1) x^2 continuidad para ambos límites.

$$f'(x) = \lim_{h \to 0} \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} = \gamma$$

por def formal:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

por lo tanto es consistente si:

I) $$\lim_{h \to 0} \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

a) Para x^2:

$$\lim_{h \to 0} \frac{-(x+2h)^2 + 4(x+h)^2 - 3x^2}{2h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{-(x^2 + 4hx + 4h^2) + 4x^2 + 8xh + 4h^2 - 3x^2}{2h} = \gamma$$

$$=$$

$$\lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} = \beta$$

$$\gamma = \lim_{h \to 0} \frac{(-1+4-3)x^2 + (-4+8)hx + (-4+4)h^2}{2h}$$

$$\gamma = \lim_{h \to 0} \frac{4hx}{2h} = 2x \equiv \beta = \lim_{h \to 0} \frac{2hx + h^2}{h} = 2x$$

b) para: $f(x) = \sin(x)$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} = \lim_{h\to 0} \frac{-\sin(x+2h) - 4\sin(x+h) - 3\sin(x)}{2h}$$

Entonces $\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h)$

$$\lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} = \gamma$$

y se conoce que $\lim_{h\to 0} \frac{\sin(h)}{h} = 1$ y $\lim_{h\to 0} \frac{1-\cos(h)}{h} = 0$

$$\lim_{h\to 0} \underbrace{\frac{\sin(x)(\cos(h) - 1)}{h}}_{-0} + \lim_{h\to 0} \cos(x) \underbrace{\left(\frac{\sin(h)}{h}\right)}_{1} = \gamma$$

$$\gamma = \cos(x) = f'(x)$$

Es decir, para que sea consistente

$$\eta = \lim_{h\to 0} \frac{-\sin(x+2h) - 4\sin(x+h) - 3\sin(x)}{2h} \equiv \cos(x)$$

$$\eta = \lim_{h\to 0} \frac{-\sin(x)\cos(2h) - \cos(x)\sin(2h) - 4\sin(x)\cos(h) - 4\cos(x)\sin(h) - 3\sin}{2h}$$

$$h = \lim_{h\to 0} \frac{\sin(x)\overbrace{(-\cos(2h) - 4\cos(h) - 3)}^{\alpha} + \cos(x)\overbrace{(-\sin(2h) - 4\sin(h))}^{\Omega}}{2h}$$

$$\eta = \lim_{h\to 0} \sin(x)\frac{\alpha}{2h} + \lim_{h\to 0} \cos(x)\frac{\Omega}{2h}$$

$$\eta = \lim_{h\to 0} \cancel{\sin(x)(\sin(2h) + 2\sin(h))} + \frac{\cos(x)(-2\cos(2h) + 4\cos(h))}{2}$$

$$\eta = \cos(x)$$

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

función de prueba:

$$x^2 = f(x)$$

$$\lim_{h \to 0} \frac{(x+h)^2 - 2x^2 + (x-h)^2}{h^2}$$

$$\lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 2x^2 + x^2 - 2hx + h^2}{h^2}$$

$$\lim_{h \to 0} \frac{(1-2+1)x^2 + (2-2)hx + 2h^2}{h^2} = \lim_{h \to 0} \frac{2\cancel{h^2}}{\cancel{h^2}} = 2$$

$$\sin(x) = f(x)$$

$$\lim_{h \to 0} \frac{\sin(x+h) - 2\sin(x) + \sin(x-h)}{h^2}$$

$$\lim_{h \to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - 2\sin(x) + \sin(x)\cos(-h) + \cos(x)\sin(-h)}{h^2},$$

$$\lim_{h \to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - 2\sin(x) + \sin(x)\cos(h) - \cos(x)\sin(h)}{h^2}$$

$$\lim_{h \to 0} \frac{\sin(x)(\cos(h) + \cancel{2} + \cos(h)) + \cos(x)(\cancel{\sin(h) - \sin(h)})}{h^2}$$

$$\lim_{h \to 0} \frac{\sin(x)(2\cos(h) + 2)}{h^2} = cte \times \lim_{h \to 0} \frac{2(\cos(h) +}{h^2}$$

L'hopital:

$$\sin(x) \cdot \lim_{h \to 0} \frac{\cancel{2}(-\sin(h))}{\cancel{2}h} = \sin(x)(-1) \equiv f''(x)$$