1.

Recurrence relation:

T(1) = C

T(n) = T(n-1) + n

Find the pattern (Closed Form):

T(n) = $T(n-1) + n$

T(n-1) = $T(n-2) + n - 1$

T(n) = $T(n-2) + n + (n - 1)$

T(n-2) = $T(n-3) + n - 2$

T(n) = $T(n-3) + n + (n - 1) + (n - 2)$

T(n) = $T(n-x) + n + (n - 1) + (n - 2) \ldots + (n - (x-1))$

T(n) = $\sum_{c=1}^{n} i = \frac{n(n+1)-2}{2}$

Conjecture: Recurrence relation is equivalent to Closed form

Base Case; n = 1;

T(1) = $\frac{1^2+1-2}{2} + C = C$

Base Case holds.

IH: If T(m) = $(\frac{m^2+m-2}{2}) + C$ then T(m+1) = $(\frac{(m+1)^2+(m+1)-2}{2}) + C$

T(m+1) = $(\frac{(m+1)^2+(m+1)-2}{2}) + C$

T(m+1) = $(\frac{m^2+2m+1+m+1-2}{2}) + C$

T(m+1) = $(\frac{m^2+3m}{2}) + C$

Inductive Step: Proving the Closed Form

T(m+1) = $(m+1) \ + T(m)$

T(m+1) = $m + 1 + \frac{m^2+m-2}{2} \ + C$

T(m+1) = $\frac{2m+2m^2+m-2}{2} \ + C$

T(m+1) = $(\frac{m^2+3m}{2}) + C$

Since Base Case and Inductive Step holds, the conjecture proves true.

2. <78, 61, 13, 45, 37, 55, 7, 89, 32, 85>

a. Selection

<78, 61, 13, 45, 37, 55, 7*, 89, 32, 85>

<7, 61, 13*, 45, 37, 55, 78, 89, 32, 85>

<7, 13, 61, 45, 37, 55, 78, 89, 32*, 85>

<7, 13, 32, 45, 37*, 55, 78, 89, 61, 85>

<7, 13, 32, 37, 45*, 55, 78, 89, 61, 85>//45 already in order

<7, 13, 32, 37, 45, 55*, 78, 89, 61, 85>//55 already in order
<7, 13, 32, 37, 45, 55, 78, 89, 61*, 85>
<7, 13, 32, 37, 45, 55, 61, 89, 78*, 85>
<7, 13, 32, 37, 45, 55, 61, 78, 89, 85*>
<7, 13, 32, 37, 45, 55, 61, 78, 85, 89>

note: This became complicated and too much work.

b. Insertion
<78, 61, 13, 45, 37, 55, 7, 89, 32, 85>
<61, 78, 13, 45, 37, 55, 7, 89, 32, 85>
<13, 61, 78, 45, 37, 55, 7, 89, 32, 85>
<13, 45, 61, 78, 37, 55, 7, 89, 32, 85>
<13, 37, 45, 61, 78, 55, 7, 89, 32, 85>
<13, 37, 45, 55, 61, 78, 7, 89, 32, 85>
<7, 13, 37, 45, 55, 61, 78, 89, 32, 85>
<7, 13, 37, 45, 55, 61, 78, 89, 32, 85>//89 stays
<7, 13, 32, 37, 45, 55, 61, 78, 85, 89>

c.Shell
<78, 61, 13, 45, 37, 55, 7, 89, 32, 85>
<32, 61, 13, 45, 37, 55, 7, 89, 78, 85>
<32, 55, 13, 45, 37, 61, 7, 89, 78, 85>
<32, 55, 7, 45, 37, 61, 13, 89, 78, 85>
<32, 55, 7, 45, 37, 61, 13, 89, 78, 85>
<13, 55, 7, 32, 37, 61, 45, 89, 78, 85>

<13, 37, 7, 32, 55, 61, 45, 89, 78, 85>

<13, 37, 7, 32, 55, 61, 45, 89, 78, 85>//no change on last gap3

<7, 37, 13, 32, 45, 61, 55, 89, 78, 85>

<7, 32, 13, 37, 45, 61, 55, 85, 78, 89>

<7, 13, 32, 37, 45, 61, 55, 85, 78, 89>

<7, 13, 32, 37, 45, 55, 61, 85, 78, 89>

<7, 13, 32, 37, 45, 55, 61, 78, 85, 89>

d. Merge

<78, 61, 13, 45, 37, 55, 7, 89, 32, 85>

<78, 61, 13, 45, 37><55, 7, 89, 32, 85>

<78, 61, 13><45, 37><55, 7, 89><32, 85>

<78, 61><13><45><37><55, 7><89><32><85>

<78><61><13><37, 45><55><7><89><32, 85>

<61, 78><13><37, 45><7, 55><89><32, 85>

<13, 61, 78><37, 45><7, 55, 89><32, 85>

<13, 37, 45, 61, 78><7, 32, 55, 85, 89>

<7, 13, 32, 37, 45, 55, 61, 78, 85, 89>

e. Quick

<78, 61, 13, 45, 37, 55, 7, 89, 32, 85>

<13, 7, 32><78, 61, 45, 55, 89, 85>//changed pivot scheme

<7><32><61, 45, 55><89, 85>

<7, 13, 32><45, 55>< ><85>

<7, 13, 32>< ><55><85, 89>

<7, 13, 32><45, 55><85, 89>

<7, 13, 32><45, 55, 61><85, 89>
<7, 13, 32><45, 55, 61, 78, 85, 89>
<7, 13, 32, 37, 45, 55, 61, 78, 85, 89>

3.

a. Selection sort is not a stable sort because it checks for the next lowest element and swaps that with the value with the value in the position in question. This can change the order of duplicate values while the array is being sorted.

b.Insertion sort is a stable sort because it only swaps values that are greater in the sorted subarray.

c.Shell is not a stable sort because the gap sequence implementation partially sorts the array and rearranges based on values with the gaps.

d.Merge is a stable sort because when comparing values to sort after the last steps of splitting, values don't swap if they are less than or equal than the compared value.

e.Quick is not a stable sort because it swaps non adjacent elements.