Effects of the US Stock Market Return and Volatility on the VKOSPI

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Abstract

The main objective of this report is to analyze how the VKOSPI (Stock Market Implied Volatility Index of South Korea) is influenced by macroeconomic and financial variables and to find the best model to predict its future values. In order to do this we compare different types of econometric models and statistical tests such as ordinary linear regression, HAR, ARCH and GARCH models.

1 Introduction

The analysis and prediction of markets volatility are of essential importance in the trading and risk management worlds. All sorts of models and approaches can be implemented in order to perfect this purpose.
A classical model of volatility dynamics is based on current market prices of tradable financial assets since they contain all available information and reflect sentiment and expectations of the market. The volatilities obtained in this way are called "implied" volatilities; they are forward-looking and have clear advantages over historical volatilities since they catch market conditions and predict future changes.
The implied volatilities were typically derived from option prices using option pricing models, such as the Black–Scholes model, but this negatively affects the models' empirical performance in forecasting future volatilities and managing the risk. Nowadays, the implied volatility indices are constructed using model-free methods.
In this report we focus our attention on using econometric models to predict the VKOSPI, which is the volatility index of South Korea's main stock index, taking into account different macroeconomic and financial variables. In particular, we will consider both Korean and American macroeconomic variables.
For example, one of the most important covariate we used is the KOSPI 200 (Korea Composite Stock Price Index) which is an index representing Korea's top 200 companies and is one of the most actively traded derivatives in the world, and whose volatility is represented by the VKOSPI.
In fact, for many years the KOSPI 200 options market has maintained the top tier position among the worldwide derivatives markets based on its trading volume and influence.
Another interesting feature of the KOSPI 200 options market is the active participation of retail investors, which is different with respect to the developed derivatives markets where the dominant market players are institutional investors. The active participation of individual investors implies that the dynamics of option prices and the derived implied volatility are more likely to be affected by market sentiment and behavioral factors, highlighting the importance of the VKOSPI as a fear gauge measure.
On the other hand for an American variable we use, for instance, the VIX, which is the most well-known volatility index of the US market and commonly used as a benchmark for the entire US stock market.
Moreover, we choose to add the inflation ratio between Korean and America inflation, in order to capture the relationship between the two markets. The inflation rate was particularly interesting also due to the period of observation (2004-2013), where it had a significant impact on the market due to the crisis.

The models we chose to use for our analysis are: linear regression, augmented heterogeneous autoregressive (HAR) models with exogenous covariates, GARCH and ARCH in order to understand the possible correlation and autocorrelation inside our timeseries.
The period that we take in consideration is 2004-2013, in which the market faced the global financial crisis and this is reflected in our analysis. In fact we noticed an increase of the volatility.

2 Literature Review

Our study is inspired by the paper "*Effects of the US Stock Market Return and Volatility on the VKOSPI*" written by Heejoon Han, Ali M. Kutan, and Doojin Ryu (2015), in which several models are presented in order to predict the evolution of the VKOSPI.
In particular they analyzed the statistical properties of the VKOSPI and the macroeconomic and financial variables that are relevant to make predictions under the elaborate HAR model framework with exogenous covariates.
This paper refers to two others studies: Corsi (2009) and Fernandes et al. (2014). The first one suggests to analyze volatilities considering their persistence and long memory properties, while Fernandes et al. (2014) examines the time-series properties of the VIX reporting that the pure heterogeneous autoregressive (HAR) model outperforms the extended HAR models, which incorporate exogenous macro-finance variables in forecasting, particularly short- term ahead forecasting.
In the *Heejoon Han, Ali M. Kutan, and Doojin Ryu* paper they modify the HAR model framework in order to mitigate endogeneity problems and measure the forecasting performance of the Corsi and Fernandes studies.
Furthermore, considering that the previous studies only refer to single markets and do not analyze the effects of market linkages and intercountry spillovers, they examine which factors, domestic versus international, might be more important in describing the timeseries properties and dynamics of the VKOSPI. In particular, they examine whether the US stock market return and implied volatility, which can be regarded as significant global market indicators, explain the dynamics of the VKOSPI, and whether they can help predict future VKOSPI levels after controlling for movements in domestic macro-finance variables.
In the paper seven different models are proposed, each one characterized by different variables based on significance of the estimated coefficients; the models are shown in 4.a.
In thepaper the judgement of the models' performance is mainly based on Mean Absolute Error (MAE) and Mean Square Error (MSE), in addition we test the best model with more tests.
Our purpose is to confirm the results obtained in the paper and, in order to extend their work, implement new models with additional macroeconomic variables and analyze more deeply the results using suitable statistical tests.
Moreover, we choose to add the relative inflation rate between South Korea and USA to improve the fitting. We choose this macroeconomic variable because we think that, especially during the crisis, it could be a significant market indicator. We notice in fact that it plays a dominant role in explaining the VKOSPI dynamics and predicting its future volatility.
Moreover, we find that numerous articles examine the fitting and forecasting ability of the US' implied volatility index and suggest its superiority over historical volatilities *(Banerjee et al., 2007; Becker et al., 2007; Carr and Wu, 2006; Corrado and Miller, 2005; Frijns et al., 2010; Jiang and Tian, 2007; Konstantinidi et al., 2008; Simon, 2003)*: in our analysis we confirm the predicting key role of the VIX.
In addition, we confirm that US stock market returns significantly improve predictions about the VKOSPI, even more than Korea's stock market returns.
All of these topics will be further explained in the next sections.

