Project 3

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1 Method

We assume that the earths orbit around the sun is circular. And from circal motion and Newtons gravitational force we have that

$$F_G = G\frac{M_E M_\odot}{r^2} = \frac{M_E v^2}{r} \tag{1}$$

We now want to show that the velocity, v, of the earth can be written as

$$v^2 r = GM_\odot = 4\pi^2 \frac{AU^3}{yr^2} \tag{2}$$

Since we say it is circular orbits can we use the centripetal force to rewrite equation 2

$$M_E \omega^2 r = G\frac{M_E M_\odot}{r^2} \tag{3}$$

where ω^2 is the angular velocity of the earth. Equation 3 can now be rewritten as

$$M_E(\frac{2\pi}{P})^2 r = G\frac{M_E M_\odot}{r^2} \tag{4}$$

$$\tag{5}$$

Where P is a period of the earth around the sun. Now can we use Keepler's third law, which says that the square of an orbital period P^2 equals the cube of the semi-major axis of its orbiting a^3

$$P^2 = a^3 \tag{6}$$

Where we substitute the circular radius r with the semi-major axis a.

1.1 Forward Euler

The forward Euler method is a algorithm to estimate the solution of a differential equation. The Forward Euler method wants to find the next point. To find the next point, it uses the point it is at, r_n, a small time step, dt, and the derivative of its position. Which can be expressed like this

$$y_{n+1} = y_n + y'_n \cdot dt \tag{7}$$

Where y_{n+1} is the next step, y_n is the current step, y'_n is the derived of the current step and dt is the time step. This algorithm is really based abbreviated version of a Taylor expansion, where we only expand the series one step at a time. But by only taking one step at a time, we will also get a local truncation error, which causes an error for each step we take.

$$
\begin{aligned}
y(t_n + dt) &= y_{n+1} \\
&= y(t_n) + y'_n \cdot dt + R(dt^2)
\end{aligned} \tag{8}
$$

Where $R(dt^2)$ is the local truncation error. Since the Forward Euler method is a first-order method, will the local truncation error be proportional to the square of the step size.

1.2 Velocity Verlet method

The velocity Verlet method is based on the kinematic equations for an moving object, which in our case is the earth's orbit around the sun. If we want to find the next time step for the velocity and position we do a approximation and uses Taylor-expansion

$$v_{t+dt} = v_t + \frac{dt}{2}(\frac{F_t}{m} + \frac{F_{t+m}}{m})R(dt^3) \tag{9}$$

We can also split the equation above and perform the calculation in several steps like this:

$$v_{t+dt} = v_t + \frac{dt}{2}(\frac{F_t}{m} + \frac{F_{t+m}}{m})R(dt^3) \tag{10}$$

We can also split the equation above and perform the calculation in several steps like this:

$$
\begin{aligned}
v(t + \frac{1}{2}\Delta t) &= v(t) + \frac{1}{2}a(t)\Delta t \\
x(t + \Delta t) &= x(t) + v(t + \frac{1}{2}\Delta t)\Delta t \\
a(t + \Delta t) &= f(x(t + \Delta t)) \\
v(t + \Delta t) &= v(t + \frac{1}{2}\Delta t) + \frac{1}{2}a(t + \Delta t)\Delta t
\end{aligned} \tag{11}
$$

1.3 Conservation of angular momentum

The definition of Keeplers law is that a area, where a connecting line between a planet and the sun, is always the same for an equal time interval. Which is illustrated in the figure below:

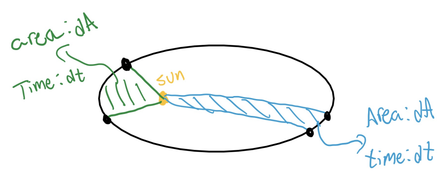

Figure 1: Keeplers second law. Where the area is the same for the same time interval.

The best way to show that the angular momentum is conserved by using Keeplers second law is to make a drawing.

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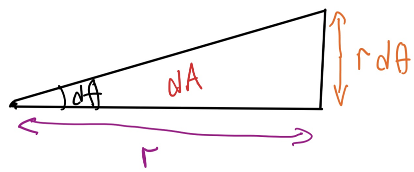

Figure 2: This square is the infinitesimal area dA that the planet has moved by a infinitesimal time interval dt

Figure 2 shows us that the area is

$$dA = \frac{1}{2} r^2 d\theta \tag{12}$$

We now want to find an infinitesimal area where the planet has moved around the sun, over an infinitesimal time step, where we use equation 12.

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \tag{13}$$

$$= \frac{1}{2} r v_\theta \tag{14}$$

And the definition of angular momentum is given as

$$L = m r v_\theta \tag{15}$$

With equation 14 and 15 we can show that the angular momentum is conserved.

2 Results

2.1 Conservation of angular momentum

If we substitute equation 15 into equation 14

$$\frac{dA}{dt} = \frac{1}{2} \frac{L}{m} \tag{16}$$

Sine $\frac{dA}{dt}$=constant and the mass m is constant means that the angular momentum also is constant and therefore also conserved.

A Calculations

References

[1] Skriv inn kilde her.