4) $\frac{du}{dt} = u^q$

Para $q = 1$ tenemos

$\frac{du}{dt} = u \rightarrow \frac{du}{u} = dt$

integrando a ambos lados:

$\int \frac{du}{u} = \int dt \rightarrow \ln(u) = t$

$u = e^t$

Si $q < 1$ podemos definir $1 - q = c$ talque c siempre es positivo; entonces:

$\int \frac{du}{u^q} = \int \frac{1}{u^{1-c}} du = \int \frac{1}{u^{-(c-1)}} du = \int u^{c-1} du$

$\rightarrow \frac{1}{c} u^c + k = t \rightarrow u = ((t + k)c)^{1/c}$

$u = [tc + ck]^{1/c}$ pero ck es una constante, a la cual le daremos valor de 1; entonces:

$u(t) = [t(1-q) + 1]^{1/1-q}$

3 $x^3 y' = x^4 y^2 - 2x^2 y - 1$; $y_1 = x^{-2}$

$$y' = xy^2 - \frac{2}{x}y - \frac{1}{x^3} \quad (1)$$

$$y = y_1 + \frac{1}{v} = \frac{1}{x^2} + \frac{1}{v}$$

Por lo tanto y':

$$y' = -\left(\frac{1}{x^3} + \frac{1}{v^3}v'\right)$$; sustituimos en (1)

$$-\left(\frac{2}{x^3} + \frac{1}{v^2}v'\right) = x\left(\frac{1}{x^2} + \frac{1}{v}\right)^2 - \frac{2}{x}\left(\frac{1}{x^2} + \frac{1}{v}\right) - \cancel{\frac{1}{x^3}}$$

$$-\left(\frac{1}{x^3} + \frac{1}{v^2}v'\right) = -\frac{2}{x}\left(\frac{v + x^2}{x^2 v}\right) + x\left(\frac{1}{x^2} + \frac{1}{v}\right)^2$$

$$'' = -\frac{2}{x^3} - \frac{2}{xv} + x\left(\frac{1}{x^4} + \frac{2}{x^2 v} + \frac{1}{v^2}\right)$$

$$'' = -\frac{2}{x^3} - \cancel{\frac{2}{xv}} + \cancel{\frac{1}{x^3}} + \cancel{\frac{2}{xv}} + \frac{x}{v^2}$$

$$-\cancel{\frac{1}{x^3}} - \frac{1}{v^2}v' = -\cancel{\frac{1}{x^3}} + \frac{x}{v^2} \rightarrow -\frac{1}{v^2}v' = \frac{x}{v^2}$$

multiplicando ambos lados por v^2:

$$-v' = x \rightarrow \frac{dv}{dx} = -x \rightarrow v = -\frac{x^2}{2} + c$$; entonces

$$y = \frac{1}{x^2} + \frac{1}{v} = \frac{1}{x^2} - \frac{2}{x^2 + 2c}$$; usando $y(\sqrt{2}) = 0$

$$y = \frac{1}{2} - \frac{2}{2 + 2c} = 0 \rightarrow c = 1$$

$$y = \frac{1}{x^2} - \frac{2}{x^2 + 2}$$