



On the relationship between Bitcoin and other assets during the outbreak of coronavirus: Evidence from fractional cointegration analysis

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ABSTRACT

This article tries to investigate the connectedness between Bitcoin and Crude Oil, S&P500 and Natural Gas with the health crisis. That is why one might apply fractional cointegration analysis on daily data over the period 01/09/2019–30/04/2020. Our results indicate the presence of fractional integration in residual series, implying the existence of a fractional cointegration relationship. A short-run joint dynamics between Bitcoin and some other assets (Crude Oil, S&P500 and Natural Gas) is nevertheless well-pronounced. Such analysis of the long and short-term dependencies between different assets could be interesting from a portfolio perspective.

The triggering of the health crisis caused by the most recently emerging coronavirus has increasingly become a global concern within a short span of time. The outbreak of such respiratory disease has spread to many continents and has killed many people around the world. The coronavirus epidemic is not obviously the first virus outbreak that poses a great challenge for individuals, policymakers and economies. Other health crises such SARS viruses, Spanish influenza, Ebola, Zika virus have led to panic episodes and general anxiety disorder. As the world has become more interconnected, the health crisis has harshly haunted the global economy and has increasingly amplified the financial markets. Even though several emergency measures have undertaken by main governments and central banks to cushion the adverse effects on global economy such as cutting the interest rates, the consequences for the overall economy and specific companies and financial markets still remain very challenging. Indeed, the Covid-19 pandemic has significantly negative effects on manufacturing healthcare system, transportation, trade, tourism, consumer demand and service. For instance, Yang et al. (2020) show the high decline in tourism sector output. The continuous spread of the health crisis has a notable worsening of the financial markets performance. For example, the S&P500 index decreased from about 3386 on February 19, 2020 to about 2481 on March 12, 2020 (Yilmazkurd, 2020). As well, the Shanghai stock market plunged 8% on February 3, 2020. As a result, investors increasingly become worried about their investment in the financial markets.

From academic standpoint, many researchers have attempted to

analyze the adverse effects on the Covid-19 pandemic on the behavior of financial markets. For instance, Albulescu (2020) attempts to examine the effects of coronavirus outbreak on the financial markets volatility index (VIX) over the period 20/01/2020–28/02/2020 based on the new case announcements and death ratio (in China and outside China), as well at the number of daily affected countries. The empirical results display that only the new cases recorded outside China and the death ratio influence positively and significantly the VIX indicator. The spread of coronavirus increasingly influences the financial market volatility, implying potential episode of international financial stress. Yilmazkurd (2020) examines the impact of the global deaths on the S&P500 index over the period 31/12/2019–12/03/2020. Using a structural vector autoregression model, the empirical results indicate that having one more global deaths leads to 0.02% of a cumulative reduction in the S&P500 index after one day, 0.06% of a cumulative reduction after one week, and 0.08% of a reduction after one month. Ramelli and Wagner (2020) analyze the impact of Covid-19 pandemic on the stock prices. The feverish and seemingly behaviorally-driven price moves are well-documented. They report that investors can initially have worried about cash flows, but applied higher discount rates as risk increased. They also indicate how the health crisis morphed into a possible financial crisis. Albulescu (2020a) examines the effects of people's number affected by coronavirus on crude oil prices after controlling for the impact of the United States (US) economic policy uncertainty and financial volatility. The empirical results indicate that the daily cases of new infections have a marginal negative impact on the crude oil prices

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in the long run. However, by increasing the financial markets volatility, the Covid-19 pandemic has an indirect impact on the recent dynamics of crude oil prices. Binder (2020) surveys the U.S consumers about their concerns about the Covid-19 pandemic, the measures undertaken by the Federal Reserve and their expectations of inflation and unemployment. Overall, the empirical results show that most U.S consumers are worried about the effects of the health crisis on the U.S economy, their health, and their personal finances. About 28% had cancelled or postponed travel and 40% purchased food or supplies in response to these concerns. About 38% were aware that the Federal Reserve had cut interest rates. Gormsen and Kojien (2020) attempt to assess how investors' expectations about economic growth across horizons evolve in response to the coronavirus outbreak and subsequent policy responses based on data from the aggregate equity market.

Bouoiyour and Selmi (2017) rather prefer to analyze the safe-haven property and volatility of Bitcoin during the spread of the Covid-19 pandemic. The empirical findings indicate that the current bullish sentiment is triggered by investors searching for Bitcoin as a safe-haven asset during crisis period. The Covid-19 outbreak seems to amplify the volatility of Bitcoin because of a search by investors for alternative asset classes amid concerns about the coronavirus. They also show that the information about the coronavirus appears to be gradually reflected in the Bitcoin price. Jana and Das (2020) examine the resilience of Bitcoin to hedge the Chinese aggregate and sectoral equity markets and the returns spillover to Alcoins during the Covid-19 pandemic. They show that Bitcoin is considered as a weak hedge asset over the overall period and a weak safe haven asset during the health crisis. Bitcoin also seems to be a weak hedge, diversifier and a weak safe haven for the sectoral equity indexes. They also report that gold clearly outperforms Bitcoin in hedging and safe haven perspectives with respect to the Chinese equity markets. The empirical findings show that the increase in Bitcoin prices is significant because of spillover from Bitcoin prices.

Based on the aforementioned (and other) studies, the cryptocurrency market tends to display a nonlinear and asymmetric relationship with the Covid-19 intensity. The linkages between Bitcoin and traditional equity markets seem to be time-varying and depend on the type of assets. But, the possible nature of association between Bitcoin and other assets is still under-explored with the outbreak of such unprecedented and unexpected event. It can be possible that the varying levels of Covid-19 intensity during the first waves of pandemic coupled with its wide scale devastation in terms of lockdowns, isolation, panic, fear, psychological distress and uncertainty in the absence of any vaccine or a sound cure may influence the relationship between Bitcoin and other assets differently, in short- and long-term. Therefore, we try to fill this research gap by analyzing the connection between Bitcoin and S&P500, Crude Oil, Natural Gas during the Covid-19 pandemic. A deeper examination in such circumstances seems to be interesting to better describe time series behavior and joint dynamics between different markets. For this end, we develop a unified framework for jointly modeling the dynamic dependencies and connectedness between different assets.

