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CS313
Assignment 4
1 March, 2019

1. Conjecture: A full binary tree has the highest number of leaf nodes among all trees with n internal nodes.

   Proof (Induction):

   Basis Step:

   All nodes have 0 or 2 children in a full binary tree

   FBT has n + 1 leaf nodes where n = # of internal nodes

   So if n = 0 then the FBT must have 1 leaf node

   (0) + 1 = 1 leaf node(s)

   A non-empty Binary Tree with 0 internal nodes must have 1 leaf node by definition

   1 = 1 leaf node(s)

   Basis Step holds

   Inductive Step:

   Inductive Hypothesis: A FBT has k + 1 internal nodes with have > leaf nodes to any other binary tree with k + l internal nodes

   A FBT F with 3 internal nodes must have 4 leaf nodes

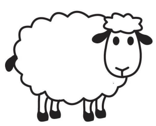
   A binary tree T with 3 internal nodes and 2 leaf nodes is still a binary tree but not a FBT.

   If we insert a node onto the binary tree T, without increasing or decreasing the number of internal nodes, we must add a leaf node giving us 3 internal nodes and 3 leaf nodes. If we add another node onto T without increasing the number of internal nodes, there is only one option which is another leaf node. This gives us 3 internal nodes and 4 leaf nodes thus changing the tree to a FBT. Therefore, the maximum amount of leaf nodes a binary tree T can have with 3 internal nodes is 3 leaf nodes while the maximum amount of leaf nodes a FBT with 3 internal nodes can have is 4 leaf nodes.

Without changing the number of internal nodes, the maximum amount of leaf nodes FBT F can have is 4 and the maximum amount binary tree T can have is 3.

Inductive step holds

Because both the basis and inductive steps hold, the conjecture holds true.



2.

Conjecture - If the full binary tree, T, has n internal nodes, I is T's internal path length and E is T's external path length, then E = I +2n for n>=0.

Basis step - I = 0, E = 0 T has 1 leaf node and 0 internal nodes

0 + 2 = 2, 0 = 0

Basis step holds

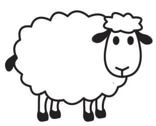
Inductive step:

IH: if E = I + 2n for n >=0 in T, then E' = I + 2(n + 1) for T with n + 1 internal nodes. d(x) = depth of new node.

E' = E - d(x) + 2(n+1), E' = E - d(x) +2(d(x) +1), E' = I +2n - d(x) + 2d(x) + 2, E'= I + d(x) + (2(n +1)

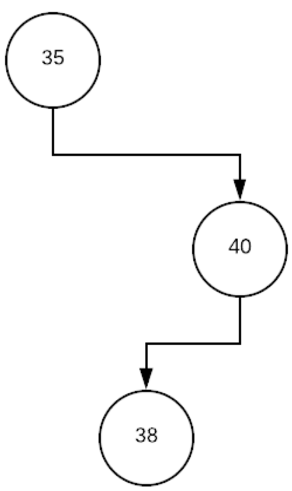
Thus, inductive step hold

Since both the basis step and inductive step holds and the conjecture is true

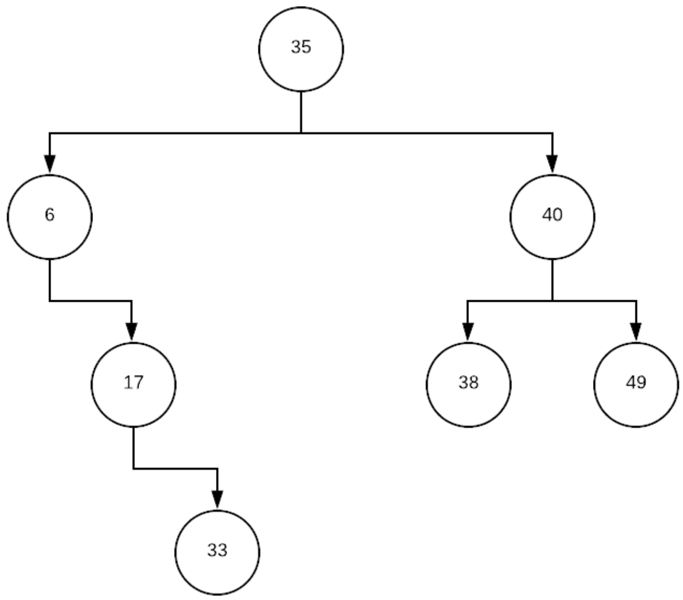


3.