3 Data

3.a Data exploration

In order to explain the behavior of the VKOSPI we considered the daily values of the following timeseries, for the time period starting from March 2002 up to December 2013, which includes 2430 daily observations.
Although the VKOSPI index has been published since April 2009, for the previous years a historical implied volatility index series can be constructed in the same manner as the VKOSPI.
The timeseries in question are:

- VKOSPI: the volatility of the South Korean Kospi200 index and our response variable.
- KOSPI200: the index tracking the top 200 largest publicly-traded common stocks traded in Korea, whose volatilty is the one represented by the VKOSPI.
- S&P500: the index representing top 500 U.S.A. companies.
- VIX: the volatilty index related to the S&P500 index.
- The exchange rate log-returns between U.S Dollar and South Korean Won.
- CD (91 days): the three month certificate of deposit rate, which is a proxy of the risk free rate.
- Term Spread (from 5 years to 91 days): the difference between the yields on the 5-year government bonds and the 3-month CD rates.
- Credit spread (BBB - AA): the yield difference between BBB and AA corporate bonds.
- STL: the Short-Term Liquidity of the KOSPI
- Volume: the total amount of shares of KOSPI..
- The ratio between South Korean and U.S. monthly measures of inflation rate.

Below, figure 1 presents the movements of the S&P 500 spot index and the VIX, while figure 2 presents the movements of the KOSPI 200 spot index and the VKOSPI during the observed time period. We can clearly notice an increase in volatility around the critical period of 2008-2010.

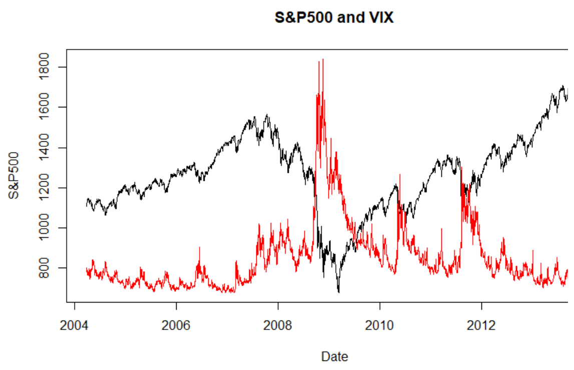


Figure 1: S&P500 and VIX

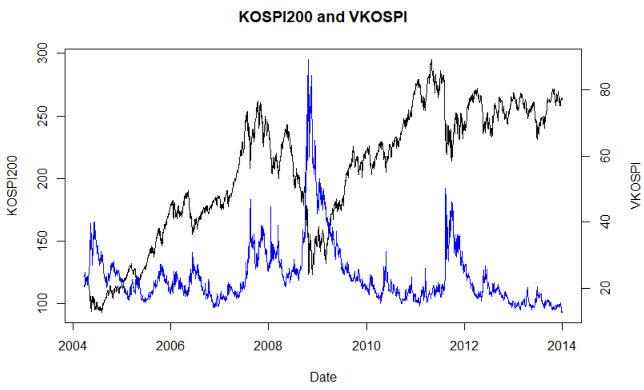


Figure 2: KOSPI200 and VKOSPI

Moreover by plotting the S&P500 together with the KOSPI200, and the VIX together with the VKOSPI we can observe how the timeseries seem highly correlated.

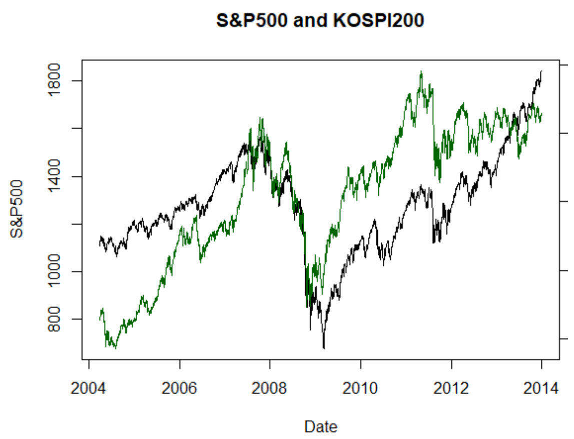


Figure 3: S&P500 and KOSPI200

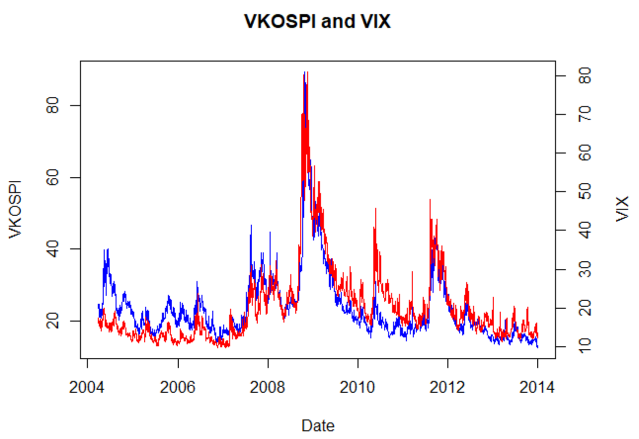


Figure 4: VKOSPI and VIX

By observing the correlation statistics in figure 5 below, we can confirm what we observed by comparing the graphs in the previous figures.
Indeed the KOSPI200 and S&P500 are positively correlated, and also the VIX with the VKOSPI.
Moreover the S&P500 and the VIX are negatively correlated, and the KOSPI200 and the VKOSPI as well. This behavior is aligned with what can be usually observed for a stock and its volatility, as higher values in the stock usually mean a lower volatility, and vice versa lower prices incentivize trading and make the volatility of the stock increase.