Our study contributes to the current literature in different ways. Our study comes to revive and therefore complements the current literature on the contagion, volatility spillover, cross-market relationships and safe-haven proprieties of different assets by revisiting such issues with the outbreak of unprecedented and unexpected event such as the Covid-19 pandemic. We analyze the association between Bitcoin and other assets from dynamic perspective. We use a formal model setup based on the fractional cointegration analysis. Such method can be suitable statistical framework to analyze and distinguish between short-term and long-term impacts in a system setting involving fractionally integrated $I(d)$ variables. A better understanding of the dynamic connectedness between cryptocurrency market and other markets can help researchers to know much more about the changing role and nature of Bitcoin in crisis times. This can lead to provide fresh insights about the sustainability of Bitcoin as an alternative asset class. It can also provide fresh insight about how the information transmission mechanism and

information spillover between cryptocurrency market and other markets seems to be during unprecedented and unexpected events. Our findings can offer insightful information for investors who search for investment alternatives. They will be useful for policymakers who have to be aware and know much more about the connectedness between markets during turbulent periods.

This paper is organized as follows. Section 2 presents literature review. Data and descriptive statistics are reported in Section 3. Section 4 presents the empirical analysis of data as well as the different empirical findings. Section 5 concludes.

1. Literature review

With the outbreak of health crisis, many researchers have increasingly focused on the behavior of cryptocurrency markets based on different econometric models. For instance, Iqbal et al. (2021) analyze the effect of Covid-19 on the daily returns of cryptocurrencies during the period 01/01/2020–15/06/2020. They report that the changing intensity levels of the health crisis influence asymmetrically the bullish and bearish phases of cryptocurrency markets. Goodell and Goutte (2020) explore that the impact of Covid-19 pandemic during the period 31/12/2019–29/04/2020. They show that the intensity of Covid-19 pandemic engenders an increased in Bitcoin prices. Arouxeta et al. (2022) examine the long-term memory in volatility and return over the period 14/11/2019–08/06/2020. They show that the long-term memory of returns was slightly influenced during the peak of health crisis (around 03/2020). Nonetheless, volatility undergoes a temporary effect in its long-range correlation structure. Apergis (2021) analyzes how the Covid-19 pandemic can determine and forecast conditional volatility returns during the period 01/02/2020–31/10/2021. The empirical results that the health crisis affects significantly and positively the conditional volatility. James et al. (2021) examine the extreme and erratic behaviors of cryptocurrency markets over the period 30/06/2018–24/06/2020. They report that cryptocurrency behavior seems to be less self-similar in returns than variance. The cryptocurrency market displays substantial homogeneity with respect to the structural breaks in variance during the pre-Covid-19 period. The health crisis affects the return extremes. Sarkodie et al., 2021 analyze the severity of Covid-19 pandemic on prices of Bitcoin, Ethereum, Bitcoin Cash and Litecoin over the period 22/01/2020–31/12/2020. They show that shocks related to Covid-19 pandemic spur digital currencies in different levels. Corbet et al. (2022) explore the association between cryptocurrency price volatility and liquidity with the advent of Covid-19 crisis. They show that cryptocurrency market liquidity tends to raise after the WHO identification of a worldwide pandemic. They also identify significant interactions between cryptocurrency price and liquidity effects.

Others researchers have rather explored the dynamic relationships between Bitcoin and other assets in order to either check the potential existence of such associations during stressful periods or understand the safe-haven nature of Bitcoin. Dutta et al. (2020) investigate the linkages between gold and oil markets and the safe-haven feature of Bitcoin during 12/2014–03/2020. They report that the time-varying linkages are well-documented, implying that gold can be considered as safe-haven for global crude oil markets. Bitcoin appears to be a diversifier for crude oil. Guo et al. (2021) examine the contagion effect between Bitcoin and the United State market, European market, Chinese market, US dollar, gold, commodity market and bond market from January 1, 2019 to May 31, 2020. They find that the contagion impact between Bitcoin and developed markets is strengthened during the pandemic. They also show that gold has contagion effect with Bitcoin whereas gold, US dollar and bond market seem to be the contagion receivers of Bitcoin. Mariana et al. (2020) examine if Bitcoin and Ethereum can be suitable as safe-havens for stocks during the period 01/07/2019–06/04/2020. They show that both virtual currencies seem to be short-term safe-havens. They report that the daily returns tend to be negatively correlated with S&P500 during the pandemic. Raheem

(2021) studies the safe-haven feature of Bitcoin against measures of uncertainty (EPU, VIX and oil shock) over the period 01/08/2019–30/05/2020. The empirical findings prove that Bitcoin cannot be considered as safe-haven during the health crisis. Therefore, the prowess of safe-haven features is sensitive to the type of shock given that Bitcoin can offer high cover against VIX and EPU shocks. Jareño et al., 2021 analyze the linkages between cryptocurrency market and oil market over the period 20/11/2018–30/06/2020. They report that the existence of strong association between cryptocurrency returns and oil shocks during crisis times. Lin et al. (2021) examine the linkages between Bitcoin and resource commodity future price in short-and long-term. They find the asymmetric long-run association between Bitcoin price and resource commodity futures price. However, the short-run asymmetry is shown in the case of silver and gold. Moussa et al. (2021) explore the short- and long-term dynamics between Bitcoin, natural gas and coal over the period 2011–2018. They show that oil Brent crude and gold substantially affect Bitcoin. Bhuiyan et al. (2021) analyze the lead-lag relationship between Bitcoin and gold, commodity, currency, stock indices, bond indices over the period 07/2014–11/2019. They find strong bidirectional causality between gold and Bitcoin and neutral lead-lag relationship between Bitcoin and the US dollar index, crude oil, the aggregate commodity index.

2. Data and descriptive statistics

We collect data on Bitcoin, Crude Oil, Gold, Natural Gas and S&P500 from the website yahoofinance over the period 01/09/2019–30/04/2020. Oil prices are approximated by West Texas Intermediate (WTI). As well, the cumulative numbers of people died (Deaths) and contaminated (Cases) by coronavirus are retrieved from the website ourworldindata on daily frequencies. The statistics (standard deviation, median, mean, skewness, Jarque-Bera statistics and kurtosis) are presented in Table 1.

