Cointegration Analysis Between BTC Price and Other Cryptocurrencies: A Time Series Approach

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1 Abstract

This paper analyzes the cointegration between Bitcoin (BTC) price and three other cryptocurrencies: Solana (SOL), Cardano (ADA), and Binance Coin (BNB). The Engle-Granger method and the Johansen test are used to determine the presence of cointegration. The results show that there is cointegration in the BTC-SOL and BTC-BNB relationships but not in the BTC-ADA relationship. These findings suggest a long-term relationship between BTC-SOL and BTC-BNB prices but not between BTC and ADA. The paper is structured as follows: the State of the Art, covers the research context related to the proposed topic, which is currently trending. The Data and Main Descriptive Statistics section explains the data collection along with a brief analysis. The Methodology section outlines the process used to generate the results analyzed in the following section. Finally, the Conclusions and Annexes sections summarize the key findings along with the code to replicate the exercise.

2 State of the Art

To delve into the topic of cointegration, we reference the work of Campbell and Shiller (1987). This study reviews some models and proposed methods for testing cointegration in the context of economic variables. Although it does not focus on cryptocurrencies, it provides an important theoretical framework for understanding cointegration concepts and time series analysis. This article highlights the importance of cointegration in analyzing long-term relationships and is relevant for understanding the theoretical basis of cointegration analysis in general.

To gain a broader understanding of how the stock market functions, the work of Chang et al. (2008) is useful. Despite focusing on short-term adjustments in the U.S. market, it is relevant to consider its discussion on market efficiency, as it warns of possible rational bubbles in the stock market, along with its analysis of market volatility. It is essential to consider these phenomena in the cryptocurrency market as well. Furthermore, to better understand the dynamics of Bitcoin (BTC), Aleti and Mizrach (2021) provide a solid context by investigating causal relationships, cointegration, and price discovery between Bitcoin spot and futures markets.

Regarding cointegration between Bitcoin and other markets, we can mention the works of Bejaoui et al. (2022) and Lee and Rhee (2022), who examine the relationship between Bitcoin and various macroeconomic variables and assets such as crude oil, the SP500, and natural gas, as well as the price of gold, etc. (The first study analyzes it during the COVID period). Both conclude with evidence of long-term and short-term dependencies between Bitcoin and various assets.

Focusing on our study's approach, there is extensive literature supporting cointegration among different cryptocurrencies. From a theoretical perspective, cointegration is primarily associated with BTC, as it was the first cryptocurrency, and many argue that it significantly influences "newer" coins. In this sense, for example, Göttfert (2019) examines whether there is cointegration between BTC's daily price and five other cryptocurrencies: Ethereum, Ripple, Bitcoin Cash, EOS, and Litecoin, over five different time periods, applying two different cointegration tests: the Engle-Granger approach with the Johansen cointegration test and a Vector Error Correction Model (VECM). The study concludes that Bitcoin's price has a statistically significant long-term impact on all the prices except EOS.

Another example is Gül (2022), who examines cointegration relationships between BTC prices and several major cryptocurrencies: BNB, ADA, DOGE, ETH, DOT, and XRP. In addition to finding cointegration and long-term relationships, the study identifies a bidirectional causal relationship between BNB and ETH. Both studies contribute to the growing body of research exploring relationships among different cryptocurrencies using time series analysis techniques.

3 Data and Main Descriptive Statistics

To explore the cointegration between BTC and other cryptocurrencies, we used weekly adjusted price data for the selected currencies: Solana (SOL), Cardano (ADA), and Binance Coin (BNB). This selection was based on comparing two currencies (ADA and BNB) with findings from previous literature and adding an additional one (SOL) that was not covered in the reviewed literature. The data were obtained from Yahoo Finance over a one-year period, starting from 08/05/2023.

BTC price was taken as $x1$, SOL as $y1$, ADA as $y2$, and BNB as $y3$, obtaining the following descriptive statistics.

```
> # Obtenemos los principales descriptivos para cada variable
> summary(x1)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  25832   28009   37221   41005   50873   71334 
> summary(y1)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  15.44   21.78   56.90   72.07  111.00  202.87 
> summary(y2)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
 0.2430  0.2902  0.3840  0.4183  0.5203  0.7282 
> summary(y3)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  208.3   234.5   247.1   326.9   342.5   606.9 
```

Figure 1: Main descriptive statistics by variable

Based on the results in Figure 1, we can generally say that ADA shows a narrower range compared to the other cryptocurrencies. Additionally, most distributions are right-skewed except for $y2$, where the mean and median are relatively close.