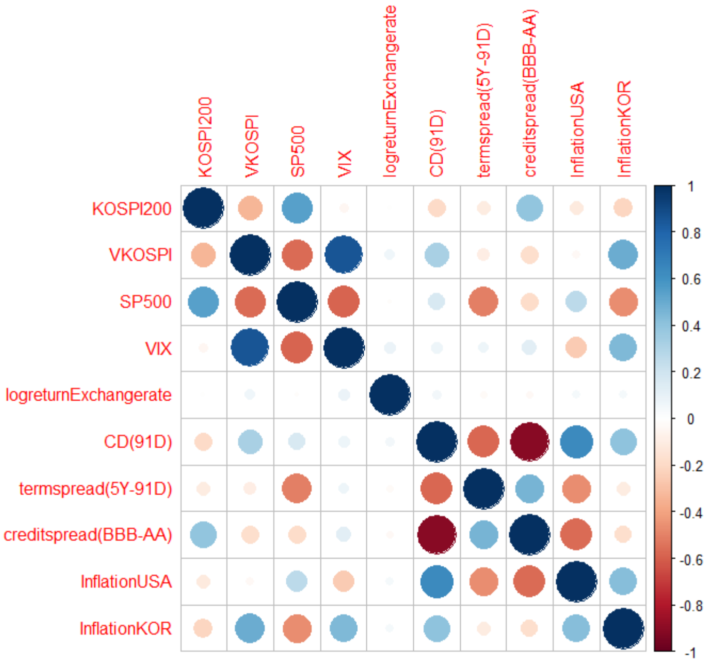


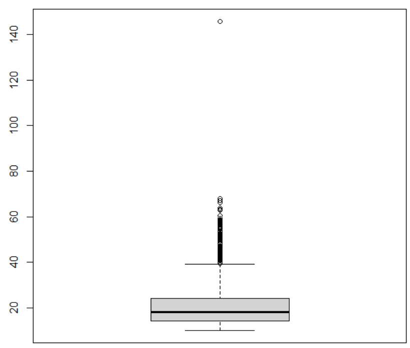
Figure 5: Correlation statistics

We also notice that the two Inflation Ratios show some notable correlation and also the three month CD rate and the credit spread are highly inversely correlated (-90%!). This high coefficient of correlation may mean that using both timeseries in the same regression model will prove to be redundant.
Finally, the only dataseries that does not appear to have relevant correlation with any other is the log-return of the exchange rate between U.S. dollar and South Korean Won.
This may mean that its explanatory value with respect to the VKOSPI might be small, but we will analyze this more in depth through our regression models, once we will have transformed our data.

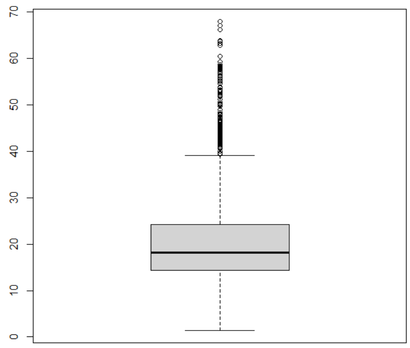
3.b Data cleaning

Given the large number of observations, we decide to analyze the dataset to make sure to remove any anomalous data.

As can be seen in the boxplot in figure 6a and in the graph in figure 7, there is a weird outlier in the STL timeseries, which is probably human error since it is 10 times higher than all others, so we remove this observation.



(a) Before cleaning data



(b) After cleaning data

Figure 6: Before/after boxplot of STL

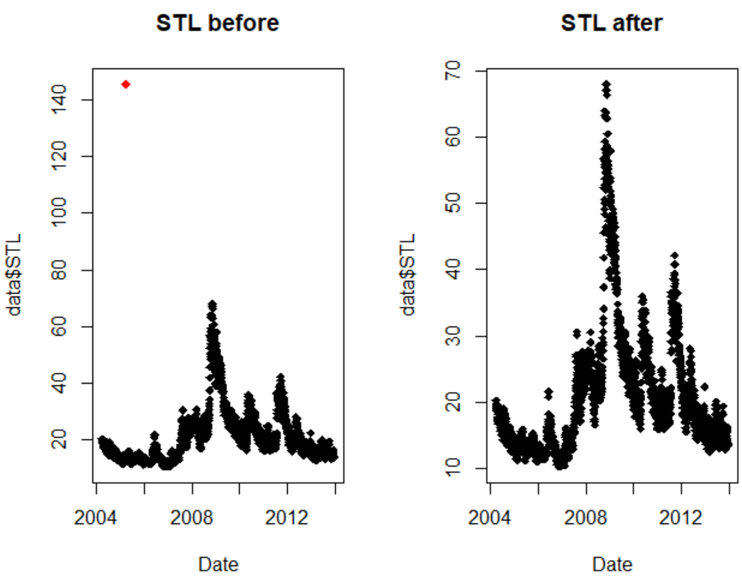


Figure 7: Before/after STL

The logreturns of the exchange rate, in figure 8, present some really high values, but by checking the historical values, we find out they are correct and correspond to the period of the crisis.

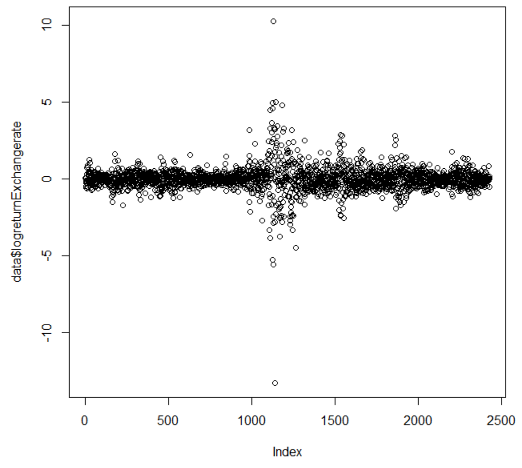


Figure 8: Logreturns of the exchange rate

Eventually, all the others variables do not present any particular outlier in the box plots, so we do not modify them.

