5.

$$\frac{du}{dt} = \alpha u \;, \quad u(0) = u_0$$

$$u_1 = u_0 + \Delta t\, u_0 \alpha = u_0 (\Delta t \alpha + 1)$$

$$u_2 = u_1 + u_0 \Delta t \alpha (1 + \alpha \Delta t) = u_1 + \Delta t \alpha u_1 = u_1 (1 + \Delta t \alpha)$$

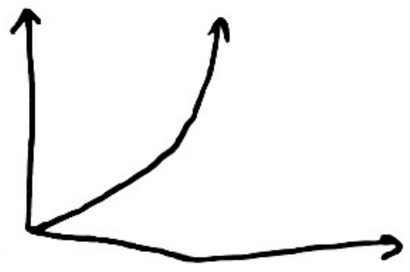
$$u_k = u_k + \Delta t \alpha u_k = u_{k-1}(1 + \Delta t \alpha) = \prod_{n=0}^{k} u_0 (1 + \Delta t \alpha)^{k-1} (1 + \Delta t \alpha)$$

$$= u_0 (1 + \Delta t \alpha)^k$$

Si: $\Delta t \leq \frac{-1}{\alpha}$

$(1 + \Delta t \alpha) \geq 0$

$\| (1 + \Delta t \alpha)^k \| \geq 0$

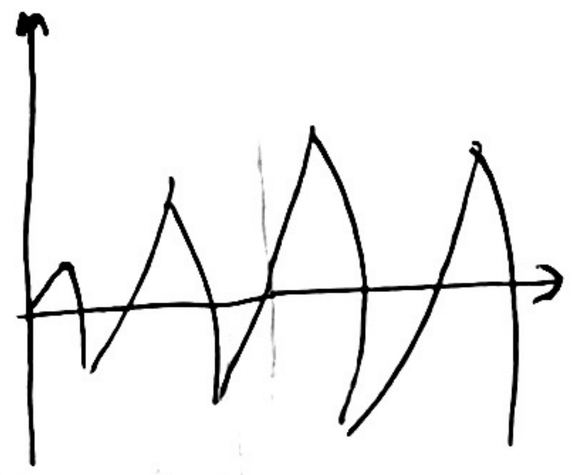


(Crece exponencialmente con k)

Si $\Delta t > \frac{-1}{\alpha}$

$(1 + \Delta t \alpha) < 0$

$(-1)^k \| (1 + \Delta t \alpha)^k \| < 0$



Es decir

$(-1)^k \| (1 + \Delta t \alpha)^k \|$

va a oscilar