Answers to Problem Set 1
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1 Question 1

1.1

A market equilibrium occurs when markets clear. This implies no excess demand (D) or supply (S) of Goods. Thus, $q_D = q_S$. This only occurs when $p_D = p_S$ (the market clearing price prevails).

$$p_D = p_S$$

using

$$p_D = a - b * q_D \text{ and } p_S = c + d * q_S$$

we get

$$a - b * q_D = c + d * q_S$$

$$0 = c + d * q_S - (a - b * q_D)$$

$$0 = c - a + d * q_S + b * q_D$$

$$0 = b * q_D + d * q_S - (a - c)$$

Since $q_D = q_S$ holds, this can be simplified even further

$$0 = (b + d) * q - (a - c) \tag{1}$$

■

1.2

Analytical computation of the equilibrium allocation. Alternative approach of previous question used. First, set quantities equal, $q_D = q_S$ and calculate the resulting equilibrium price p^*. By inserting the equilibrium price into both quantity functions, we get the equilibrium quantity and can show that $q_D = q_S$ in fact holds.

$$q_D = q_S$$

$$\frac{a-p}{b} = \frac{c-p}{d}$$

$$d(a-p) = b(p-c)$$

$$da + bc = p(d+b)$$

$$\frac{da+bc}{d+b} = p^*$$

Now, insert into the quantity functions:

$$q_D = \frac{a-p^*}{b} \qquad q_S = \frac{c-p^*}{d}$$

$$q_D = \frac{a - \frac{da+bc}{d+b}}{b} \qquad q_S = \frac{c - \frac{da+bc}{d+b}}{d}$$

$$q_D = \frac{a-c}{d+b} = q \qquad q_S = \frac{a-c}{d+b} = q$$

which can also be computed by rearranging (1):

$$0 = (b+d) * q - (a-c)$$

$$(a-c) = (b+d) * q$$

$$\frac{a-c}{b+d} = q$$

1.3

The LU decomposition. The application of this procedure can be found in the MATLAB file PS1Q1.m.

1. Rearrange the equations given in the problem set so that, when solving for x, we solve for $x = [p, q]'$.

   $$a = p + bq$$

   $$c = p - dq$$

   Which gives the system

   $$\begin{pmatrix} 1 & b \\ 1 & -d \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

2. Decompose the matrix A into the two factors L and U:

   $$A = L * U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & -b-d \end{pmatrix}$$

   Which then gives the following system of equations:

$$\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}1 & b\\0 & -b-d\end{pmatrix}\begin{pmatrix}p\\q\end{pmatrix}=\begin{pmatrix}a\\c\end{pmatrix}$$

3. Solve this system of equations.

   (a) First solve $Ly = b$ by forward induction.

$$\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}a\\c\end{pmatrix}$$

$$y_1 = a$$

$$y_1 + y_2 = c$$

   which gives

$$y_1 = a$$

$$y_2 = c - a$$

   (b) Then solve $Ux = y$ by backward induction.

$$\begin{pmatrix}1 & b\\0 & -(b+d)\end{pmatrix}\begin{pmatrix}p\\q\end{pmatrix}=\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}a\\c-a\end{pmatrix}$$

$$-(b+d)q = y_2 = c - a$$

$$p + bq = a$$

   which gives

$$q = \frac{a-c}{b+d}$$

$$p = \frac{ad+bc}{b+d}$$

1.4

```
%% Subquestion 4 - LU-Decomposition
clear;
clc;
close all;

a=3;
b=0.5;
c=1;
d=c;

A=[1,b;1 -d];

y=[a; c];

[L,U]=lu(A);

t=L\y;
x=U\t;

disp(['LU Result: The market clearing price ', num2str(x(1,1)), ' clears the mar
```

LU Result: The market clearing price 2.3333 clears the market at the quantity 1.3333!

