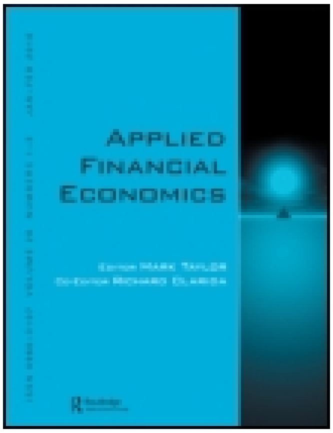

Applied Financial Economics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rafe20

Nonlinear short-run adjustments in US stock market returns

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Published online: 07 Jul 2008.

Applied Financial Economics, 2008, **18**, 1075–1083

Routledge
Taylor & Francis Group

Nonlinear short-run adjustments in US stock market returns

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Using the considerably powerful nonparametric cointegration tests proposed by Bierens (1997, 2004), we do not find any evidence indicative of the existence of rational bubbles in the US stock market during the long period of 1871 to 2002. In addition, with the application of a logistic smooth transition error-correction model designed to detect the nonlinear short-run adjustments to the long-run equilibrium, we also obtain substantial empirical evidence in favour of the so-called noise trader models where arbitrageurs are reluctant to immediately engage in trading when stock returns deviate insufficiently from their fundamental value.

I. Introduction

During the past few decades, a wealth of studies has been undertaken to investigate the relationship between stock prices and dividends from both the theoretical and empirical points of view (see, for example, Campbell and Shiller, 1987; Han, 1996; Taylor and Peel, 1998; Caporale and Gil-Alana, 2004; McMillan, 2004). From a theoretical perspective, the stock valuation model is based on the premise that stock prices are dependent upon the present value of the discounted future dividends, where the discount rate is equal to the required rate of return. Based on the proposed theory, stock prices and dividends would be cointegrated with log returns, which depend on the log dividends minus the log stock prices. Thus, it can be interpreted that stock returns can be predicted from the dividend yields. This relationship, however, cannot be expected to exactly hold true for long especially since deviations commonly arise because of the time-varying required rate of return, speculative bubbles and fads or the omission of other relevant variables such as retained earnings. For this reason, it is obvious that returns can be modelled with a linear error-correction (EC) approach (Campbell and Shiller, 1987).

Recent research has advocated that the relationship between stock prices and dividends may best be characterized by using a nonlinear model. For example, theoretical models studying the interaction between arbitrageurs and noise traders have generally suggested that small and large deviations from long-run equilibrium may exhibit very different return dynamics since arbitrageurs must constantly be wary of the possibility that noise traders may drive returns far away from equilibrium. More specifically, the issue emphasized here is the difference between the dynamics governing small deviations from the fundamental equilibrium and those governing large deviations.

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Applied Financial Economics ISSN 0960–3107 print/ISSN 1466–4305 online © 2008 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/09603100701408148

The major contribution of this research is to compare the performance of the linear EC models with that of the nonlinear EC models, including a logistic smooth transition error-correction (LSTEC) model, for US stock market returns over the long period of 1871 to 2002. The LSTEC model is capable of capturing the market dynamics that differentiate between small and large deviations from long-run equilibrium, and more importantly it also allows for a gradual transition between regimes, which is consistent with the 'stylized facts' of a slow mean reversion in asset returns (see, Campbell *et al.*, 1997; McMillan, 2004).

The remainder of this article is organized as follows. Section II describes the sample data. Section III presents the research methodology and discusses the empirical results. Section IV summarizes the conclusions.

II. Sample Data

We analyse the US Standard and Poor's stock price index and dividend data over the period of 1871 to 2002, which was collected from Professor Shiller's Web site http://aida.econ.yale.edu/~shiller. The variables of log dividends and log stock prices do not follow the normal distribution and are time serially correlated. The descriptive statistics of the sample data are summarized in Table 1.

III. Research Methodology and Empirical Results

Unit root tests

A significant consensus has been emerging in the recent research, i.e. the financial time series data may exhibit nonlinearities; thus the conventional tests for stationarity such as the Augmented Dickey–Fuller (ADF) unit root tests may not be able to detect the mean-reverting tendency of financial time series variables. Should this indeed be the case, it would be necessary to perform the stationary tests in a nonlinear framework. Therefore, we adopt the nonlinear stationary test advanced by Kapetanios *et al.* (2003) (henceforth, the KSS test) in our study.

