# Reasoning and Logic, Tantalizing TA-check 7

## **Deadline:** the 29th of October 23:59

### Introduction

This assignment is about lectures 1 through 13, with an emphasis on lectures 11 to 13, which cover chapter 4 of the book. Every question has an indication for how long the question should take you in an exam-like setting. In total this set of exercises should take you about 128 minutes.

### Questions of Helpful Homework 5 (Monday 29$^{th}$ August, 2022, 15:55)

0. (5 min.) **Splitting the work**

   This TA-check should be done in pairs. However, as those of you using the skill circuits probably noticed, we do not recommend doing all of the TA-check in one go. Instead we recommend you do specific questions from this TA-check after studying specific concepts. To make sure you can both work through the material at your own pace, we recommend you first divide the work between the two of you.

   We recommend you both do the first subquestion of each question and then split the remaining subquestions (one taking the odd ones, the other the even ones) and do those individually. Since the difficulty increases, make sure you alternate and do not divide first half vs second half! Then in question 4 you should discuss your answers and merge them into one set of answers to request feedback on.

1. (a) *Old exam question* Consider the set $\mathbb{Z}$ of integers.
   Let $A = \{x \in \mathbb{Z} \mid \exists y \in \mathbb{Z}(x = 2y)\}$ and $B = \{x \in \mathbb{Z} \mid \exists y \in \mathbb{Z}(x = 2y + 1)\}$.

   i. (2 min.) Formulate a bijective function $f : A \to B$.

   > **Solution:** Take for instance $f(x) = x + 1$. This maps every element in $A$ to exactly one unique element in $B$.

   ii. (10 min.) Consider the cardinalities of $A$ and $B$, formulate a claim about how they relate and prove it.

   > **Solution:** $|A| = |B|$
   >
   > *Proof.* TP: $|A| = |B|$
   > Consider the function $f(x)$ presented in question a.
   > This function is a bijection, which means it is both injective and surjective.
   > Thus for every $y \in B$ there is one element $x \in A$, such that $f(x) = y$ (surjective), thus $|A| \geqslant |B|$. Also for every $x \in A$ there is one element $y \in B$, such that $f(x) = y$ (injective), thus $|B| \geqslant |A|$. Thus $|A| = |B|$ must hold. QED

   iii. (5 min.) Formulate a bijective function $g : A \to \mathbb{Z}$ and explain what this means for the cardinality of $A$ compared to that of $\mathbb{Z}$.

   > **Solution:** Take for instance $g(x) = x/2$. As such a bijection exists, this means that these sets must also be of equal cardinality.

Question 1: 17 min.

2. **Functions on Trees and Graphs**

   (a) (5 min.) Create a function $f : \mathit{TREE} \to \mathbb{N}$ that returns the number of odd values in a tree whose domain $D = \mathbb{N}$.

**Solution:**

$$f(t) = \begin{cases} 0 & \text{if } t = \varnothing \\ 1 & \text{if } t = (x, \varnothing) \wedge 2 \nmid x \\ 0 & \text{if } t = (x, \varnothing) \wedge 2 \mid x \\ 1 + \sum\limits_{1 \leqslant i \leqslant k} f(T_i) & \text{if } t = (x, (T_1, \ldots, T_k)) \wedge 2 \nmid x \\ \sum\limits_{1 \leqslant i \leqslant k} f(T_i) & \text{else} \end{cases}$$

(b) (5 min.) Create a function $f : \mathit{TREE} \times \mathbb{N} \to \mathit{TREE}$ that returns a tree $t'$ so that $t = t'$ except that every value has been increased by $n$.

**Solution:**

$$f(t, n) = \begin{cases} \varnothing & \text{if } t = \varnothing \\ (x + n, \varnothing) & \text{if } t = (x, \varnothing) \\ (x + n, (f(T_1, n), \ldots, f(T_k, n))) & \text{else} \end{cases}$$

(c) (10 min.) Create a function $f : \mathit{TREE} \to \mathbb{R}$ that takes a parse tree $t$ and returns the value of the expression. For trees that are not a parse tree, your function does not have to work properly. You may assume the domain of the trees to be $D = \mathbb{R} \cup \{+, -, /, *\}$, the input to be a well-formed parse tree and that $f(\varnothing) = 0$.

