# Project 3

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# 1 Method

## 1.1 Forward Euler

The forward Euler method is a algorithm to estimate the solution of a differential equation. The Forward Euler method wants to find the next point. To find the next point, it uses the point it is at, $r_n$, a small time step, $dt$, and the derivative of its position. Which can be expressed like this

$$y_{n+1} = y_n + y'_n \cdot dt \tag{1}$$

Where $y_{n+1}$ is the next step, $y_n$ is the current step, $y'_n$ is the derived of the current step and $dt$ is the time step.
This algorithm is really based abbreviated version of a Taylor expansion, where we only expand the series one step at a time. But by only taking one step at a time, we will also get a local truncation error, which causes an error for each step we take.

$$y(t_n + dt) = y_{n+1} = y(t_n) + y'_n \cdot dt + R(dt^2) \tag{2}$$

Where $R(dt^2)$ is the local truncation error.
Since the Forward Euler method a first-order method, will the local truncation error be proportional to the square of the step size.

## 1.2 Verlet method

$$v_{t+dt} = v_t + \frac{dt}{2}(\frac{F_t}{m} + \frac{F_{t+m}}{m})R(dt^3) \tag{3}$$