1. First of all, let us short notation in terms of $r_L := r_{low}$ and $r_H := r_{high}$. In order to check concavity of the objective function, consider its SOC:

$$-\gamma p\,(w_0(1+r^f+\alpha(r_L-r^f)))^{-(1+\gamma)}\,(r_L-r^f)^2+-\gamma(1-p)\,(w_0(1+r^f+\alpha(r_H-r^f)))^{-(1+\gamma)}\,(r_H-r^f)^2$$

Obviously its sign is ambiguous. Therefore, even though the (two) inequality constraints are (weakly) concave and no equality constraints arise, the first "convexity-condition" is not satisfied generally. That is, KTK-conditions are just necessary (as usual) but not sufficient for an optimum.

2. a)

FOC:

$$p\,(w_0(1+r^f+\alpha(r_L-r^f)))^{-\gamma}\,(r_L-r^f)+(1-p)\,(w_0(1+r^f+\alpha(r_H-r^f)))^{-\gamma}\,(r_H-r^f)=0 \tag{1}$$

Assume the opposite, i.e., that the optimal portfolio share α^* (the value of α for which the equation above holds) depends on initial wealth w_0. Denote this maximizer contingent on w_0 by $\alpha(w_0)$. As the optimal portfolio share responds on w_0, the first order derivative of the LHS of (1) (setting $\alpha = \alpha(w_0)$) with respect to wealth should be equal zero. Namely, the total differentiation of the FOC w.r.t. α and w_0:

$$\begin{aligned}\frac{\mathrm{d}\mathbb{E}u(w_1)}{\mathrm{d}\alpha\,\mathrm{d}w_0}= & -\gamma w_0^{-(1+\gamma)}\,p\,(1+r^f+a(r_L-r^f))^{-\gamma}\,(r_L-r^f)+w_0^{-\gamma}\,p\,\alpha'(w_0)\,(r_L-r^f)^2 \\ & -\gamma w_0^{-(1+\gamma)}\,(1-p)\,(1+r^f+a(r_H-r^f))^{-\gamma}\,(r_H-r^f)+w_0^{-\gamma}\,p\,\alpha'(w_0)\,(r_H-r^f)^2=0.\end{aligned}$$

Clearly, this equality can be rearranged as follows:

$$w_0^{-\gamma}\,p\,\alpha'(w_0)\,(r_L-r^f)^2+w_0^{-\gamma}\,p\,\alpha'(w_0)\,(r_H-r^f)^2$$
$$=\tfrac{1}{\gamma}\,w_0^{-(1+\gamma)}\,p\,(1+r^f+\alpha(r_L-r^f))^{-\gamma}\,(r_L-r^f)+\tfrac{1}{\gamma}\,w_0^{-(1+\gamma)}\,(1-p)\,(1+r^f+\alpha(r_H-r^f))^{-\gamma}\,(r_H-r^f)$$

and now, the RHS of this equality corresponds to the FOC multiplied by γw_0^{-1} which has to be still equal zero. Thus, dividing the equality by the LHS (except $\alpha'(w_0)$) yields

$$\alpha'(w_0)=\frac{0}{w_0^{-\gamma}\,p\,(r_L-r^f)^2+w_0^{-\gamma}\,p\,(r_H-r^f)^2}=0$$

which contradicts that the optimal portfolio share depends on w_0.

Alternatively, notice that the initial objective (which we aim to maximize) can be divided by w_0^{ϕ} and thus, wealth cancels out. Clearly, the optimal portfolio shares for the old resp. the new problem, will coincide.

b) The associated FOC becomes:

$$p\,(1+r^f+\alpha(r_L-r^f))^{\phi-1}\,(r_L-r^f)+(1-p)\,(1+r^f+\alpha(r_H-r^f))^{\phi-1}\,(r_H-r^f)=0 \tag{2}$$

using $(r_L-r^f)=-(r_H-r^f)=-0.1$ and rearranging we obtain

$$p^{\frac{1}{\phi-1}}\,(1+r^f+\alpha\,(-0.1))=(1-p)^{\frac{1}{\phi-1}}\,(1+r^f+\alpha\,0.1) \tag{3}$$

$$\Leftrightarrow \alpha=10(1+r^f)\,\frac{p^{\frac{1}{\phi-1}}-(1-p)^{\frac{1}{\phi-1}}}{p^{\frac{1}{\phi-1}}+(1-p)^{\frac{1}{\phi-1}}}=2.73308... \tag{4}$$

evaluated at $p=.1$, $r^f=.02$ and $\phi=3$.

3. a) Intuitively , $\alpha \geq 0$ prevents the case where the household supplies the portfolio and $\alpha \leq 1$ rules out that the household borrows money in order to invest more in the portfolio.