1.5

```
%% Subquestion 5 - Gauss-Seidel fixed-point iteration

clear;


a=3;
b=0.5;
c=1;
d=c;

%initial guess of quantity
q=0.1;
%Set up difference criterion to a value higher than in the while loop
q_difference=100;

%Set up empty vectors for storage of historical values
q_difference_hist=nan(100,1);
q_Dp_hist=nan(100,1);
q_Dq_hist=nan(100,1);
```

```
q_Sq_hist=nan(100,1);
q_Sp_hist=nan(100,1);
q_Time=nan(100,1);

%Iteration index
i=1;

%Begin iteration
while q_difference>0.01

    q_Dp=a-b*q;          %Demand-price from initial quantity
    q_Dp_hist(i,1)=q_Dp;
    q_Dq_hist(i,1)=q;
    q_Sq=(q_Dp-c)/d;     %Supply-quantity from Demand-price
    q_Sq_hist(i,1)=q_Sq;
    q_Sp=c+d*q_Sq;       %Supply-price for difference
    q_Sp_hist(i,1)=q_Sp;

    if i>1
    q_difference=abs(q_Sp_hist(i,1)-q_Dp_hist(i-1,1));
    q_difference_hist(i,1)=q_difference;
    end
    q=q_Sq;              %Quantity for next guess set
    q_Time(i,1)=i;
    i=i+1;

end
disp(['Gauss-Seidel Iteration Result (using quantity as initial guess): The mark
```

Gauss-Seidel Iteration Result (using quantity as initial guess): The market clearing price 2.3357 clears the market at the quantity 1.3357 after 9 iterations!

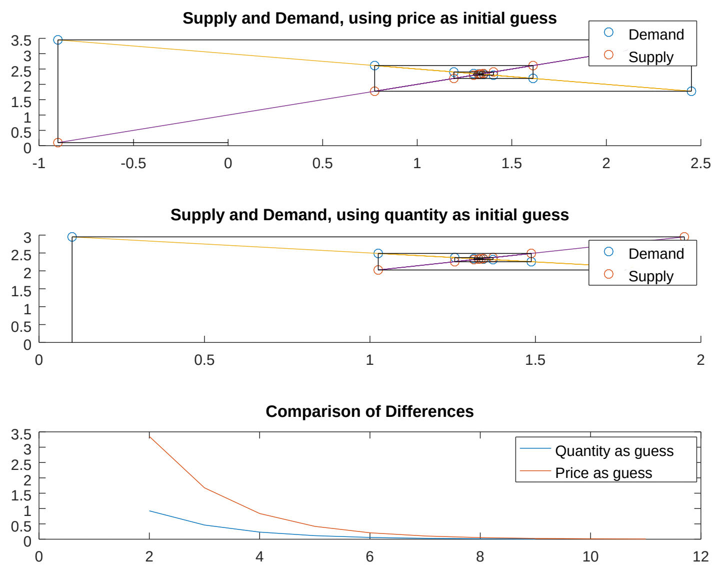
```
%Alternatively: Using the price as a first guess

%Initial Guess
p=0.1;

%Set up difference criterion to a value higher than in the while loop
difference=100;

%Set up empty vectors for storage of historical values
difference_hist=nan(100,1);
Dp_hist=nan(100,1);
Dq_hist=nan(100,1);
```

```
Sq_hist=nan(100,1);
Sp_hist=nan(100,1);
Time=nan(100,1);

%Iteration index
i=1;

while difference>0.01

    Sq=(p-c)/d;     %Supply-quantity from price
    Sq_hist(i,1)=Sq;
    Sp=p;         %Supply-price for difference
    Sp_hist(i,1)=Sp;
    Dp=a-b*Sq;        %Demand-price from initial quantity
    Dp_hist(i,1)=Dp;
    Dq_hist(i,1)=Sq;

    if i>1
    difference=abs(Sq_hist(i-1,1)-Dq_hist(i,1));
    difference_hist(i,1)=difference;
    end
    p=Dp;              %Quantity for next guess set
    Time(i,1)=i;
    i=i+1;

end

disp(['Gauss-Seidel Iteration Result (using price as initial guess): The market
```

Gauss-Seidel Iteration Result (using price as initial guess): The market clearing
price 2.3312 clears the market at the quantity 1.3312 after 11 iterations!

