

$$b.a) \vec{r}(t+\Delta t) = \sum_{n=1}^{\infty} \frac{r^{(n)}(t) \Delta t^n}{n} + r(t)$$

Expandamos hasta el término cuadrático:

$$r(t+\Delta t) \approx r(t) + v(t)\Delta t + \frac{a(t)}{2} \Delta t^2 \quad (2,89a)$$

$$\vec{v}(t) = \frac{r(t+\Delta t) - r(t-\Delta t)}{2\Delta t}$$

$$\vec{v}(t+\Delta t) = \frac{r(t+2\Delta t) - r(t-\Delta t+\Delta t)}{2\Delta t} = \frac{r(t+2\Delta t) - r(t)}{2\Delta t}$$

$$\vec{v}(t+\Delta t) = v(t) + \frac{r(t+2\Delta t) - r(t)}{2\Delta t}$$

$$r(t+2\Delta t) = 2\vec{r}(t+\Delta t) - r(t) + a(t+\Delta t)\Delta t^2$$

$$\vec{v}(t+\Delta t) = \frac{2\vec{r}(t+\Delta t) - r(t) + a(t+\Delta t)\Delta t^2 - r(t)}{2\Delta t}$$

$$\vec{v}(t+\Delta t) = \frac{2\vec{r}(t+\Delta t) - 2r(t)}{2\Delta t} + \frac{a(t+\Delta t)\Delta t}{2}$$

$$\vec{v}(t+\Delta t) = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} + \frac{1}{2} (a(t+\Delta t)) \Delta t$$

$$\frac{\vec{r}(t+\Delta t) - r(t)}{\Delta t} = \vec{v}(t) + \frac{1}{2} a(t) \Delta t$$

$$\vec{v}(t+\Delta t) = \vec{v}(t) + \frac{1}{2} (a(t)\Delta t + a(t+\Delta t)\Delta t) \quad (2,89b)$$