Diffusion

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1 Introduction

2 Theory

2.1 Heat equation

The general heat equation can be written as,

$$\frac{\partial T(\mathbf{x},t)}{\partial t} = \frac{k}{C\rho}\nabla^2 T(\mathbf{x},t). \tag{1}$$

where $\mathbf{x}$ is the spatial vector, t is time, c_p is the specific heat capacity, ρ is the density and k is the thermal conductivity. We can then gather all the constants in the diffusion constant, $D = \frac{k}{C\rho}$. For the first part of the project we will just set the diffusion constant equal to one. The heat equation in one dimension then becomes,

$$\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2}, \tag{2}$$

or

$$T_{xx} = T_t. \tag{3}$$

2.2 Numerical methods for solving the heat equation

We set the initial conditions of equation (2) at $t = 0$ to,

$$T(x,0) = 0, \quad 0 < x < L \tag{4}$$

where $L = 1$ is the length of the x-region of interest. We set the boundary conditions to

$$T(0,t) = 0, \quad t \geq 0, \tag{5}$$

and

$$T(L,t) = 1, \quad t \geq 0. \tag{6}$$

Equation (2) with the mentioned initial conditions and boundary conditions can be solved numerically using the forward Euler method, the backward Euler method and the implicit Crank-Nicholson scheme.

3 Method

4 Implementation

5 Results

6 Discussion

7 Concluding remarks

A Appendix

References

References

[1] Jensen, M.H., 2015, Computational Physics Lecture Notes Fall 2015

[2] Jensen, M.H., 2017, Computational Physics Lectures: Statistical physics and the Ising Model