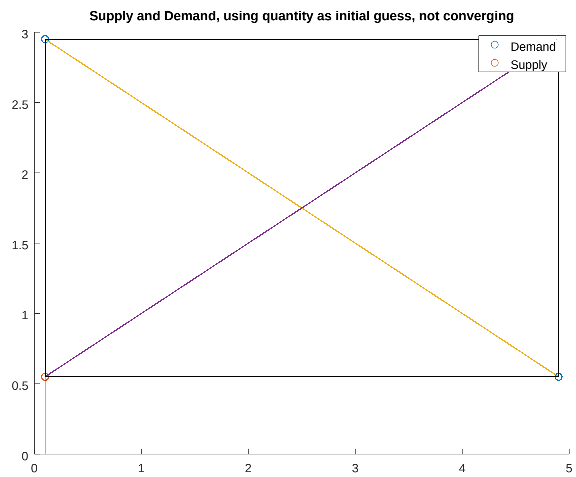
```
figure
subplot(3,1,1);

scatter(Dq_hist,Dp_hist)
hold on
scatter(Sq_hist,Sp_hist)
plot(Dq_hist,Dp_hist,Sq_hist,Sp_hist)
line([Sq_hist(1,1) 0], [Sp_hist(1,1) Sp_hist(1,1)])

%TO SHOW HOW THE ALGORITHM WORKS!
for j=1:(i-1)
line([Dq_hist(j,1) Sq_hist(j+1,1)], [Dp_hist(j,1) Dp_hist(j,1)])
```

```
end

for j=1:(i-1)
line([Sq_hist(j,1) Sq_hist(j,1)], [Dp_hist(j,1) Sp_hist(j,1)])
end

title('Supply and Demand, using price as initial guess')
legend('Demand','Supply')
hold off

subplot(3,1,2);
scatter(q_Dq_hist,q_Dp_hist)
hold on
scatter(q_Sq_hist,q_Sp_hist)
plot(q_Dq_hist,q_Dp_hist,q_Sq_hist,q_Sp_hist)
line([q_Dq_hist(1,1) q_Dq_hist(1,1)], [q_Dp_hist(1,1) 0])
%TO SHOW HOW THE ALGORITHM WORKS!
for j=1:(i-1)
line([q_Dq_hist(j,1) q_Sq_hist(j,1)], [q_Dp_hist(j,1) q_Dp_hist(j,1)])
end

for j=1:(i-1)
line([q_Sq_hist(j,1) q_Sq_hist(j,1)], [q_Dp_hist(j,1) q_Sp_hist(j+1,1)])
end
title('Supply and Demand, using quantity as initial guess')
legend('Demand','Supply')
hold off

%To show convergence
%figure
%plot(q_Time,q_difference_hist)
%title('Difference, using quantity as initial guess')

subplot(3,1,3);
plot(q_Time,q_difference_hist,Time,difference_hist)
title('Comparison of Differences')
legend('Quantity as guess','Price as guess')



%Non convergent case

a=3;
b=0.5;
c=b;
d=c;
```




```
%initial guess of quantity
q=0.1;
%Set up difference criterion to a value higher than in the while loop
nq_difference=100;

%Set up empty vectors for storage of historical values
```

```
nq_difference_hist=nan(100,1);
nq_Dp_hist=nan(100,1);
nq_Dq_hist=nan(100,1);
nq_Sq_hist=nan(100,1);
nq_Sp_hist=nan(100,1);
nq_Time=nan(100,1);

%Iteration index


%Begin iteration
for i=1:100
    nq_Dp=a-b*q;        %Demand-price from initial quantity
    nq_Dp_hist(i,1)=nq_Dp;
    nq_Dq_hist(i,1)=q;
    nq_Sq=(nq_Dp-c)/d;    %Supply-quantity from Demand-price
    nq_Sq_hist(i,1)=nq_Sq;
    nq_Sp=c+d*nq_Sq;        %Supply-price for difference
    nq_Sp_hist(i,1)=nq_Sp;

    if i>1
    nq_difference=abs(nq_Sp_hist(i,1)-nq_Dp_hist(i-1,1));
    nq_difference_hist(i,1)=nq_difference;
    end
    q=nq_Sq;            %Quantity for next guess set
    nq_Time(i,1)=i;

end

figure
scatter(nq_Dq_hist,nq_Dp_hist)
hold on
scatter(nq_Sq_hist,nq_Sp_hist)
plot(nq_Dq_hist,nq_Dp_hist,nq_Sq_hist,nq_Sp_hist)
line([nq_Dq_hist(1,1) nq_Dq_hist(1,1)], [nq_Dp_hist(1,1) 0])
%TO SHOW HOW THE ALGORITHM WORKS!
for j=1:(i-1)
line([nq_Dq_hist(j,1) nq_Sq_hist(j,1)], [nq_Dp_hist(j,1) nq_Dp_hist(j,1)])
end
for j=1:(i-1)
line([nq_Sq_hist(j,1) nq_Sq_hist(j,1)], [nq_Dp_hist(j,1) nq_Sp_hist(j+1,1)])
end
title('Supply and Demand, using quantity as initial guess, not converging')
legend('Demand','Supply')
hold off
```