3.c Data testing

Before proceeding with our models we must test for some key assumptions, most importantly stationarity, in order to use ARMA models, and homoskedasticity which is an assumption of linear regression models, but also test for cointegration and for the possible presence of structural changes.
By running the Augmented Dickey-Fueller test on all of our timeseries, we find that we cannot reject the presence of an unit root at a level of 5% for any of them, with the exception of the log-returns of the exchange rate.
Furthermore, by running the Chow test on our covariates, it shows the presence of structural changes for our timeseries during the observation period. We show for example in figure 9 the plot of our response variable VKOSPI together with the progressive value of the F-statistic of the Chow test

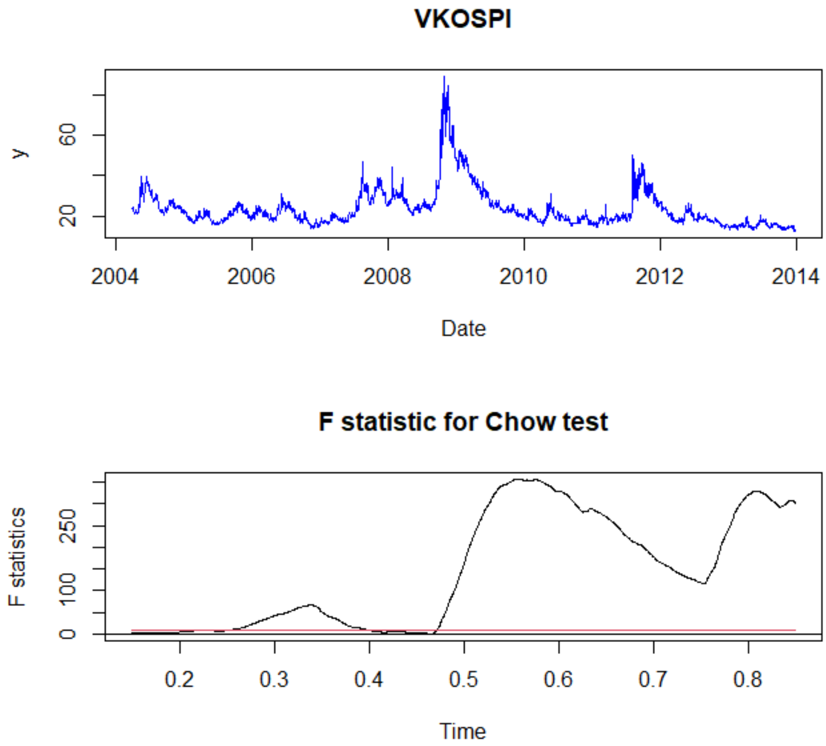


Figure 9: Test for structural breaks on VKOSPI

By looking at the second plot in figure 9, the constant red line indicates the value of the f-test corresponding to a p-value of 5%, therefore when the black line is above the red (which is most of the times) we reject the null hypothesis of not having structural changes.

To try to tackle these issues we perform a log transformation on our time series, then perform the same tests as before.
Regarding the Augmented Dickey-Fueller test we find a pvalue of 0.193 for the transformed VKOSPI, so we still cannot reject the presence of a unit root.
By performing the the Chow test we find the same result as before.

We show in figure 10 the plot of our new response variable log(VKOSPI) together with the progressive value of the F-statistic of the Chow test, the results are a little better than the ones of VKOSPI without the logarithm transformation.

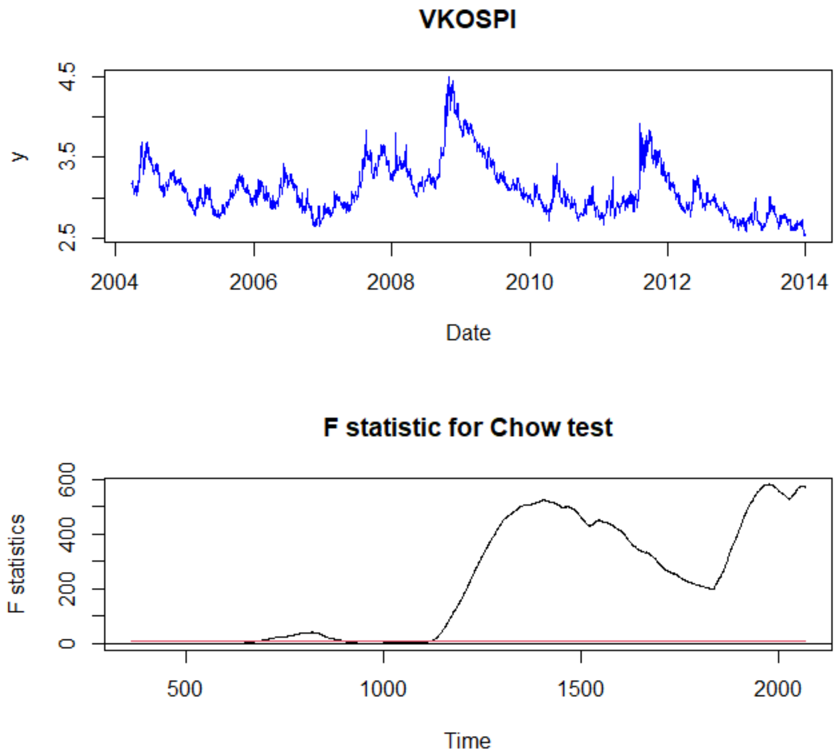


Figure 10: Test for structural breaks on log(VKOSPI)

Nonetheless, since our focus is not just ARMA models and in the paper the log transformation of the VKOSPI was deemed good enough based on the results of the models, we proceed.

