Model and Observer-Based Controller Design for a Quanser Helicopter with Two DOF

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Model and Observer-Based Controller Design for a Quanser Helicopter with two DOF

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***Abstract*—In this work, the dynamic model and the prototype control for a helicopter of 2 Degrees of Freedom (DOF) are presented. The model of the system is nonlinear and open loop unstable, which is obtained by using the equations of movement of Euler-Lagrange. The resulting linear model is function of an operating point. The controller design consists of two control loops, one for the elevation angle (Pitch) and the other for rotational motion (Yaw). For the control system, state feedback techniques are used and an observer is designed to estimate all states. The equations from the state feedback control scheme with observer are described and from them, a central controller is designed. This is an observer-based control. The controller achieves stability and good performance in different operating points. The model was found by considering additional parameters that were not taken into account in the model provided by the manufacturer, obtaining a more accurate model.**

***Keywords*-Modelling; LQR control; Integral effect; Quanser; Observer.**

I. Introduction

In the control applications of chemical processes, robotic, control of aircraft, among others, it is required to control different physical variables, such as temperature, pressure, flux, level, etc., to carry out a determined purpose. The concept of feedback is used, which consists on comparing the desired value of a physical variable in front of the real value and based on the error, running a determined control action. In the design of feedback control systems it is necessary to find a controller to keep or get the stability to diminish the perturbances that are signals that tend to change the value of a signal or output variable.

The two DOF helicopter consists of two rotors in quadrature, pivoted around the Pitch and Yaw axes. To find the dynamic model of the plant, the method of Euler-Lagrange was used, which describes the equations of movement and implicated forces. The resulting model is nonlinear, from which a linear model that is in function of the pitch angle. The control system in closed loop uses a LQR controller; the LQR control is a state feedback technique that finds an optimum solution to a minimization problem.

The document is divided into a part of modeling, another from the control system design, and finally it shows the results obtained in the real plant.

II. Model

The helicopter with two degrees of freedom is pivoted around the Pitch axis by the θ angle and around the Yaw axis by the ψ angle [11][12][15] as shown in Figure 1; the Pitch angle is defined positive when the nose of the helicopter rises and the Yaw angle is defined positive for rotations in a clockwise direction.

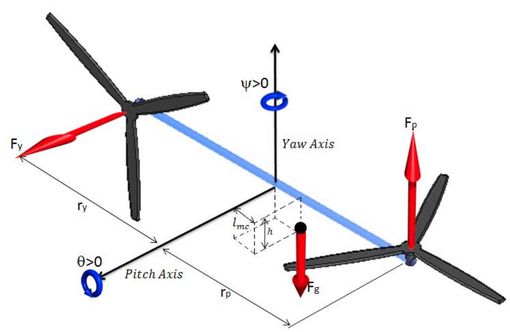


Fig. 1. Dynamics of 2DOF Helicopter

The gravitational force F_g generates a torque in the Pitch axis that makes the nose of the helicopter drop because the center of mass is not in the pivot but at a distance l_{mc} along the length of the fuselage at a height h below this. Figure 2 shows the direct kinematics of the helicopter.

To find the equations of motion the Euler-Lagrange method was used, which is based on the analysis of kinetic and potential energy of the system.

The position of center of mass of the system in relation to the fixed coordinate system (pivot) is found by using homogeneous transformation matrices $(Rot_{z_0,\psi})$ $(Rot_{y_1,\theta})$, [9][2][13] and the Cartesian position of the mass center is obtained.

$$X_{mc} = (l_{mc}\ \cos\theta + h\ \sin\theta)\ \cos\psi \quad (1)$$
$$Y_{mc} = (-l_{mc}\ \cos\theta - h\ \sin\theta)\ \sin\psi \quad (2)$$
$$Z_{mc} = l_{mc}\ \sin\theta - h\ \cos\theta \quad (3)$$

