# Exercise 1.14

September 9, 2021 20:10 PM

**Exercise 1.14:** Draw the tree illustrating the process generated by the `count-change` procedure of Section 1.2.2 in making change for 11 cents. What are the orders of growth of the space and number of steps used by this process as the amount to be changed increases?

```
(define (count-change amount) (cc amount 5))
(define (cc amount kinds-of-coins)
  (cond ((= amount 0) 1)
        ((or (< amount 0) (= kinds-of-coins 0)) 0)
        (else (+ (cc amount
                     (- kinds-of-coins 1))
                 (cc (- amount
                        (first-denomination
                         kinds-of-coins))
                     kinds-of-coins)))))
(define (first-denomination kinds-of-coins)
  (cond ((= kinds-of-coins 1) 1)
        ((= kinds-of-coins 2) 5)
        ((= kinds-of-coins 3) 10)
        ((= kinds-of-coins 4) 25)
        ((= kinds-of-coins 5) 50)))
```

For space, the order of growth is proportional to the depth of the tree, which is the sum of the number of denominations + the amount = O(n + d) where n is the amount and d is the number of denominations.

The number of steps can be analyzed through the use of a decision tree. Change can be made for any amount by first deciding how many coins of the largest denomination should be used. Then, the same decision can be made with the remaining amounts and the next largest denomination until all denominations are used. The total number of steps will be, at most, the product of the number of possibilities at each stage.

For 11c,

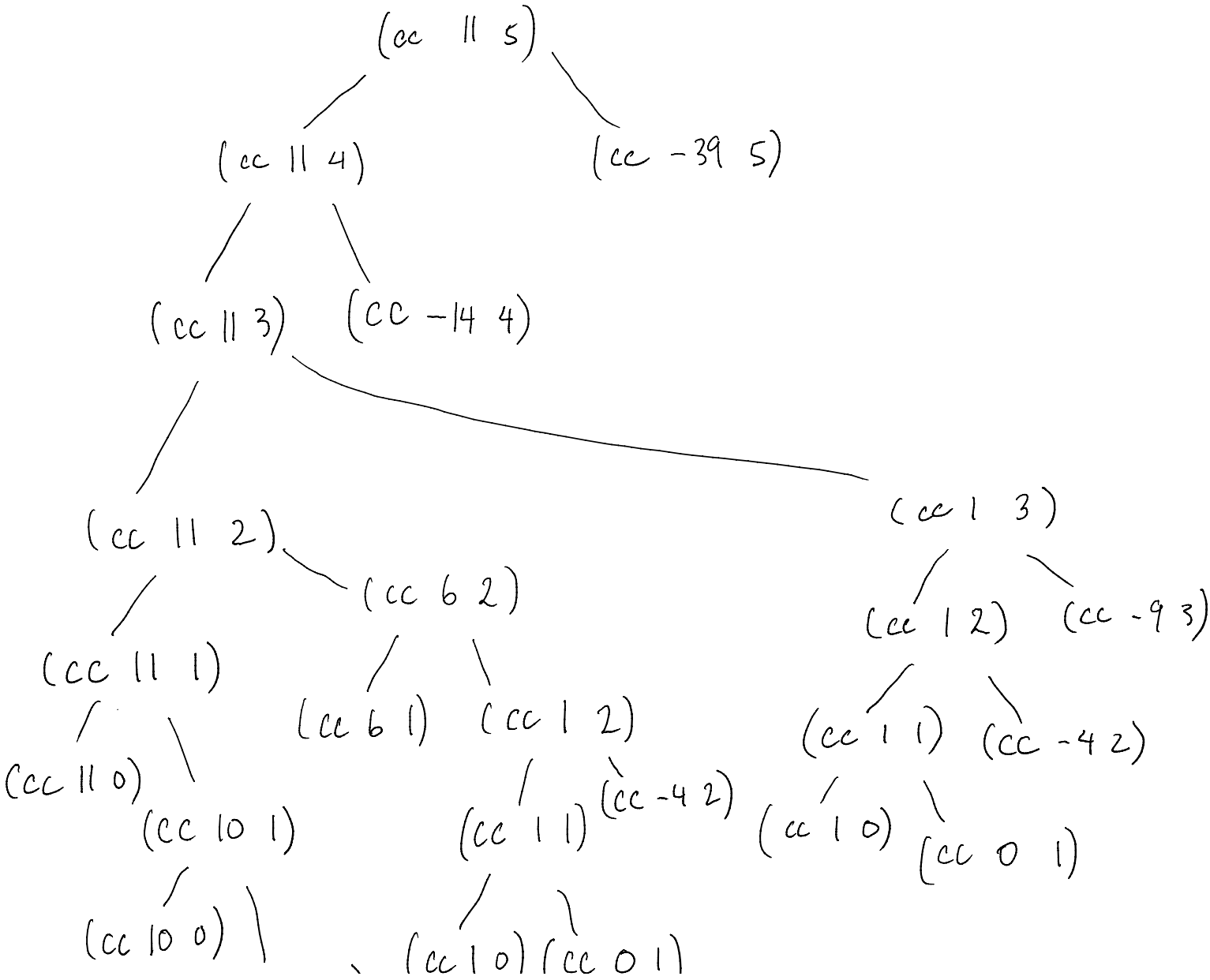
There is 1 = $\lfloor 11/50 \rfloor + 1$ way to make change with 50c (use 0 50c coins)
There is 1 = $\lfloor 11/25 \rfloor + 1$ way to make change with 25c (use 0 25c coins)
There are 2 = $\lfloor 11/10 \rfloor + 1$ ways to make change with 10c (use 1 or 0 10c coins)
There are 3 = $\lfloor 11/5 \rfloor + 1$ ways to make change with 5c (use 2, 1, or 0 5c coins)
There are 12 = $\lfloor 11/1 \rfloor + 1$ ways to make change with 1c (use 11, 10, 9, … , 0 1c coins)

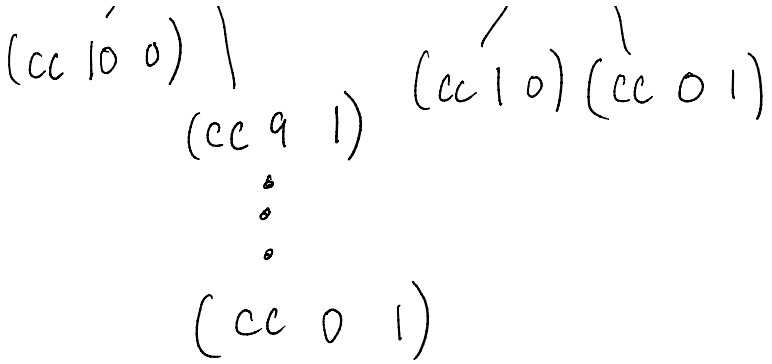
The total number of possibilities will less than 1 x 1 x 2 x 3 x 12 = 72. This is because after each stage in the decision tree, the number of possibilities for the remaining amounts will be less than what it was for the total amount (i.e. After making change using 5c coins, you will be left with either 11c, 6c, or 1c, which do not all have 12 ways of making change for each)

In general,

Total steps < $\left\lfloor \frac{n}{50} \right\rfloor * \left\lfloor \frac{n}{25} \right\rfloor * \left\lfloor \frac{n}{10} \right\rfloor * \left\lfloor \frac{n}{5} \right\rfloor * \left\lfloor \frac{n}{1} \right\rfloor + 5$

Which means this algorithm is $O(n^5)$ or in general $O(n^d)$.

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(cc 9 1)
(cc 0 1)
(cc 1 0)
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