4 Methodology & Results

4.a HAR

At first, we replicate all the models presented by the paper. They estimate the pure HAR model and the various versions of the HAR-X model with different exogenous variables.
In order to avoid multicollinearity problems, each model is characterized by different variables based on significance of the estimated coefficients.

$$M1 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{10,t-1} + \epsilon_t$$

$$M2 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{10,t-1} + \gamma_1 Ex_{t-1} + \gamma_2 Rf_{t-1} + \gamma_3 Credit_{t-1} + \gamma_4 Term_{t-1} + \epsilon_t$$

$$M3 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{5,t-1} + \gamma_1 Ex_{t-1} + \gamma_2 ln(VIX_{t-1}) + \epsilon_t$$

$$M4 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{10,t-1} + \gamma_1 Rf_{t-1} + \gamma_2 Credit_{t-1} + \gamma_3 ReturnUS_{t-1} + \epsilon_t$$

$$M5 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{10,t-1} + \gamma_1 Ex_{t-1} + \gamma_2 Rf_{t-1} + \gamma_3 Credit_{t-1} + \gamma_4 Term_{t-1} + \gamma_5 ReturnKOR_{t-1} + \epsilon_t$$

$$M6 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{10,t-1} + \gamma_1 Rf_{t-1} + \gamma_2 ln(VIX_{t-1}) + \gamma_3 ReturnUS_{t-1} + \epsilon_t$$

$$M7 : y_t = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{10,t-1} + \gamma_1 Ex_{t-1} + \gamma_2 ln(VIX_{t-1}) + \gamma_3 ReturnKOR_{t-1} + \epsilon_t$$

The terms $y_{5,t-1}$ and $y_{10,t-1}$ represent the moving average at 5 and 10 days time lag, which are the medium-term weekly (w) realized volatility and 10 days realized volatility at time t, respectively. The key motivation for including these heterogeneous components is that agents with different time horizons perceive, react to, and cause different types of volatility components as explained in Corsi (2009).

The first model, M1, is the pure HAR model and it is statistically significant with a very low pvalue and high R-squared.

In the second model, M2, we add four macroeconomic domestic variables: the USD/KRW exchange rate return (Ex), the interest rate (Rf), the credit spread yield (Credit), and the term spread yield (Term). The result is a model statistically significant, but depending on the threshold of significance the Term Spread regressor fails to reject the null hypothesis of it being equal to zero.

In the three next models, all the variables related to the US and Korean market are incorporated, namely, the logarithm of the US implied volatility index measured by the VIX (ln(VIX)), the US stock market return measured by the S&P 500 spot return (ReturnUS), and the Korean stock market return measured by the KOSPI 200 spot return (ReturnKOR).
By mixing and maxing variables in different way and excluding not significant coefficients, we obtain Model 3 (M3), Model 4 (M4), and Model 5 (M5).

At this point, the only statistically significant terms, $y_{1,t}, y_{5,t}, y_{10,t}$,and Korea's macroeconomic variables, are included.
In the last steps, both ln(VIX) and ReturnUS are added to the model from the second step, which gives Model 6 (M6).
Then we add both ln(VIX) and ReturnKOR to the model from the second step, which gives Model 7 (M7).

We notice that the joint presence of residual autocorrelation and lagged dependent variable among the regressors induces inconsistent coefficient estimates. Therefore, in each case, we ensure that the residuals are not serially correlated by adding lagged dependent variables up to lag k (k = 1, 5, and 10) to the model.
For the pure HAR model, the coefficients of the daily and biweekly components, $y_{1,t-1}$ and $y_{10,t-1}$, are significantly estimated at the 1% significance level, while those of the weekly and monthly components, $y_{5,t-1}$ and $y_{22,t-1}$, are not significant.
In conclusion, we confirm that all the models presented in the paper are statistically significant with a low p-value and an high R^2.
In order to rank the models we study the MSE (mean squared error) and the MAE (mean absolute error) of each model, reported in table 1 below.

model	MAE	MSE
M1	0.074	0.011
M2	0.073	0.010
M3	0.076	0.011
M4	0.073	0.010
M5	0.073	0.010
M6	0.071	0.010
M7	0.076	0.011

Table 1: MAE and MSE of model

The best model according to MSE and MAE criteria is the 6^{th} model presented. We can check the assumptions by looking at residual vs fitted plot in figure 11.
Moreover to assess the correctness of the best model we performed more tests that are not presented in the paper.
The Breush-Pagan test gives a very low p-value, meaning there is an heteroskedastic structure.
Then, the Shapiro Test shows that the normal assumption on the residuals does not hold and normality can be rejected.
Finally, the box test's result is an high p-value 0.217, so H0 can be accepted and

the residuals are independent.