**Solution:**

$$f(t) = \begin{cases} 0 & \text{if } t = \varnothing \\ x & \text{if } t = (x, \varnothing) \\ f(T_1) + f(T_2) & \text{if } t = (+, (T_1, T_2)) \\ f(T_1) - f(T_2) & \text{if } t = (-, (T_1, T_2)) \\ f(T_1) * f(T_2) & \text{if } t = (*, (T_1, T_2)) \\ f(T_1)/f(T_2) & \text{if } t = (/, (T_1, T_2)) \\ -f(T_1) & \text{if } t = (-, (T_1)) \end{cases}$$

(d) (3 min.) Create a directed graph $G = (V, E)$ with $|V| = 8$ and $10 \leqslant |E| \leqslant 15$.

(e) (5 min.) Create a function $f : V \times V \to \{0, 1\}$ that returns 1 iff there is a path between the vertices. You should give this function in set-notation.

3. **Repetition**

(a) *Old exam questions*

i. (4 min.) For this question you need to translate a claim from natural language to predicate logic. Make sure to define all predicates you introduce and to include all of the information from the original statement in your translation.
Only puzzles are amazing and cool.

**Solution:** $\forall x((\mathit{Amazing}(x) \wedge \mathit{Cool}(x)) \to (\mathit{Puzzle}(x)))$

ii. (6 min.) For this question you need to translate a claim from natural language to predicate logic. Make sure to define all predicates you introduce and to include all of the information from the original statement in your translation.
All coatis love exactly one other coati.

**Solution:** $\forall x(\mathit{Coati}(x) \to \exists y(x \neq y \wedge \mathit{Loves}(x, y) \wedge \mathit{Coati}(y) \wedge \forall z((z \neq y \wedge \mathit{Coati}(z)) \to \neg \mathit{Loves}(x, z))))$

iii. (8 min.) Consider the following ternary operator $p \xrightarrow{q} r$, with the truth table:

| p | q | r | $p \xrightarrow{q} r$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Consider now the following two formulas:

- $q \xrightarrow{(q \wedge \neg p)} p$
- $\neg(p \leftrightarrow \neg q)$
- For 3 points create a single truth table containing the truth values of both statements.
- For 1 point explain how your truth table shows the statements are (not) equivalent, refer to specific lines from your truth table in your explanation where appropriate.

**Solution:**

| p | q | $\overbrace{q \wedge \neg p}^{Z}$ | $q \xrightarrow{Z} p$ | $\neg(p \leftrightarrow \neg q)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |

The statements are not equivelant, as row 2 shows they have different values in that scenario.

iv. (20 min.) The owls, coatis, and snails have long held Korfball tournaments (a sport of Dutch origin) in which all three species send their best teams. Each of them plays each other team exactly once in the tournament and you are responsible for maintaining the score sheet.

Unfortunately you dozed off for a bit and missed all the matches! Panicking, you interview some of the animals present to figure out the scores. After 20 minutes of running around, you get the following information from some owls:

- Coati And Proud (CAP) played two games so far, out of which they lost one and won the other, and have scored a total of 2 goals.
- Slow And Steady (SAS) played one game so far, which they won, and have scored a total of 3 goals, whilst getting also getting 3 against them.
- Finally Owlness Forever (ONF) won no games, scored 2 goals, and got no goals against them so far.

Some coatis tell you however, that everything you just wrote down is wrong. The owls lied about it all! Furthermore they tell you that each team has played at least one match so far, and that in no match the teams scored more than 4 goals together. You know you can trust the coatis, as they always tell the truth, so what do you do now?

Give the real scores in the matches, or state that the match has not yet been played.

