Reasoning and Logic, Tantalizing TA-check 1

Deadline: the 10th of September 23:59

Introduction

This assignment is about module 1 and the start of module 2. Every question has an indication for how long the question should take you in an exam-like setting. In total this set of exercises should take you about 196 minutes, but as you will find out soon this work is to be split!

Questions of Tantalizing TA-check 1 (Wednesday 10th August, 2022, 15:07)

0. (5 min.) **Splitting the work**

   This TA-check should be done in pairs. However, as those of you using the skill circuits probably noticed, we do not recommend doing all of the TA-check in one go. Instead we recommend you do specific questions from this TA-check after studying specific concepts. To make sure you can both work through the material at your own pace, we recommend you first divide the work between the two of you.

   We recommend you both do the first subquestion of each question and then split the remaining subquestions (one taking the odd ones, the other the even ones) and do those individually. Since the difficulty increases, make sure you alternate and do not divide first half vs second half! Then in question 7 you should discuss your answers and merge them into one set of answers to request feedback on.

1. **Formulating precisely**

   Consider the following claims and arguments written in English. Formulate a logically precise claim in propositional logic using logical operators to represent the claim. For instance for "I like puzzles and I like tea", one might use $p \wedge q$ where p is "I like puzzles" and r is "I like tea.".

   (a) (2 min.) If I do this homework, then I have a greater chance at passing the course.

   (b) (2 min.) Only if I do this homework will I get feedback from TA's.

   (c) (2 min.) I can choose to do the MC-test in week 3 or I can choose not to.

   (d) (4 min.)

   If I pass the course, then I have practiced well.
   If I pass the course, then I have passed the endterm.
   I have practiced well and passed the endterm.
   ___
   Therefore, I have passed the course.

   (e) (4 min.)

   It is not true that: I do the homework and I do not get feedback from TA's.
   I do the homework or I do not have a greater chance of passing the course.
   I do not get feedback from the TA's
   ___
   Therefore, I do not have a greater chance of passing the course.

   Question 1: 14 min.

2. **Validity of arguments**

   Answer each of the following questions in at most 5 lines of text (excluding truth tables if required). For each of the questions (which consist of just an argument) indicate if the argument is valid or not. If they are valid, prove it using a truth table. If they are invalid, prove it using a counterexample and explain how this counterexample shows the argument to be invalid.

   (a) (3 min.) Explain why we need a full truth table to show an argument is valid, but why only a counterexample suffices to show an argument is invalid.

   (b) (3 min.) $p \to q, \neg q \therefore \neg p$.

   (c) (4 min.) Consider the argument from question 1d.

   (d) (4 min.) Consider the argument from question 1e.

(e) (2 min.) Consider an argument without any premises (like the argument of the next question). Explain when such an argument is valid and when such an argument is invalid.

(f) (3 min.) $\therefore (p \to q) \to \neg(p \land \neg q)$

(g) (5 min.) *Old exam question* Consider a new ternary operator $p \xrightarrow{q} r$, such that when q is false $p \xrightarrow{q} r$ is equivalent to $p \leftrightarrow r$, and when q is true $p \xrightarrow{q} r$ is equivalent to $p \land \neg r$. Give a truth table for $p \xrightarrow{q} r$.

Question 2: 24 min.

3. **Equivalence of statements** For each of the following pairs of statements, indicate if they are equivalent and explain why (not) using a truth table. In case of unequivalent statements an incomplete truth table suffices.

(a) (5 min.) $(p \lor (q \land r)) \stackrel{?}{\equiv} (\neg p \to (q \land r))$

(b) (5 min.) $(p \lor q) \to (r \lor q) \stackrel{?}{\equiv} (\neg r \lor \neg q) \to (\neg p \lor \neg q)$

(c) (6 min.) $(q \lor (p \to q)) \stackrel{?}{\equiv} \neg(\neg q \land (p \land \neg q))$

(d) (6 min.) $((\neg s \leftrightarrow p) \land (r \to p)) \stackrel{?}{\equiv} ((\neg p \land \neg r \land \neg s) \lor (p \land s))$

Question 3: 22 min.

4. For every formula in propositional calculus it is possible to rewrite it to a formula in disjunctive normal form (DNF). A formula in this form consists of a disjunction of conjunctions, with the conjunctions consisting of loose atoms and/or their negations. Formula $F = (p_1 \land \neg p_2)$ contains a single conjunction and is therefore in disjunctive normal form. A slightly less trivial example is formula $G = (p_1 \land (p_2 \to p_3))$, which can be rewritten as follows:

$$\begin{aligned} G &= (p_1 \land (p_2 \to p_3)) & \\ &\equiv (p_1 \land (\neg p_2 \lor p_3)) & (p \to q) \equiv (\neg p \lor q) \\ &\equiv ((p_1 \land \neg p_2) \lor (p_1 \land p_3)) & \text{distributive law} \end{aligned}$$

Rewrite the following formulas to DNF.