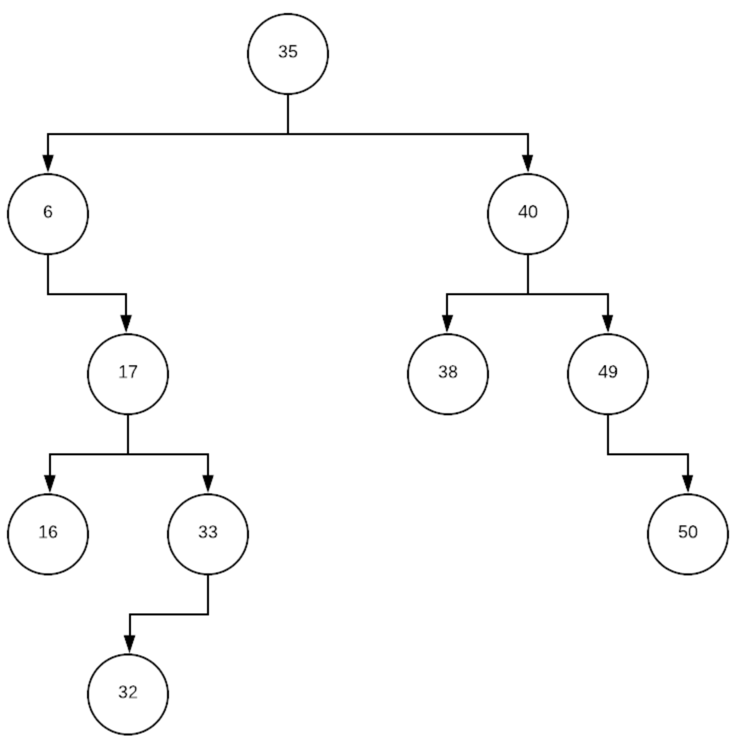
a. After insertion of 38



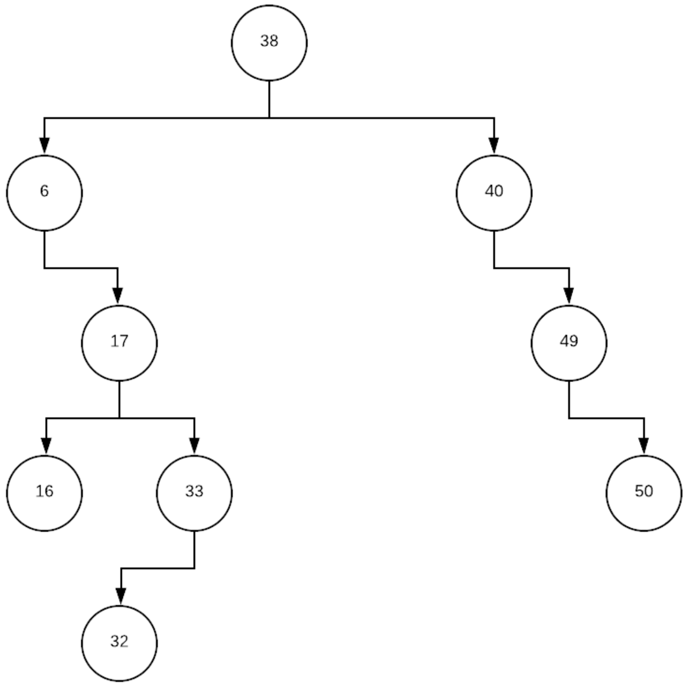
After insertion of 33



After insertion of 50



b. After deletion of 35

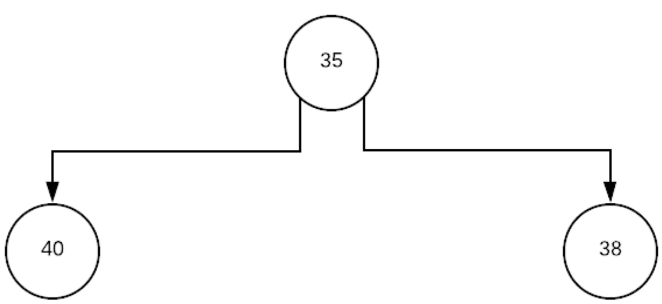


