# Performance measurement - basic tools

Section 1

# Performance measurement

# Profit and Loss

For a single instrument, define the Profit & Loss (P&L):

$$\Pi_T = \Pi(t_0, T) = V_T - V_{t_0} - P_T + S_T + C_T$$

where $V$ represents the value of the investment, $P$ and $S$ are namely purchases and sells between $t_0$ and $T$, $C$ is the value of the cash flows generated by the instrument.
The balance condition leads to the following equation for the P&L of a portfolio:

$$\Pi_T = V_T - V_{t_0} - D_T + W_T$$

where $V$ represents the value of the portfolio, $D$ and $W$ are respectively deposits and withdrawals from $t_0$ to $T$.

# Portfolio return - money weighted

For a portfolio, let $F_T = D_T - W_T$ be the net deposits. Several metrics for returns are available, a few examples follow.

- (Simple) IRR. $r_T = r(t_0, T)$ s.t. the following equation holds:

$$V_T = (1 + r_T)V_{t_0} + \sqrt{(1 + r_T)}F_T$$

- (Simple) Dietz

$$r(t_0, T) = \frac{\Pi_T}{V_{t_0} + 0.5 \cdot F_T}$$

- Modified Dietz

$$r(t_0, T) = \frac{\Pi_T}{V_{t_0} + \sum f_t \cdot w_t}$$

where $f_t$ are the cash flows on day $t$ and $w_t = (T - t)/T$

# Portfolio return - time weighted

Time weighted returns are defined by:

$$r(t_0, T) = \prod_{t=t_0+1}^{T} (1 + \bar{r}_t) - 1$$

where $\bar{r}_t = \frac{V_t - f_t}{V_{t-1}}$ and the time grid includes all the times in which a cash flow takes place.
Every times a cash flow takes place, the portfolio value must be computed. Since this is quite expensive or even impossible a hybrid methodology is employed, replacing $\bar{r}_t$ by a money weighted return, e.g. $\tilde{r}_t = \frac{\Pi}{V_{t-1} + \alpha F_t}$ with $\alpha \in [0, 1]$, which is a slight generalization of the Dietz return.

Section 2

# Performance analysis

# Brinson model

The model decomposes the arithmetic excess return with respect to the benchmark for a single period. Let the returns of the benchmark and the portfolio be:

$$b = \sum W_i \cdot b_i$$
$$r = \sum w_i \cdot r_i$$

where $W_i$ and $w_i$ are the weights of an asset class and $b_i$, $r_i$ the respective returns. Three components are computed.

- Asset allocation: $A_i = (w_i - W_i)b_i$
- Stock picking: $S_i = (r_i - b_i)W_i$
- Interaction effect: $I_i = (w_i - W_i)(r_i - b_i)$

# Brinson and Fachler

The Brinson model has a positive asset allocation term for each class with a positive return. Knowing which class performed better than the benchmark can be a more valuable information. If $\sum w_i = \sum W_i$ this can be achieved substituting the asset allocation formula as follows:

$$A_i = (w_i - W_i)(b_i - b)$$

Models are also available for multi currency attribution, fixed income attribution,...

# Multi period attribution

Since the return composition is non linear, the sum over time of excess returns is not equal to the excess return over the whole period. Several approaches can be followed to project the A, S and I components over several periods, so that they sum to the excess return.
The GRAP proposed a method based only on algebraic manipulation of the returns. Let $a_i = r_i - b_i$, then $1 + r_i = 1 + b_i + a_i$. Plugging in into $1 + r = \prod 1 + r_i$ one can obtain:

$$a = \sum_{k=1}^{T} \left( a_k \cdot \prod_{t=1}^{k-1} (1 + r_t) \cdot \prod_{t=k+1}^{T} (1 + b_t) \right)$$

from which you can obtain the desired result substituting back $a$ with $r - b = A + S + I$.

# Riferimenti bibliografici


- G. Brinson and N. Fachler, Measuring non-US equity portfolio performance, Journal of Portfolio Management (1985)
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- GRAP (Groupe de Recherche en Attribution de Performance) Synthèse des modèles d'attribution de performance (1997)