As shown in Table 1, the mean return varies from -0.11% (Natural Gas) to 2.39% (Crude Oil). The average cases and deaths are equal to 286615.4 and 18242.87 around the world. Interestingly enough, Crude Oil appears to be the riskiest asset whereas S&P500 seems to be more stable asset. The values of standard deviation are equal to 695313.8 and 48471.28 for cases and deaths, respectively. As well, the daily returns for all assets (except for Oil and Gold) are negatively skewed during the sample period, implying that the left tail is particularly extreme (i.e. negative values or losses are much more likely). The leptokurtic feature of return distribution is very salient in our sample. Based on the Jarque-Bera test, all the variables seem to be not normally distributed.

Fig. 1 illustrates the evolution of the daily returns over time. At first glance, Fig. 1 shows not only cyclical movements of all returns time series but also volatility clustering behavior of different variables. The inspection of such graphs clearly shows that Bitcoin returns seem to be instable over time, with different decreasing and increasing trends during the period 01/09/2019–30/04/2020. As well, we display the presence of some different patterns in the evolution of the other assets' returns with different bearish and bullish market phases. Such instability can be related to the outbreak of coronavirus. The common patterns and instable evolution call upon not only to examine correlations between

assets but also to employ models accommodating nonlinearity and asymmetry in the joint dynamics of the variables. Table A in Appendix 1 reports linear relationships among variables based on variance-covariance matrix and correlation matrix.

3. Estimation results and interpretation

Overall, the methodological approach used in this paper can be summarized in Fig. 2.

One might afterwards test if indicators are (non)sationnary using two unit root tests: Phillips and Perron (1988) test and Dickey and Fuller (1979, 1981) test. Table 2 presents the empirical results.

From Table 2, we show that each series is I(1). That is, all series seem to have a unit root in level based on Dickey-Fuller test and Phillips-Perron test. After first-differencing, variables become stationary given the values of T-Statistics and Z-alpha. The first differenced variables do not have a unit root. We then investigate the presence of the long memory for different variables using the Hurst test based on the R/S tests.

From Table 3, the results from the Hurst exponent show the existence of the issue of long memory. The value of the Hurst exponent indicates that these variables clearly display the propriety of long memory given $0.5 < H < 1$. The coefficient of long memory "d" is estimated based on the method of Geweke and Porter-Hudak (1983) (GPH). The GPH estimators of the memory parameter d of the series and the asymptotic standard deviation of each variable are reported in Table 3. The estimated long memory parameter for each series ranges between 0 and 0.5. After estimating the long memory parameter d, the autocorrelation degree in the fractionally differenced series is analyzed using the autocorrelation and partial autocorrelation functions as shown in Fig. 2 in Appendix 2.

Each series can be modeled by the Autoregressive Fractionally Integrated Moving Average (ARFIMA) models. We use the estimated long memory coefficient for each series and one might identify which the suitable ARFIMA(p,d,q) generating process based on AIC criterion. The estimation results are reported in Table 4.

From Table 4, the optimal lag lengths related to the AR and MA models are chosen based on the AIC criterion. Seven candidate models are obtained. Out of these models, a parsimonious model is chosen which is characterized by the lowest value of AIC criterion. The optimal model for the fractionally difference series is reported (in bold) in Table 4. For instance, the empirical results show that ARFIMA(3,d,0) model outperforms given the lowest value of AIC criterion (equal to 324.7701). After model identification, we estimate the model parameters (Table 5) for each time series. From Table 6, the results clearly show that having fitted a model to the fractionally differenced time series is well-documented.

As previously shown, all of the variables under study contain unit roots and are integrated of order one. That is why it seems be interesting to perform a cointegration analysis amongst these variables and test for the existence of a stable long-term relationship between these time series. In particular, given that time series display long memory propriety, we use fractional cointegration analysis. As reported by Lardic and Mignon (2004), the classical cointegration analysis allows for an integer

Table 1
Descriptive statistics of variables.

Variables	Bitcoin	Oil	Gold	Gas	Cases	Deaths	S&P500
Observations	243	243	243	243	243	243	243
Mean	-0.04	2.39	0.05	-0.11	286615.4	18242.87	0.03
Standard deviation	4.52	61.12	1.2	3.79	695313.8	48471.28	2.01
Median	-0.1	0	0	0	27	0	0
Skewness	-3.98	12.92	0.84	-0.05	2.66	2.91	-0.9
Kurtosis	45.33	195.23	9.7	3.62	6.04	7.48	12.9
Jarque-Bera	21,822	399,290	1001.8	136.78	669.59	927.88	1752.8
p-value	2.2×10^{-16}	2.2×10^{-16}	2.2×10^{-16}	2.2×10^{-16}	2.2×10^{-16}	2.2×10^{-16}	2.2×10^{-16}

Note: (.) is the p-value of the Jarque-Bera test.

