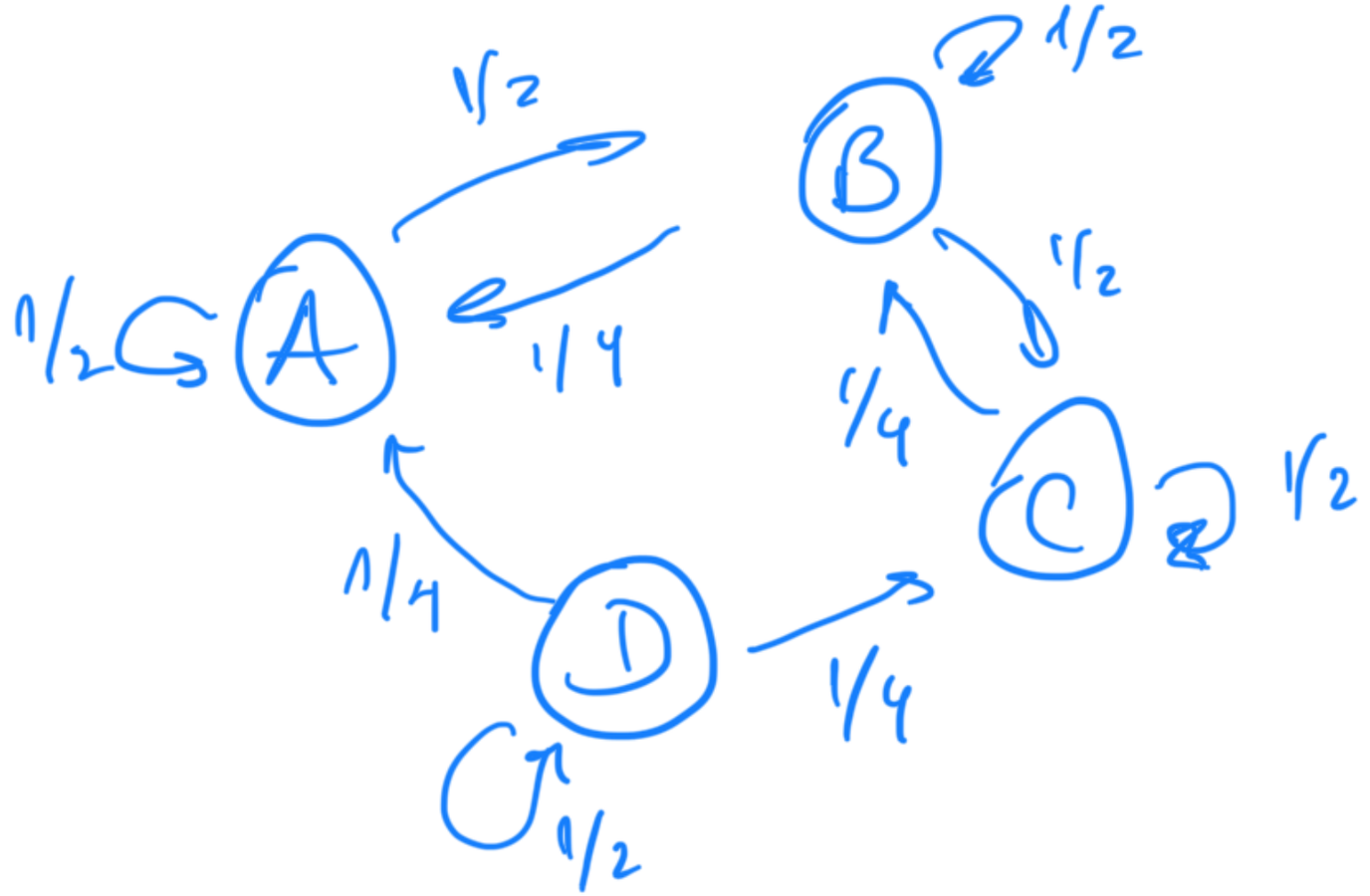
Ejemplo Markov



$$A = \begin{pmatrix} 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/2 & 1/2 & 0 \\ 0 & 1/4 & 1/2 & 1/4 \\ 0 & 0 & 0 & 1/2 \end{pmatrix}$$

$$\chi_A(\lambda) = \det \begin{pmatrix} \lambda - 1/2 & -1/4 & 0 & -1/4 \\ -1/2 & \lambda - 1/2 & -1/2 & 0 \\ 0 & -1/4 & \lambda - 1/2 & -1/4 \\ 0 & 0 & 0 & \lambda - 1/2 \end{pmatrix} = (\lambda - \tfrac{1}{2}) \det \begin{pmatrix} \lambda - \frac{1}{2} & -\frac{1}{4} & 0 \\ -\frac{1}{2} & \lambda - \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{4} & \lambda - \frac{1}{2} \end{pmatrix} =$$