4 Methodology

The methodology described below was applied to each cointegration analysis, i.e., SOL-BTC, ADA-BTC, and BNB-BTC. However, we will explain the SOL-BTC case in detail since the methodology remains the same, with only the results varying in the next section. The results are presented in various figures in **??**, along with the code used in **??**.

First, we transformed the data into a time series, each containing 54 observations (weeks), as shown in Figure 2. Analyzing BTC's order, we performed the Augmented Dickey-Fuller (ADF) unit root test and found that the series is non-stationary, as indicated by the results in Figure 3, where the tau statistic is -1.9435, not statistically significant at even a 10% level.

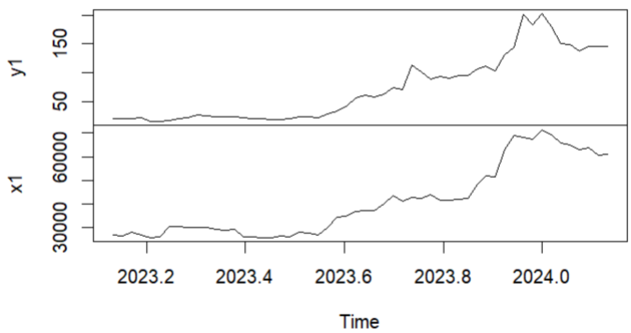

Figure 2: Time series: BTC (x1) and SOL (y1)

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```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max 
-4014.4 -1204.8  -326.8   872.4 10653.1 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)  
(Intercept) 2053.41011 1167.42828   1.759   0.0850 .
z.lag.1       -0.10757    0.05535  -1.944   0.0578 .
tt           104.16058   54.33494   1.917   0.0612 .
z.diff.lag     0.21129    0.13872   1.523   0.1343  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2549 on 48 degrees of freedom
Multiple R-squared:  0.1037,	Adjusted R-squared:  0.04768 
F-statistic: 1.851 on 3 and 48 DF,  p-value: 0.1505


Value of test-statistic is: -1.9435 2.0482 1.9624 

Critical values for test statistics: 
      1pct  5pct 10pct
tau3 -4.04 -3.45 -3.15
phi2  6.50  4.88  4.16
phi3  8.73  6.49  5.47
```

Figure 3: Augmented Dickey-Fuller Test for the BTC time series

Next, we verify that the series are first-order stationary. To do this, we apply a differentiation and use the unit root test again. In Figure 4, we can visually observe the differentiation for BTC-SOL, and after performing the ADF test on BTC prices, we obtain Figure 5, confirming that the series is first-order stationary at a significance level of 1%, as the new tau statistic is $-4.1534 < -2.6$.

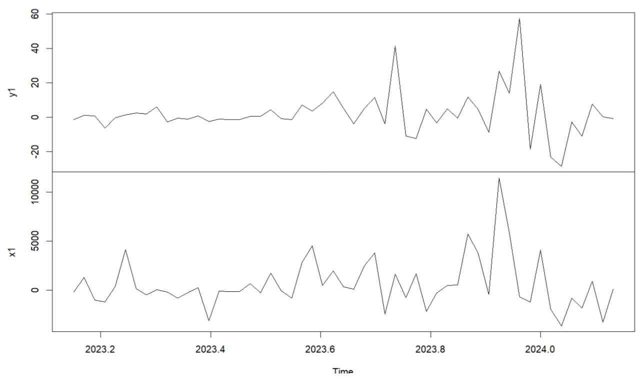

Figure 4: First-order series: BTC (x1) and SOL (y1)

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```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max
-3385.2  -919.6    40.2  1723.2 11345.7

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
z.lag.1    -0.74061    0.17831  -4.153 0.000131 ***
z.diff.lag -0.04491    0.14453  -0.311 0.757343
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2670 on 49 degrees of freedom
Multiple R-squared:  0.3901,    Adjusted R-squared:  0.3652
F-statistic: 15.67 on 2 and 49 DF,  p-value: 5.48e-06


Value of test-statistic is: -4.1534

Critical values for test statistics:
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

Figure 5: ADF Test: First-order BTC series

The rest of the unit root tests for the remaining series can be found in **??**. Since both series are of the same order, we can analyze their cointegration. We use the Engle and Granger method, which establishes that: since we are estimating the coefficient, we first use ordinary least squares, then we can perform unit root tests on the residuals, and if they are stationary, we can say that both variables are cointegrated or that a long-term relationship exists between them.

Figure 6 shows the residuals graph of the SOL-BTC model. Visually, we can infer that it meets the condition and that the residuals are stationary. However, to verify this, we perform the ADF test, obtaining in Figure 7 that the residuals are stationary or order 0, since being less than 2, we reject the null hypothesis of non-stationarity.

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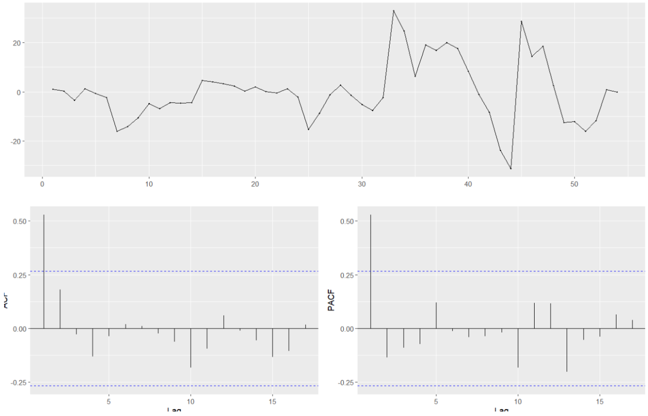

Figure 6: Residuals: model `y1~x1`