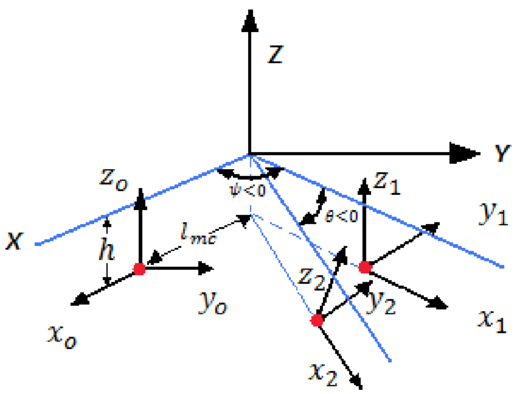


Fig. 2. Kinematics of 2DOF Helicopter

A. Kinetic and Potential Energy

The potential energy due to gravity is [10][11][14]

$$P = m_{heli}\, g\, Z_{mc} = m_{heli}\, g\, (l_{mc}\ \sin\theta - h\ \cos\theta) \quad (4)$$

Where m_{heli} is the total moving mass of the helicopter.

The total kinetic energy

$$T = T_{r,p} + T_{r,y} + T_t \quad (5)$$

Is the addition of rotational kinetic energy acting on Pitch $T_{r,p}$ and Yaw $T_{r,y}$ with translational kinetic energy generated by the movement of the center of mass T_t. The rotational kinetic energy on Pitch and Yaw is

$$T_{r,p} = \frac{1}{2} J_{eqp}\dot{\theta}^2 \qquad T_{r,y} = \frac{1}{2} J_{eqy}\dot{\psi}^2 \quad (6)$$

Where J_{eqp} and J_{eqy} are the moments of inertia equivalent respectively to Pitch and Yaw. Translational kinetic energy is

$$T_t = \frac{1}{2} m_{heli}(\dot{X_{mc}}^2 + \dot{Y_{mc}}^2 + \dot{Z_{mc}}^2) \quad (7)$$

B. Nonlinear Equations of Motion

The equations of motion of Euler-Lagrange [2][6][9] are defined as

$$\begin{aligned} \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q_1}} - \frac{\partial L}{\partial q_1} &= Q_1 \\ \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q_2}} - \frac{\partial L}{\partial q_2} &= Q_2 \end{aligned} \quad (8)$$

Where L is the Lagrange variable, which corresponds to the difference between kinetic and potential energy of the system

$$L = T - P \quad (9)$$

The generalized coordinates are

$$q = [q_1 \quad q_2 \quad \dot{q_1} \quad \dot{q_2}]^T = [\theta \quad \psi \quad \dot{\theta} \quad \dot{\psi}]^T$$

and the generalized forces are

$$Q_1 = \tau_p(V_{mp}, V_{my}) - B_p\dot{\theta} \qquad Q_2 = \tau_y(V_{mp}, V_{my}) - B_y\dot{\psi}$$

Before Equation includes the rotational viscous friction acting about the Pitch and Yaw axes B_p y B_y . The torques generated at the Pitch and Yaw axes are function of the voltages applied to the motors.

$$\tau_p(V_{mp}, V_{my}) = \tau_{pp} + \tau_{py} = (K_{pp}V_{mp} + F_{cpp}) + (K_{py}V_{my} + F_{cpy})$$

$$\begin{aligned} \tau_y(V_{mp}, V_{my}) = \tau_{yp} + \tau_{yy} = (K_{yp}V_{mp} + F_{cyp})\cos\theta \\ +(K_{yy}V_{my} + F_{cyy})\cos\theta \end{aligned}$$

where V_{mp} is the input Pitch motor voltage and V_{my} is the input Yaw motor voltage. The torques acting on the axes of Pitch and Yaw are coupled.

The functions of torque are represented by τ_{pp}, τ_{yy}, τ_{py} y τ_{yp} K_{pp}, while K_{yy}, K_{py} and K_{yp} are the constants for voltage to torque of the Pitch and Yaw motor/propeller actuators that were found experimentally. $F_{cpp}, F_{cpy}, F_{cyp}$ y F_{cyy} are the constant terms appearing in the linear regression due to Coulomb friction, it is the voltage needed for the system to initiate movement.