Central to the KSS test is the goal to detect the presence of nonstationarity against a nonlinear but globally stationary exponential smooth transition autoregressive (ESTAR) process. The model is expressed as follows.

$$\Delta Y_t = \gamma Y_{t-1}\{1 - \exp(-\theta Y_{t-1}^2)\} + \nu_t \tag{1}$$

where Y_t is the time series data studied, ν_t is an independently identically distributed error term with a zero mean and constant variance, and $\theta \geq 0$ is the transition parameter of the ESTAR model and governs the speed of transition. Under the null hypothesis, Y_t follows a linear unit root process, but under the alternative hypothesis, Y_t follows a nonlinear stationary ESTAR process. One shortcoming in this framework is that the parameter γ is not identified under the null hypothesis. Thus, Kapetanios *et al.* (2003) used a first-order Taylor series approximation for $\{1 - \exp(-\theta Y_{t-1}^2)\}$ under the null hypothesis of $\theta = 0$ and then approximated equation 1 by using the following auxiliary regression:

$$\Delta Y_t = \xi + \delta Y_{t-1}^3 + \sum_{i=1}^{k} b_i \Delta Y_{t-i} + \nu_t, \quad t = 1, 2, \ldots, T \tag{2}$$

Under this framework, the null hypothesis and the alternative hypothesis are expressed as $\delta = 0$ (nonstationarity) against $\delta < 0$ (nonlinear ESTAR stationarity). Table 2 presents the KSS nonlinear stationarity test results, and these results clearly indicate that both the US stock prices and dividend series are nonstationary, and become stationary in the first difference. The KSS nonlinear stationarity test is further applied to test whether the EC term between the US stock prices and dividend series follows a nonlinear stationary process. The results of the KSS test in Table 2 also indicate that the EC term follows a nonlinear stationary process, which suggests the possible existence of the cointegration relationship between the stock prices and dividends.

For the sake of comparison, we also incorporate the ADF tests, the Phillips and Perron (1988, PP) tests, and the Kwiatkowski *et al.* (1992, KPSS) tests into our study and the results are shown in Table 3. The results imply that the US stock prices and dividends are both nonstationary in levels but become stationary in the first differences, further signifying that stock prices and dividends are integrated of order one, I(1). On the basis of these results, we proceed to test whether these two variables are cointegrated by using the considerably powerful nonparametric cointegration tests proposed by Bierens (1997, 2004).

Table 1. Descriptive statistics of sample data

	Log dividends		Log stock prices	
Mean	0.089301		3.199521	
Median	−0.371064		2.531693	
Maximum	2.814810		7.262341	
Minimum	−1.714798		1.178655	
SD	1.370746		1.609908	
Skewness	0.594452		0.842605	
Kurtosis	2.096241		2.616387	
Jarque–Bera	12.26651	(0.002170)***	16.42902	(0.000271)***
Ljung–Box $Q(4)$	527.0010***		530.4523***	
Ljung–Box $Q(8)$	1037.3965***		1053.6474***	
Ljung–Box $Q^2(4)$	527.7738***		512.8241***	
Ljung–Box $Q^2(8)$	1023.7277***		975.3679***	

Notes: Numbers in parentheses indicate the *p*-value for the Jarque–Bera normality test statistics.
***Denotes significance at the 1% level.

Table 2. The nonlinear KSS unit root tests

	Log stock prices		Log dividends		EC
KSS	Level	1st Diff	Level	1st Diff	Level
t-Statistics of $\hat{\delta}$	0.674838 (3)	−1.946172 (6)*	0.175361 (1)	−2.568610 (10)**	−2.1204* (1)

Notes: Critical values for the t statistics of $\hat{\delta}$ are tabulated in Kapetanios *et al*. (2003).Critical values for 10, 5 and 1% are −1.92, −2.22 and −2.82, respectively.
* and ** Denote significance at the 10 and 5% levels, respectively.
Numbers in parentheses indicate the lag length (k) of the following testing model.

$$\Delta Y_t = \xi + \delta Y_{t-1}^3 + \sum_{i=1}^{k} b_i \Delta Y_{t-i} + v_t, t = 1, 2, \ldots, T$$

Nonparametric cointegration test of Bierens (1997, 2004)

Various studies have documented discrepancies between the conventional Johansen's test and Bierens' nonparametric cointegration approach. Coakley and Fuertes (2001) found that the results from the nonparametric cointegration approach support a equilibrium relationship between the spot exchange rates and their relative prices (consumer price index and wholesale price index) based on the purchasing power parity (PPP) for 18 OECD economies, whereas the standard Johansen's tests yield mixed evidence. Moreover, Davradakis (2005) also demonstrated that although the Johansen's tests did not find a long-run relationship between monetary fundamentals and the dollar spot exchange rates for 19 countries, the nonparametric cointegration approach indicates there is a cointegrating relationship for the majority of the countries studied. These discrepancies can be interpreted as a consequence of the nonlinearities in the underling variables. The conventional cointegration framework presents a misspecification problem when the true nature of the adjustment process is nonlinear and the speed of adjustment varies with the magnitude of the disequilibrium, see Coakley and Fuertes (2001).