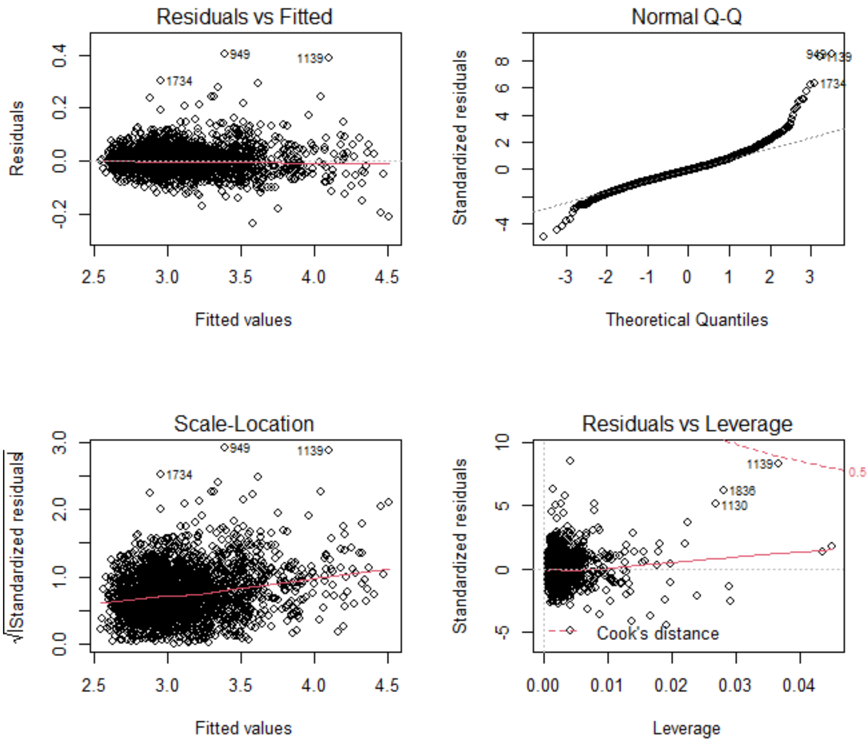


Figure 11: Model 6

In figure 11, we can see the main plots of the residuals of model 6 and below in figure 12 how good the 5 steps ahead prediction is.

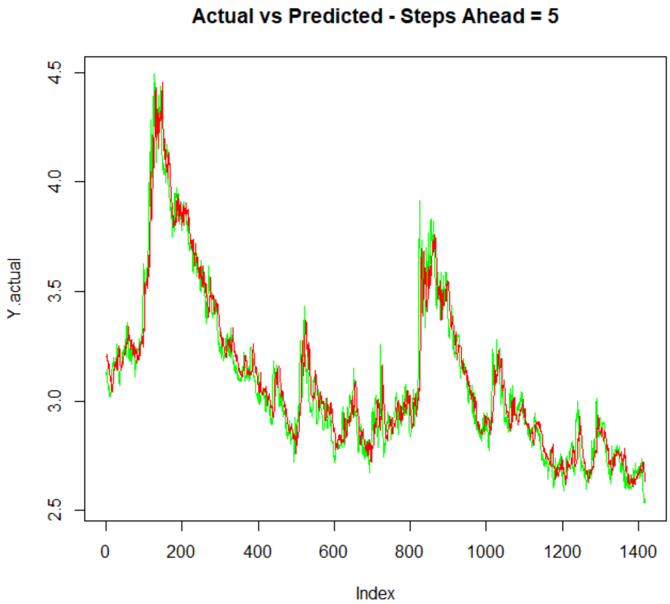


Figure 12: 5 steps prediction of Model 6

4.b Linear Regression

We decide to take into consideration in our study also a simple multiple regression over all the variables.
In multiple regression, the relationship between Y and a number of explanatory variables $X_1, X_2, ..., X_k$ is

$$Y_i = \beta_0 + \sum_i \beta i X_i + \epsilon_i$$

At first, we run a linear regression including all the variables in our dataset, in order to find the less significant ones and focus our study to the most important ones. For the model with all teh variables together we obtain a low collective pvalue and a quite high R^2, about 0.78, but there are many variables with high pvalue that can be excluded. We can see in residuals vs fitted plot in figure 13, the independence and gaussianity of the data is not verified.

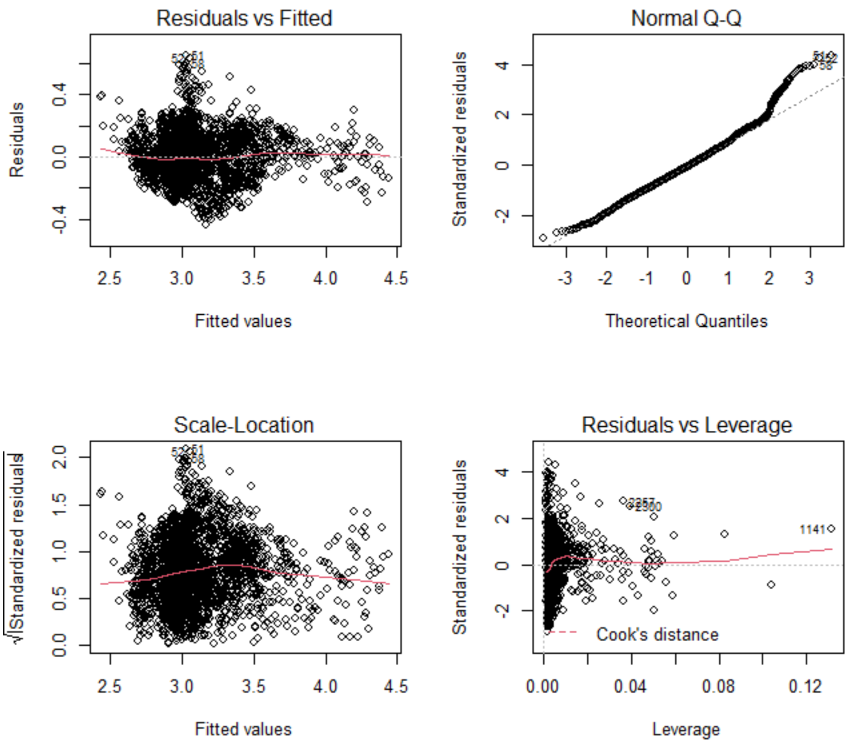


Figure 13: Global linear regression

We further investigate our model we perform the following test:

- Breusch-Pagan Test in order to check the homoschedasticity. We obtain a pvalue of $3.6 * 10^{-6}$ so we have eteroschedasticity..
- Box test on the residuals in order to check their indipendence. We obtain a pvalue smaller than $2 * 10^{-16}$ so the residuals are not indipendent.
- Shapiro test on the residuals in order to check their normality. We obtain a pvalue of $6.3 * 10^{-16}$ so the residuals are not normally distrubuted, as can be guessed observing figure 13.