```
###############################################################
# Augmented Dickey-Fuller Test Unit Root / Cointegration Test #
###############################################################

The value of the test statistic is: -4.0086
```

Figure 7: ADF: residuals of the model `y1~x1`

Since β in the model is an estimate, the ADF test is not the best option to determine whether there is cointegration between the variables, as its critical values would not be properly calculated. Therefore, we complement the analysis using the Johansen test, which provides more reliable results in large samples. The amount of data obtained is considered sufficient to apply this methodology and compare its results.

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```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.242101218 0.002971283

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 |  0.15  6.50  8.18 11.65
r = 0  | 14.57 15.66 17.95 23.52

Eigenvectors, normalised to first column:
(These are the cointegration relations)

             y1.l2       x1.l2
y1.l2  1.000000000 1.000000000
x1.l2 -0.003785911 0.004783697

Weights W:
(This is the loading matrix)

          y1.l2        x1.l2
y1.d -0.4506509 -0.003507529
x1.d 26.1484996 -1.066094238
```

Figure 8: Johansen Test: model `y1~x1`

5 Results Analysis

The results show that all selected time series are of order 1. Therefore, we can apply the Engle and Granger method. Additionally, for each cointegration analysis, we verify whether a long-term relationship exists between both variables using the Johansen procedure, based on the explanation of the results generated by the command found in QuantStart (n.d.).

For SOL-BTC, as shown in Figure 8 in the section "Values of test statistic and critical values of test", the test statistic value for the null hypothesis $r <= 1$ (0.15) is lower than the corresponding critical values at significance levels, indicating that the null hypothesis of $r <= 1$ is not rejected. On the other hand, the test statistic value for the null hypothesis $r = 0$ (14.57) is also lower than the critical values but is close to the critical value at a 10

The cases of ADA-BTC and BNB-BTC are somewhat different. As seen in Figure 21 and Figure 22, the ADF test indicates that there is no stationarity in the residuals for either model. To enhance the analysis, the Johansen procedure is also applied in both cases.

In the case of ADA-BTC, as shown in Figure 23, the statistics 8.42 and 1.46 are significantly lower than the critical values. Therefore, we do not have statistical evidence to

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reject the null hypothesis of no cointegration between the variables.

Finally, in the case of BNB-BTC, using the results shown in Figure 24, we see that the statistic 16.03 for the test $r = 0$ exceeds the critical value at the 10% significance level, providing statistical evidence of cointegration between these variables or a long-term relationship, unlike the results obtained from the ADF test.