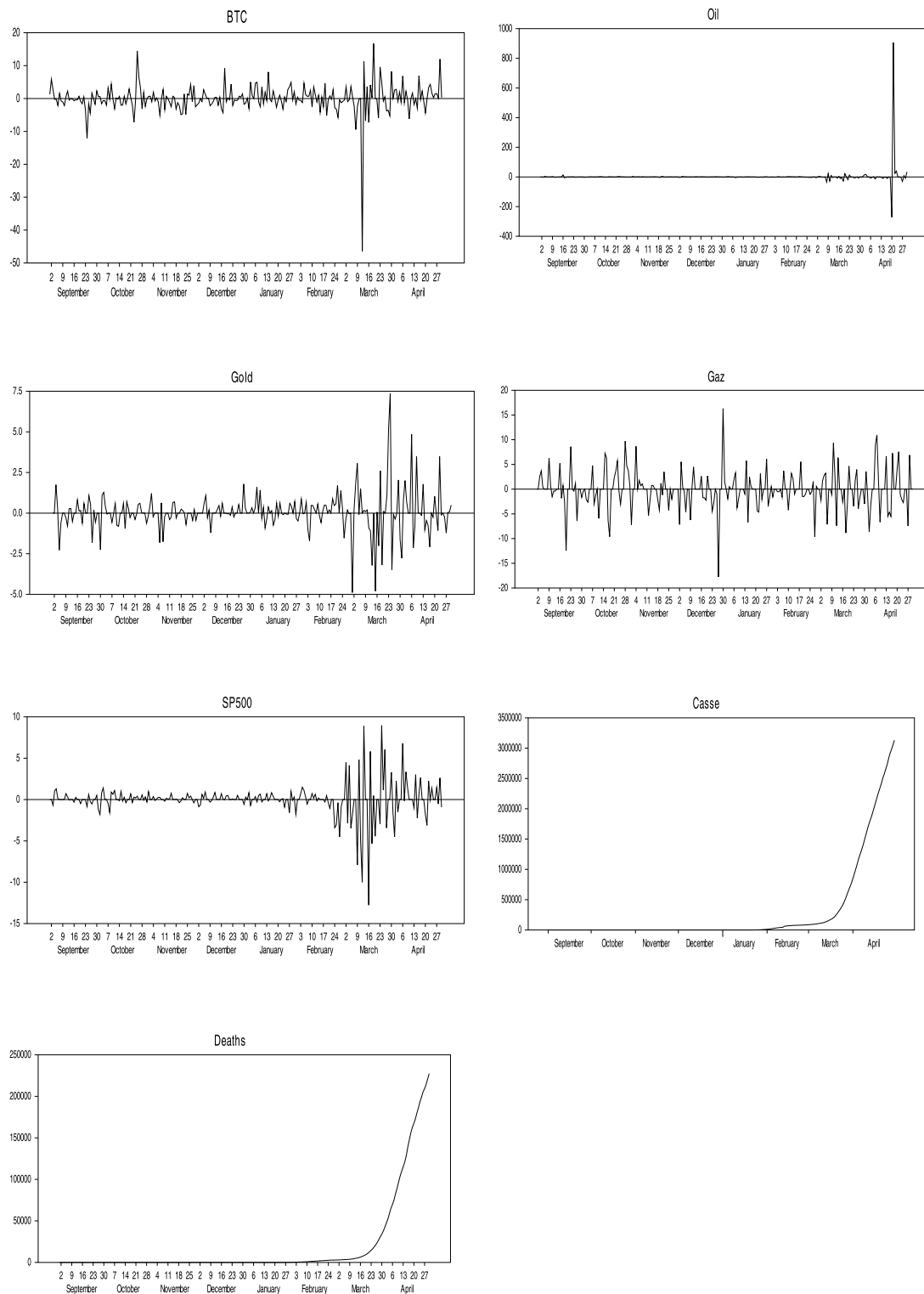


Fig. 1. Different time paths of variables.

order of integration in the equilibrium error process, which could be *ad hoc* assumption. Unlike such method, fractional cointegration enables the integration order of the error in the equilibrium relationship to have any real value between 0 and 1, i.e. to be fractionally integrated. This implies more various mean-reverting behaviors and offers insights on how the equilibrium relationship between the variables reacts to exogenous shocks. In particular, a fractionally integrated error term displays the existence of equilibrium relationship between time series in

long-term. So, we propose here to examine fractional cointegration between Bitcoin, S&P500, Gold and Oil when controlling for other variables which might affect this relationship. We indeed include the cumulative number of individuals passed away and contaminated by health crisis as control variables in the fractional cointegration analysis in order to account for the health crisis. Table 6 reports the estimation results of long-term relationship between variables.

From Table 6, Oil and Gold have positive and significant impact on

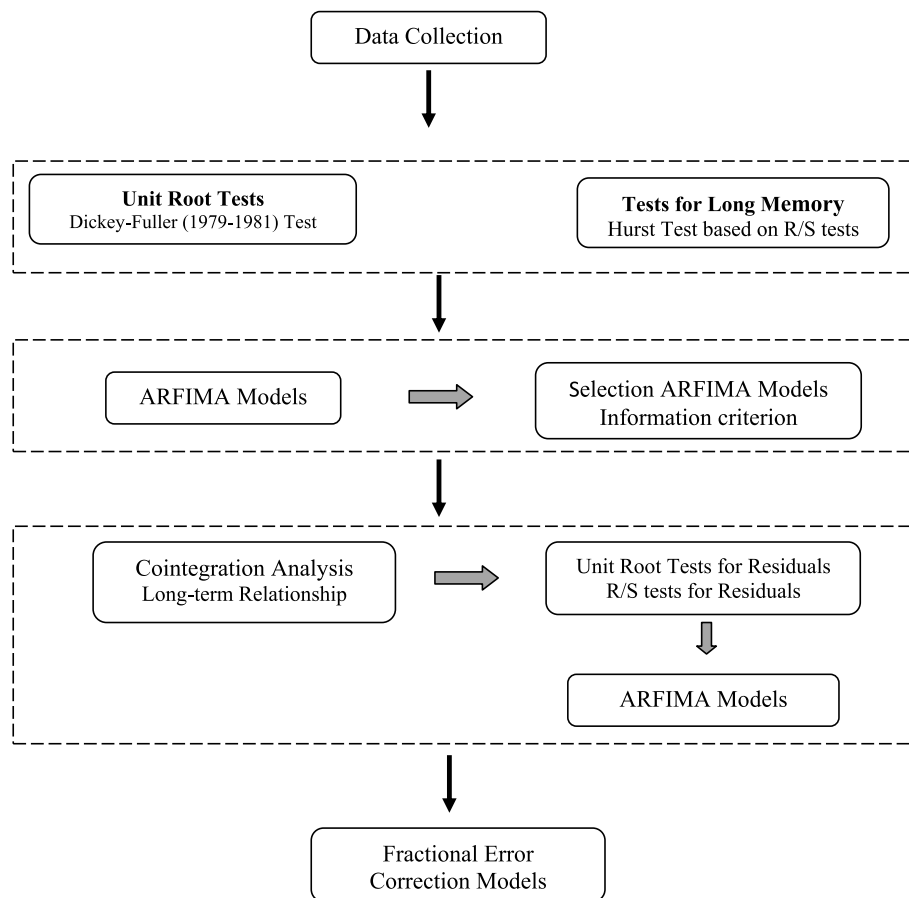


Figure 2. Research Methodology

Table 2
Unit root tests.

