Delft University of Technology

Reasoning & Logic
CSE1300

Assignment: TA-check 2

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2.1 (a)

In Tarski's world, it is possible to describe situations using formulas whose truth can be evaluated, which are expressed in a first-order language that uses predicates such as Rightof(x,y), which means that x is situated—somewhere, not necessarily directly—to the right of y, or Blue(x), which means that x is blue. In the world in Figure 2.9 in Delftse Foundations of Computation (p. 30), for instance, the formula∀x(Triangle(x) → Blue(x)) holds, since all triangles are blue, but the converse of this formula, ∀x(Blue(x) → Triangle(x)), does not hold, since object c is blue but not a triangle.

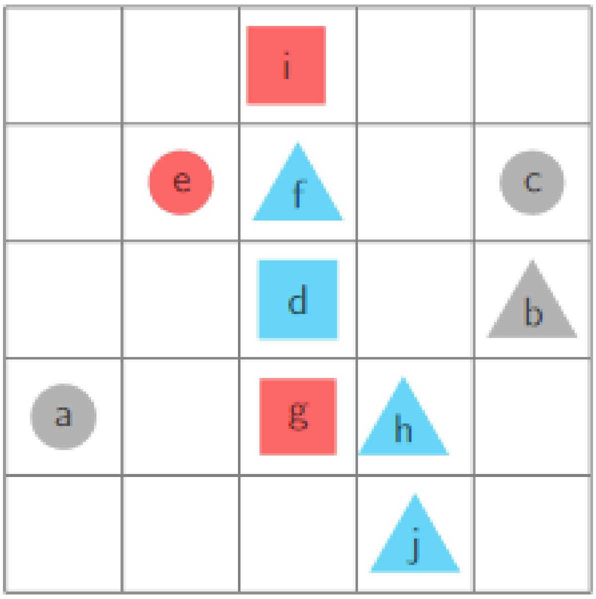

Figure 1: An instance of a Tarski World.

Alfred Tarski contributed much to the semantics of first-order languages and the systematic development of the concept of truth. In the program Tarski's World, described in the book Language, Proof and Logic by Barwise and Etchemendy, a 'Tarski World' is a visualization of a 'mathematical structure,' a core concept in that theory: a structure is a description of a 'situation' wherein one can evaluate whether a statement in a given first-order language (say, T) is true or false. A structure contains a domain with objects, plus an indication of all (combinations of) objects with the properties denoted by the predicate symbols in the language T: Blue(a) is true in a structure (e.g. named A) in which the object with the name 'a' belongs to the set of objects with the property of being blue. That set is called B^A: the set in structure A that indicates which objects have the property indicated by predicate symbol B. In Tarski's World, color indicates to which set each object belongs, which could be notated more generally by describing the set B as B^A = {a,c,e,g}. The relation that corresponds to the predicate RightOf is indicated by the positioning between the objects, but this can be denoted more generally by describing the relation as the set (see chapter 1) of all pairs (x,y) ∈ D x D for which x is situated right of y in the picture: $RightOf^A$ = {(b,a), (c,a), (d,a), (c,f),...}.

2.1.1 (I)

Give the structure that is visualized in Figure 1 (of this document) in the abstract way as described above. Take another good look at section 2.4.4 in the book, where you can find explanations about what this should entail. You may leave out LeftOf, AboveOf, and BelowOf.

D = {a,b,c,d,e,f,g,h,i,j}
$Blue^A$ = {d,f,h,j}
Red^A = {e,g,i}
$Grey^A$ = {a,b,c}
$Square^A$ = {d,g,i}

$Circle^A$ = {a,c,e}
$Triangle^A$ = {b,f,h,j}
$RightOf^A$ = {(b,a), (c,a), (d,a), (e,a), (f,a), (g,a), (h,a), (i,a), (j,a), (b,e), (c,e), (d,e), (f,e), (g,e), (h,e), (i,e), (j,e), (b,i), (c,i), (h,i), (j,i), (b,f), (c,f), (h,f), (j,f), (b,d), (c,d), (h,d), (j,d), (b,g), (c,g), (h,g), (j,g), (b,h), (c,h), (b,j), (c,j)}

2.1.2 (II)

D = {a,b}
$Blue^B$ = {a}
$Grey^B$ = {b}
$Square^B$ = {b}
$Triangle^B$ = {a}

Arguments:

- ∃x(Triangle(x) ∨ Blue(x)), see a
- ∃x(Square(x) ∧ Grey(x)), see b
- ∀y(Circle(y) → Red(y)), no circles
- ∀z(Triangle(x) → ¬Grey(z)), see a

2.1.3 (III)

A counter example is a Tarski World with a single entity. An entity can never be left or right of itself, so in the claim ∀x∀y(RightOf(x,y) ↔ LeftOf(x,y)) one side is always false and the other is always true (due to the not). This means that the claim is false.

2.2 (b)

For the following invalid arguments, give suitable counterexamples. A counterexample is a structure as in question 'a', in which all premises (if any) hold, while the conclusion does not hold. You also need to specify which sets in your structure correspond to which predicate symbols in the formulas. Note that the domain of a structure cannot be empty. Each time, explain why your structure forms a counterexample.

