The ising model

Sebastian Amundsen, Marcus Berget and Andreas Wetzel

November 13, 2020

1 Introduction

2 Method

The analytical expression

We have the normalization constant Z which defines the partition function:

$$Z(\beta) = \sum_s e^{(-\beta E_s)} \tag{1}$$

With $\beta = 1/T$, where T is temperature. We can use this partition function to find the probability P_s of finding a system in a state s:

$$P_s = \frac{e^{-(\beta E_s)}}{Z} \tag{2}$$

Where E_s is the energy in a given state. We have that the mean energy E_m is given by:

$$E_m = \sum_s \frac{E_s e^{-(\beta * E_s)}}{Z} \tag{3}$$

We have that the mean magnetization $\langle M \rangle$ is given by:

$$\langle M \rangle = \sum_s^M M_s P_s(\beta) = \frac{1}{Z} \sum_s^M M_s e^{-(\beta E_s)} \tag{4}$$

Where M_s is the different magnetizations.

2.1 Metropolis algorithm

The metropolis algorithm only considers ratios between probabilities, which means that we do not need to calculate the partition function at all when we are using the algorithm.

3 Results

Table 1: Energy and magnetization given number of up spins.

$N_{\text{spins up}}$	Degeneracy	E	M
4	1	-8 J	4
3	4	0	2
2	4	0	0
2	2	8 J	0
1	4	0	-2
0	1	-8 J	-4

4 Discussion

5 Concluding remarks

A Calculations

We find the mean magnetization by using equation 4:

$$\begin{aligned} \langle M \rangle &= 4 \times P(-8J) + 4 \times 2 \times P(0) + 4 \times 0 \\ &+ 2 \times 0 + 4 \times 2 \times P(0) + 4 \times 1 \times P(-8) \end{aligned} \tag{5}$$

References

[1] kilder