Dickey and Fuller (1979, 1981) Test					
Level		First Difference			
Variables	Lags	T-Statistics	p-value	T-Statistics	p-value
Bitcoin	6	-2.8372	0.1978	-5.6421	0.01
Oil	6	-2.5668	0.2231	-6.7842	0.01
Gold	6	-2.9965	0.163	-6.8534	0.01
Gas	6	-2.3935	0.2417	-6.1645	0.01
Cases	6	-3.0606	0.1297	-5.8672	0.01
Deaths	6	-2.5253	0.2206	-5.7431	0.01
S&P500	6	-2.076	0.2732	-5.023	0.01
Phillips and Perron (1988) Test					
Level		First Difference			
Variables	Truncation lag	Z(alpha)	p-value	Dickey-Fuller	p-value
Bitcoin	4	7.2316	0.99	-283.53	0.01
Oil	4	8.6523	0.99	-292.34	0.01
Gold	4	7.7852	0.99	-236.76	0.01
Gas	4	6.9632	0.01	-195.01	0.01
Cases	4	7.3162	0.99	-169.28	0.01
Deaths	4	8.2478	0.99	-505.33	0.01
S&P500	4	8.5341	0.99	-512.45	0.01

Bitcoin. Also, S&P500 returns have a positive and significant effect on Bitcoin returns. On the other hand, Gas negatively and significantly affects Bitcoin. The variables Cases and Deaths seem not to have an impact on Bitcoin returns. Such empirical findings provide insightful characterization of the dynamic dependencies and interrelatedness between different markets. So, there is an evidence of a long-run fractional cointegration relationship between cryptocurrency market, stock market and commodity markets.

Table 3
Test for long memory.

Variables	Simple R/S Hurst estimation	Corrected R over S Hurst exponent	Empirical Hurst exponent	Corrected empirical Hurst exponent	Theoretical Hurst exponent
dBitcoin	0.6435	0.7117	0.7515	0.6973	0.5533
dOil	0.5825	0.6147	0.6761	0.6210	0.5533
dGold	0.7300	0.8623	0.7047	0.7310	0.5533
dGas	0.5526	0.6026	0.8947	0.8184	0.5533
dCases	0.8160	1.4320	0	0	0.5533
dDeaths	0.8049	NA	0	0	0.5533
dS&P500	0.7631	0.9789	0.7180	0.7475	0.5533
Estimation of d for Geweke and Porter-Hudak (1983) Test					
Variables	\hat{d}	Asymptotic Standard Deviations			
dBitcoin	0.2768	0.2197			
dOil	0.1512	0.2197			
Gold	0.4696	0.2197			
Gas	0.2603	0.2197			
dCases	0.1136	0.2197			
dDeaths	0.3469	0.2197			
dS&P500	0.1208	0.2197			

For this widely-used approach in cointegration tests, Table 8 reports results based on Dickey and Fuller (1979, 1981) test and Phillips and Perron (1988) test applied on the residuals of the fractional cointegration analysis. According Lardic and Mignon (2004), such tests are characterized by asymptotic features under which both the null and alternative hypotheses seem to be known. The empirical results clearly display that the error term is stationary at the 1% significance level, implying the existence of cointegration between different markets. We then apply different R/S tests on residuals in first difference to estimate

Table 4
Result of AIC criterion for ARFIMA (p,d,q).

AIC	(1,d,0)	(2,d,0)	(3,d,0)	(4,d,0)	(0,d,1)	(1,d,1)	(1,d,2)
dBitcoin	741.5442	718.2953	712.5303	713.9396	716.3325	711.7022	711.9224
dOil	2081.562	2039.692	2028.509	2014.883	1968.041	1967.508	1969.645
dGold	97.5058	89.1233	89.7643	86.4443	97.4701	90.5813	90.3321
dGas	645.9278	633.1674	634.8387	636.8055	645.713	642.5339	634.7539
dCases	3997.792	3981.792	3980.346	3959.721	4468.525	3973.558	3981.172
dDeaths	3015.108	3006.317	3007.778	2997.144	3107.924	2990.305	2980.52
dS&P500	363.5455	363.9813	324.7701	326.7275	351.526	353.1249	332.2541

Table 5
Results of the estimated ARFIMA (p,d,q) model parameters.

Coefficients	dBTC	dOil	dGold	dGas	dCases	dDeaths	dS&P500
phi(1)	−0.2864		−0.0647	0.0352	0.652	0.9831	0.5298
phi(2)			−0.2459	−0.2480	0.08		−0.28
phi(3)			−0.0982		−0.086		0.3963
phi(4)			−0.1519		0.3482		
theta(1)	−0.7674	−0.9999				0.5712	
theta(2)						0.1885	
d.f	0.2768	0.1513	0.4696	0.2603	0.1136	0.3469	0.1208
zbar	3.7942	13.7281	1.0354	3.6487	12779.9	923.28	1.5649
sigma^2	18.9739	3586.36	1.3702	13.7019	1.4762	2.4738	3.7466

Table 6
Estimation Results of long-term Relationship between Variables.

Variables	Coefficients	Std. Dev	t-value	Pr(> t)
Intercept	−0.1045	0.0138	−7.5725	0.000
Oil	0.4035	0.0482	8.3714	0.000
Gold	0.1264	0.0234	5.4017	0.000
Gas	−0.1450	0.0169	−8.5799	0.000
Cases	−0.2238	0.1067	−0.550	0.583
Deaths	0.3803	0.5837	0.652	0.515
S&P500	0.9266	0.1355	6.839	0.000

the value of the Hurst exponent and the long memory coefficient. From [Table 7](#), the statistics of the R/S tests for residuals of long-term relationship between variables range between 0.5 and 1, implying the presence of a long memory.

The estimation of ARFIMA models on residuals in first difference is based on two alternative hypotheses. The null hypothesis of unit root ($d = 1$) against the alternative hypothesis of fractional integration ($d < 1$), i.e. $d' = d - 1 = 0$ under the null hypothesis against $d' = d - 1 < 0$ under the alternative hypothesis (the alternative of fractional integration). Recall that “d” is the integration coefficient of residuals in level and “d’” is the integration parameter of residuals in first difference.

Using [Geweke and Porter-Hudak \(1983\)](#) method is based on choosing the number of periodogram ordinates “m” ($m =$ and T : number of observations). The number of periodogram ordinates is selected using the interval $[T^{0.4}, T^{0.55}]$. Such choice makes it possible to stabilize residuals of this long-term relationship when the number of periodogram ordinates varies. The estimation results of these residuals in first

Table 7
Results from applying unit root tests and R/S tests on residual series.

Unit Root Tests on Residual Series					
Tests	Dickey and Fuller (1979, 1981) Test		Phillips and Perron (1988) Test		
	T-Statistic	P-Value	Z(alpha)	P-Value	
Residuals	−7.2888	0.01	−271.74	0.01	
R/S Tests on Residual Series in First Difference					
	Simple R/ S Hurst estimation	Corrected R over S Hurst exponent	Empirical Hurst exponent	Corrected empirical Hurst exponent	Theoretical Hurst exponent
Residuals	0.7406	0.6818	0.5527	0.5243	0.5531

difference using the [Geweke and Porter-Hudak \(1983\)](#) method and exact maximum likelihood procedure are reported in [Table 8](#).

