

Problem 1:

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insert(string):void
    temp <-- head
    node <-- string
    success <-- false
    if (head is null)
        head <-- node
    else
        while (head.next is not null AND success is false)
            count <-- 0
            while (string.charAt(count) = head.charAt(count))
                increment count
            if (string.charAt(count) < head.charAt(count))
                node.next <-- head
                head <-- node
                success <-- true
            else
                if (string.charAt(count) > head.charAt(count))
                    if (head.next = null)
                        head.next <-- node
                        success <-- true
                    head <-- head.next
        if (head != node)
            head <-- temp

```

Problem 2:

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isPalindrome(string):boolean
    counter <-- 0
    original <-- ""
    while counter is less than length of string
        character <-- string.charAt(counter)
        if ((character is greater than 64 AND character is less than 91) OR
            (character is greater than 96 AND character is less than 123))
            stack.push(character)
            original <-- original + character
        increment counter
    reverse <-- ""
    counter <-- 0

```

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while counter is less than the length of the original
    reverse <-- reverse + stack.pop()
return reverse.equals(original)

```

Problem 3:

Assuming that the gender of b is unknown.

Reflexive : No, a can't be his own brother

Symmetric : Maybe, if b is male then b is also a's brother

Antisymmetric : No, not reflexive

Transitive : Maybe, if there is a third sibling c and b is a male. a is b's brother, b is c's brother and a is c's brother.

Therefore, R is not an equivalence relation.

Problem 4:

Reflexive : yes, all numbers are divisible by themselves

Symmetric: no, for example 3 divides 9 but 9 does not divide 3

Antisymmetric : yes

Transitive : yes, for example 3 divides 9, 9 divides 18, 3 divides 18

Therefore, R is not an equivalence relation

Problem 5:

Solving the Tower of Hanoi works recursively by acknowledging the number of disks and pegs so that the solution is found by assigning the correct moves based on number of disks and the moves before.

Problem 6:

Contrapositive: x^2 is odd, then x is odd

$$x = 2k + 1$$

$$x^2 = 4x^2 + 4x + 1$$

$$= 2(2x^2 + 2x) + 1 \text{ is odd}$$

Therefore the conjecture is true

Problem 7:

Conjecture: $\sum_{i=1}^n i^2 = \frac{2n^3 + 3n^2 + n}{6}$

Proof (induction):

Basis step: $\sum_{i=1}^1 i^2 = 1 = \frac{2(1)^3 + 3(1)^2 + 1}{6} = 1 \quad 1=1$

Inductive step: if $\sum_{i=1}^k i^2 = \frac{2k^3 + 3k^2 + k}{6}$, then $\sum_{i=1}^{k+1} i^2 = \frac{2(k+1)^3 + 3(k+1)^2 + k+1}{6} = \frac{2n^3 + 9n^2 + 13n + 6}{6}$

Assume $\sum_{i=1}^k i^2 = \frac{2k^3 + 3k^2 + k}{6}$, add $k+1$ to both sides

$$\sum_{i=1}^k i^2 + (k+1) = \frac{2k^3 + 3k^2 + k}{6} + \frac{6(k+1)^2}{6} = \frac{2n^3 + 9n^2 + 13n + 6}{6}$$

Since $\sum_{i=1}^{k+1} i^2 = \frac{2n^3 + 9n^2 + 13n + 6}{6}$, the inductive step holds

Since the Basis and the inductive steps both hold, then the conjecture is true.