

4.

a. countEven(int[] list, int index)

```
    if(list.length == index) { (3 steps)
```

```
        return 0 (1 step)
```

```
    } else if(list[index] mod 2 == 0) { (3 steps)
```

```
        return 1 + countEven(list, index + 1) (
```

```
    } else {
```

```
        return countEven(list, index + 1)
```

```
    }
```

b. $T(n) = T(n - 1) + c_2$

c. $T(1) = c_1$

$$n - x = 1$$

$$x = n - 1$$

$$T(n - 1) = T(n - 1) + c_2$$

$$T(n) = T(n - 2) + c_2 + c_2$$

$$T(n) = T(n - x) + c_2 * x$$

$$= T(n - (n - 1)) + c_2(n - 1)$$

$$= T(1) + c_2 n - c_2$$

Closed Form

$$T(n) = c_1 + c_2 n - c_2$$

d. Conjecture: $T(n) = c_1 + c_2n - c_2$ is equivalent to the Recurrence Relation $T(n) \{$

$$T(n - 1) + c_2 \text{ if } n > 1, c_1 \text{ if } n = 1 \}$$

$$\text{Basis Step: } T(1) = c_1 + c_2n - c_2$$

$$n = 1$$

$$T(1) = c_1 + c_2(1) - c_2$$

$$T(1) = c_1 + c_2 - c_2$$

$$T(1) = c_1$$

The basis step holds

Inductive Step:

$$\text{IH: If } T(k) = c_1 + c_2(k) - c_2, \text{ then } T(k + 1) = c_1 + c_2k$$

$$\text{Assume by IH that } T(k) = c_1 + c_2k - c_2$$

then we know that

$$T(k + 1) = T((k + 1) - 1) + c_2$$

$$T(k + 1) = T(k) + c_2$$

$$= c_1 + c_2k - c_2 + c_2$$

$$T(k + 1) = c_1 + c_2k$$

Therefore inductive step holds.

Because both the inductive and basis steps hold true, the conjecture is true.

Q^{ED}