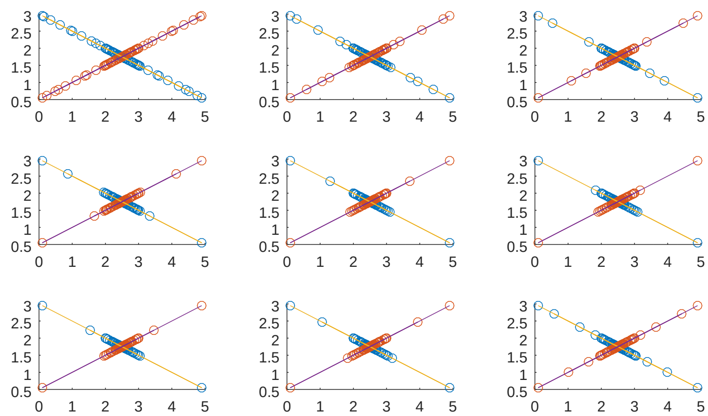


1.6

```
%% Subquestion 6 - Using a dampening factor

lambda=linspace(0.1,0.9,9);
iteration_count=nan(9,1);

a=3;
b=0.5;
c=b;
d=c;
```

```
%Set up empty vectors for storage of historical values
d_difference_hist=nan(100,1);
d_Dp_hist=nan(100,1);
d_Dq_hist=nan(100,1);
d_Sq_hist=nan(100,1);
d_Sp_hist=nan(100,1);
d_Time=nan(100,1);


figure
for j=1:9
%initial guess of quantity
q=0.1;
%Set up difference criterion to a value higher than in the while loop
d_difference=100;
i=1;
while d_difference>0.01

    d_Dp=a-b*q;         %Demand-price from initial quantity
    d_Dp_hist(i,1)=d_Dp;
    d_Dq_hist(i,1)=q;
    d_Sq=(d_Dp-c)/d;    %Supply-quantity from Demand-price
    d_Sq_hist(i,1)=d_Sq;
    d_Sp=c+d*d_Sq;        %Supply-price for difference
    d_Sp_hist(i,1)=d_Sp;

    if i>1
    d_difference=abs(d_Sp_hist(i,1)-d_Dp_hist(i-1,1));
    d_difference_hist(i,1)=d_difference;
    q=lambda(1,j)*d_Sq_hist(i,1)+(1-lambda(1,j))*d_Sq_hist(i-1,1);
    else
    q=d_Sq;
    end
    %Quantity for next guess set
    d_Time(i,1)=i;
    i=i+1;

end
iteration_count(j,1)=i;

subplot(4,3,j)
scatter(d_Dq_hist,d_Dp_hist)
```

```
hold on
scatter(d_Sq_hist,d_Sp_hist)
plot(d_Dq_hist,d_Dp_hist,d_Sq_hist,d_Sp_hist)
hold off
end
[M,I] = min(iteration_count);
Size_I=size(I);
c = categorical({'0.1','0.2','0.3','0.4','0.5','0.6','0.7','0.8','0.9'});

%bar(c,iteration_count)
subplot(4,3,[10 11 12]);
for i=1:Size_I(1,2) %allows for multiple minima
b = bar(c,iteration_count);
title({'Number of iterations needed to find the solution';'The lowest value is c
set(get(gca,'title'),'Position',[5.5 60 1])
b.FaceColor = 'flat';
b.CData(I(1,i),:) = [.5 0 .5];
end
```



2 Question 2

2.1

```
%% Part 1 - loading and computing the logarithms
clear all;
clc;
```

```
A=xlsread('/Users/sebastiankuhnl/Desktop/GSEFM/Year 1/Semester 1.2/Mathematical
Germany=A(1,:);
Greece=A(2,:);

LogGermany=log(Germany);
LogGreece=log(Greece);
```

2.2

```
%% Part 2 - Applying HP filter

%This version requires machine learning and statistics toolbox
smoothing = 1600; %unecessary since it is the default value of the function but
TrendGermany = hpfilter(LogGermany,smoothing);
TrendGreece = hpfilter(LogGreece,smoothing);
```