- Coati And Proud (CAP) vs Slow And Steady (SAS)
- Slow And Steady (SAS) vs Owlness Forever (ONF)
- Coati And Proud (CAP) vs Owlness Forever (ONF)

Explain how you derived your answer by showing your logic!

**Solution:**

| | Games Played | Won | Lost | Tied | Goals scored | Goals against |
|---|---|---|---|---|---|---|
| Coati And Proud (CAP) | 2 | 1 | 1 | | 2 | 0 |
| Slow And Steady (SAS) | 1 | 1 | | | 3 | 3 |
| Owlness Forever (ONF) | | 0 | | | 2 | 0 |

Starting with the number of mathes, we know all teams have played at least one match. Hence Coati And Proud (CAP) must have played 1 and not 2 matches, Slow And Steady (SAS) must have played 2 and not 1 matches, and from this we know that 3 matches have been played in total and so Owlness Forever (ONF) must have played 1.

| | Games Played | Won | Lost | Tied | Goals scored | Goals against |
|---|---|---|---|---|---|---|
| Coati And Proud (CAP) | 1 | | | | | |
| Slow And Steady (SAS) | 2 | | | | | |
| Owlness Forever (ONF) | 1 | | | | | |

Next, let's look at the results. Coati And Proud (CAP) has not won one match, nor have they lost one. Since they played only one, they must have tied in their match against Slow And Steady (SAS). Since Owlness Forever (ONF) did not win zero matches, they must have won theirs against Slow And Steady (SAS).

| | Games Played | Won | Lost | Tied | Goals scored | Goals against |
|---|---|---|---|---|---|---|
| Coati And Proud (CAP) | 1 | | | 1 | | |
| Slow And Steady (SAS) | 2 | | 1 | 1 | | |
| Owlness Forever (ONF) | 1 | 1 | | | | |

Finally, let's consider the scores. Coati And Proud (CAP) must have tied, and since no match had more than 4 goals altogether, the score was 0-0, 1-1, or 2-2. Since Coati And Proud (CAP) did not score 2 goals, and did not get 0 goals against, the score for this match must have been 1-1. The other match is a win for Owlness Forever (ONF) so they must have scored more goals than Slow And Steady (SAS). However they did not score 3 goals, and Slow And Steady (SAS) did not score 0 in that match. If Slow And Steady (SAS) had scored 2, Owlness Forever (ONF) could not have won (as they would have required 3 goals, bringing the total to 5), so Slow And Steady (SAS) must have scored 1. This means that Owlness Forever (ONF) must have scored 2, or 3 goals. However since they did not score 2, they must hav scored 3.

| | Games Played | Won | Lost | Tied | Goals scored | Goals against |
|---|---|---|---|---|---|---|
| Coati And Proud (CAP) | 1 | | | 1 | 1 | 1 |
| Slow And Steady (SAS) | 2 | | 1 | 1 | 2 | 4 |
| Owlness Forever (ONF) | 1 | 1 | | | 3 | 1 |

- Coati And Proud (CAP) vs Slow And Steady (SAS): 1-1
- Slow And Steady (SAS) vs Owlness Forever (ONF): 1-3
- Coati And Proud (CAP) vs Owlness Forever (ONF): unplayed

(b) (10 min.) **Do this question together. Reflection:**

- How did you use the feedback from the teaching assistants on the previous assignment in this weeks' helpful homework?
- What questions do you have for a teaching assistant when you go and discuss your work with them?
- What was the hardest question for you to answer and what will you do to improve your skills in answering that type of question on an exam?

**Solution:** It is generally good to ask yourself these questions when you are done with an(y) assignment. We will make it explicit in this course for a bit, but even if other courses do not, ask yourself these questions!

Question 3: 48 min.

4. (30 min.) **Combining the work**

Having each done half of the homework, you should now briefly discuss the work you did. For each question pick at least one subquestion each to discuss with your partner. If there are other answers you are not sure about, discuss those too. Make changes, and update the answers.

Now scan your answers and submit them on Brightspace as a group. Next week you can then book a time slot with a TA to get your feedback!