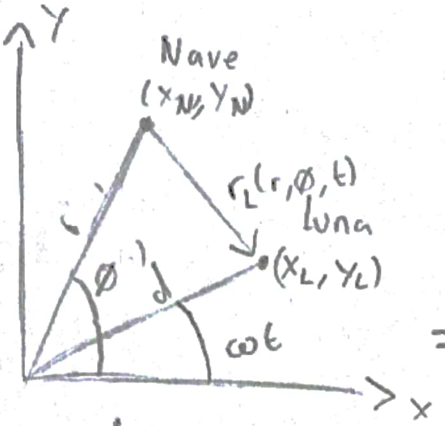
c)



$$r_L = \|(x_L - x_N, y_L - y_N)\|$$

$$= \sqrt{(x_L - x_N)^2 + (y_L - y_N)^2}$$

$$= \sqrt{x_L^2 - 2x_L x_N + x_N^2 + y_L^2 - 2y_L y_N + y_N^2}$$

$$= \sqrt{d^2 \cancel{\cos^2(\omega t)} - 2dr\cos(\omega t)\cos\phi + r^2 \cancel{\cos^2\phi} + d^2 \cancel{\sin^2 \omega t} - 2d\sin(\omega t)\sin\phi + r^2 \cancel{\sin^2\phi}}$$

$$x_L = d\cos(\omega t)$$
$$y_L = d\sin(\omega t)$$
$$x_N = r\cos\phi$$
$$y_N = r\sin\phi$$

$$= \sqrt{d^2 + r^2 - 2dr\cos(\omega t)\cos\phi - 2dr\sin(\omega t)\sin\phi}$$

Utilizando $\cos(a-b) = \cos a \cos b + \sin a \sin b$

$$r_L = \sqrt{d^2 + r^2 - 2dr\cos(\phi - \omega t)}$$

como $r \to r(t)$

$$r_L = \sqrt{r(t)^2 + d^2 - 2dr(t)\cos(\phi - \omega t)}$$

d) En coordenadas polares definimos el Hamiltoniano como:

$$H = -\mathcal{L}(\dot r, \dot\phi, r, \phi) + \frac{\partial \mathcal{L}}{\dot r}\dot r + \frac{\partial \mathcal{L}}{\dot \phi}\dot\phi$$

donde

$$\mathcal{L} = K - V$$

K, la energía cinética, está dada por:

$$K = \frac{1}{2} m (|\dot{\vec r}|)^2 \qquad \text{con } |\dot{\vec r}| = \sqrt{\dot r^2 + r^2 \dot\phi^2}$$

$$K = \frac{1}{2} m \dot r^2 + \frac{1}{2} m r^2 \dot\phi^2$$

V, la energía potencial, en este caso, viene dada por;

$$V = V_T + V_L = -\frac{G m m_T}{r} - \frac{G m m_L}{r_L}$$

Así,

$$\mathcal{L} = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 + \frac{Gmm_T}{r} + \frac{Gmm_L}{r_L}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = m\dot{r} = p_r \quad , \quad \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = mr^2\dot{\phi} = p_\phi$$

$$\dot{r} = \frac{p_r}{m} \qquad \dot{\phi} = \frac{p_\phi}{mr^2}$$

Entonces,

$$H = -\left[\frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} + \frac{Gmm_T}{r} + \frac{Gmm_L}{r_L}\right] + \frac{p_r^2}{m} + \frac{p_\phi^2}{mr^2}$$

$$H = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{Gmm_T}{r} - \frac{Gmm_L}{r_L} \quad , \text{con } r_L \to r_L(r, \phi, t)$$

Métodos computacionales

e) Tenemos que $H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - G\frac{m m_T}{r} - G\frac{m m_L}{r_L(r,\phi,t)}$

$$\dot{r} = \frac{\partial H}{\partial P_r} = \frac{\partial}{\partial P_r}\left(\frac{P_r^2}{2m}\right) + 0 - 0 - 0$$

$$\boxed{\dot{r} = \frac{2P_r}{2m} = \frac{P_r}{m}}$$

$$\dot{\phi} = \frac{\partial H}{\partial P_\phi} = \frac{\partial}{\partial P_\phi}\left(\frac{P_\phi^2}{2mr^2}\right) + 0 - 0 - 0$$

$$\boxed{\dot{\phi} = \frac{\partial H}{\partial P_\phi} = \frac{2P_\phi}{2mr^2} = \frac{P_\phi}{mr^2}}$$

$$\dot{P}_r = -\frac{\partial H}{\partial r} = -\left(-\frac{P_\phi^2}{2mr^3} - \left(-\frac{G m m_T}{r^2}\right) - \left(-G\frac{m m_L}{r_L(r,\phi,t)^2} \cdot \frac{\partial r_L}{\partial r}\right)\right)$$

$$\frac{\partial r_L}{\partial r} = \frac{1}{2}\underbrace{\left(r_{(t)}^2 + d^2 - 2r_{(t)} d\cos(\phi - \omega t)\right)}_{r_L}^{-\frac{1}{2}} \cdot \left(2r - 2d\cos(\phi - \omega t)\right)$$

$$\dot{P}_r = \left(\frac{P_\phi^2}{2mr^3} - G\frac{m m_T}{r^2} - G\frac{m m_L}{r_L^2} \cdot \frac{(r - d\cos(\phi - \omega t))}{\sqrt{r^2 + d^2 - 2rd\cos(\phi - \omega t)}}\right)$$

$$\boxed{\dot{P}_r = \left(\frac{P_\phi^2}{2mr^3} - G\frac{m m_T}{r^2} - G\frac{m m_L}{r_L^3}(r - d\cos(\phi - \omega t))\right)}$$