Much like the properties in the Johansen's approach (Johansen, 1988; Johansen and Jueslius, 1990), the cointegration test of Bierens (1997, 2004) is also derived from the solutions to a generalized eigenvalue problem. The main difference is that in the Bierens' nonparametric cointegration approach, the problem associated with the generalized eigenvalue is formulated on the basis of two random matrices which are constructed independently of the data generating process (DGP). In this research, we construct these matrices which consist of the weighted means of the system variables in levels and in the first

Table 3. The conventional unit root tests for log stock prices and log dividends

	ADF		PP		KPSS	
	Level	1st Difference	Level	1st Difference	Level	1st Difference
Panel A: The conventional unit root tests for log stock prices						
Intercept	1.2693 (0)	−10.2672*** (0)	2.0097 (11)	−10.2264*** (6)	1.3069*** (9)	0.4661 (6)
Trend	−1.3368 (0)	−9.6395*** (1)	−1.0495 (7)	−10.8291*** (12)	0.3156*** (9)	0.0466 (11)
Panel B: The conventional unit root tests for log dividends						
Intercept	0.8164 (2)	−8.2875*** (1)	1.1727 (15)	−8.5167*** (17)	1.3274*** (9)	0.3397 (12)
Trend	−2.8124 (1)	−8.5091*** (1)	−1.9938 (15)	−9.3137*** (23)	0.3053*** (9)	0.0701 (18)

Note: ***Denotes significance at the 1% level.

differences such that their generalized eigenvalues share the similar properties to those in the Johansen's approach.

The Bierens' nonparametric cointegration test considers the general framework as follows.

$$z_t = \pi_0 + \pi_1 t + y_t \tag{3}$$

where π_0 and π_1 are the optimal mean and trend terms, respectively, and y_t is a zero-mean unobservable process such that Δy_t is stationary and ergodic. Apart from these conditions of regularity, the method does not require any further specifications of DGP for z_t, and in this sense, it is completely nonparametric.

Bierens' method is based on the generalized eigenvalues of the matrices A_m and $(B_m + cT^{-2}A_m^{-1})$, where A_m and B_m are defined as follows.

$$A_m = \frac{8\pi^2}{T}\sum_{k=1}^{m} k^2\left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(\frac{2k\pi(t-0.5)}{T}\right)z_t\right) \times \left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(\frac{2k\pi(t-0.5)}{T}\right)z_t\right)' \tag{4}$$

$$B_m = 2T\sum_{k=1}^{m}\left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(\frac{2k\pi(t-0.5)}{T}\right)\Delta z_t\right) \times \left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(\frac{2k\pi(t-0.5)}{T}\right)\Delta z_t\right)' \tag{5}$$

which are computed as the sums of the outer-products of the weighted means of z_t and Δz_t, where T is the sample size and c is a positive constant. To ensure the invariance of the test statistics to the drift terms, the weighted functions of $\cos(2k\pi(t-0.5)/T)$ are recommended here. Similar to the properties of the Johansen's likelihood ratio test, the ordered generalized eigenvalues that we obtain from this nonparametric method are the solutions to the problem of $\det[P_T - \lambda Q_T] = 0$ when the pair of random matrices P_T and Q_T are defined as $P_T = A_m$ and $Q_T = (B_m + cT^{-2}A_m^{-1})$. Thus, this method can be used to test the hypothesis of cointegration of rank r. To estimate r, Bierens (1997, 2004) proposed two statistics the λ min and $g_m(r_0)$. The λ min statistic, which corresponds to the Johansen's maximum likelihood procedure, tests the hypothesis of $H_0(r)$: cointegration of rank r against $H_1(r+1)$: cointegration of rank $r+1$. The critical values for this test are tabulated in Bierens (2004). The $g_m(r_0)$ test statistic is computed from Bierens' generalized eigenvalues as follows.