$$= (\lambda - \tfrac{1}{2})\left((\lambda - \tfrac{1}{2}) \left((\lambda - \tfrac{1}{2})^2 - \tfrac{1}{8} \right) + \tfrac{1}{4}\left(-\tfrac{1}{2}\right)(\lambda - \tfrac{1}{2}) \right) =$$

$$= (\lambda - \tfrac{1}{2})^2 \left((\lambda - \tfrac{1}{2})^2 - \tfrac{1}{8} - \tfrac{1}{8} \right)$$

$$(\lambda - \tfrac{1}{2})^2 - \tfrac{1}{4} = 0 \iff |\lambda - \tfrac{1}{2}| = \tfrac{1}{2} \begin{cases} \lambda = 0 \\ \lambda = 1 \end{cases}$$

$$= (\lambda - \tfrac{1}{2})^2 (\lambda - 1)\lambda \rightarrow \text{autovalores: } 0, \tfrac{1}{2}, 1$$

multip 2

Busco autovectores y llegamos a

$$A \begin{pmatrix} -1/2 \\ 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad A \begin{pmatrix} 1/4 \\ 1 \\ 1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 1 \\ 1/2 \\ 0 \end{pmatrix}$$

$$A \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$E_0 = \langle (-\tfrac{1}{2}, 1, -\tfrac{1}{2}, 0) \rangle = \langle (1, -2, 1, 0) \rangle$$

$$E_1 = \langle (\tfrac{1}{4}, 1, \tfrac{1}{2}, 0) \rangle = \langle (1, 4, 2, 0) \rangle$$

$$E_{1/2} = \langle (-1, 0, 1, 0), (0, -1, 0, 1) \rangle$$

donc $r^{(0)} = \begin{pmatrix} 300 \\ 300 \\ 300 \\ 300 \end{pmatrix}$

exemple pour $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{4}{7}\begin{pmatrix} 1 \\ 4 \\ 2 \\ 0 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} - \frac{2}{7}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

$$\Rightarrow \underbrace{\begin{pmatrix} 300 \\ 300 \\ 300 \\ 300 \end{pmatrix}}_{} = \frac{1200}{7}\begin{pmatrix} 1 \\ 4 \\ 2 \\ 0 \end{pmatrix} + \frac{300}{7}\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} - \frac{600}{7}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + 300\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow A^k \begin{pmatrix} 300 \\ 300 \\ 300 \\ 300 \end{pmatrix} = A^k \left[\frac{1200}{7} \begin{pmatrix} 1 \\ 4 \\ 2 \\ 0 \end{pmatrix} + \frac{300}{7} \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} - \frac{600}{7} \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + 300 \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix} \right]$$

$$= \frac{1200}{7} A^k \begin{pmatrix} 1 \\ 4 \\ 2 \\ 0 \end{pmatrix} + \frac{300}{7} A^k \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix} - \frac{600}{7} A^k \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + 300 A^k \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

$$1^k \begin{pmatrix} 1 \\ 4 \\ 2 \\ 0 \end{pmatrix} \qquad 0^k \begin{pmatrix} 1 \\ -2 \\ 0 \\ -1 \end{pmatrix} \qquad \left(\frac{1}{2}\right)^k \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad \left(\frac{1}{2}\right)^k \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow v^{(\infty)} = \lim_{k \to \infty} v^{(k)} = \lim_{k \to \infty} A^k \begin{pmatrix} 300 \\ 300 \\ 300 \\ 300 \end{pmatrix} = \frac{1200}{7} \begin{pmatrix} 1 \\ 4 \\ 2 \\ 0 \end{pmatrix}$$

Finalmente hozía 1200 visitantes

→ Cuando el límite hay $\frac{1200}{7} + 4 \cdot \frac{1200}{7} + 2\,\frac{1200}{7} = 1200$

$$E_1 = \langle (1,4,2,0) \rangle = \langle (1/7, 4/7, 2/7, 0) \rangle$$

$$A^{\infty} = \begin{pmatrix} 1/7 & 1/7 & 1/7 & 1/7 \\ 4/7 & 4/7 & 4/7 & 4/7 \\ 2/7 & 2/7 & 2/7 & 2/7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

⇒ como existe A^{∞} ⟶ $r^{(\infty)} = A^{\infty} r^{(0)}$

$$A^{\infty} \begin{pmatrix} 300 \\ 300 \\ 300 \\ 300 \end{pmatrix} = \begin{pmatrix} 1/7 & 1/7 & 1/7 & 1/7 \\ 4/7 & 4/7 & 4/7 & 4/7 \\ 2/7 & 2/7 & 2/7 & 2/7 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 300 \\ 300 \\ 300 \\ 300 \end{pmatrix} = \begin{pmatrix} \frac{1200}{7} \\ 4 \cdot \frac{1200}{7} \\ 2 \cdot \frac{1200}{7} \\ 0 \end{pmatrix}$$