From [Table 8](#), the long memory coefficient is statistically different from zero when the number of ordinates is equal to $T^{0.4}$. However, it is equal to zero for other ordinates and the estimated long memory parameter is not statistically significant for these ordinates. Hence, residual series seem to be only fractionally integrated at an ordinate equal to $T^{0.4}$. Nonetheless, one might reject the fractional cointegration for other periodogram ordinates. We then retain the ARFIMA(p,d,q) model which minimized the values of Akaike Information Criterion (AIC) and Schwartz Information Criterion (SIC). From [Table 8](#), the estimation results show that residual series in first difference can be modeled using ARFIMA (2,d,0) model and ARFIMA (1,d,0) model based on AIC and SIC criteria, respectively.

It is noteworthy that residuals (in first difference) of long-term relationship can be modeled using the ARFIMA (2, d, 0) model and ARFIMA (1, d, 0) model based on AIC and SIC criteria. One might use the ARFIMA(2,d,3) model using the Exact Maximum Likelihood (EML) procedure given the value of the SIC information criterion. Therefore, one might perform three ARFIMA processes on residuals (in first difference). [Table 8](#) reports the estimated parameters of long memory structure which ranges between 0 and 0.5. So, residual series are fractionally integrated. Such empirical finding implies that this is evidence of a long-run cointegration relationship between Bitcoin market, stock market and commodity markets when controlling the number of confirmed cases and deaths caused by the coronavirus.

Finally, the estimation results of the Fractional Error Correction (FEC) model applied on endogenous (Bitcoin), exogenous (Oil, Gas and S&P500) and control (Death and Cases) variables are reported in [Table 9](#). The FEC model is estimated using the Ordinary Least Squares technique. We also model the residuals of long-term relationship using the ARFIMA(p,d,q) models which are retained by minimizing the AIC and SIC criteria.

We consider the residuals in first difference with a long memory coefficient equal to 0.4 for both FEC models. More precisely, the residuals are modeled using an ARFIMA (2, 0.4, 0) model and ARFIMA(1, 0.4, 0) model according to AIC (FEC model 1) and SIC (FEC model 2) criteria, respectively. The FEC models include two equilibria: a short-term (resp. long-run) equilibrium where the variables are stationary using first difference (resp. linear combination showed that the first difference residuals of this relationship as a power of “d” ($0 < d < 1$)).

Table 8
Results for residuals series.

ARFIMA Estimation Model on Residual series using Geweke and Porter-Hudak (1983) Method							
	T ^{0.4}	T ^{0.5}	T ^{0.6}	T ^{0.7}	T ^{0.8}		
\hat{d}	0.4472	0.4758	0.4880	0.4014	0.4156		
Standard deviation	0.1448	0.3197	0.3540	0.3100	0.2826		
T-Student	3.0875	1.4883	1.3788	1.2949	1.4707		
Result of AIC and SIC Criteria for ARFIMA(p,d,q) Models on Residual Series							
	(1,d,0)	(2,d,0)	(3,d,0)	(4,d,0)	(0,d,1)	(1,d,1)	(1,d,2)
AIC	694.6649	693.3719	693.8955	701.7232	695.808	691.6225	699.4151
SIC	708.6207	710.8166	714.8291	726.1457	709.7638	709.0672	720.3488
ARFIMA Estimation Model on Residual Series using Exact Maximum Likelihood Procedure							
Methods	EML			AIC		SIC	
Coefficients	ARFIMA(2,d,3)			ARFIMA(2,d,0)		ARFIMA (1,d,0)	
phi(1)	−0.1779			−0.0825		−0.1091	
phi(2)	0.5809			0.1230			
theta(1)	0.4539						
theta(2)	0.6918						
theta(3)	−0.1459						
d.f	0.4465			0.3997		0.3991	
Fitted mean	0.0018			0.0017		0.0013	
sigma^2	16.0936			16.6733		16.8137	

Notes: EML: Exact Maximum Likelihood procedure;
- Akaike Information Criterion (AIC);
- Schwartz Information Criterion (SIC).

Table 9
Estimation results of the fractional error correction models (ECMF).

Models	FEC Model 1		FEC Model 2	
	Coefficients	P-Value	Coefficients	P-Value
Intercept	−0.8148	0.0000	0.6068	0.0000
ΔOil_t	0.4996	0.9865	0.4657	0.0008
ΔGold_t	−0.1160	0.58600	0.1586	0.1051
ΔGas_t	0.3252	0.0002	0.8867	0.0294
ΔCases_t	−0.3785	0.4953	0.3747	0.1414
ΔDeaths_t	0.4873	0.47067	−0.4644	0.1346
$\Delta \text{S\&P500}_t$	0.1161	0.0000	0.1161	0.0000
$\Delta^d z_{t-1}$	0.0163	0.0000	−0.6066	0.0000

become stationary). From Table 9, the estimation results of the FEC model 1 clearly show significant and positive short-term relationships between Bitcoin, S&P500 and Natural Gas. On the other hand, the estimation results of FEC model 2 display a positive and significant short-term relationship between Bitcoin, Oil, Gas and S&P500. From Table 9, the error-correction term is negative and significant for the FEC model 2. This implies that Bitcoin market tends to move to restore equilibrium. The sign of the error-correction term also shows that changes in the Bitcoin returns adjust in opposite direction to the previous period's deviation from equilibrium. In particular, the coefficient of −0.6066 indicates that about 60% of the disequilibria of the previous period shock adjusting back to the long-run equilibrium in the current period.

4. Conclusion

In this paper, we attempt to investigate the dynamic relationships between Bitcoin and other assets (S&P500, Crude Oil and Natural Gas) during the Covid-19 pandemic. To do so, we use fractional cointegration analysis to better examine short- and long-run dependencies inherent in the daily data during the period 01/09/2019–30/04/2020. Overall, the fractional integration offers a suitable statistical framework for capturing long- and short-term dependences in time series (e.g. Baillie, 1996). The one of statistical advantages of using the ARFIMA models is dealing with non-stationary data and better performing other models such as ARIMA models in terms of result estimation and forecasting accuracy due to the fractional difference parameter which explains correlation structure in data. In this respect, Bhardwaj and Swanson

(2004) stipulate that using ARFIMA models is more suitable for data (in our case, daily data) which display very slowly decaying autocorrelations given that it is based on the hyperbolic autocorrelation decay patterns. Nevertheless, we use such model without checking the heteroscedasticity of variances. Therefore, it is interesting to complete it, in further research, by using nonlinear models such as Markov-switching models.

