FYS3150-Project 1

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Abstract

1 Introduction

2 Method

By multiplying **A** and **v** we get

$$\begin{bmatrix} 2 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & & \\ 0 & \vdots & \vdots & -1 & 2 & -1 \\ 0 & \vdots & \vdots & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ \vdots \\ v_n \\ v_{n+1} \end{bmatrix} = \begin{bmatrix} 2v_0 - v_1 \\ -v_0 + 2v_1 - v_2 \\ -v_1 + 2v_2 - v_3 \\ \vdots \\ -v_{n-1} + 2v_n - v_{n+1} \\ -v_n + 2v_{n+1} \end{bmatrix}$$

$$= \begin{bmatrix} h^2 f_0 \\ h^2 f_1 \\ h^2 f_2 \\ \vdots \\ h^2 f_n \\ h^2 f_{n+1} \end{bmatrix} = \widetilde{\mathbf{b}}$$

2.1 General algorithm

2.2 Algorithm for specific tri-diagonal matrix

In our special case we can implement a solver that is even simpler than what is described previously. We will exploit the fact that the matrix has identical matrix elements along the diagonal and identical values for the non diagonal elements $\vec{e}_i$. In this case we can precalculate the new values for the updated matrix elements d_i without taking into account the values for $\vec{e}_i$:

$$d_i = 2 - \frac{1}{\tilde{d}_{i-1}} = \frac{i+1}{i} \tag{1}$$

Here the initial value is $\tilde{d}_1 = 2$. The new righthand side solution $\tilde{f}_i$ is given by:

$$\tilde{f}_i = f_i + \frac{(i-1)\tilde{f}_{i-1}}{i} \tag{2}$$

Here the initial value is $\tilde{f}_1 = f_1$. The last step is to make a backward substitution which gives the final solution u_i:

$$u_{i-1} = \frac{i-1}{i}(\tilde{f}_{i-1} + \tilde{u}) \tag{3}$$

This method requires that we know the last value u_n in the u_i array. This value is given by $u_n = \tilde{f}_n/\tilde{b}_n$.

3 Implementation

All programs used is available at:
`https://github.com/Sebamun/FYS3150_Projekter`

We implement the algorithms numerically with varying values of grid points n.

3.1 c)

4 Results

5 Concluding remarks