Punto 2

$\Omega = \{(t_{n-2}, F_{n-2}), (t_{n-1}, F_{n-1}), (t_n, F_n)\}$; $t_n - t_{n-1} = h$

Entonces empecemos por L_n

$$L_n = F_n \frac{(t - t_{n-1})(t - t_{n-2})}{(t_n - t_{n-1})(t - t_{n-2})} = \frac{F_n}{2h^2}(t - t_{n-1})(t - t_{n-2})$$

Integremos la parte derecha desde t_n hasta t_{n+1}

$$\int_{t_n}^{t_{n+1}} L_n = \frac{F_n}{2h^2}\int_{t_n}^{t_{n+1}} (t - t_{n-1})(t - t_{n-2})$$

Pero sabemos que $t_{n+1} - t_n = h$, así que podemos cambiar los limites de 0 a h y $t_{n-1} = -h$; $t_{n-2} = -2h$

Entonces

$$\int_{t_n}^{t_{n+1}} L_n = \frac{F_n}{2h^2}\int_0^h (t + h)(t + 2h)$$

$$\int_0^h t^2 + 2ht + th + 2h^2 = \int_0^h t^2 + 3ht + 2h^2$$

$$\frac{t^3}{3} + \frac{3}{2}ht^2 + 2h^2t \Big|_0^h = \frac{h^3}{3} + \frac{3h^2}{2} + 2h^3 = h^3\frac{23}{6}$$

$$\rightarrow \frac{F_n}{2h^2} \cdot h^3\frac{23}{6} = \frac{23}{12}hF_n$$

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• Para $L_{n-1} = \frac{(t-t_n)(t-t_{n-2})\, F_{n-1}}{(t_{n-1}-t_n)(t_{n-1}-t_{n-2})} = \frac{F_{n-1}}{-h^2}(t-t_n)(t-t_{n-2})$

la integral es por lo tanto:

$$\int_0^h t(t+2h) = \int_0^h t^2 + 2ht = \frac{t^3}{3} + ht^2 \Big|_0^h$$

$$\rightarrow \frac{h^3}{3} + h^3 = \frac{4}{3}h^3 = \frac{16h^3}{12}$$

Entonces tenemos $\frac{16}{12} h F_{n-1}$

• Para $L_{n-2} = \frac{(t-t_n)(t-t_{n-1})F_{n-2}}{(t_{n-2}-t_n)(t_{n-2}-t_{n-1})} = \frac{F_{n-2}}{2h^2}(t-t_n)(t-t_{n-1})$

La integral queda:

$$\int_0^h t(t+h)\,dt = \frac{t^3}{3} + \frac{ht^2}{2}\Big|_0^h = \frac{h^3}{3} + \frac{h^3}{2} = \frac{5}{6}h^3$$

Entonces tenemos: $\frac{5}{12} h F_{n-2}$

como $y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} P_k(t)\,dt$

y $P_k(t) = \frac{h}{12}(23F_n - 16F_{n-1} + 5F_{n-2})$; Entonces

$$y_{n+1} = y_n + \frac{h}{12}(23F_n - 16F_{n-1} + 5F_{n-2})$$

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Para orden 4 tenemos:

$$\Omega = \{(t_{n-3}, F_{n-3}), (t_{n-2}, F_{n-2}), (t_{n-1}, F_{n-1}), (t_n, F_n)\}$$

Para $L_n: \frac{(t - t_{n-2})}{(t_n - t_{n-2})} \cdot \frac{(t - T_{n-3})}{t_n - t_{n-3}} \cdot \frac{(t - t_{n-1})}{(t_n - t_{n-1})} F_n$

$$L_n: (t - T_{n-2}) \cdot (t - t_{n-3}) \cdot (t - t_{n-1}) \cdot \frac{F_n}{6h^3}$$

Resolviendo en python tenemos: $\frac{55}{24} h$.

Para $L_{n-1}: \frac{(t - T_n)}{(t_{n-1} - T_n)} \cdot \frac{(t - T_{n-2})}{(t_{n-1} - T_{n-2})} \frac{(t - T_{n-3})}{(t_{n-1} - T_{n-3})} F_{n-1}$

que nos da: $-\frac{59}{24} h F_{n-1}$

Para $L_{n-2}: \frac{(t - T_n)}{(T_{n-2} - T_n)} \cdot \frac{(t - T_{n-1})}{(T_{n-2} - T_{n-1})} \cdot \frac{(t - T_{n-3})}{(t_{n-2} - T_{n-3})} F_{n-2}$

que nos da: $\frac{37}{24} h F_{n-2}$

Para $L_{n-3}: \frac{(t - T_n)}{(t_{n-3} - t_n)} \cdot \frac{(t - t_{n-1})}{(t_{n-3} - t_{n-1})} \cdot \frac{(t - t_{n-2})}{(t - t_{n-2})} F_{n-3}$

que nos da: $-\frac{9}{24} F_{n-3}$

Entonces para orden 4 tenemos:

$$y_{n+1} = y_n + \frac{h}{24}(55 F_n - 59 F_{n-1} + 37 F_{n-2} - 9 F_{n-3})$$