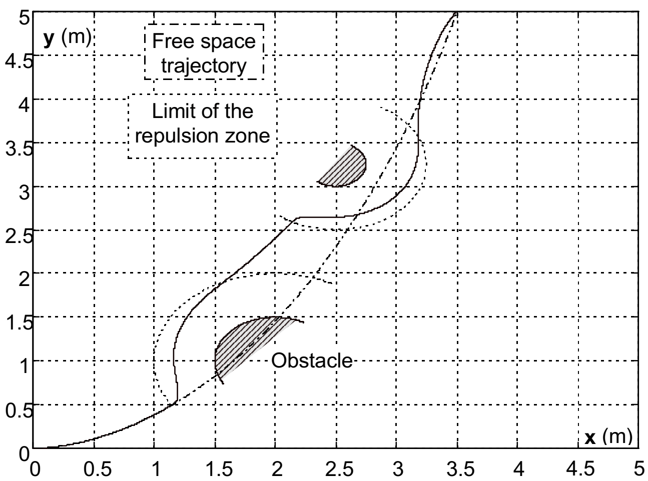

Figure 6. Trajectory described by the mobile robot to avoid two obstacles on its path.

Figure 6 shows the trajectory described by the mobile robot, for a case in which two obstacles appear on its original trajectory towards the target.

6.2 Motion Control II : Positioning with prescribed orientation (x_d=3.5, y_d=5, φ_d=0)

Figure 7 shows the trajectory described by the mobile robot, for a case in which an obstacle appears on its original trajectory towards the target. Figure 8 shows the corresponding linear and angular velocities along that trajectory.

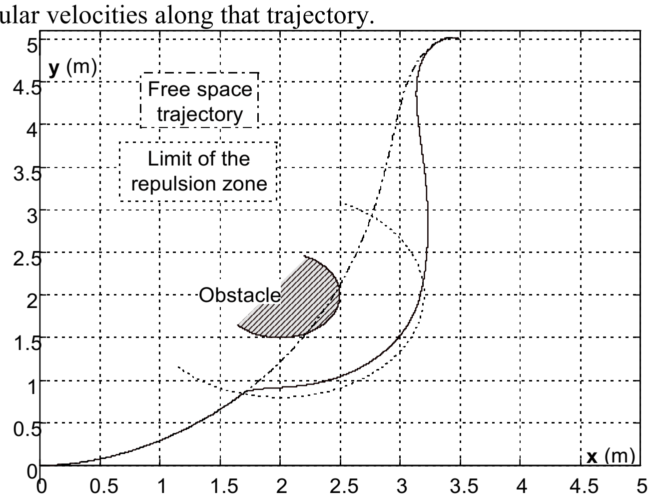

Figure 7. Trajectory described by the mobile robot to avoid an obstacle on its path.

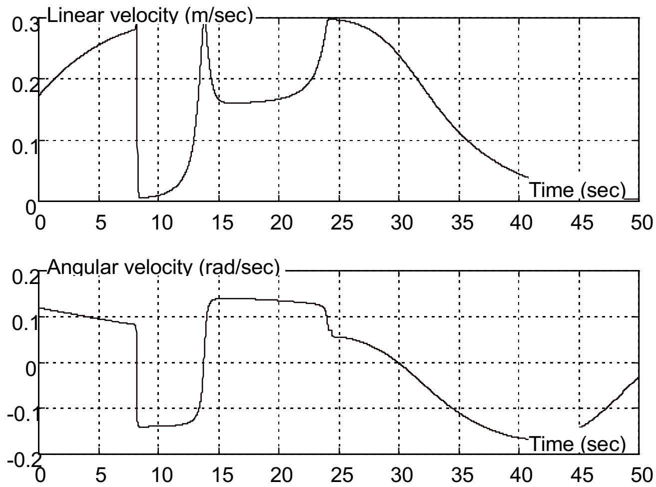

Figure 8. Linear and angular velocities of the mobile robot when avoiding obstacles.

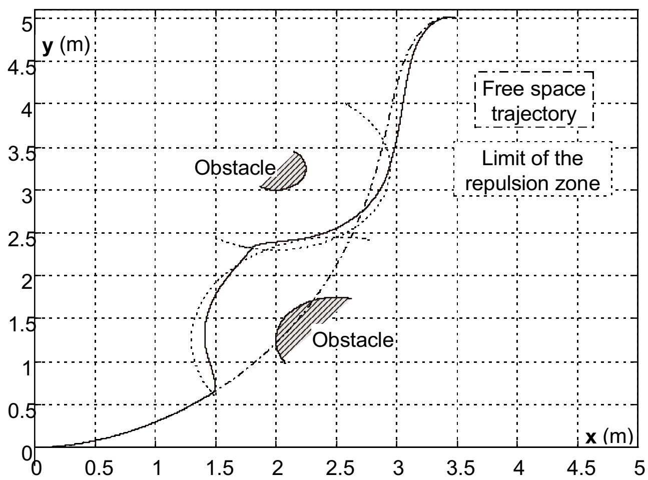

Figure 9. Trajectory described by the mobile robot to avoid two obstacles on its way.

Figure 9 shows the trajectory described by the mobile robot, for a case in which two obstacles appear on its original trajectory towards the target.

For this example, the impedance control loop is active only when the mobile robot finds an obstacle at less than 0.7 m.

7. Conclusions

This paper presents a simple and effective closed loop control law for a unicycle-like vehicle, combined with an effective control law for obstacle avoidance. Two control objectives are considered: positioning with and without final orientation of the vehicle. The control systems is structured based in two loops, the position control loop and the impedance control loop. Impedance is defined in reference to a fictitious force as a function of the sensed distance to any obstacles close to the robot path. The controller keeps position error *e* within admissible bounds in order to avoid saturation of control actions. The control system is proved to globally and asymptotically drive the control errors towards zero. Simulations have been carried out in order to show the good performance properties of the proposed control system.

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