6 Conclusions

1. Based on the Johansen procedure, there is cointegration in the BTC-BNB price relationship, implying that the prices of these cryptocurrencies tend to move together in the long run. Possible cointegration is also suggested in the BTC-SOL relationship, as its statistic is close to the critical value at a 10

2. There is no cointegration between BTC and ADA prices (something to contrast with the literature review findings), implying that the prices of these cryptocurrencies do not tend to move together in the long run.

3. The results suggest that BTC has a greater influence on SOL and BNB prices than on ADA prices. These findings could have significant implications for investors seeking a better understanding of the cryptocurrency market.

4. As an observation, it is important to mention that the data sample taken may not have been large enough as initially considered, which could affect the obtained results. Additionally, the literature review mentions how high volatility in these markets could change the results over time.

5. Since there is cointegration between the two variables, an error correction model may exist. That is, this long-term relationship can be modeled using a Vector Error Correction Model (VECM), which could help predict future movements or understand market dynamics.

References

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7 Appendix 1

The code used in R software, along with the data utilized for this study, can be found at the following GitHub link:

https://github.com/Sebas20031-DS/Research-and-Writings/tree/main/Time%20Series%20Cointegration%20Analysis%20BTC%20and%20Altcoins

File name: Cointegration_BTC_cryptocurrencies.R

8 Appendix 2

Original Time Series Compared to BTC:

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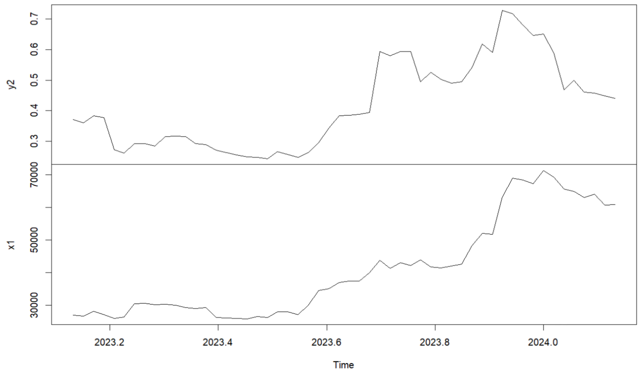

Figure 9: Time series: BTC (x1) and ADA (y2)

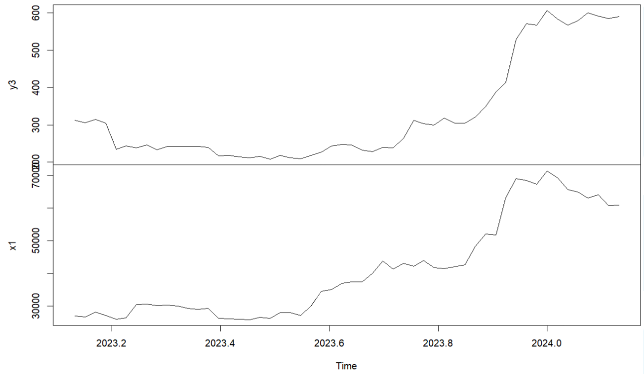

Figure 10: Time series: BTC (x1) and BNB (y3)

ADF Test for Original Series:

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```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max
-25.756  -6.212  -1.721   2.851  57.592

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.56571    4.19875  -0.611   0.5440
z.lag.1     -0.17240    0.07938  -2.172   0.0348 *
tt           0.63265    0.29488   2.145   0.0370 *
z.diff.lag  -0.02420    0.14130  -0.171   0.8648
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.12 on 48 degrees of freedom
Multiple R-squared:  0.1004,	Adjusted R-squared:  0.04413
F-statistic: 1.785 on 3 and 48 DF,  p-value: 0.1626


Value of test-statistic is: -2.172 2.3113 2.4403

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -4.04 -3.45 -3.15
phi2  6.50  4.88  4.16
phi3  8.73  6.49  5.47
```

Figure 11: Augmented Dickey-Fuller test: SOL time series

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max
-0.11360 -0.01988 -0.00590  0.01639  0.19191

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0316551  0.0215071   1.472   0.1476
z.lag.1     -0.1285419  0.0745025  -1.725   0.0909 .
tt           0.0008588  0.0007109   1.208   0.2329
z.diff.lag   0.0668504  0.1452668   0.460   0.6475
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0492 on 48 degrees of freedom
Multiple R-squared:  0.05903,	Adjusted R-squared:  0.0002144
F-statistic: 1.004 on 3 and 48 DF,  p-value: 0.3994


Value of test-statistic is: -1.7253 1.0208 1.5055

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -4.04 -3.45 -3.15
phi2  6.50  4.88  4.16
phi3  8.73  6.49  5.47
```