$$\hat{g}_m(r_0) = \begin{cases} \left(\prod_{k=1}^{n}\hat{\lambda}_{k,m}\right)^{-1}, \text{ if } r_0 = 0 \\ \left(\prod_{k=1}^{n-r}\hat{\lambda}_{k,m}\right)^{-1}\left(T^{2r}\prod_{k=n-r+1}^{n}\hat{\lambda}_{k,m}\right) \\ \quad \text{if } r_0 = 1, \ldots, n-1 \\ T^{2n}\prod_{k=1}^{n}\hat{\lambda}_{k,m}, \text{ if } r_0 = n \end{cases} \tag{6}$$

This statistic employs the tabulated optimal values in Bierens (1997) for m when $r_0 < n$, while $m = n$ when $r_0 = n$, where n is the number of system variables. This verifies $\hat{g}_m(r_0) = O_p(1)$ if $r = r_0$ and will approach infinity in probability if $r \neq r_0$. A consistent estimate of r is therefore derived from $\hat{r}_m = \arg\min_{r_0 < n}\{\hat{g}_m(r_0)\}$. This statistic is valuable when reconfirming the determination of r. Moreover, as pointed out by Bierens (1997), one of the major advantages of this nonparametric cointegration test lies in its superiority to detect cointegration especially when the EC mechanism is nonlinear. The nonlinear DGP of the EC term may be due to the existence of transaction costs, [Coakley and Fuertes (2001) and Davradakis (2005)].

Table 4. Bierens' nonparametric cointegration tests for log stock prices and log dividends

Hypotheses	λ min Test statistics	5% Critical value	10% Critical value
Conclusion $r=1$			
H_0: $r=0$	0.00335**	(0, 0.017)	(0, 0.050)
H_1: $r\geq 1$			
H_0: $r\leq 1$	12.718	(0, 0.054)	(0, 0.111)
H_1: $r\geq 2$			
Cointegration rank (r)		$g_m(r_0)$	
$g_m(r_0)$ Test statistics			
$r_0=0$		23.911	
$r_0=1$		4.437	
$r_0=2$		12.317	

Notes: ** Denote significance at the 5% level, respectively. The λ min test is based on Bierens' generalized eigenvalues of the matrices of P_T and Q_T, where $P_T=A_m$ and $Q_T=(B_m+cT^{-2}A_m^{-1})$ and A_m and B_m are computed as the sums of the outerproducts of the weighted means of Z_t and ΔZ_t, where $z_t=\pi_0+\pi_1 t+y_t$. T is the sample size, and c is a positive constant. The value of c is 1, as suggested in Bierens (2004).The critical values are from Bierens (2004). If the value of the λ min statistic is outside the critical region, we do not reject the null hypothesis. However, if the value of the λ min statistic is within the critical region, we would reject the H_0. If both of the null hypotheses are not rejected, we conclude that $r=0$, i.e. there is no cointegration, where r denotes the number of cointegrating vectors (see Bierens, 2004).

Table 4 presents the results of both the λ min and $g_m(r_0)$ test statistics. The λ min test results imply that there is a long-run relationship between log stock prices and log dividends, a finding which is further supported by the $g_m(r_0)$ test results. The minimum value of the $g_m(r_0)$ statistics appears in the cointegration rank of $r=1$. Thus, the long-run cointegration equilibrium relationship between stock prices and dividends indicates a sign of the absence of rational bubbles in the US stock market during the period of 1871 to 2002.

Nonlinear tests and estimations from the logistic STEC model

Stock valuation models customarily assume that log stock returns are determined by a linear relationship between the cointegrated log dividends and log stock prices and that any deviations from this fundamental equilibrium are most likely short-lived. After identifying a long-run equilibrium relationship between stock prices and dividends, we are now able to describe the stock returns using an EC model stated below.

$$r_t=\alpha_0+\alpha_1 z_{t-1}+\sum_{i=1}^{k}\alpha_{i+1}r_{t-i}+\varepsilon_t \tag{7}$$

where r_t stands for stock returns; $z_{t-1}=(p_{t-1}-\theta_0-\theta_1 d_{t-1})$ represents the EC term; α_1 measures the speed of adjustment to equilibrium; p_t and d_t represent log stock prices and log dividends, respectively. The optimal lag length k in $\sum_{i=1}^{k}\alpha_{i+1}r_{t-i}$ is chosen to ensure there are no serial correlations in the residuals (ε_t).