Then we consider the model after removing the regressors which are not significant. In particular we keep only the log returns of VKOSPI, the log returns of S&P500, the log returns of VIX, CD and the Inflation Ratio. We perform the same test as before and we find a different result only in the Breusch-Pagan Test in which we obtain a pvalue of 0.1339866 so at least now homoschedasticity holds.

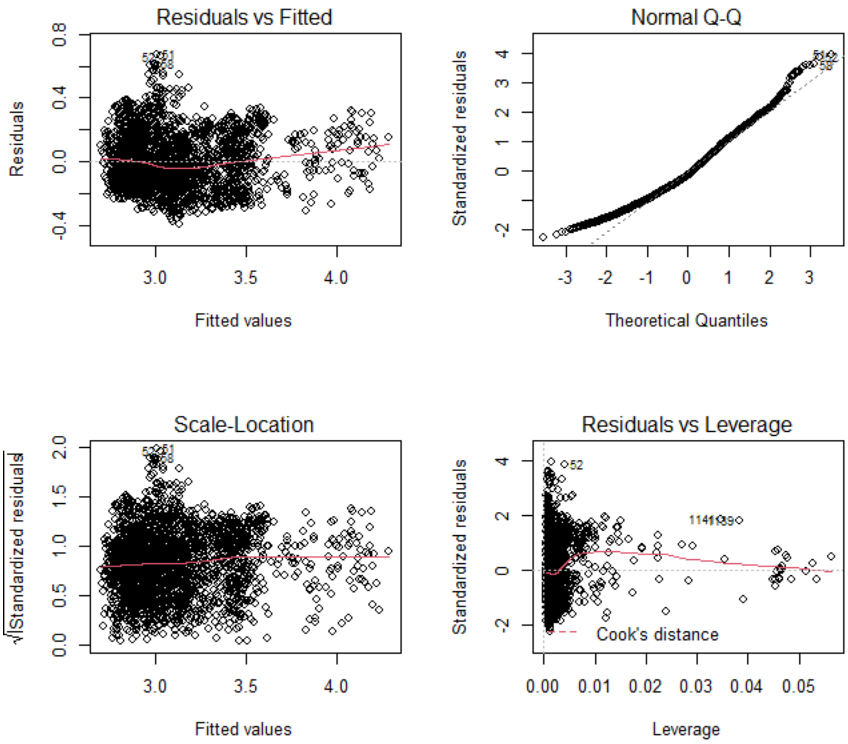


Figure 14: Linear regression model

Then we consider in alternative a model with only the logreturns of VIX, termspread, inflation ratio between KOREA and USA, credit spread and the logreturns of the S&P500 as regressors.
We obtain a linear model with a low p-value ($< 2 * 10^-16$), $R^2 = 0.7461$, and with all the coefficients that are statistically significant with a low p-value. In this case the pvalue of the Breush Pagan test is 0.09757374 (>5%) so we can accept H0, the homoschedasticy holds, although all the other tests lead to the same results of previous models.

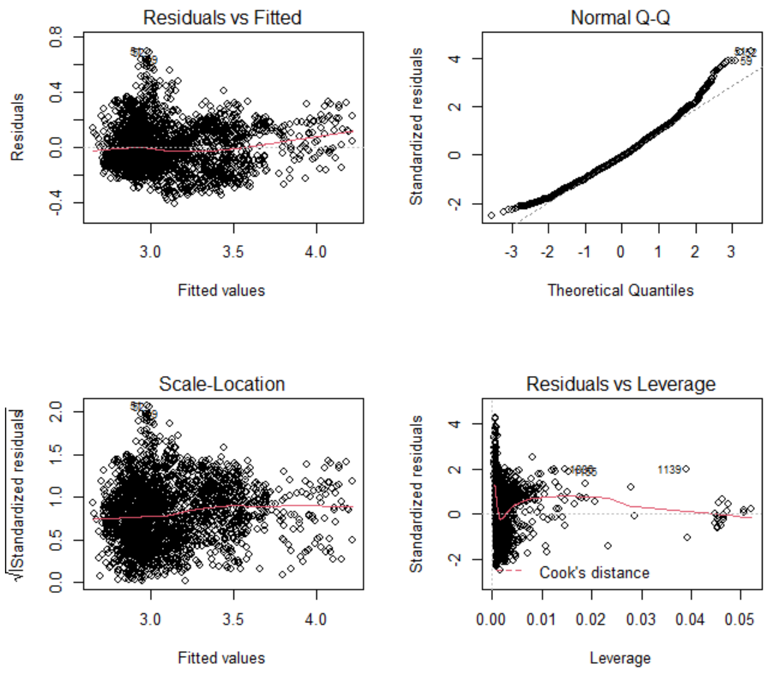


Figure 15: Linear regression model

In figure 16 we can observe the graph of the 5 steps ahead prediction of the linear regression homoskedastic model we have found.