As an example consider the following argument:
$\exists$x(P(x) $\wedge$ Q(x)), $\forall$x(Q(x) $\rightarrow$ R(x)) $\therefore$ $\forall$x(P(x) $\rightarrow$ R(x))

What's being claimed is that if there is an object with property P that it also has property Q, and all objects that have Q also have R, then all all objects with P have R as well. This is clearly not the case, as is shown by the following structure C with domain D = {4, 5}. Let P^c = {4, 5} and $Q^c = R^c$ = {4}. Now the first premise holds (take x = 4), the second one holds as $Q^c = R^c$, but the conclusion does not hold: take x = 5.

2.2.1 (I). $\exists$x(P(x)) $\therefore$ $\forall$x(P(x))

Structure A with domain D = {a,b}
Let P^A = {a}
The first premise holds, but the conclusion does not: take x = b.

2.2.2 (II). $\therefore$ $\forall$xQ(x)

Structure B with domain D = {a}
The (non-existent) premises hold, but the conclusion does not because it is not a tautology.

2.2.3 (III). $\forall$x(P(x,x)) $\therefore$ $\forall$x$\forall$y(P(x,y))

Structure C with domain D = {a,b}
Let P^C = {(a, a),(b, b)}
The first premise holds, but the conclusion does not: take x = a & y = b.

2.2.4 (IV). $\neg\forall$x(P(x) $\wedge$ $\exists$y($\neg$Q(x,y))), $\forall$x($\neg$P(x) $\rightarrow$ $\exists$y(Q(x,y))) $\therefore$ $\forall$x$\exists$y(Q(x,y))

Structure D with domain D = {a,b}
For the first premise, let P^D = {b} and Q^D = {(a,a),(a,b)}.
For the second premise, let P^D = {b} and Q^D = {(a, a)}.
The premises hold, but the conclusion does not: (a, a), (b, ?)

2.2.5 (V). $\neg\forall$x(P(x) $\rightarrow$ $\exists$yQ(x,y)), $\forall$x$\forall$y(R(x,y) $\rightarrow$ (P(y) $\vee$ Q(x,y))) $\therefore$ $\exists$z($\neg$P(z) $\vee$ $\forall$y(Q(z,y) $\wedge$ $\neg$R(y,z)))

Structure E with domain D = {a,b,c}
For the first premise, let P^E = {a,b,c} and Q^E = {}.
For the second premise, let R^E = {(a,a),(a,b),(a,c)} P^E = {a,b,c}, Q^E = {(a,a),(b,a),(c,a)}.
The premises hold, but the conclusion does not: take z = a & y = b.

3 Question 3

4 Question 4

In the previous homework assignment we saw the disjunctive normal form that every formula can be rewritten to. Every formula from propositional calculus can also be written as a conjunction of disjunctions, in conjunctive normal form (CNF). So the formula (p ∨ q) is already in conjunctive normal form as the conjunction of one disjunction, but ((p ∧ q) ∨ r) is not in CNF because it is not a conjunction of disjunctions; rewriting it to ((p ∨ r) ∧ (q ∨ r)) gives a conjunctive normal form of this formula. Now rewrite each of the following formulas to an equivalent form in conjunctive normal form. You need to be able to justify every step you take, so write down every step explicitly. Also make truth tables for the formulas labeled with '*' to show that the formula in CNF that you have found is equivalent to the formula in the assignment that you started with.

4.1 (a). ((a ∧ b) → c)*

(¬(a ∧ b) ∨ c)
(¬a ∨ ¬b ∨ c)

a	b	c	$((a \wedge b) \to c)$	$(\neg a \vee \neg b \vee c)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

4.2 (b). ¬(a → (b ∧ c))*

¬(¬a ∨ (b ∧ c))
a ∧ ¬(b ∧ c)
a ∧ (¬b ∨ ¬c)

a	b	c	$\neg(a \to (b \wedge c))$	$a \wedge (\neg b \vee \neg c)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

4.3 (c). ((a ∨ b) → (c ∨ d))

(¬(a ∨ b) ∨ (c ∨ d))
(¬a ∧ ¬b) ∨ (c ∨ d)
(¬a ∨ c ∨ d) ∧ (¬b ∨ c ∨ d)

4.4 (d). ¬(p → ¬(q ∨ (¬r ∧ s))

¬(¬p ∨ ¬(q ∨ (¬r ∧ s)))
p ∧ (q ∨ (¬r ∧ s))
p ∧ (q ∨ ¬r) ∧ (q ∨ ¬s)

5 Question 5

5.1 (a)

In your own words, explain the difference between a function and a predicate
This depends on the definition of a function. When a function is an algorithm, then a function is a formal definition, which uses predicates, of said algorithm. On the other hand, a function can be a way to express how a predicate interacts with a structure's domain. For example, a domain of discourse is a set, A, and a predicate is a function from A.

5.2 (b)

Give an example of a vacuously true statement about the real-world