2.3

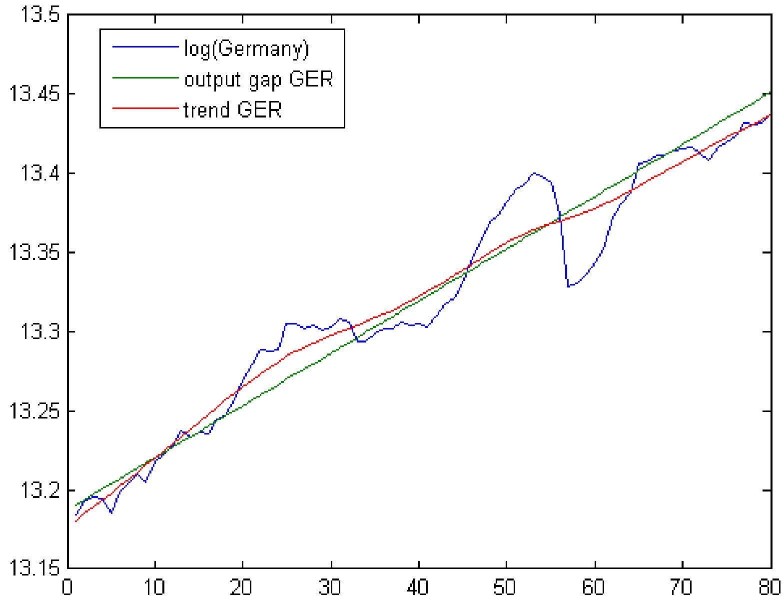
```
%% Part 3 - Applying OLS

%Creating time variable
Time = zeros(size(Greece));
Size = size(Greece);
for i=1:Size(1,2)
    Time(1,i)=i;
end

%OLS

Var_GRE=var(LogGreece);
Var_GER=var(LogGermany);
Cov_GRE=cov(Time,LogGreece);
Cov_GER=cov(Time,LogGermany);

b_1_GER=Var_GER/Cov_GER(1,2);
b_1_GRE=Var_GRE/Cov_GRE(1,2);

Mean_Time=mean(Time);
Mean_GER=mean(LogGermany);
Mean_GRE=mean(LogGreece);

b_0_GER=Mean_GER-Mean_Time*b_1_GER;
b_0_GRE=Mean_GRE-Mean_Time*b_1_GRE;

Yhat_GER=b_0_GER+Time*b_1_GER;
Yhat_GRE=b_0_GRE+Time*b_1_GRE;
```