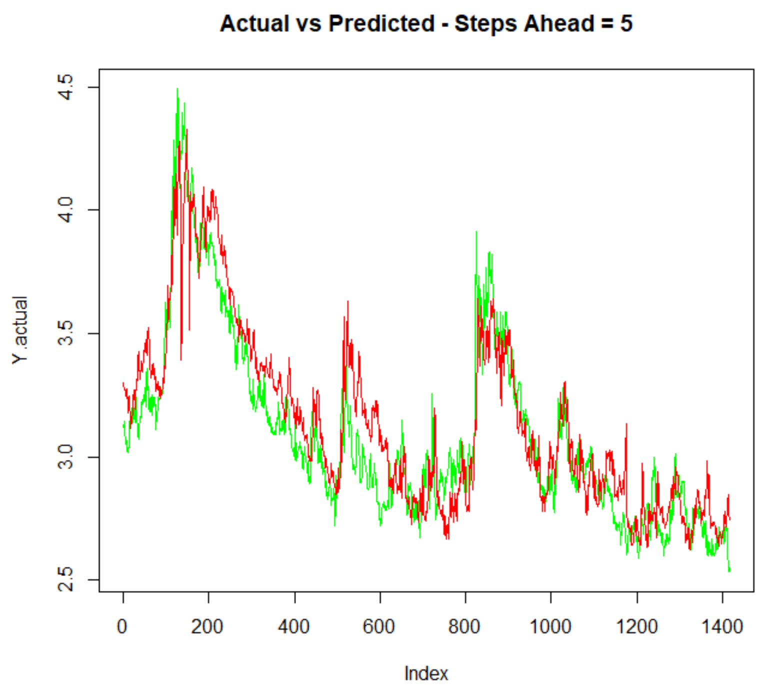


Figure 16: Linear regression 5 steps prediction

4.c GARCH and ARCH

In this section, our goal is to check for the presence of autoregressive conditional heteroskedastic (ARCH) effects in our model.
For this reason, we performed Augmented Dickey-Fuller test and Fstats on the log-returns of VKOSPI and its differential time series in order to verify the presence of a structural change.
Regarding the Fstats, for the VKOSPI time series we obtain a small p-value so we do not have stationarity, while if we consider its first differences time series we have evidence of it.
The test we performed is the Chow test, which is the same as the one presented in 3.c.
Regarding the Augmented Dickey-Fuller test, we test both the time series and we get for the log return a pvalue of 0.1933, so we do not reject the null hypotesis of having a unit root. On the other hand, in the case of first differences we get a pvalue of 0.01, so we can assume stationarity.
Then, we perform the GARCH test on the first differences time series of VKOSPI and we get a p-value lower than $2 * 10^{-16}$, so we have evidence of Garch effect.
Therefore we estimate the mean equation performing the regression of squared residuals on one-lagged squared. The result is a statistically significant model but with a very low R^2, due to high volatility.
Then we tried to fit a ARMA(0,0) + GARCH(1,1) model, but as it can be seen in figure 17, the result is not satisfactory: the prediction is pretty much constant. So we can conclude that what we want to predict is too volatile to fit this model.

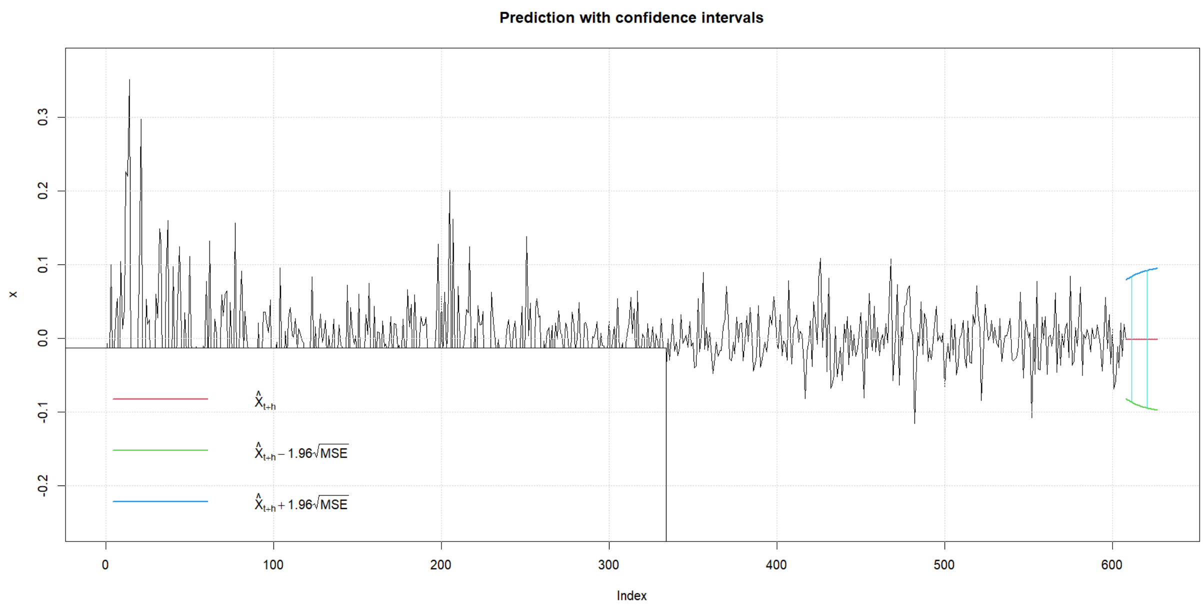


Figure 17: Garch model

5 Conclusions

Many studies focused on advanced markets do not consider the influence of global market factors in predicting market volatility indices in emerging markets.
Our empirical results show that the features of the VKOSPI are well captured by the HAR framework and that Korea's macroeconomic variables can explain the VKOSPI dynamics.
Moreover, we find that US stock market return and implied volatility index of the US market play a key role in explaining the dynamics of the VKOSPI and predicting its future values. In fact we can confirm that, when two global factors, both the US stock market returns and the US implied volatility index, are incorporated into the HAR framework such as in the linear regression one, the model exhibits the best performance in terms of both in-sample fitting and out-of-sample forecasting ability.
In particular, in the linear regression models, we show that, if we add suitable variables such as the Inflation Ratio which has a market key role during the period we are taking into consideration, we obtain even better results.
Regarding the ARCH and GARCH, unfortunately, using these frameworks we do not get a good predictor of the VKOSPI due to its high volatility.
Finally, we are confident that since this study is based on the Korean market, its results can be extended and replicated to other markets with similar characteristics.

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