c. Preorder: 35, 6, 17, 16, 33, 32, 40, 38, 49, 50 Inorder: 6, 16, 17, 32, 33, 35, 38, 40, 49, 50

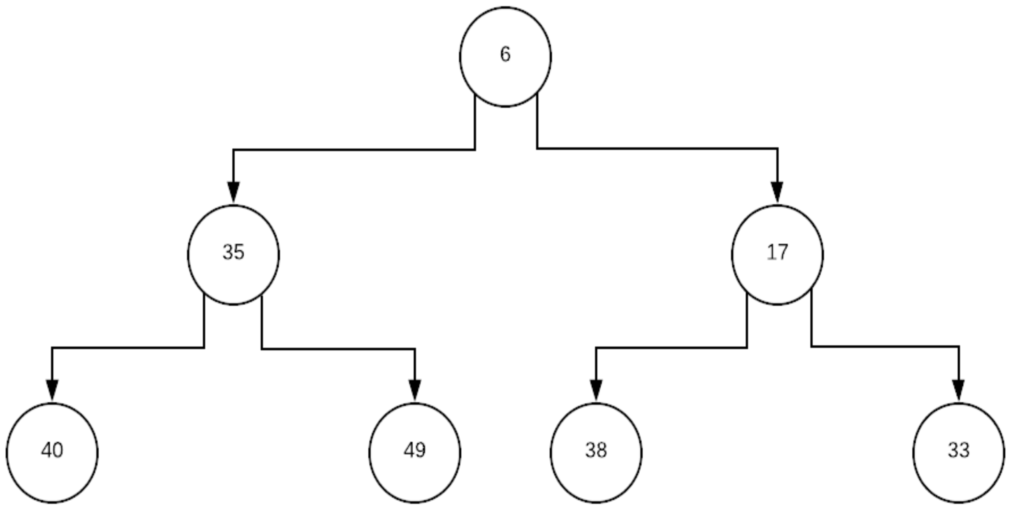
Postorder: 16, 32, 33, 17, 6, 38, 50, 49, 40, 35

4.