To fully capture the different dynamics of both small and large deviations from long-run equilibrium, we apply the smooth transition error-correction (STEC) model which allows for different types of return behaviour in different regimes. Thus, we rewrite Equation 7 as follows.

$$\begin{aligned} r_t=&\left(\alpha_0+\alpha_1 z_{t-1}+\sum_{i=1}^{k}\alpha_{i+1}r_{t-i}\right)\\ &+\left(\beta_0+\beta_1 z_{t-1}+\sum_{i=1}^{k}\beta_{i+1}r_{t-i}\right)\\ &F(z_{t-d}:\gamma,\tau)+\varepsilon_t \end{aligned} \tag{8}$$

The STEC model is theoretically more appealing than the threshold model in that the latter imposes an abrupt switch in the parameter values, and it would be the observed outcome only when all traders act simultaneously. In other words, for a market with numerous traders behaving heterogeneously in time, the STEC model is considerably more appropriate. The STEC model is governed by the continuous transition function $F(z_{t-d}$: γ, $\tau)$, where z_{t-d} is the transition variable; d is the optimal lag length for the transition variable z_{t-d}; γ is the smoothness parameter measuring how fast the transition is from one regime (small deviations) to the other (large deviations), and τ is the threshold parameter determining where the transition occurs.

As in Teräsvirta (1994), we consider two alternative specifications for the transition function in Equation 8:

$$F(z_{t-d}:\gamma,\tau)=\left\{1+\exp\left[-\gamma\frac{z_{t-d}-\tau}{\sigma^2_{z_{t-d}}}\right]\right\}^{-1},\ \gamma>0 \tag{9}$$

$$F(z_{t-d}:\gamma,\tau)=1-\exp\left[-\gamma\frac{(z_{t-d}-\tau)^2}{\sigma_{z_{t-d}}}\right],\ \gamma>0 \tag{10}$$

Equation 8 with the transition function (9) is called the logistic STEC (LSTEC) model, where $F(z_{t-d}: \gamma, \tau) = 0 \sim 1$ as $z_{t-d} = -\infty \sim +\infty$. The LSTEC model specifies different dynamics for the two different return regimes with a smooth transition between them. This specification allows the parameters of α's and β's of the STEC model in Equation 8 to change with the different values of the transition variable z_{t-d}. If $\gamma \rightarrow 0$, the model is reduced to a linear EC (EC) model. If $\gamma \rightarrow +\infty$, then $F(z_{t-d}: \gamma, \tau) = 1$ for $z_{t-d} > \tau$, and $F(z_{t-d}: \gamma, \tau) = 0$ for $z_{t-d} \leq \tau$, and accordingly the STEC model becomes a two-regime threshold model. The LSTEC model can, therefore, be viewed as an error correction threshold (ECT) model with one threshold value τ to distinguish between two regimes including the small and large deviations from the equilibrium. Since $F(z_{t-d}: \gamma, \tau)$ is not symmetric about τ, the LSTEC model is capable of generating the asymmetric short-run dynamics in two forms. The short-run dynamics will take on the form, $r_t = (\alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i}) + (\beta_0 + \beta_1 z_{t-1} + \sum_{i=1}^{k} \beta_{i+1} r_{t-i}) + \varepsilon_t$ during a period of expansion with $z_{t-d} > \tau$. However, the dynamics will switch into $r_t = (\alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i}) + \varepsilon_t$ during a period of recession with $z_{t-d} \leq \tau$. The transition from one state to the other is smooth and takes on the form of $r_t = (\alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i}) + (\beta_0 + \beta_1 z_{t-1} + \sum_{i=1}^{k} \beta_{i+1} r_{t-i}) F(z_{t-d} : \gamma, \tau) + \varepsilon_t$.

Equation 8 with the transition function (10) is called the exponential STEC (ESTEC) model. The ESTEC model assumes that there are similar dynamics in the extreme regimes but different dynamics in the transition period since $F(z_{t-d}: \gamma, \tau) = 1$ as $|z_{t-d}| = +\infty$. The ESTEC model allows the parameters to change symmetrically about τ with the transition variable z_{t-d}. In the extreme case, when $\gamma \rightarrow 0$, the model is reduced to a linear EC model with $\mathrm{r}_t = (\alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i}) + \varepsilon_t$. When $\gamma \rightarrow +\infty$, the model switches to the other regime with $r_t = (\alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i}) + (\beta_0 + \beta_1 z_{t-1} + \sum_{i=1}^{k} \beta_{i+1} r_{t-i}) + \varepsilon_t$. Since $F(z_{t-d}: \gamma, \tau)$ is symmetric about τ, the ESTEC model gives similar short-run dynamics between the periods of expansion and recession. This model implies that there is a symmetric transition from one state to the other. The ESTEC model may be viewed as a generalization of the ECT model with two threshold values to distinguish among three regimes including one within the equilibrium and two outside the equilibrium.