Figure 12: Augmented Dickey-Fuller test: ADA time series

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```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max
-52.994 -13.081   0.365   7.257  98.060

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.00563    8.37110   0.120   0.9049
z.lag.1     -0.06411    0.03652  -1.755   0.0856 .
tt           0.87781    0.33013   2.659   0.0106 *
z.diff.lag   0.18390    0.13858   1.327   0.1908
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 22.55 on 48 degrees of freedom
Multiple R-squared:  0.1942,	Adjusted R-squared:  0.1439
F-statistic: 3.857 on 3 and 48 DF,  p-value: 0.01496


Value of test-statistic is: -1.7555 2.9171 3.5811

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -4.04 -3.45 -3.15
phi2  6.50  4.88  4.16
phi3  8.73  6.49  5.47
```

Figure 13: Augmented Dickey-Fuller test: BNB time series

Differentiated Series:

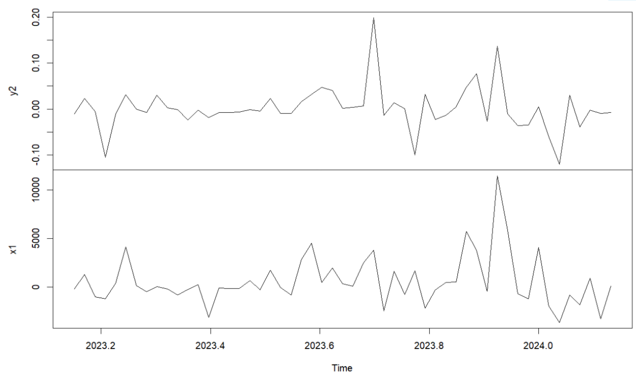

Figure 14: First-order series: BTC (x1) and ADA (y2)

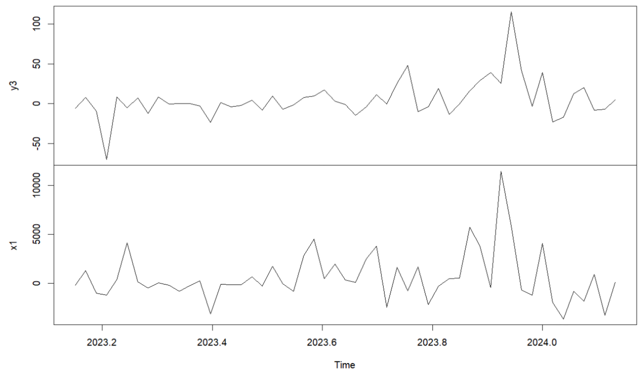

Figure 15: First-order series: BTC (x1) and BNB (y3)

ADF Test for Differentiated Series:

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max 
-34.728  -2.445   1.152   4.889  50.692 

Coefficients:
           Estimate Std. Error t value Pr(>|t|)    
z.lag.1     -0.7642     0.1997  -3.827 0.000368 ***
z.diff.lag  -0.2784     0.1372  -2.029 0.047889 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.35 on 49 degrees of freedom
Multiple R-squared:  0.5661,	Adjusted R-squared:  0.5483 
F-statistic: 31.96 on 2 and 49 DF,  p-value: 1.31e-09


Value of test-statistic is: -3.8271 

Critical values for test statistics: 
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

Figure 16: ADF Test: SOL first-order series

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```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max
-0.120708 -0.012995 -0.003873  0.012061  0.198697

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
z.lag.1    -0.91985    0.20099  -4.577 3.25e-05 ***
z.diff.lag -0.07712    0.14216  -0.542     0.59
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.04997 on 49 degrees of freedom
Multiple R-squared:  0.5024,    Adjusted R-squared:  0.4821
F-statistic: 24.73 on 2 and 49 DF,  p-value: 3.75e-08


Value of test-statistic is: -4.5765

Critical values for test statistics:
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

Figure 17: ADF Test: ADA first-order series

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max
-68.083  -7.453   0.159  12.851 103.618

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
z.lag.1     -0.6155     0.1670  -3.686  0.00057 ***
z.diff.lag  -0.1107     0.1419  -0.780  0.43886
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 24.08 on 49 degrees of freedom
Multiple R-squared:  0.3544,    Adjusted R-squared:  0.3281
F-statistic: 13.45 on 2 and 49 DF,  p-value: 2.208e-05


Value of test-statistic is: -3.6861

Critical values for test statistics:
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

Figure 18: ADF Test: BNB first-order series

Model Residuals:

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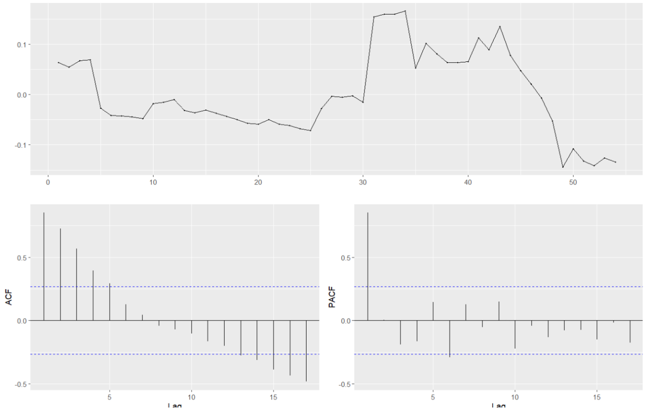

Figure 19: Residuals: Model $y2 \sim x1$

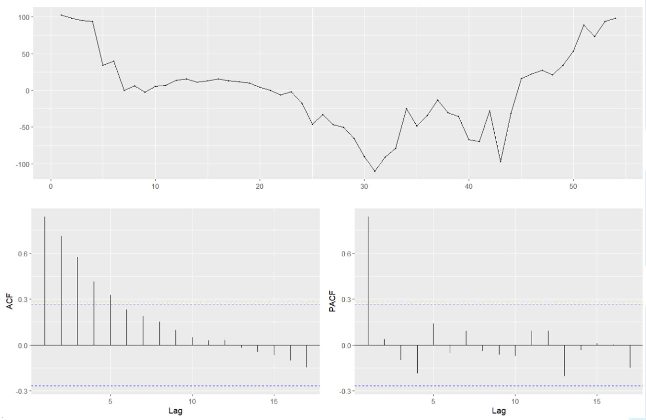

Figure 20: Residuals: Model $y3 \sim x1$

ADF Test for Residuals:

```
###############################################################
# Augmented Dickey-Fuller Test Unit Root / Cointegration Test #
###############################################################

The value of the test statistic is: -1.522
```

Figure 21: ADF Test: Residuals of model $y2 \sim x1$

```
###############################################################
# Augmented Dickey-Fuller Test Unit Root / Cointegration Test #
###############################################################

The value of the test statistic is: -1.7438
```

Figure 22: ADF Test: Residuals of model $y3 \sim x1$

Johansen Process for Models:

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```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.12522653 0.02774589

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 | 1.46  6.50  8.18 11.65
r = 0  | 8.42 15.66 17.95 23.52

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                y2.l2         x1.l2
y2.l2  1.000000e+00  1.000000e+00
x1.l2 -5.644381e-06 -1.826931e-05

Weights W:
(This is the loading matrix)

              y2.l2        x1.l2
y2.d   -0.1047916 4.229377e-02
x1.d 4224.3194049 2.397945e+03
```

Figure 23: Johansen Test: Model $y2 \sim x1$

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.22140267 0.05630886

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 |  3.01  6.50  8.18 11.65
r = 0  | 16.03 15.66 17.95 23.52

Eigenvectors, normalised to first column:
(These are the cointegration relations)

             y3.l2        x1.l2
y3.l2  1.00000000  1.000000000
x1.l2 -0.01269204 -0.005985037

Weights W:
(This is the loading matrix)

           y3.l2        x1.l2
y3.d -0.1303542  -0.02821743
x1.d -1.9103076 -10.69694405
```

Figure 24: Johansen Test: Model $y3 \sim x1$

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