Based on such analysis, there is substantial evidence that there exists fractional integration in residual series based on different tests. Such finding leads to the existence of a fractional cointegration relationship. That is, a stable relationship between Bitcoin and other assets is well-documented. A short-run joint dynamics between Bitcoin and some other assets (Crude Oil, S&P500 and Natural Gas) is nevertheless well-documented. Our results also show that the outbreak of coronavirus (during the first waves) cannot be viewed as totally catalyst in shaping the joint dynamics of different markets. Rather, such health crisis can lead to substantial very short-term persistence to shocks, but cannot play crucial role in mean-reverting behavior for the long-run. Such results further invite to high-frequency intraday analysis of the market behaviors during the health crises. Our analysis thus shows the usefulness of the fractional cointegration method as alternative econometric methods (compared to the NARDL model and DCC-GARCH model) for modeling the joint dynamics between cryptocurrency and other markets.

This study thus provides fresh and insightful understandings of the potential substantial nature of Bitcoin from short- and long-term perspectives. It also assesses which Bitcoin market can potentially be less/more suitable to be paired with gold, stock market and oil market to reduce the maximum financial risk with the advent of unexpected and unprecedented event. From portfolio management perspective, the investment decisions seem to be more challenging and critical during the health crisis period. That is why this study analyzes the dynamic connectedness among different assets and lays the initial foundations for understanding the potential safe-haven nature of Bitcoin. A deeper examination of the dependencies between assets still remains interesting during different (normal and turbulent) periods. The empirical findings of this paper can have fruitful implications for investors, policymakers and risk managers for using Bitcoin in optimal hedging or investment strategies while accounting for the heterogeneity in the horizons of investors. The heterogeneous patterns of connectedness between Bitcoin and other assets in short- and long-term could thus have an important impact on investor portfolio.

One important limitation that needs to be mentioned is the non-

consideration of the role of media hype and social media in determining the joint dynamics between markets during turbulent periods. Therefore, as interesting and potential research path, we would suggest to conduct an empirical analysis on the media coverage influence and how information spillover from pandemic-related news to different markets can determine the risk contagion path. This can provide fresh insights on how the markets incorporate information into the price system and the degree of synchronization between markets varies according to the announcement of unprecedented news.

Ethical approval

This article does not contain any studies with human participants

Appendix 1

Table A reports the variance-covariance matrix between different variables. The off-diagonal elements refer to the covariances between all possible pairs of variables whereas the diagonal elements of the matrix contain the variances of the variables (in bold). As well, Table 1 also presents the correlation matrix. Overall, some asymmetry patterns in variances and covariances between variables are well-documented. There is a positive relationship between Bitcoin and Gold. On the other hand, there is a negative relationship between Gold and Oil.

Table A
Relationship between Variables

Variance-Covariance Matrix							
Variables	Bitcoin	Oil	Gold	Gas	Cases	Deaths	S&P500
Bitcoin	20.422	6.313	0.223	0.863	3.103×10^5	2.213×10^4	3.711
Oil	6.313	3.736×10^3	-0.125	24.647	6.145×10^6	4.456×10^5	-12.5
Gold	0.223	-0.125	1.440	-0.156	2.504×10^4	1.269×10^3	0.050
Gas	0.863	24.647	-0.156	14.34	7.805×10^4	5.505×10^3	1.033
Cases	3.103×10^5	6.145×10^6	25039.1	78042.975	4.835×10^{11}	3.355×10^{10}	96442.69
Deaths	2.213×10^4	4.456×10^6	1268.667	5504.706	3.355×10^{10}	2.349×10^9	6452.91
S&P500	3.711	-12.5	0.050	1.033	96442.69	6452.91	4.037389
Correlation Matrix							
Variables	Bitcoin	Oil	Gold	Gas	Cases	Deaths	S&P500
Bitcoin	1.000	0.023	0.041	0.050	0.099	0.101	0.409
Oil	0.023	1.000	-0.002	0.106	0.145	0.150	0.102
Gold	0.041	-0.002	1.000	-0.034	0.030	0.022	0.021
Gas	0.050	0.106	-0.034	1.000	0.030	0.031	0.136
Cases	0.099	0.145	0.030	0.031	1.000	0.995	0.069
Deaths	0.101	0.150	0.022	0.03	0.995	1.000	0.066
S&P500	0.409	-0.102	0.021	0.136	0.069	0.066	1.000

Appendix 2

performed by any of the authors.

Declaration of competing interest

We declare no conflict interest between all authors in this paper.

Data availability

Data will be made available on request.

