Punto 3

tenemos

$$U_{ij}^{l+1} = \gamma^2 \Big[U_{(i+1)j}^{l} - 2U_{ij}^{l} + U_{(i-1)j}^{l} + \frac{\Delta\rho}{\rho[i]}$$

$$\left(U_{ij}^{l} - U_{(i-1)j}^{l}\right) + \left(\frac{\lambda^2}{\rho[i]}\right)\left(U_{i(j+1)}^{l} - 2U_{ij}^{l} + U_{i(j-1)}^{l}\right)\Big]$$

$$+ 2U_{ij}^{l} - U_{ij}^{l-1}$$

Ahora definimos λ y γ así

$$\lambda = \frac{\Delta\rho}{\Delta\theta} \qquad \gamma = \frac{\alpha\,\Delta t}{\Delta\rho}$$

tomando ρ con γ en polares se tiene

$$\rho = (x^2 + y^2)^{1/2} \quad y \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\rightarrow \quad x = \rho\cos(\theta) \;;\; y = \rho\,\mathrm{Sen}(\theta)$$

Derivando parcialmente tenemos

$$\frac{\partial\rho}{\partial x} = \frac{\rho\cos\theta}{\rho} = \cos(\theta) \quad y \text{ entonces:}$$

$$\frac{\partial\rho}{\partial y} = \mathrm{Sen}\,\theta$$

para θ tenemos .

$$\frac{\partial \theta}{\partial y} = \frac{\cos\theta}{\rho} \quad y \quad \frac{\partial \theta}{\partial x} = \frac{-\operatorname{sen}\theta}{\rho}$$

usando el operador de Laplace:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \nabla^2 u$$ y usando las bi...

das anteriores tenemos

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial x}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \rho^2}\cos^2(\theta) + \frac{\partial^2 u}{\partial \theta^2}\cdot\frac{\operatorname{sen}^2\theta}{\rho^2} - \frac{\partial^2 u}{\partial \rho\,\partial \theta}\frac{\operatorname{sen}(2\theta)}{\rho} + \frac{\partial u}{\partial \theta}\frac{\operatorname{sen}(2\theta)}{\rho^2} + \frac{\partial u}{\partial \rho}\frac{\operatorname{sen}^2\theta}{\rho}$$

y haciendo lo mismo para $\partial^2 u / \partial y^2$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial \rho^2}\operatorname{sen}^2\theta + \frac{\partial^2 u}{\partial \theta^2}\frac{\cos^2(\theta)}{\rho^2} + \frac{\partial u^2}{\partial \rho\,\partial \theta}\frac{\operatorname{sen}(2\theta)}{\rho} + \frac{\partial u}{\partial \rho}\frac{\cos^2(\theta)}{\rho} - \frac{\partial u}{\partial \theta}\frac{\operatorname{sen}(2\theta)}{\rho^2}$$

Al sumar ambos ecuaciones tenemos:

$$\nabla^2 U = \frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho}$$

usando

$$\frac{\partial^2 U}{\partial t^2} = \delta^2 \nabla^2 U \quad , \text{ tenemos}$$

$$U_{ij}^{L+1} = \left(\frac{\delta \Delta t}{\Delta \rho}\right)^2 \Big[U^L_{(i+1)j} - 2U^L_{ij} + U^L_{(i-1)j} + \left(\frac{\Delta \rho}{\rho[i]}\right)\left(U^L_{ij} - U^L_{i(j-1)}\right) + \left(\frac{\Delta \rho^2}{\Delta \theta^2 \rho[i]}\right)\left(U^L_{i(j+1)} - 2U_{ij}\right.$$

$$\left. + U^L_{(j-1)}\right)\Big] + 2U^L_{ij} - U^{L-1}_{ij}$$

usando

$$\lambda = \frac{\Delta \rho}{\Delta \theta} \qquad r = \frac{\delta \Delta t}{\Delta \rho}$$

$$U^{L+1}_{ij} = r^2 \Big[U^L_{i(j+1)} - 2U^L_{ij} + U_{i(j-1)j} + \left(\frac{\Delta \rho}{\rho[i]}\right)\left(U^L_{ij} - U^L_{i(j-1)}\right) + \left(\frac{\lambda^2}{\rho[i]}\right)\left(U^L_{i(j+1)} - 2U^L_{ij}\right.$$

$$\left. + U^L_{i(j+1)}\right] + 2U^L_{ij} - U^{L-1}_{ij}$$