

$$r_{n+1} = 2r_n + (-) r_{n-1} + h^2 a_n$$

$$hV_n = (r_{n+1} - r_{n-1}) / 2$$

$$-2hV_n + r_{n+1} = r_{n-1}$$

$$r_{n+1} = 2r_n + hV_n - r_{n-1} + h^2 a_n$$

$$2r_{n+1} = 2r_n + 2hV_n + h^2 a_n$$

$$r_{n+1} = r_n + hV_n + \underbrace{\frac{h^2}{2} a_n}_{\text{Error}}$$

$$hV_{n+1} = r_{n+1} - r_n + \frac{h^2}{2} \sum d_p a_n - q + 2$$

Expanding in series:

$$r_{n+1} = r_n + hV_n + \frac{h^2}{2} \frac{1}{3} a_n - \frac{h^2}{2} \frac{a_{n-1}}{3}$$

$$a_n = \frac{3(r_{n+1} - r_n - hV_n + \frac{h^2}{2} \frac{a_{n-1}}{3})}{4h^2}$$

$$r_{n+1} = r_n + hV_n + \frac{h^2}{4 \cdot 2 h^2} (a_{n+1} + 3(r_{n+1} - r_n - hV_n + \frac{h^2}{2} \frac{a_{n-1}}{3}))$$

$$hV_{n+1} = \cancel{r_{n+1}} + hV_n + \frac{h^2}{6} (a_{n+1} + 2a_n) - \cancel{r_n} + \frac{h^2}{6} (2a_{n+1} + a_n)$$

$$hV_{n+1} = hV_n + \frac{h^2}{6} (3a_{n+1} + 3a_n) = hV_n + \frac{h^2}{2} (a_{n+1} + a_n)$$