Evaluating the Euler-Lagrange expressions results in the nonlinear equations of motion

$$\begin{aligned} (J_{eqp} + M_{heli}(l_{mc}^2 + h^2))\ddot{\theta} + M_{heli}\left[\tfrac{\sin 2\theta(l_{mc}^2 - h^2)}{2} - l_{mc}h\cos 2\theta\right]\dot{\psi}^2 \\ +M_{heli}g(l_{mc}\cos\theta + h\sin\theta) + B_p\dot{\theta} \\ = (K_{pp}V_{mp} + F_{cpp}) + (K_{py}V_{my} + F_{cpy}) \end{aligned}$$

$$\begin{aligned} \left[J_{eqy} + M_{heli}\left[\cos^2\theta(l_{mc}^2 - h^2) + l_{mc}h\sin 2\theta + h^2\right]\right]\ddot{\psi} \\ +M_{heli}\left[\sin 2\theta(h^2 - l_{mc}^2) + 2l_{mc}h\cos 2\theta\right]\dot{\theta}\dot{\psi} + B_y\dot{\psi} \\ = (K_{yp}V_{mp} + F_{cyp})\cos\theta + (K_{yy}V_{my} + F_{cyy})\cos\theta \end{aligned}$$

C. Linear State-Space Model

If we define as state variables [14][1][8]

$$X = \begin{bmatrix}\theta & \psi & \dot{\theta} & \dot{\psi}\end{bmatrix}^T = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$$

And as control signals

$$U = [V_{mp} \quad V_{my}]^T = [u_1 \quad u_2]^T$$

Linearizing around the equilibrium induced by $\bar{x_1} = \bar{\theta}$ constant, obtain the linear model parameterized by $\bar{\theta}$ [5].

$$A(\bar{\theta}) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{M_{heli}g(l_{mc}\sin\bar{\theta} - h\cos\bar{\theta})}{[J_{eqp}+M_{heli}(l_{mc}^2+h^2)]} & 0 & \frac{-B_p}{[J_{eqp}+M_{heli}(l_{mc}^2+h^2)]} & 0 \\ 0 & 0 & 0 & \frac{-B_y}{J_{eqy}+M_{heli}[\cos^2\bar{\theta}(l_{mc}^2-h^2)+l_{mc}h\sin 2\bar{\theta}+h^2]} \end{bmatrix}$$

$$B(\bar{\theta}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{[J_{eqp}+M_{heli}(l_{mc}^2+h^2)]} & \frac{K_{py}}{[J_{eqp}+M_{heli}(l_{mc}^2+h^2)]} \\ \frac{K_{yp}\cos\bar{\theta}}{J_{eqy}+M_{heli}[\cos^2\bar{\theta}(l_{mc}^2-h^2)+l_{mc}h\sin 2\bar{\theta}+h^2]} & \frac{K_{yy}\cos\bar{\theta}}{J_{eqy}+M_{heli}[\cos^2\bar{\theta}(l_{mc}^2-h^2)+l_{mc}h\sin 2\bar{\theta}+h^2]} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

III. LQR Control with Integral Effect

For the LQR controller design with integral effect two new states are defined (Integral Pitch and Yaw error)[3][4][8][12]. The LQR Control minimizes the cost function

$$J = \int x^T Q x + u^T R u \tag{10}$$

The cost function indicates that we can give priority to certain states, and give weight to the control signals. The closed loop system with the LQR controller is a model of order 6, since the linearized plant model is of order 4 and increased system becomes of order 6 by the two integrators in Pitch and Yaw error.

LQR control design provides a payoff vector for the increased system, which must be separated into two: the first four elements of the vector are the proportional gains K and the other two are integral gains K_i.