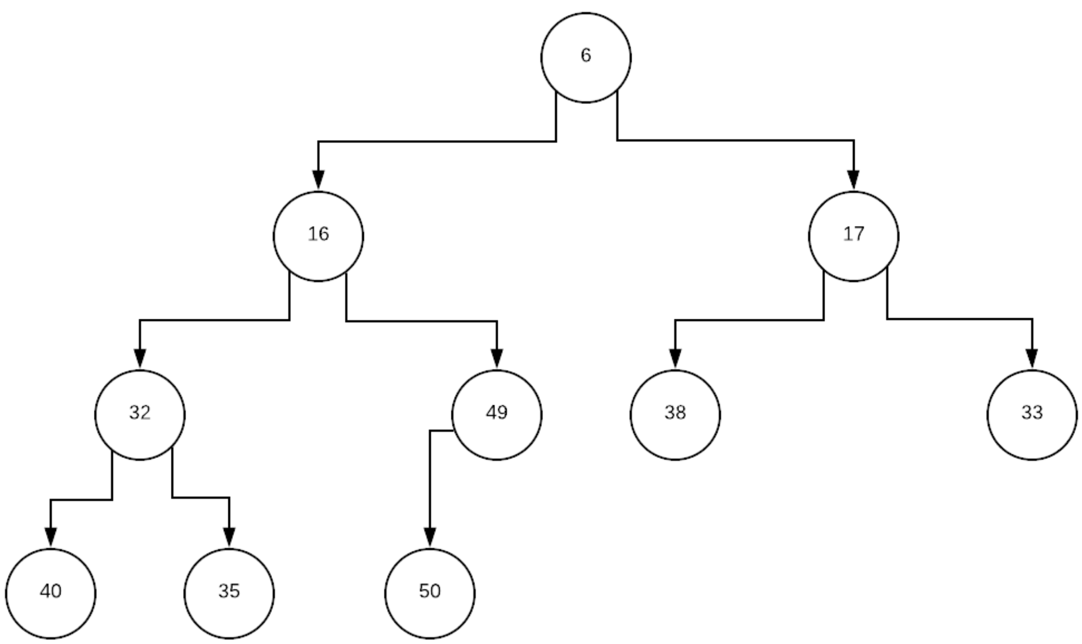
a. After insertion of 38



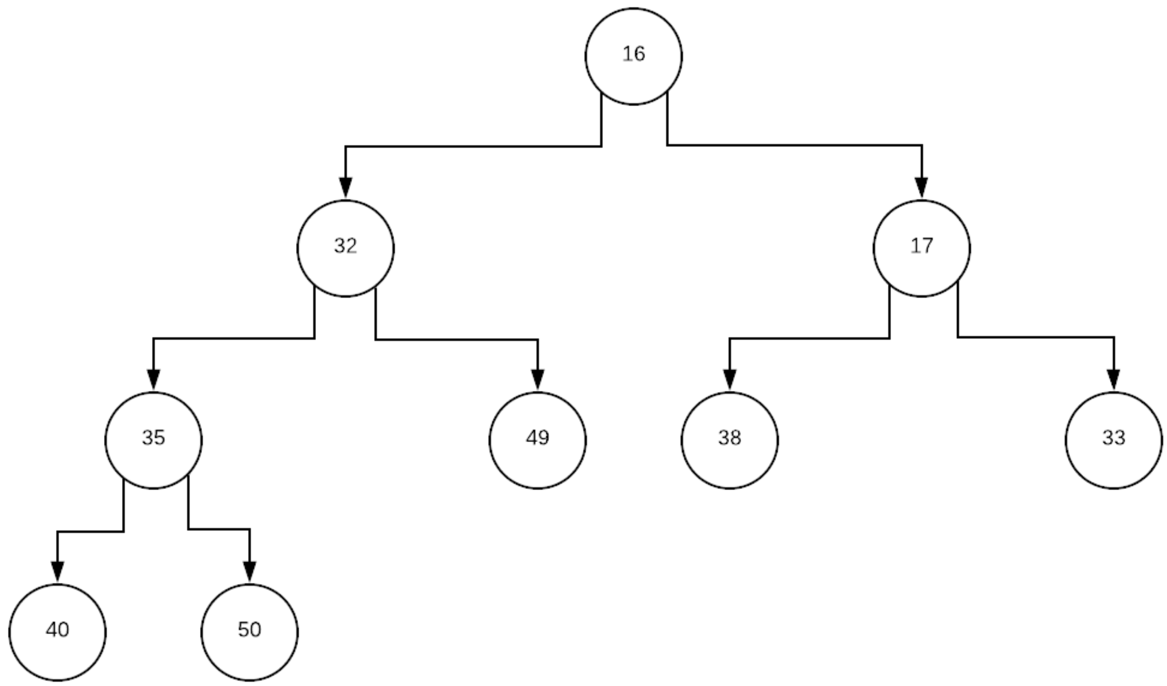
After insertion of 33



After insertion of 50



b. getMin operation



5.printRange(node, low, high):void

if node is null

return

if low is less than node

printRange(node.left, low, high)