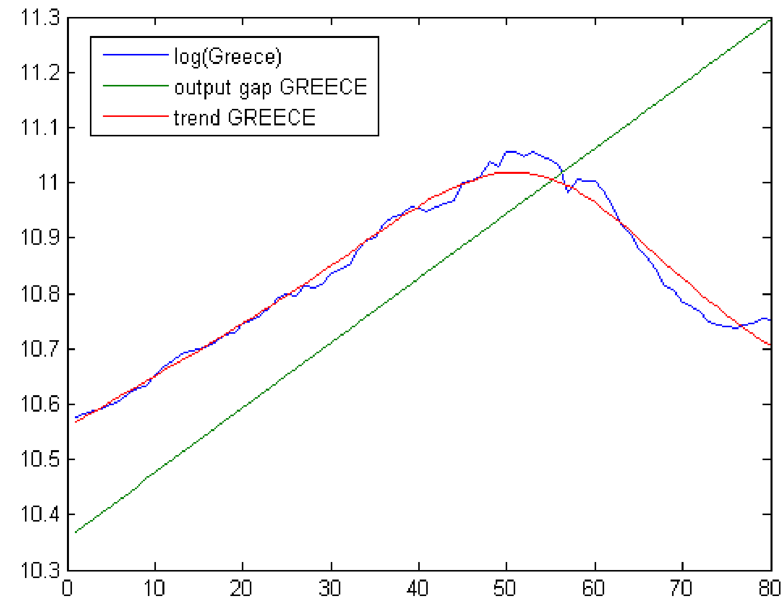
2.4

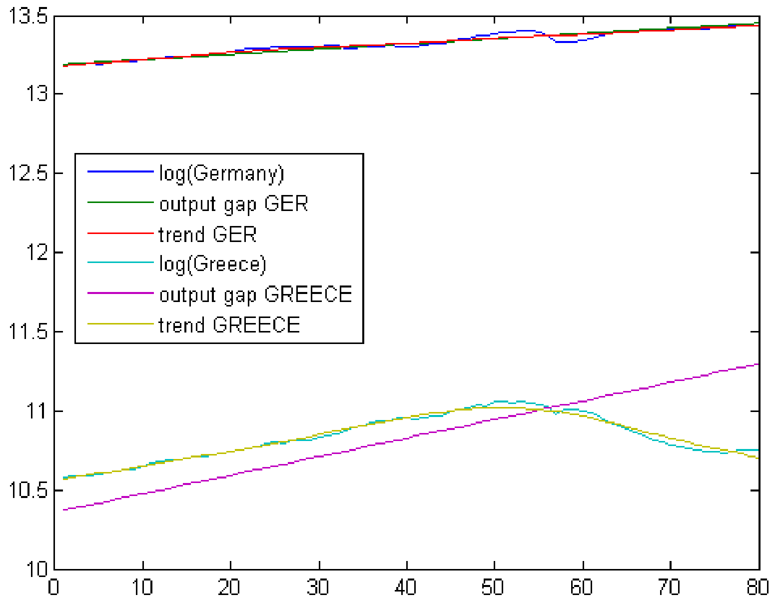
```
%% Part 4 - Output Gap

Y_GER_trend_HP=exp(TrendGermany);
Y_GER_trend_OLS=exp(Yhat_GER);

Y_GRE_trend_HP=exp(TrendGreece);
Y_GRE_trend_OLS=exp(Yhat_GRE);

Y_Gap_GER_HP=Germany-transpose(Y_GER_trend_HP);
Y_Gap_GER_OLS=Germany-Y_GER_trend_OLS;

Y_Gap_GRE_HP=Greece-transpose(Y_GRE_trend_HP);
Y_Gap_GRE_OLS=Greece-Y_GRE_trend_OLS;
```

2.5

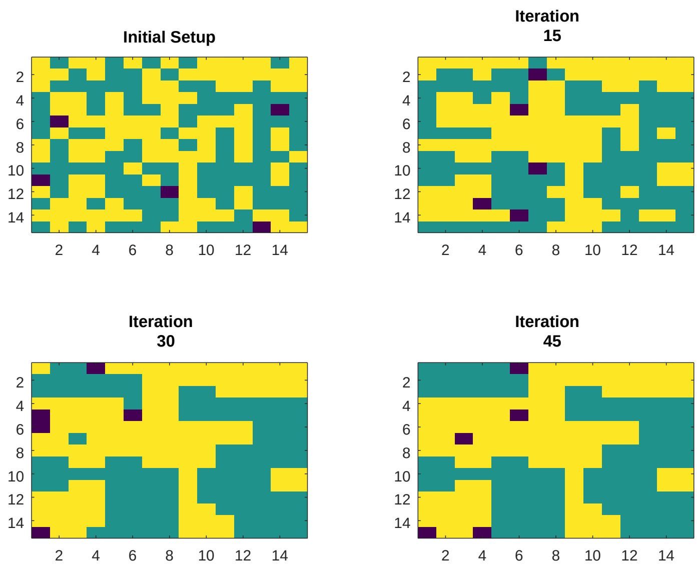
```
%% Part 5 - Plot

figure
plot(Time,LogGermany,Time,Yhat_GER,Time,TrendGermany);
legend('log(Germany)','output gap GER','trend GER');

figure
plot(Time,LogGreece,Time,Yhat_GRE,Time,TrendGreece);
legend('log(Greece)','output gap GREECE','trend GREECE');
```


13.5
13.45
13.4
13.35
13.3
13.25
13.2
13.15
0
10
20
30
40
50
60
70
80
log(Germany)
output gap GER
trend GER


11.3
11.2
11.1
11
10.9
10.8
10.7
10.6
10.5
10.4
10.3
0
10
20
30
40
50
60
70
80
log(Greece)
output gap GREECE
trend GREECE


13.5
13
12.5
12
11.5
11
10.5
10
0
10
20
30
40
50
60
70
80
log(Germany)
output gap GER
trend GER
log(Greece)
output gap GREECE
trend GREECE

