

Reasoning and Logic, Tantalizing TA-check 6

Deadline: the 22nd of October 23:59

Introduction

This assignment is about lectures 1 through 13, with an emphasis on lectures 11 to 13, which cover chapter 4 of the book. Every question has an indication for how long the question should take you in an exam-like setting. In total this set of exercises should take you about 155 minutes.

Questions of Helpful Homework 6 (Monday 29th August, 2022, 15:55)

0. (5 min.) Splitting the work

This TA-check should be done in pairs. However, as those of you using the skill circuits probably noticed, we do not recommend doing all of the TA-check in one go. Instead we recommend you do specific questions from this TA-check after studying specific concepts. To make sure you can both work through the material at your own pace, we recommend you first divide the work between the two of you.

We recommend you both do the first subquestion of each question and then split the remaining subquestions (one taking the odd ones, the other the even ones) and do those individually. Since the difficulty increases, make sure you alternate and do not divide first half vs second half! Then in question 3 you should discuss your answers and merge them into one set of answers to request feedback on.

1. Relations and functions

(a) Suppose we have a function $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, defined as:

$$g(a, b) = \begin{cases} \left(\frac{-a}{b}, \frac{b}{-a}\right) & \text{if } a < 0. \\ (-b, b) & \text{if } a = 0; \\ (a\sqrt{b}, b\sqrt{a}) & \text{if } a > 0; \end{cases}$$

- i. (2 min.) The function g is a function, i.e., a set. What format do the elements of this set have?
- ii. (3 min.) Determine $g(2, 5)$, $g(-5, 2)$, $g(0, 4)$ and $g(2, 8)$, and explain your answers.
- (b) Suppose we have the sets $X = \{a, b, c, d, e\}$ and $Y = \{k, m, n, p, q\}$ and the function $h : X \rightarrow Y$ that is defined as $h(a) = k$, $h(b) = m$, $h(c) = q$, $h(d) = n$, $h(e) = p$.
 - i. (2 min.) Draw a directed graph for h .
 - ii. (3 min.) Suppose we have the following sets: $A = \{k, p\}$, $C = \{a, e\}$, $D = \{n\}$, and $E = \{k, p, q\}$. Determine $h^{-1}(A)$, $h(X)$, $h(C)$, $h^{-1}(D)$, $h^{-1}(E)$, and $h^{-1}(Y)$.
- (c) For each of the following functions, indicate whether it is injective and whether it is surjective. Motivate your answer.
 - i. (3 min.) $f_1 : \mathbb{Z} \rightarrow \mathbb{N}$, with $f_1(x) = 0$.
 - ii. (3 min.) $f_2 : \mathbb{R} \rightarrow \mathbb{Z}$, with $f_2(x) = \lfloor x \rfloor$ ¹.
 - iii. (3 min.) $f_3 : \mathbb{N} \rightarrow \mathbb{R}$, with $f_3(x) = 2^x$.
- (d) Suppose we have the sets $A = \{a, b, c, d, e\}$ and $B = \{c, d, e, f, g\}$. For each of the following relations $R \subseteq A \times B$, draw a directed graph (see Epp, p. 446). Also indicate whether R is a function, and explain why each time.
 - i. (2 min.) $R = \{(a, c), (d, g)\}$
 - ii. (3 min.) $R = \{(a, e), (b, e), (e, g)\}$
 - iii. (3 min.) $R = \{(a, d), (a, f), (b, f), (d, d)\}$
 - iv. (3 min.) $R = \{(a, c), (c, d), (d, e), (b, d), (b, e)\}$
- (e) Suppose we have the set $A = \{0, 1, 2, 3, 4\}$.
 - i. (2 min.) Explain when a 'relation R on A ' is (I) transitive, (II) reflexive and (III) symmetric. Provide examples of elements that need to be in the different relations to show the relations have these properties.

¹ $\lfloor x \rfloor$ is x rounded down to the closest integer

- ii. (3 min.) How do elements of an equivalence class for a relation R relate to each other—in terms of R ?
- iii. (15 min.) Give four relations R_1 to R_4 on A . The first three R_1 to R_3 must each have exactly one of the properties mentioned at e.i, and the last R_4 must have all three of the properties. Give the relations in set-roster notation. For R_4 , also give all equivalence classes.
- (f) A relation R on A is symmetric iff for all $x, y \in A$: $xRy \rightarrow yRx$. A relation R on A is 'anti-symmetric' iff for all $x, y \in A$: $(xRy \wedge yRx) \rightarrow (x = y)$.
 - i. (10 min.) For each of the following two claims, give a proof or a counterexample.
 - I. If a relation R on a set A is transitive, then R is cannot be anti-symmetric.
 - II. If a relation R on a set A is reflexive, then R is symmetric.
- (g) Transitivity
 - i. (2 min.) Give a systematic method that, given a binary relation R on a set A , constructs a graph G_R representing the relation.
 - ii. (5 min.) We will prove the following theorem: A relation R on a set A is transitive iff (If a vertex b can be reached in G_R from a , then (a, b) is also an edge in G_R).
First, show the structure of the proof as much as you can, based on the structural elements of the theorem—that is, without using the definitions of the terms used.
 - iii. (20 min.) Now, fill in the details of your proof, i.e., give the complete proof.

Question 1: 87 min.

2. Repetition

(a) Old exam questions

- i. (3 min.) Give a Venn Diagram for the set $(A - B) \cup (B - C)$.
- ii. (5 min.) Claim: For all sets A if $|A|$ is odd and $\emptyset \notin A$ then there is **no** set B such that $\mathcal{P}(B) = A$. If the claim is true, prove it. If it is false, give a counterexample.
- iii. (15 min.) Four young apprentices broke into a temple to steal four sacred element crystals. When the alarm went off, they panicked, and each of them swallowed the crystal they held right before they were caught. You must determine who ate which crystal. The elements compel their masters. Those who ate the earth and water crystals must speak the truth, while those who consumed fire and air must lie. The youths are too scared to confess their own transgressions. Instead, they fall to accusing each other. This is what they said:
 - Coyote Caoti said something before you arrived (but you don't know what)
 - Dr. Whoo says that Coyote Caoti is telling the truth about that statement
 - Donna says that she didn't eat the earth crystal
 - Marty says that if Coyote Caoti ate the air crystal, then Dr. Whoo ate the earth crystal

Remember that:

- Each student ate a different elemental crystal
- The earth and water crystals force their owner to tell the truth
- The fire and air crystals force their owners to lie

Who ate what crystal? Explain how you derived your answer.

(b) (10 min.) **Do this question together. Reflection:**

- How did you use the feedback from the teaching assistants on the previous assignment in this weeks' helpful homework?
- What questions do you have for a teaching assistant when you go and discuss your work with them?
- What was the hardest question for you to answer and what will you do to improve your skills in answering that type of question on an exam?

Question 2: 33 min.

3. (30 min.) Combining the work

Having each done half of the homework, you should now briefly discuss the work you did. For each question pick at least one subquestion each to discuss with your partner. If there are other answers you are not sure about, discuss those too. Make changes, and update the answers.

Now scan your answers and submit them on Brightspace as a group. Next week you can then book a time slot with a TA to get your feedback!