1)

$$\dot{q}(t) = f(q,p) = \frac{dq}{dt}$$

$$\dot{p}(t) = g(q,p) = \frac{dp}{dt}$$

$$\frac{dE}{dt} = \frac{\partial E}{\partial q}\frac{dq}{dt} + \frac{\partial E}{\partial p}\frac{dp}{dt}$$

$$\frac{dE}{dt} = \left[\frac{\partial E}{\partial q} + \frac{\partial E}{\partial p}\right]\begin{bmatrix}\frac{dq}{dt} \\ \frac{dp}{dt}\end{bmatrix} = \begin{pmatrix} \frac{\partial \frac{dq}{dt}}{\partial q} & \frac{\partial \frac{dq}{dt}}{\partial p} \\ \frac{\partial \frac{dp}{dt}}{\partial q} & \frac{\partial \frac{dp}{dt}}{\partial p} \end{pmatrix} E$$

$$\frac{dE}{dt} = \begin{pmatrix} \frac{\partial f}{\partial q} & \frac{\partial f}{\partial p} \\ \frac{\partial g}{\partial q} & \frac{\partial g}{\partial p} \end{pmatrix} E = ME$$

b)

$$x' = 2x - y$$

$$y' = x + 2y$$

$$M = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\frac{dE}{dt} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} E$$

$$E = E_0 e^{\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} t}$$

c) Eigenvalues:

$$\det(M - \lambda I) = 0$$

$$\det\begin{pmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{pmatrix} = 0$$

$(\lambda - 2)^2 + 1 = 0$

$\lambda^2 - 4\lambda + 5 = 0$

$\lambda_1 = 2 + i$

$\lambda_2 = 2 - i$

Eigenvectors:

$(2 - 2 + i) v_1 - v_2 = 0$

$v_1 + (2 - 2 + i) v_2 = 0$

$i v_1 = v_2$

$v_1 + i v_2 = 0$

$v_1 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$(2 - 2 - i) v_1 - v_2 = 0$

$v_1 + (2 - 2 - i) v_2 = 0$

$v_2 = \begin{bmatrix} 1 \\ i \end{bmatrix}$