In the light of our pursuit to estimate the parameters of γ, τ and d, it is essential here to test the linearity with $F(z_{t-d}: \gamma, \tau) = 0$ in Equation 8 for various values of d before estimating the nonlinear STEC model. The null hypothesis of linearity H_0: $\gamma = 0$ is tested against the alternative hypothesis of nonlinearity H_1: $\gamma > 0$. Since the nonlinear STEC model can only be identified under the alternative hypothesis, it would render the application of the conventional Lagrange multiplier (LM) test of linearity invalid. Faced with this problem, we turn to Luukkonen *et al.* (1988) who suggested that the transition function $F(z_{t-d}: \gamma, \tau)$ be replaced with its third-order Taylor approximation about $\gamma = 0$. Thus, the STEC model in Equation 8 can be reformed as follows.

$$\begin{aligned} r_t &= \pi_0 + \pi_1' W_t + \kappa_1' W_t(z_{t-d}) \\ &\quad + \kappa_2' W_t(z_{t-d})^2 + \kappa_3' W_t(z_{t-d})^3 + \eta_t \end{aligned} \tag{11}$$

where $W_t = (z_{t-1}, r_{t-1}, r_{t-2}, r_{t-3}, \ldots, r_{t-k})$ in our case. If it is assumed that the delay parameter d is known, then the linearity test is equivalent to the test of the hypothesis

$$H_0 : \kappa_1' = \kappa_2' = \kappa_3' = 0 \tag{12}$$

An auxiliary regression can be defined as:

$$\begin{aligned} \varepsilon_t &= \pi_0 + \pi_1' W_t + \kappa_1' W_t(z_{t-d}) \\ &\quad + \kappa_2' W_t(z_{t-d})^2 + \kappa_3' W_t(z_{t-d})^3 + \nu_t \end{aligned} \tag{13}$$

where ε_t is the residual obtained from Equation 7 under the null hypothesis of linearity. Thus, the LM test of linearity against the nonlinear STEC model can then be performed by computing the following statistic

$$\mathrm{LM} = \frac{(\mathrm{SSR}_0 - \mathrm{SSR}_1)/(3(k+1))}{\mathrm{SSR}_1/(T - 4(k+1) - 1)} \tag{14}$$

where SSR_0 is the sum of the squared residuals ε_t, while SSR_1 is the sum of the squared residuals ν_t obtained from Equation 13. The statistic has an asymmetric F-distribution with $3(k+1)$ and $T - 4(k+1) - 1$ degrees of freedom under the null hypothesis of linearity. One possible way to identify the appropriate model between LSTEC and ESTEC models is through a sequence of tests on Equation 13. Thus, we consider a sequence of the null hypotheses as follows.

$$\begin{aligned} &H_{03} : \kappa_3' = 0 \\ &H_{02} : \kappa_2' = 0 \big| \kappa_3' = 0 \\ &H_{01} : \kappa_1' = 0 \big| \kappa_2' = \kappa_3' = 0 \end{aligned} \tag{15}$$

We would select the LSTEC model provided that H_{03} is rejected. If H_{03} is not rejected but H_{02} is rejected, we would adopt the ESTEC model. If both H_{03} and H_{02} are not rejected but H_{01} is rejected, we would select the LSTEC model [see Granger and Teräsvirta (1993) and Teräsvirta (1994)].

Table 5 shows the results of the LM test of linearity against the nonlinear STEC model, and we find strong evidence of nonlinearity in the stock returns. In order to specify d, we estimate Equation 13 across a range of values for d ($1 \leq d \leq 6$), where the nonlinearity test statistic with the minimum p-value determines the optimal value for d ($d = 5$) in the subsequent estimation of Equation 8. The results in Table 6 show that H_{03} is rejected for $d = 5$. Thus, it indicates that the LSTEC model would be the more appropriate model.

Finally, we attempt to make a comparison between the linear EC model and the nonlinear LSTEC model, including the parameter estimates, model specification tests, and residual tests for both models. Not surprisingly, the results in Table 7 consistently suggest that the LSTEC model is superior to the linear alternative based on all the different criteria used. More specifically, the LSTEC model has a relatively higher adjusted R^2, lower residual variance as well as lower AIC and SBC values, while showing no evidence of the ARCH effects. Moreover, the variance ratio also shows a reduction of 16% in the residual variance of the nonlinear LSTEC model, when compared with that of the linear model.