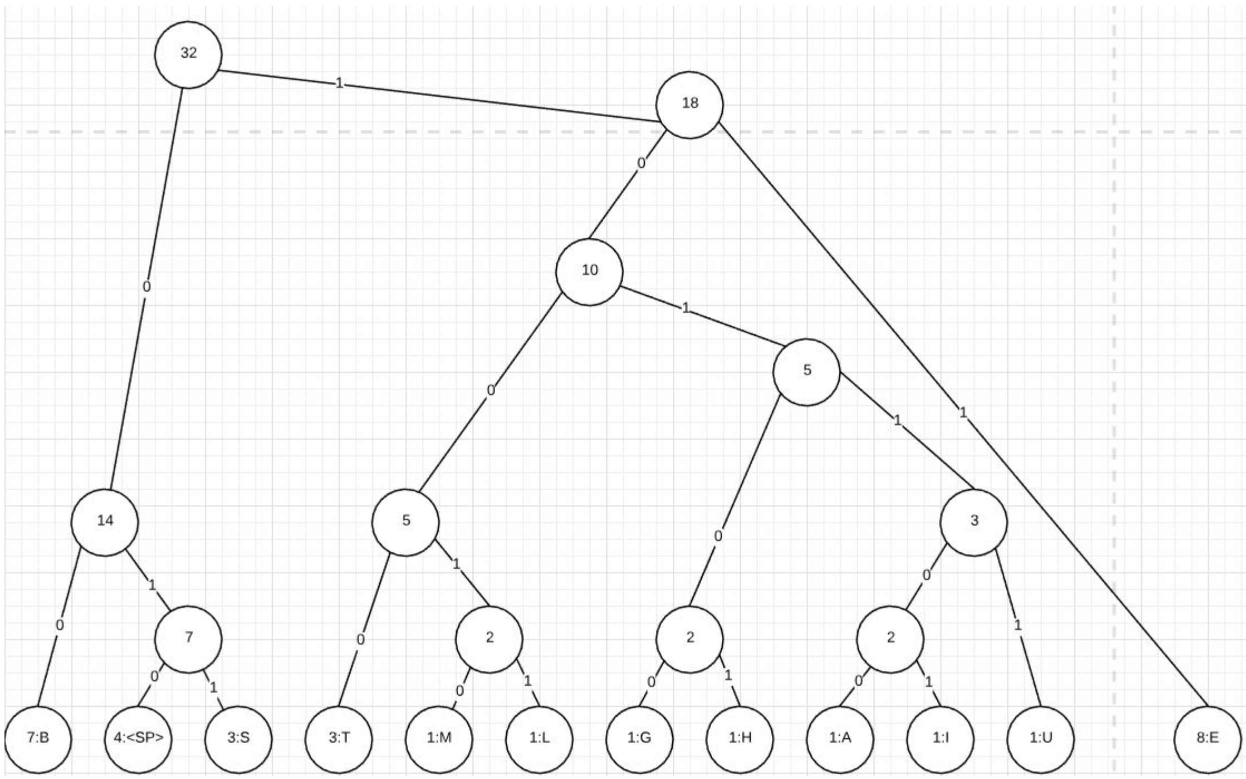
if low is less than or equal to node AND high is greater than or equal to node

print node

if high is greater than node

printRange(node.right, low, high)

6.



a. BUMBLEBEES BEGET THE BEST BABIES

B U M L E S G T H A I <sp>

7 1 1 1 8 3 1 3 1 1 1 4

E <sp> S T U M L G H A I B

Letter	Frequency	Encoding
E	8	00
B	7	11
<sp>	4	010
S	3	101
T	3	0110
U	1	01110
M	1	011110
L	1	011111
G	1	10000
H	1	10001
A	1	10010
I	1	10011

b. $8 * 2^5 = 8 * 32 = 256$ bits

c. $8 * 2 = 16$

$7 * 2 = 14$

$4 * 3 = 12$

$3 * 3 = 9$

$3 * 4 = 12$

$1 * 5 = 5$

$1 * 6 = 6$

$1 * 6 = 6$

1 * 5 = 5

1 * 5 = 5

1 * 5 = 5

1 * 5 = 5

= 100 bits