Computational Mathematics TC2020

Assignment 01

Sebastián A. Saldaña
A01570274
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Preliminaries: Sets, relations and functions

1. Calculate the result of the following operations (4 marks):

   (a) $\{a, b, c\} \times \{1, 2, 3, 4\}$
   $S = \{(a, 1), (a, 2), (a, 3), (a, 4), (b, 1), (b, 2), (b, 3), (b, 4), (c, 1), (c, 2), (c, 3), (c, 4)\}$

   (b) $\{a, \{b\}, \{\{c\}\}\} \times \emptyset$
   $S = \emptyset$

   (c) $\mathcal{P}(\{x : x \in \mathbb{N}, x < 4\})$
   $S = \{\emptyset, 1, 2, 3, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

   (d) $|\mathcal{P}(\{y : y \in \mathbb{Z}, 0 < y < 10\})| = 512$

2. The following are relations from $\{1, 2, 3, 4\}$ to $\{1, 2, 3, 4\}$. Indicate which relations are **transitive**, **reflexive**, or **symmetric**. Also indicate which are **functions** and which one are only **relations**. In case of being functions, indicate if they are **total** or **partial** functions, and which ones are **injective**, **surjective** and **bijective** (4 marks).

   (a) $\{(2, 2), (3, 3), (1, 1), (4, 4)\}$ ***This is a reflexive, transitive, and symmetric relation which is also a total, bijective function.***

   (b) $\{(1, 1), (2, 2), (3, 3), (4, 3)\}$ ***This is a transtivie relation and a total,surjective function.***

   (c) $\{(1, 1), (3, 4), (2, 2), (3, 3)\}$ ***This is only a relation***

   (d) $\{(1, 1), (2, 2), (3, 3)\}$ ***This is a reflexive, transitive, and symmetric relation which is also a partial, surjective function.***

3. Using the information from the lecture, investigate what is the **transitive closure**. Then, write its definition in your own words. Finally, find the transitive closure for each of the relations of the previous problem (2 marks)[1].
   A **transitive closure** is the set of individual relations between elements of a relation that encapsulates all of these elements that will make it transitive.

   (a) $R^* = \{(2, 2), (3, 3), (1, 1), (4, 4)\}$

   (b) $R^* = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$

   (c) $R^* = \{(1, 1), (3, 4), (2, 2), (3, 3), (4, 3)\}$

   (d) $R^* = \{(1, 1), (2, 2), (3, 3)\}$

[1] Do not forget to cite the sources properly. Consider that they are reliable sources and, if possible, list two or three resouces; do not consider just the first definition you find.

Bibliography

[1] Udacity. (2015). *Transitive Closure - Georgia Tech - Computability, Complexity, Theory: Algorithms.* Recovered from `https://www.youtube.com/watch?v=NM0mAmylfMg`

[2] GVSUMath. (2015). *Transitive Closure.* Recovered from: `https://www.youtube.com/watch?v=0O8Jfs9uZnc`