(a) (3 min.) $(p \lor \neg(q \land r))$

(b) (4 min.) $\neg(p \land (q \lor r))$

(c) (5 min.) $(\neg(p \to q) \land (p \to r))$

(d) (8 min.) $\neg((p \land q) \leftrightarrow (r \lor s))$

Question 4: 20 min.

5. **First-order logic statements to and from natural language**

(a) Translate the following statements to a first-order language. Each time, you should first name the symbols you will use, along with their meaning, such as predicate symbols (usually capital letters such as $B(x, y)$ or Brother(x, y) for 'x is the brother of y,' or $W(x, y)$ for 'x wants y'), and constants (small letters without argument, such as j for 'John'). You can reuse symbols in later subquestions, as long as you do not define new symbols with the same name but a different meaning.

You should think about what objects should be constants, and what objects should not be. For example, if you were to translate: "Stefan is a lecturer", then "Stefan" refers to a specific object in your domain and should thus be a constant. On the other hand, being a lecturer is a property that objects can (not) have and thus should be a predicate. Thus you could translate this as $Lecturer(s)$ where s is Stefan and $Lecturer(x)$ means "x is a lecturer".

Remember that predicates should define only a single property, and not secretly introduce new objects. E.g., $A(x)$ for 'x owns an aviation license' is incorrect! This predicate now does two things, both positing that an object exists which is an aviation license, and also showing a relation between x and this license.

i. (1 min.) All lawyers are awesome.

ii. (1 min.) Phoenix is a lawyer.

iii. (3 min.) Maya eats a burger.
iv. (3 min.) Maya eats burgers. *(Note the subtle difference to the previous statement, how do we show this in the translation?)*
v. (4 min.) Maya was helped by Phoenix, who also helped Maya's cousin.
vi. (4 min.) CSE1300 students are the only ones who know about Phoenix.
vii. (5 min.) Phoenix is never someone's only help.
viii. (5 min.) There is no lawyer without a badge who only defended one case.

(b) Suppose we have the domain $D = \{a_1, a_2, a_3, p_1, p_2, p_3, p_4, p_5, p_6, u_1, u_2, u_3, u_4\}$, the predicates $D(x, y)$, $P(x, y, z)$, $U(x, y)$ and $A(x, y)$. The interpretations of these symbols is as follows: $D(x, y)$ means paper x is stored in database y, $P(x, y, z)$ means x has published paper y at university z, $U(x, y)$ means university x uses database y and lastly $A(x, y)$ means author x wrote paper y.

Now form natural language sentences for the following first-order sentences. You are allowed to use the constants from the domain in your sentences, or you can assign (consistent) natural names to them. E.g. you can make a_1 be Stefan and a_2 be Neil if you so choose.

i. (1 min.) $P(a_1, p_1, u_3)$
ii. (3 min.) $\exists x(P(a_2, x, u_2))$
iii. (3 min.) $\exists x\forall y\forall z(\neg(P(y, z, x)))$
iv. (5 min.) $\forall w\forall x\forall y\forall z((P(w, x, y) \wedge D(x, z)) \rightarrow U(y, z))$
v. (5 min.) $\forall x\exists y\exists z(A(y, x) \wedge A(z, x) \wedge \neg(y = z))$

Question 5: 43 min.

6. **Essay questions, do this question together.**

(a) (5 min.) In your own words, describe the principle of explosion. Also provide an example of an argument that uses it. Hint: you are welcome to google this principle, learn about it, and then report back in your own words! (*Yes we know you may not have seen this yet in the course! Above all else we teach you, we want to teach you how to learn about new things!*)

(b) (5 min.) Prove that with just $\{\neg, \rightarrow\}$ we can emulate the $\{\wedge, \vee, \leftrightarrow\}$ operators. In other words, show that $\{\neg, \rightarrow\}$ is functionally complete.

(c) (8 min.) The operator $p \xrightarrow{q} r$ from earlier in this homework is functionally complete. Prove this.

(d) (5 min.) Consider the truth table for a compound proposition with 7 unique atoms and 143 connectives. How many different configurations can the column for the main connective have?

(e) (5 min.) Does the order of quantifiers matter? Give an example to support your answer.

(f) (10 min.) **Reflection:**

- What questions do you have for a teaching assistant when you go and discuss your work with them?
- What was the hardest question for you to answer and what will you do to improve your skills in answering that type of question on an exam?

Question 6: 38 min.

7. (30 min.) **Combining the work**

Having each done half of the homework, you should now briefly discuss the work you did. For each question pick at least one subquestion each to discuss with your partner. If there are other answers you are not sure about, discuss those too. Make changes, and update the answers.

Now scan your answers and submit them on Brightspace as a group. Next week you can then book a time slot with a TA to get your feedback!