3 Question 3




```
%% Set up the grid

 clear;
 clc;
```

```
Board=zeros(15,15);
Iteration_Count=0;
%110 White (1 indicates white)
i=1;
while i<111

Col = randi(size(Board, 2));
Row = randi(size(Board, 1));
if Board(Col,Row)==0
    Board(Col,Row)=1;
    i=i+1;
else
end
end

%110 Black (2 indicates black)

j=1;
while j<111

Col = randi(size(Board, 2));
Row = randi(size(Board, 1));
if Board(Col,Row)==0
    Board(Col,Row)=2;
    j=j+1;
else
end
end

%Plot situation after zero iterations
figure
subplot(2,2,1)
Plot_0=imagesc(Board);
title('Initial Setup')

for ij=1:45
%% Spot unoccupied houses

Non_Occupied=zeros(5,2);
Non_Index=1;

for i=1:15
    for j=1:15
        if Board(i,j)==0

            Non_Occupied(Non_Index,1)=i;
```

```
                Non_Occupied(Non_Index,2)=j;
                if Non_Index<5
                Non_Index=Non_Index+1;

                end
            else
            end
        end
    end

%% Evaluate position

Movers_Location=zeros(15,15);
Movers_Count=0;

for i=1:15
    for j=1:15
        Self=Board(i,j);
        if Self ~= 0 %Check if the considered house is actually occupied

        %If possible, get neighbor values, alternatively, set equal to zero

            if i==1
            Upper_N=0;
            else
            Upper_N=Board(i-1,j);
            end

            if i==15
            Lower_N=0;
            else
            Lower_N=Board(i+1,j);
            end

            if j==1
            Left_N=0;
            else
            Left_N=Board(i,j-1);
            end

            if j==15
            Right_N=0;
            else
            Right_N=Board(i,j+1);
            end
```

```
            %Compare to neighbors

             Neigh=zeros(4,1);
             Neigh(1,1)=double((Self==Upper_N));
             Neigh(2,1)=double((Self==Lower_N));
             Neigh(3,1)=double((Self==Left_N));
             Neigh(4,1)=double((Self==Right_N));

            %at least 35% must be equal to not move
             Neigh_Eval=sum(Neigh)/4;

            if Neigh_Eval<0.35
                Movers_Location(i,j)=1; %everyone with a one wants to move
                Movers_Count=Movers_Count+1;
            end

        end

    end
end

%% Only five households are allowed to move each period (since only five houses

Allowed_Movers=zeros(5,1);
Allowed_Movers_Pos=zeros(5,2);
Want_to_Move=zeros(Movers_Count,2);
k=1;

for i=1:15
    for j=1:15
        if Movers_Location(i,j)==1
            %Store to movers

             Want_to_Move(k,1)=i;
             Want_to_Move(k,2)=j;

             k=k+1;
        end
    end
end

%Pick five movers

m=1;

if Movers_Count>=5
```

```
    Movers_Counter=6;
else
    Movers_Counter=Movers_Count+1;
end
while m<Movers_Counter
    New_Mover=randi(Movers_Count);
    if ismember(New_Mover, Allowed_Movers)==0
        Allowed_Movers(m,1)=New_Mover;
        m=m+1;
    end
end

for i=1:Movers_Counter-1
    Allowed_Movers_Pos(i,1)=Want_to_Move(Allowed_Movers(i,1),1);
    Allowed_Movers_Pos(i,2)=Want_to_Move(Allowed_Movers(i,1),2);
end

%% Let the five households move to a random unoccupied house

Moves_To=zeros(5,2);
Unoccupied_NowOccupied=zeros(5,1);
i=1;

while i<Movers_Counter

    Row=randi(Movers_Counter-1);
    if ismember(Row, Unoccupied_NowOccupied)==0
    Unoccupied_NowOccupied(i,1)=Row;

    Moves_To(i,1)=Non_Occupied(Row,1);
    Moves_To(i,2)=Non_Occupied(Row,2);
    i=i+1;
    end

end

%Set new location of selected movers to zero, set unoccupied houses to new
%values

for i=1:Movers_Counter-1
    Board(Non_Occupied(i,1),Non_Occupied(i,2))=Board(Allowed_Movers_Pos(i,1),All
    Board(Allowed_Movers_Pos(i,1),Allowed_Movers_Pos(i,2))=0;
end

Iteration_Count=Iteration_Count+1;
Title= ['Iteration ' num2cell(Iteration_Count)];
```