When examining the parameter estimates of the nonlinear LSTEC model, we found that although the estimated value of γ is large, it is not statistically significantly different from zero. However, Teräsvirta (1994) asserted that this should not be interpreted as evidence of weak nonlinearity. Besides, Sarantis (1999) further demonstrated the difficulty of estimating γ, while Sarno (2000) argued that the statistical significance of γ is, in essence, simply not a question because the linearity has already been rejected in the earlier tests. To estimate γ more accurately, many observations in the immediate neighborhood of τ's are typically required. Nevertheless, it may not be appropriate since we would probably end up with a higher standard error for the γ estimates from the fitted model. The large estimated value of γ found in our study implies a fast transition (a sharp switch) from one regime to the other. The following logistic

Table 5. LM test of linearity against the nonlinear STEC model

d	1	2	3	4	5	6
LM	2.635488	3.810711	2.909224	3.762206	4.042098	1.254534
p-Value	0.008215	0.000305	0.003846	0.003846	0.000161	0.269607

Notes: The LM statistics are computed to test the H_0: $\kappa_1' = \kappa_2' = \kappa_3' = 0$ in the equation of $r_t = \pi_0 + \pi_1' W_t + \kappa_1' W_t(z_{t-d}) + \kappa_2' W_t(z_{t-d})^2 + \kappa_3' W(z_{t-d})^3 + \eta_t$,

$$\mathrm{LM} = \frac{(\mathrm{SSR}_0 - \mathrm{SSR}_1)/(3(k+1))}{\mathrm{SSR}_1/(T - 4(k+1) - 1)}$$

where SSR_0 is the sum of the squared residuals ε_t in $r_t = \alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i} + \varepsilon_t$, and SSR_1 is the sum of the squared residuals ν_t in $\varepsilon_t = \pi_0 + \pi_1' W_t + \kappa_1' W(z_{t-d}) + \kappa_2' W_t(z_{t-d})^2 + \kappa_3' W(z_{t-d})^3 + \nu_t$.

Table 6. Model specification for the LSTEC vs. the ESTEC models

D	F-statistics for testing H_{03}	p-Value	F-Statistics for testing H_{02}	p-Value	F-statistics for testing H_{01}	p-Value
1	1.895576	0.134193	3.428539	0.019381	2.311591	0.079522
2	1.852243	0.141601	5.488635	0.001445	3.503100	0.017557
3	1.611504	0.190500	4.340042	0.006111	2.465585	0.065486
4	4.190785	0.007443	5.748425	0.001050	1.423999	0.239109
5	6.014993	0.000768	2.935798	0.036278	1.655090	0.180381
6	0.502394	0.681391	2.405564	0.070916	0.867424	0.460128

Note: The F-statistics are computed to test a sequence of the null hypotheses: H_{03}, H_{02} and H_{01} for the equation of

.
$$\varepsilon_t = \pi_0 + \pi_1' W_t + \kappa_1' W_t(z_{t-d}) + \kappa_2' W_t(z_{t-d})^2 + \kappa_3' W_t(z_{t-d})^3 + \nu_t$$

$$H_{03} : \kappa_3' = 0$$
$$H_{02} : \kappa_2' = 0 \middle| \kappa_3' = 0$$
$$H_{01} : \kappa_1' = 0 \middle| \kappa_2' = \kappa_3' = 0$$

Table 7. Comparison between the linear EC and the nonlinear LSTEC models

Variables	Coefficients	Linear EC model		Nonlinear LSTEC model	
Constant	α_0	0.0694 (0.0555)		0.0565 (0.0523)	
z_{t-1}	α_1	−0.0802 (0.0914)		0.0276 (0.1081)	
r_{t-1}	α_2	0.7861 (0.0709)***		0.9047 (0.0735)***	
r_{t-3}	α_4	0.1541 (0.0710)***		0.0528 (0.0726)	
Constant	β_0	–		1.7869 (0.5172)***	
z_{t-1}	β_1	–		−0.9624 (0.3025)***	
r_{t-1}	β_2	–		−1.1534 (0.2802)***	
r_{t-3}	β_4	–		0.1398 (0.2358)	
Transition speed	γ	–		37.7945 (28.8646)	
Threshold parameter	τ			0.2167 (0.0102)***	
	Centered R^2	0.8503		0.8818	
Model R^2	Uncentered R^2	0.9666		0.9732	
	Adjusted R^2	0.8467		0.8723	
AIC		309.0638		285.77579	
SBC		320.5031		314.21766	
LM test for ARCH effects		4.226710	[0.041858]	0.308048	[0.57888042]
Ljung–Box $Q(4)$		1.5759		3.4563	
Ljung–Box $Q(8)$		2.6956		4.7196	
SSR		10.3174		8.1070	
Variance ratio				0.8395	

