Delft University of Technology

Reasoning & Logic
CSE1300

Assignment: TA-check 3

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Contents

1 Question 1

2 Question 2. First steps in recursive functions

(a). Consider the following recursive function f(n). List the first 7 values of f(n), so for n = 0 to n = 6

$$f(n) = \begin{cases} 1 & \text{if } n \leqslant 0 \\ f(n-2) + f(3) & \text{if } n > 3 \\ f(n-1) + 3 & \text{else} \end{cases}$$

- f(0) = 1
- f(1) = f(0) + 3 = 4
- f(2) = f(1) + 3 = 7
- f(3) = f(2) + 3 = 10
- f(4) = f(2) + f(3) = 17
- f(5) = f(3) + f(3) = 20
- f(6) = f(4) + f(3) = 27

(b). Consider the following recursive function g(n). List the first 7 values of g(n), so for n = 0 to n = 6

$$g(n) = \begin{cases} 1 & \text{if } n \leqslant 1 \\ g(n/2) + 3 & \text{if } n > 1 \text{ and } n \text{ is even} \\ g(3n+1) - 2 & \text{else} \end{cases}$$

- g(0) = 1
- g(1) = 1
- g(2) = g(1) + 3 = 4
- g(3) = g(10) - 2 = 12
- g(4) = g(2) + 3 = 7
- g(5) = g(16) - 2 = 11
- g(6) = g(3) + 3 = 15
- g(8) = g(4) + 3 = 10
- g(10) = g(5) + 3 = 14
- g(16) = g(8) + 3 = 13

(c). Formulate a recursive function hpnq that computes the number of odd digits in a number

$$h(n) = \begin{cases} n & \text{if } n \leq 0 \\ h(abs(n/10)) & \text{if } n \bmod 2 = 0 \\ h(abs(n/10)) + 1 & \text{else} \end{cases}$$

3 Question 3. A first induction proof

Prove the following theorem using mathematical induction: **Theorem.** For all integers n $\geq$ 1: $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$.
P(n) $= \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$
Proof. We will prove this theorem by induction
Base case: Consider the case n = 1. P(1) $= \sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1+1}$. Since each side of the equation is equal to $\frac{1}{2}$, this is true.
Inductive step: Let k $\geq$ 2 be arbitrary. Assume that P(k) is true. We want to show that P(k + 1) is true. P(k + 1) is the statement $\sum\limits_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{(k+1)}{(k+1)+1}$. But, we can compute that
$\sum\limits_{i=1}^{k+1} \frac{1}{i(i+1)} = (\sum\limits_{i=1}^{k} \frac{1}{i(i+1)}) + \frac{1}{(k+1)((k+1)+1)}$.
Since P(k) is true, we can replace the first sum with $\frac{k}{k+1}$.
This gives us $\frac{k}{k+1} + \frac{1}{(k+1)((k+1)+1)} = \frac{k+1}{(k+1)+1}$
which is what we wanted to show. This completes the induction.

4 Question 4. Satisfiability of propositions

(a). Explain when a proposition is satisfiable and how you can show that a proposition is satisfiable
A proposition is satisfiable when there exsists a truth assignment that makes the proposition true. This can be shown by using a truth table.

(b). What property does a proposition that is not satisfiable have?
A proposition that is not satisfiable has the property that no truth assignment can make the proposition true, thus a contradiction.

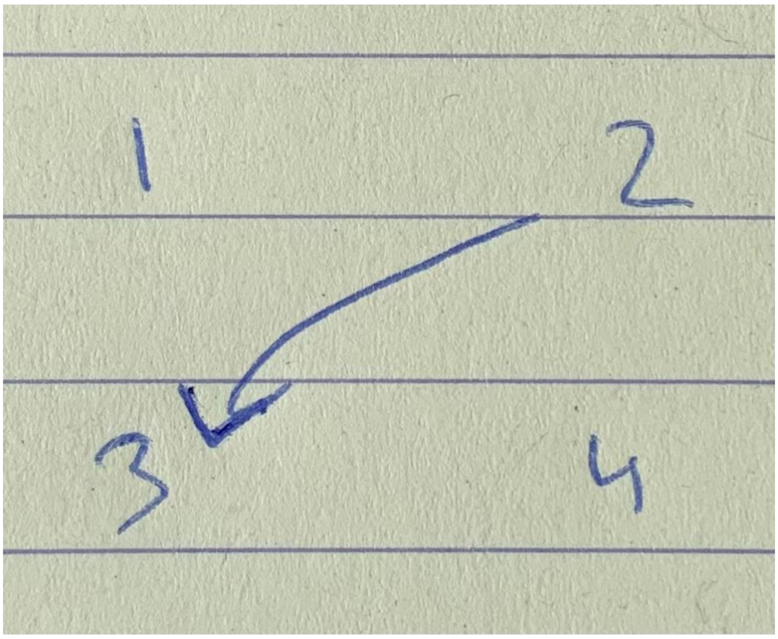
(c). How can you show that a proposition is not satisfiable?
A proposition is not satisfiable when it is a contradiction.