$$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = -\left(+G\frac{m m_L}{r_L^2}\frac{\partial r_L}{\partial \phi}\right) \quad \Bigg| \quad \frac{\partial r_L}{\partial \phi} = \frac{1}{2}\left(r^2 + d^2 - 2rd\cos(\phi - \omega t)\right)^{-\frac{1}{2}} \cdot 2rd(+\sin(\phi - \omega t))$$

$$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = -G\frac{m m_L}{r_L^2} \cdot \frac{rd\sin(\phi - \omega t)}{r_L}$$

$$\boxed{\dot{P}_\phi = -\frac{G m m_L}{r_L^3} rd\sin(\phi - \omega t)}$$

Primavera

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f) Las variables normalizadas: $\tilde{r} = \frac{r}{d}$, $\tilde{P}_r = \frac{P_r}{md}$, $\tilde{P}_\phi = \frac{P_\theta}{md^2}$

$$\dot{r} = d\dot{\tilde{r}} = \frac{P_r}{m} \rightsquigarrow \boxed{\dot{\tilde{r}} = \frac{P_r}{md} = \tilde{P}_r}$$

$$\boxed{\dot{\phi} = \frac{P_\phi}{mr^2} = \frac{P_\phi}{md^2\tilde{r}^2} = \frac{\tilde{P}_\phi}{\tilde{r}^2}}$$

$\tilde{r} = \frac{r}{d}$, se reemplaza r gorro.

$$\dot{\tilde{P}}_r = \frac{1}{md}\dot{P}_r = \frac{1}{md}\left(\frac{P_\phi^2}{mr^3} - G\frac{mm_T}{r^2} - G\frac{mm_L}{r_L^3}(r - d\cos(\phi-\omega t))\right)$$

Primero, haré la conversión $r = d\tilde{r}$, $r_L = d\tilde{r}'$

$$\dot{\tilde{P}}_r = \frac{1}{md}\left(\frac{P_\phi^2}{md^3\tilde{r}^3} - G\frac{mm_T}{d^2\tilde{r}^2} - G\frac{mm_L}{d^3\tilde{r}'^3}(d\tilde{r} - d\cos(\phi-\omega t))\right)$$

$$\dot{\tilde{P}}_r = \frac{1}{md}\left(md\frac{\tilde{P}_\phi^2}{\tilde{r}^3} - md\frac{\Delta}{\tilde{r}^2} - md\frac{\Delta}{m_T}\frac{m_L}{\tilde{r}'^3}(\tilde{r} - \cos(\phi-\omega t))\right)$$

$$\dot{\tilde{P}}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \frac{\Delta}{\tilde{r}^2} - \frac{\Delta\mu}{\tilde{r}'^3}(\tilde{r} - \cos(\phi-\omega t))$$

$$\boxed{\dot{\tilde{P}}_r = \frac{\tilde{P}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}(\tilde{r} - \cos(\phi-\omega t))\right]}$$

$$\dot{\tilde{P}}_\phi = \frac{1}{md^2}\dot{P}_\theta = \frac{1}{md^2}\left(-G\frac{mm_L}{r_L^3}rd\sin(\phi-\omega t)\right)$$

cambio $r = d\tilde{r}$, $r_L = d\tilde{r}'$

$$\dot{\tilde{P}}_\phi = \frac{1}{md}\left(-G\frac{mm_L}{d^3\tilde{r}'^3}d\tilde{r}\sin(\phi-\omega t)\right) = \frac{1}{md}\left(-md\frac{\Delta}{m_T}\frac{m_L}{\tilde{r}'^3}\tilde{r}\sin(\phi-\omega t)\right)$$

$$\boxed{\dot{\tilde{P}}_\phi = -\frac{\Delta\mu\tilde{r}}{\tilde{r}'^3}\sin(\phi-\omega t)}$$

Momentos canónicos iniciales:

$$\tilde{P}_{r_0} = \frac{P_{r_0}}{md} = \frac{m}{md}\frac{dr}{dt} = \frac{1}{d}\frac{d}{dt}(x^2+y^2)^{+\frac{1}{2}} = \frac{x\dot{x}+y\dot{y}}{rd}$$

$\dot{x} = v\cos\theta$, $\dot{y} = v\sin\theta$.

$$\Rightarrow P_{r_0} = \frac{v(x\cos\theta + y\sin\theta)}{rd} = \frac{v}{rd}(r\cos(\theta)\cos\phi + r\sin\theta\sin\phi)$$

$$\Rightarrow \tilde{P}_{r_0} = \tilde{v}(\cos\theta\cos\phi + \sin\theta\sin\phi) = \tilde{v}\cos(\theta-\phi)$$

$$\tilde{P}_\phi = \frac{P_\phi}{md^2} = \tilde{r}^2\frac{d}{dt}\tan^{-1}\left(\frac{y}{x}\right) = \tilde{r}^2\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{d}{dt}\left(\frac{y}{x}\right)$$

$$\tilde{P}_\phi = \tilde{r}^2\frac{\cancel{x^2}}{x^2+y^2}(\dot{y}x - \dot{x}y)\frac{1}{\cancel{x^2}} = \frac{\tilde{r}^2}{r^2}(\dot{y}x-\dot{x}y)$$

$$\tilde{P}_\phi = \frac{\tilde{r}^2}{r^2}(v\sin\theta\, r\cos\phi - v r\sin\theta\sin\phi) = \frac{\tilde{r}^2}{r}v\sin(\theta-\phi)$$

$$\tilde{P}_\phi = \tilde{r}\tilde{v}\sin(\theta-\phi)$$

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