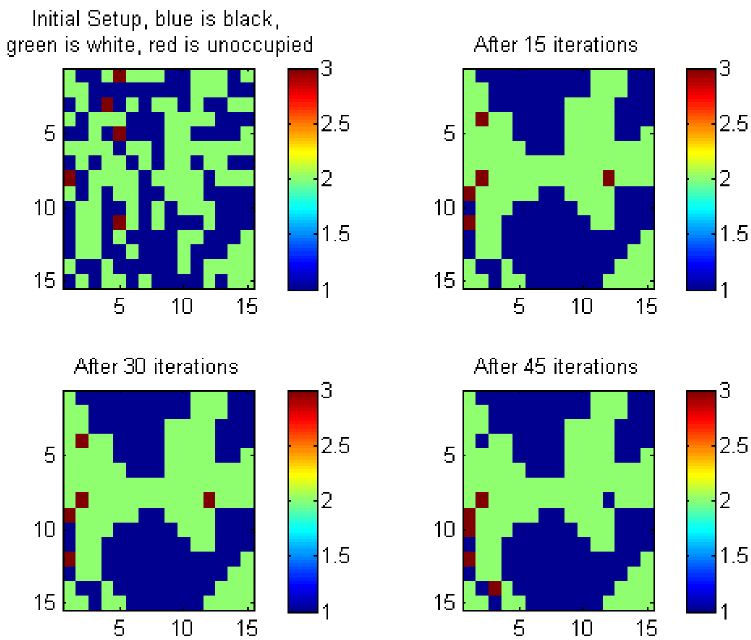
```
if ij==15
    subplot(2,2,2)
    Plot_1=imagesc(Board);
    title(Title)
elseif ij== 30
    subplot(2,2,3)
    Plot_2=imagesc(Board);
    title(Title)
elseif ij== 45
    subplot(2,2,4)
    Plot_3=imagesc(Board);
    title(Title)
end

end
```

A Alternative Moving Understanding P3

```
clear;
%create the hood
Hood=zeros(15,15);
%now generate your people
n=110; %Blacks
m=110; %whites
p=5;   %unoccupied
%move your people into the hood
%1: Black
%2: White
%3: unoccupied
plots=0;
for i=1:15
  for j=1:15
     sum=n+m+p;
      x=rand;
     if x<n/sum
         Hood(i,j)=1;
         n=n-1;
     else
       if x<(n+m)/sum
           Hood(i,j)=2;
           m=m-1;
       else
           Hood(i,j)=3;
```

```
            p=p-1;
          end
        end
    end
  end
%now this is the hood (black is blue, white is green, unoccupied is red
plots=plots+1;
figure
subplot(2,2,plots)
imagesc(Hood)
title({'Initial Setup, blue is black,';' green is white, red is unoccupied'})
colorbar
iterations=0;
%start iteration
for iterations=1:45
%detect mover
Neighbors=zeros(15,15);
Same=zeros(15,15);
movers=0;
Mover=[0 0 0];
Free=[0 0];
%check same skin middle
for i=1:15
    for j=1:15
        Skin=Hood(i,j);
        if Skin==3
            Free=[Free;i j];
        else
        for z=1:3
        for w=1:3
            if (i+z-2>=1) && (i+z-2<=15)
              if (j+w-2>=1) && (j+w-2<=15)
                if Hood(i+z-2,j+w-2)~=3
                    Neighbors(i,j)=Neighbors(i,j)+1; %number neighbors (including o
                end
              if isequal(Hood(i+z-2,j+w-2),Skin)
                if (z~=2) || (w~=2) %dont count yourself
                  Same(i,j)=Same(i,j)+1; %number same skin
                end
              end
              end
            end
        end
        end
        Neighbors(i,j)=Neighbors(i,j)-1; %deduct oneself
        if Same(i,j)/Neighbors(i,j)<=0.35
```

```
                movers=movers+1;
                Mover=[Mover;i j Skin];
                Free=[Free;i j];
            end
            end
        end
    end
    Mover=Mover(2:end,:);
    Free=Free(2:end,:);
    %move into new house
    sz = size(Mover);
    for k=1:sz(1,1) %in this loop there is an error if movers<3
        newpos=unidrnd(length(Free),1,1);
        Hood(Free(newpos,1),Free(newpos,2))=Mover(k,3);
        Free(newpos,:)=[];
    end
    Mover=[0 0 0];
    %make free houses unoccupied
    for l=1:length(Free)
        Hood(Free(l,1),Free(l,2))=3;
    end
    if (iterations==30) || (iterations==15)
    plots=plots+1;
    subplot(2,2,plots)
    imagesc(Hood)
    title(['After ' num2str(iterations) ' iterations'])
    colorbar
    end
    end
    %end of iteration
    plots=plots+1;
    subplot(2,2,plots)
    imagesc(Hood)
    title('After 45 iterations')
    colorbar
```



Initial Setup, blue is black,
green is white, red is unoccupied
After 15 iterations
After 30 iterations
After 45 iterations
5
10
15
1
1.5
2
2.5
3