(d). For each of the following domains and propositions, show that they are either satisfiable or not. If the proposition is satisfiable, show this using a formal structure using all elements in the domain. If the proposition is not satisfiable, explain that is it not satisfiable using the method described in your answer of c.

i. (5 min.) $D^i = \{1, 2, 3, 4\}$

(1) $\forall x, y(P(x, y) \to \neg P(y, x))$
(2) $\forall x, y, z(P(x, y) \land P(y, z) \to P(x, z))$
(3) $P(2, 3)$

Structure I with domain $D^i = \{1, 2, 3, 4\}$ is satisfiable.



ii. (10 min.) $D^{ii} = \{1, 2, 3\}$

(1) $\forall x, y(P(x, y) \rightarrow \neg P(y, x))$
(2) $\forall x, y, z(P(x, y) \wedge P(y, z) \rightarrow P(x, z))$
(3) $\forall x \exists y(P(x, y))$
(4) $P(1, 2)$
(5) $P(2, 3)$

Structure II with domain $D^{ii} = \{1, 2, 3\}$ is not satisfiable due to predicate 3.

5 Question 5. Revision

(a). Consider the following argument written in propositional logic. If it is invalid provide a counterexample and explain how it shows the argument is invalid. If it is valid, prove it.

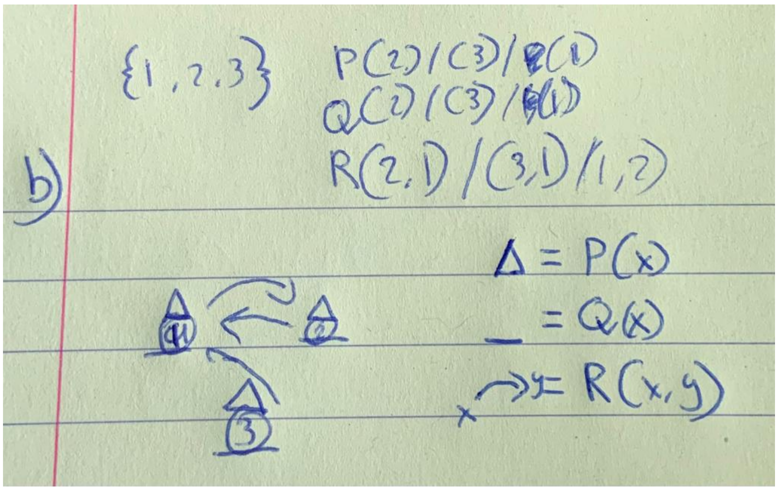
$$\begin{array}{l} p \rightarrow ((q \wedge r) \vee (r \rightarrow q)) \\ \neg q \leftrightarrow \neg(r \vee p) \\ q \\ \hline \therefore p \end{array}$$

p	q	r	$p \rightarrow ((q \wedge r) \vee (r \rightarrow q))$	$\wedge$	$\neg q \leftrightarrow \neg(r \vee p)$	$\wedge$	q	p
0	0	0	1	1	1	1	0	0
0	0	1	1	1	0	0	0	0
0	1	0	0	1	0	0	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	1	0	0	0	0
1	0	1	1	0	0	0	0	0
1	1	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1

The last two situations of the truth table show that the argument is valid.

(b). Consider the following argument written in predicate logic. Provide a counterexample in the form of a formal structure.

$$\begin{array}{l} \forall x(P(x) \leftrightarrow (Q(x) \wedge \exists y(R(x,y)))) \\ P(3) \\ \exists x(x \neq 3 \wedge P(x)) \\ \exists x, y(x \neq y \wedge R(x,y)) \\ \hline \therefore \neg\exists y(\forall x R(x,y)) \end{array}$$



6 Question 6. Essay questions

(a). In your own words, describe what a theorem prover such as Z3 does and why this tool can be useful.
Z3 theorem prover is a program that can analyse and prove theorems digitally. It can be useful because it can prove theorems that are difficult to prove by hand.

(b). In your own words, explain the idea and structure of an induction proof.
An induction proof is a proof that uses induction to prove a statement. It is structured by first proving the base case, then proving the inductive step.