IV. Observer Based Control

Based on the LQR controller with integral effect proposed in the previous section is designed observer. An observer is a dynamic system that simulates the behavior of the real system and it is used to estimate the states of the same [14][10][15]. The observer-based control shown in Figure 3 can be described as a matrix form

$$\begin{bmatrix} \dot{X}_\delta \\ \dot{X}_i \end{bmatrix} = \begin{bmatrix} A - BK - LC & BK_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_\delta \\ X_i \end{bmatrix} + \begin{bmatrix} 0 & L \\ I & -I \end{bmatrix} \begin{bmatrix} R \\ Y \end{bmatrix}$$

$$U_\delta = \begin{bmatrix} -K & K_i \end{bmatrix} \begin{bmatrix} \hat{X}_\delta \\ X_i \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} R \\ Y \end{bmatrix}$$

I is the identity matrix. The controller can be viewed as a controller of two degrees of freedom (2DOF).

With

$$Y = [I - PC_2]^{-1} PC_1 R$$

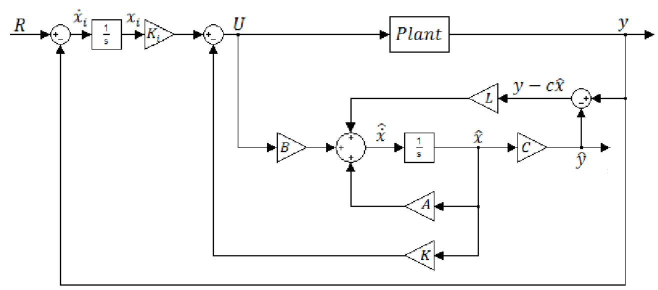


Fig. 3. Block diagram of observer-based controller

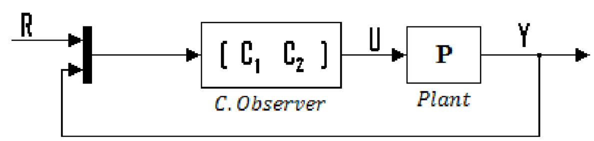


Fig. 4. Control block diagram of the observer-based 2DOF controller seen

V. LQR Control Design and Experimental Results

The parameter values were found experimentally, which can be seen in table 1.

Table 1. Parameter values

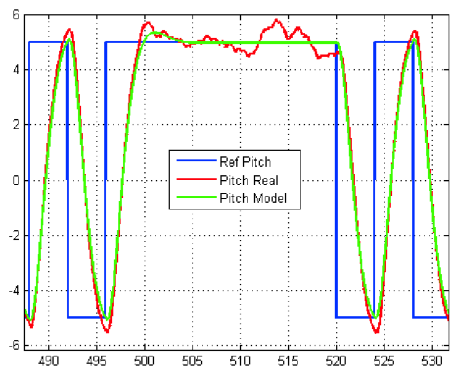
Symbol	**IDENT**	Units
K_{pp}	0.02638	N.m/V
K_{py}	0.00189	N.m/V
K_{yp}	0.002096	N.m/V
K_{yy}	0.01871	N.m/V
B_p	0.01325	N/V
B_y	0.8513	N/V
J_{eqp}	0.0332	kg.m^2
J_{eqy}	0.0371	kg.m^2
l_{mc}	0.0122	m
h	0.00714	m
M_{heli}	1.3872	kg

The matrices R and Q to $\theta = 0$ and $\theta = 30$ that were used are

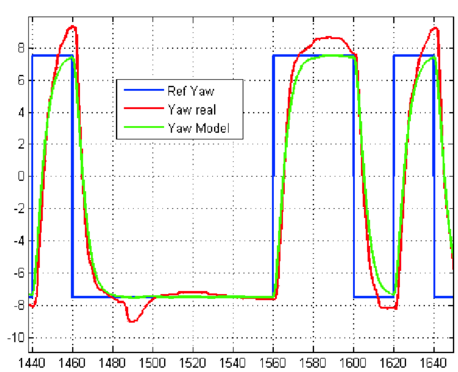
$$\begin{aligned} R &= diag([0.1 \quad 0.1]) \\ Q_{\theta=0} &= diag([2 \quad 40 \quad 10 \quad 5 \quad 8 \quad 2.5]) \\ Q_{\theta=30} &= diag([2 \quad 40 \quad 10 \quad 15 \quad 8 \quad 1.5]) \end{aligned}$$