Notes: ***Denote significance at the 1% level, respectively.
SSR stands for the sum of the squared residuals for each model.
Variance ratio is the ratio of the variance of the nonlinear model relative to the variance of the linear model.
The models were estimated based on the following equation, with $F(z_{t-d}: \gamma, \tau) = 0$ for the linear model

$$r_t = \left(\alpha_0 + \alpha_1 z_{t-1} + \sum_{i=1}^{k} \alpha_{i+1} r_{t-i}\right) + \left(\beta_0 + \beta_1 z_{t-1} + \sum_{i=1}^{k} \beta_{i+1} r_{t-i}\right) F(z_{t-d} : \gamma, \tau) + \varepsilon_t$$

The numbers in parentheses are the SEs of the estimates.

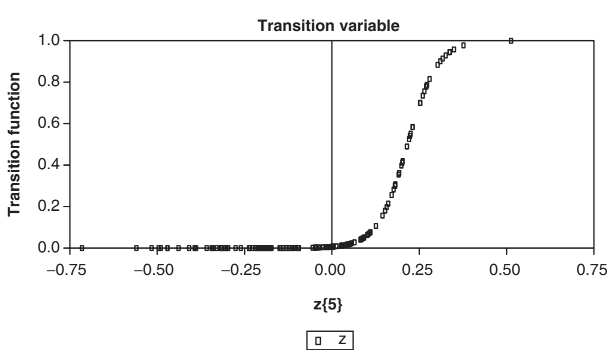

Fig. 1. Relationship between the logistic transition function and the transition variable $F(z_{t-5}: \gamma, \tau) = \{1 + \exp[-37.7945(z_{t-5} - 0.2167)/0.2841]\}^{-1}$

transition function is further estimated and illustrated in Fig. 1.

$$F(z_{t-5} : \gamma, \tau) = \left\{1 + \exp\left[-37.7945 \frac{(z_{t-5} - 0.2167)}{0.2841}\right]\right\}^{-1}$$

Figure 1 shows that the transition from the lower regime (smaller deviations) to the upper regime (larger deviations) is almost instantaneous at the threshold values of $z_{t-5} = 0.0$ and 0.44. The short-run dynamics of the stock returns reach the lower regime as $(z_{t-5} - \tau) \to -\infty$ and $F(z_{t-5}: \gamma, \tau) \to 0$, whereas returns reach the upper regime as $(z_{t-5} - \tau) \to \infty$ and $F(z_{t-5}: \gamma, \tau) \to 1$. Not to be ignored, the stock return dynamics are asymmetric, with the significantly negative coefficient (−0.9624) of the EC term z_{t-1} included in the upper regime. It suggests that there is no sign of a mean reversion to equilibrium in the lower regime but a quick mean reversion to equilibrium in the upper regime. These results indicate that the dynamics governing the small deviations from the long-run equilibrium differ from those governing the large deviations. Theoretical models of studying the interaction between arbitrageurs and noise traders have suggested that small and large deviations may exhibit different return dynamics given that arbitrageurs must always be aware of the potential for noise traders to drive returns further away from equilibrium. Needless to say, our results confirm the implications of the noise trader models, and therefore, acknowledge the potentially harmful behaviour of such noise traders. Let's come straight to the point. Large deviations are characterized by quick mean reversion because arbitrageurs have more confidence in being able to move the market in the appropriate direction and their risk exposure to the adverse price movements is lower. However, small deviations are characterized by persistence and slow reversion since arbitrageurs are reluctant to immediately act upon the mispricings due to the fact that they are now exposed to greater price risks and adverse market movements, which might be induced by the noise traders. Consequently, our findings from the LSTEC model are different from those reported by McMillan (2004). McMillan (2004) adopted an exponential smooth transition threshold EC model to examine the return dynamics in UK stock market and found small return deviations are characterized by quick mean reversion, whereas large return deviations are characterized by persistent deviations from equilibrium and slow mean reversion.

IV. Conclusions

In this study, using the more powerful nonparametric cointegration tests of Bierens (1997, 2004), we demonstrate that no rational bubbles existed in the US stock market throughout the period of 1871 to 2002. Our application of a LSTEC model, designed to detect the nonlinear short-run adjustments to the long-run equilibrium, provides substantive empirical evidence in favour of noise trader models where arbitrageurs are reluctant to instantaneously engage in trading when stock returns deviate insufficiently from their fundamental value.

Acknowledgements

The authors gratefully acknowledge the helpful comments and suggestions of the editor, co-editor, and the anonymous referee from *Applied Financial Economics*.

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