1 Question 1

1.1 1.1

A market equilibrium occurs when markets clear. This implies no excess demand (D) or supply (S) of Goods. Thus, $q_D = q_S$. This only occurs when $p_D = p_S$ (the market clearing price prevails).

$$p_D = p_S$$

using

$$p_D = a - b * q_D \text{ and } p_S = c + d * q_S$$

we get

$$a - b * q_D = c + d * q_S$$

$$0 = c + d * q_S - (a - b * q_D)$$

$$0 = c - a + d * q_S + b * q_D$$

$$0 = b * q_D + d * q_S - (a - c)$$

Since $q_D = q_S$ holds, this can be simplified even further

$$0 = (b + d) * q - (a - c) \tag{1}$$

■

1.2 1.2

Analytical computation of the equilibrium allocation. Alternative approach of previous question used. First, set quantities equal, $q_D = q_S$ and calculate the resulting equilibrium price p^*. By inserting the equilibrium price into both quantity functions, we get the equilibrium quantity and can show that $q_D = q_S$ in fact holds.

$$q_D = q_S$$

$$\frac{a-p}{b} = \frac{c-p}{d}$$

$$d(a - p) = b(p - c)$$

$$da + bc = p(d + b)$$

$$\frac{da+bc}{d+b} = p^*$$

Now, insert into the quantity functions:

$$q_D = \frac{a-p^*}{b} \qquad q_S = \frac{c-p^*}{d}$$

$$q_D = \frac{a-\frac{da+bc}{d+b}}{b} \qquad q_S = \frac{c-\frac{da+bc}{d+b}}{d}$$

$$q_D = \frac{a-c}{d+b} = q \qquad q_S = \frac{a-c}{d+b} = q$$

which can also be computed by rearranging (**??**):

$$0 = (b+d) * q - (a-c)$$

$$(a-c) = (b+d) * q$$

$$\frac{a-c}{b+d} = q$$

$$\frac{a-c}{d+b} = q$$

1.3 1.3

The LU decomposition. The application of this procedure can be found in the MATLAB file PS1Q1.m.

1. Rearrange the equations given in the problem set so that, when solving for x, we solve for $x = [p, q]'$.

$$a = p + bq$$

$$c = p - dq$$

   Which gives the system

$$\begin{pmatrix} 1 & b \\ 1 & -d \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

2. Decompose the matrix A into the two factors L and U:

$$A = L * U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & -b-d \end{pmatrix}$$

   Which then gives the following system of equations:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & -b-d \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

3. Solve this system of equations.

   (a) First solve $Ly = b$ by forward induction.

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

$$y_1 = a$$

$$y_1 + y_2 = c$$

which gives

$$y_1 = a$$

$$y_2 = c - a$$

(b) Then solve $Ux = y$ by backward induction.

$$\begin{pmatrix} 1 & b \\ 0 & -(b+d) \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a \\ c - a \end{pmatrix}$$

$$-(b+d)q = y_2 = c - a$$

$$p + bq = a$$

which gives

$$q = \tfrac{a-c}{b+d}$$

$$p = \tfrac{ad+bc}{b+d}$$