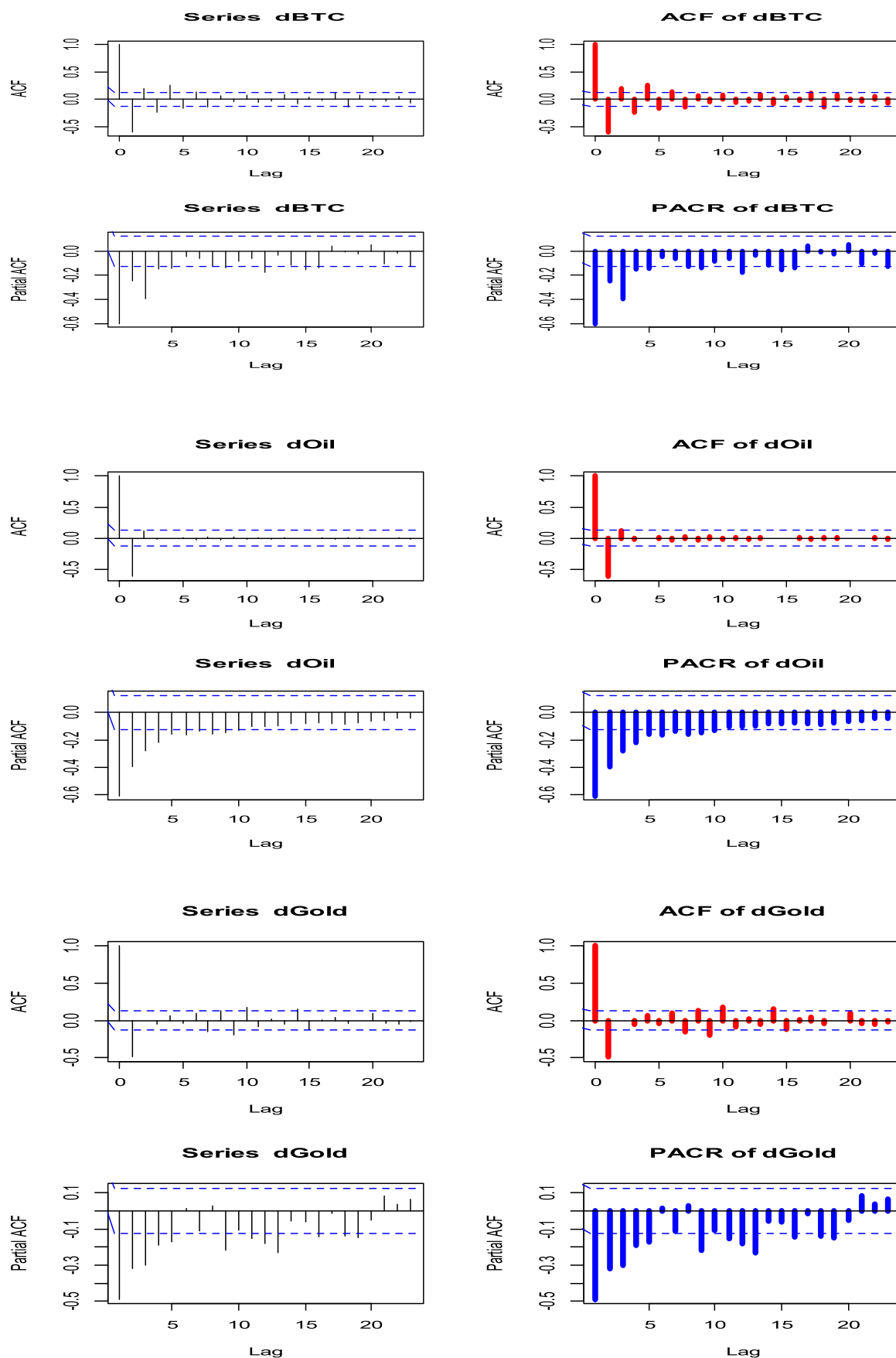


Fig. 2. ACF and PACF of Fractionally Differenced Series.

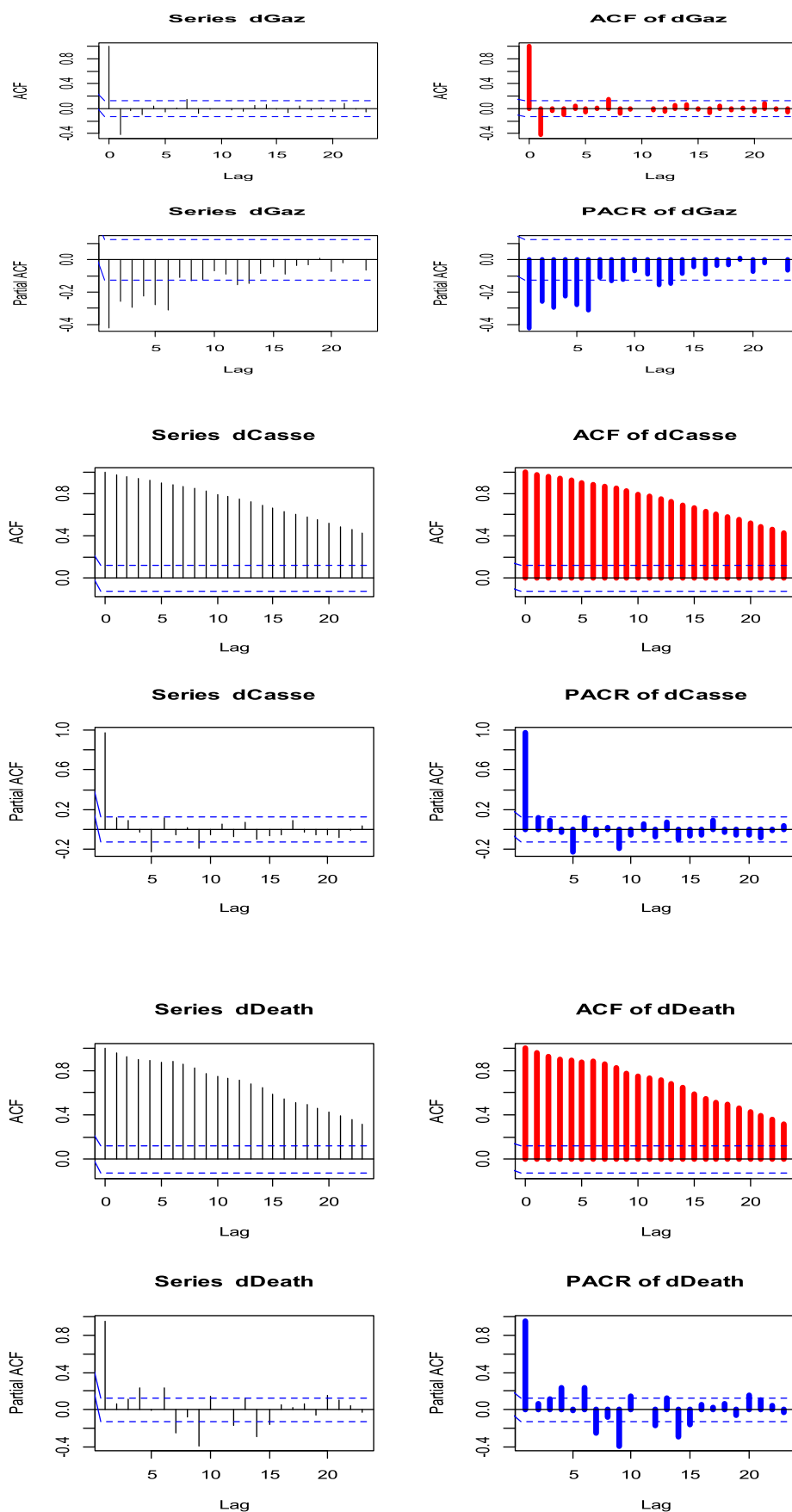


Fig. 2. (continued).

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