The observer poles are located far away from the dominant poles of the closed loop system, (were located in -100, -110, -120 and -130) so that the closed loop system with the observer approaching a system of order 6, it is, like a LQR with integral effect without observer. The poles of the closed loop system are in $\lambda_1 = -0.2510$, $\lambda_2 = -0.7635 - 0.6053j$, $\lambda_3 = -0.7635 + 0.6053j$, $\lambda_4 = -2.9456$, $\lambda_5 = -7.7058$, $\lambda_6 = -9.6742$.

Figure 5 and 6 show the system response Vs. the excitation signal around the equilibrium points $\theta = 0$ and $\theta = 30$. It is seen that the time response is well behaved, since it does not present excessive overshoot and always reaches the reference value due to the integral effect.

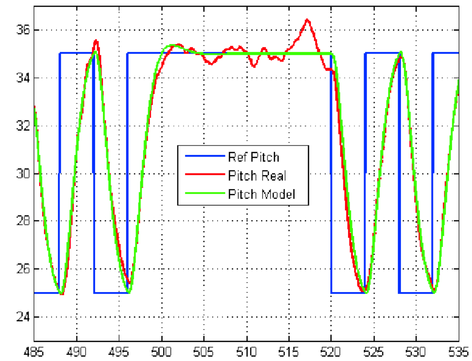


(a) Pitch $(around \quad \theta = 0^o)$

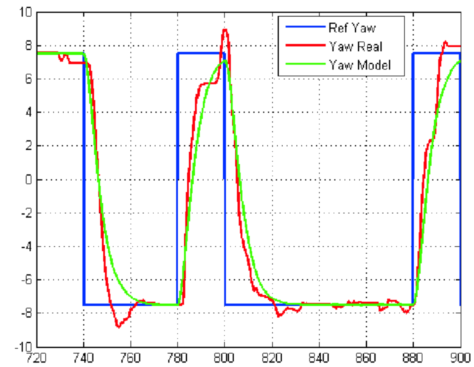


(b) Yaw $(around \quad \theta = 0^o)$

Fig. 5. Reference Vs Output $(around \quad \theta = 0^o)$



(a) Pitch $(around \quad \theta = 30^o)$



(b) Yaw $(around \quad \theta = 30^o)$

Fig. 6. Reference Vs Output $(around \quad \theta = 30^o)$

VI. Conclusion

The system mathematical model was found by considering additional parameters that were not taken into account in the model provided by the manufacturer Quanser [11], obtaining a more accurate model.

When making the linearization was found that it depends on the desired equilibrium point; it is, the system gain and the location of the poles change according to operating point.

We obtained a linear model parameterized by θ, which lets find the model in the desired operating point.

The system is open loop unstable. By analyzing the location of the poles from the parameters found experimentally, it was found that one pole goes to the right half plane, which makes the system more difficult to control. An unstable pole imposes algebraic constraints on the system's sensitivity function regardless of the control technique used; which implies restrictions on the achievable performance of the step response of closed loop system. When there is an unstable pole, there is necessarily overshoot in the step response, and this will be greater, the longer the settling time in closed loop is.

The time response in the Yaw axis is different when the movement in Yaw changes its direction; this is mainly due to the propeller is designed to generate thrust in just one direction, then the result is that the gain is different when moving in each direction.

The dynamic response in the Yaw axis is strongly affected by Coulomb friction because as the actual position is closer to the reference, control signal tends to zero and it is not sufficient to overcome the Coulomb friction, reason why there is no motion; in addition, each time the position reaches the desired value, the control signal tends to zero, then, when the reference signal is changed, we again must overcome the coulomb friction to start moving.

The observer-based control fulfilled stability and good performance at all operating points.

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