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DOCENTE: Benavidez

Discusión problemas 4.5.2 y 4.5.3

4.5.2: Write down the generic state vector for the system of two particles with spin. Generalize it to a system with n particles (this is important: it will be the physical realization for quantum registers!).

4.5.2

$|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad |\Psi_1\rangle = C_{10}|\uparrow\rangle + C_{11}|\downarrow\rangle = \begin{bmatrix} C_{10} \\ C_{11} \end{bmatrix} \in \mathbb{C}^2$

$|\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad |\Psi_2\rangle = C_{20}|\uparrow\rangle + C_{21}|\downarrow\rangle = \begin{bmatrix} C_{20} \\ C_{21} \end{bmatrix} \in \mathbb{C}$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = \begin{bmatrix} C_{10}C_{20} \\ C_{10}C_{21} \\ C_{11}C_{20} \\ C_{11}C_{21} \end{bmatrix} \in \mathbb{C}^4$$

$$= C_{10}C_{20}|\uparrow\rangle\otimes|\uparrow\rangle + C_{10}C_{21}|\uparrow\rangle\otimes|\downarrow\rangle + C_{11}C_{20}|\downarrow\rangle\otimes|\uparrow\rangle + C_{11}C_{21}|\downarrow\rangle\otimes|\downarrow\rangle$$

En general un vector de estado genérico con n partículas permanecerá a $\mathbb{C}^{2^n}$

4.5.3: Assume the same scenario as in Example 4.5.2 and let |ϕ>=| x0>⨂|y1>+|x1>⨂|y1>. Is this state separable?

4.5.3

$|\emptyset\rangle = |x_0\rangle \otimes |y_1\rangle + |x_1\rangle \otimes |y_1\rangle$ -

$|\psi_1\rangle = C_0|x_0\rangle + C_1|x_1\rangle = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$

$|\psi_2\rangle = C_0'|y_0\rangle + C_1'|y_1\rangle = \begin{bmatrix} C_0' \\ C_1' \end{bmatrix}$

$|\emptyset\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} C_0 C_0' \\ C_0 C_1' \\ C_1 C_0' \\ C_1 C_1' \end{bmatrix}$

$|\phi\rangle = C_0C_0'|x_0\rangle \otimes |y_0\rangle + C_0C_1'|x_0\rangle \otimes |y_1\rangle + C_1C_0'|x_1\rangle \otimes |y_0\rangle + C_1C_1'|x_1\rangle \otimes |y_1\rangle$

- para que sea $= |\emptyset\rangle$;

$C_0C_0' = 0$
$C_0C_1' = 1$
$C_1C_0' = 0$
$C_1C_1' = 1$

$\rightarrow$ luego una solución es que:

$C_0' = 0$
$C_0 = 1$
$C_1 = 1$
$C_1' = 1$

$\rightarrow$ de esta manera si son separables y una posible solución es que

$|\psi_1\rangle = |x_0\rangle + |x_1\rangle = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$|\psi_2\rangle = |y_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$