Final Project: SII

Insurance and Econometrics

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Contents

1 Text of the Project

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:

ASSETS

- there is a single fund all made of equity, Ft = St
- at the beginning (t=0) the value of the fund is equal to the invested premium F0 = C0 = 100,000
- equity features
  - listed in the regulated markets in the EEA
  - no dividend yields
  - to be simulated with a Risk Neutral GBM (sigma=25%) and a time varying instantaneous rate r

LIABILITIES

- contract terms
  - whole Life policy, single premium already cashed in
  - benefits
    - in case of lapse, the beneficiary gets the value of the fund at the time of lapse, without penalties applied
    - in case of death, the beneficiary gets the maximum between the 110% of the invested premium and the value of the fund
    - when a benefit is paid to the beneficiary, a fixed cost of 20 euro is applied reducing the paid benefit
  - others
    - Regular Deduction, RD of 2.00%
    - Commissions to the distribution channels, COMM (or trailing) of 1.40%
    - No External Fees
- model points
  - just 1 model point
  - male with insured aged x=60 at the beginning of the contract
- operating assumptions
  - mortality: rates derived from the life table SI2021 (`https://demo.istat.it/index_e.php`)
  - lapse: flat annual rates lt=15%
  - expenses: constant unitary (i.e. per policy) cost of 50 euros per year, that grows following the inflation pattern
- economic assumption
  - risk free: rate r derived from the yield curve (EIOPA IT without VA 31.03.22), supposing linear interpolation of the zero rates and using the formula DFt+dt = DFt * exp[-rt*dt]
  - inflation: flat annual rate of 2%

Other specifications

- Time horizon for the projection: 50 years.
  In case there still was an outstanding portfolio in T=50, let all the people leave the contract with a massive surrender.
- The interest rates dynamic is deterministic, while the equity one is stochastic

QUESTIONS

1. Code a Matlab script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained. The risks to be considered are:
    - Market Interest
    - Market equity
    - Life mortality
    - Life lapse
    - Life cat
    - Expense
2. Calculate the Macaulay BEL duration in all the cases and provide comments on the results obtained.
3. Split the BEL value into its main PV components: premiums (=0), death benefits, lapse benefits, expenses, and commissions.
4. Replicate the same calculations in an Excel spread sheet using a deterministic projection. Do the results differ from 1? If so, what is the reason behind?
5. Open questions:
    - what happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components.
    - what happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

2 Summary Tables of Results

	ASSETS	LIABILITIES	BOF	ΔBOF	dur_L
BASE	100.000	109.237,63	-9.23763	//	6,2592
IR_{up}	100.000	108.696,10	-8.696,10	0	6,2248
IR_{dw}	100.000	109.456,70	-9.456,70	219,07	6,2748
EQUITY	61.000	69.397,45	-8.397,45	0	6,3526
MORTALITY	100.000	109.432,62	-9.432,36	194,73	6,2058
LAPSE_{up}	100.000	105.310,74	-5.310,74	0	4,3113
LAPSE_{dw}	100.000	119.950,20	-19.950,20	10.712,57	10,6392
MASS	100.000	81.070,89	17.929,11	0	5,9413
CAT	100.000	109.249,63	-9.249,63	12,00	6,2507
EXPENSES	100.000	109.288,15	-9.288,15	50,52	6,2600

	IR	Equity	Market
SCR	219,07	0	219,07

	Mortality	Lapse	CAT	Expenses	Life
SCR	194,73	10.712,57	12,00	50,52	10.742,98

BSCR = 10.799,83

3 Calculations Procedure & Comments

To compute the Basic Solvency Capital Requirement we must first find the value of the assets and liabilities of the insurance company:

ASSETS

Since our fund is fully composed by equity, we simulate its values at each future year with a Risk Neutral GBM and using a Monte-Carlo simulation

$$S_{t+dt} = S_t \; e^{(f_{t+dt,t} \; - \; \frac{\sigma^2}{2})dt \; + \; \sigma\sqrt{dt}W_1}$$

where St is the value of the equity at the previous time step, $ft + dt, t$ is the Forward Rate at time t_0 for the period $(t, t + dt)$, σ is the volatility and W_t is a Wiener process (simulated like a Standard Normal since $W_1 \sim N(0, 1)$.
To simulate the equity we chose a time step of 4 months and Nsim = 10^6, which gave a sufficient accuracy and a reasonable run time.

LIABILITIES

The potential benefits to be paid to the policy holder at each year depend on the value of the fund after having deducted the fees (RD), both for the lapse benefit and for the death benefit.
Moreover we have the maintenance costs which grow following the inflation rate and the annual commission, which depends on the value of the fund as well.

For t = 1:50 :

$$fees_t = F_{t-dt} \cdot 2\%$$
$$F_t' = F_t \ - \ fees_t$$
$$C_{lapse,t} = F_t' \ - \ 20Eur$$
$$C_{death,t} = max(F_t' \ , \ 110\% \cdot F_0) \ - \ 20Eur$$
$$C_{expenses,t} = 50Eur \cdot (1 \ + \ inflation\_rate)^{t-dt}$$
$$C_{commissions,t} = F_{t-dt} \cdot 1.4\%$$

To evaluate Ct, since we used the Monte-Carlo simulation, we computed the capital for each of the scenarios and then took the mean.
Then we discounted the cash flows of each year back to t_0, by multiplying by the financial discount, the cumulative survival probability up to each year, the probability of not having early lapse (we will call this first part the base discount) and a demographic discount which is unique to lapse and death benefits:

$$BaseDiscount_t = (1 + r_t)^t \cdot_t p_x \cdot (1 - l_x)^{t-dt}$$
$$C^{discounted}_{lapse,t} = C_{lapse,t} \cdot BaseDiscount_t \cdot (1 - q_{x+t}) l_x$$
$$C^{discounted}_{death,t} = C_{death,t} \cdot BaseDiscount_t \cdot q_{x+t}$$
$$C^{discounted}_{expenses,t} = C_{expenses,t} \cdot BaseDiscount_t$$
$$C^{discounted}_{commissions,t} = C_{commissions,t} \cdot BaseDiscount_t$$

Where r_t is the risk free rate at each year, q_{x+t} is the probability to die between $t - dt$ and t, ${}_t p_x$ is the survival probability between x and t, l_x is the annual lapse rate.
To compute the liabilities we sum every discount cash flow in the period.
In order to measure the level of risk of the liabilities, we calculated also the Macaulay Duration

$$Duration = \frac{\sum_{i=1}^{T} C_i^{discounted}(t_i - t_0)}{\sum_{i=1}^{T} C_i^{discounted}}$$

Basic Solvency Capital Requirement (BSCR)
Afterward, in order to compute the Basic Solvency Capital Requirement we computed the BOF (Basic Own Funds) which is defined as the difference between the assets and the liabilities:

$$BOF = F_0 \ - \ V$$

Later, for any possible risk (Market Interest, Market Equity, Life Mortality, Life Lapse, MASS, Life Cat and Expenses), we created a stressed scenario in which we recomputed all the above quantities under a particular stressed situation, to control

how the company will react to a possible negative future scenario.
We calculated the ΔBOF as:

$$\Delta BOF = max(BOFbase \ - \ BOFstressed, 0)$$

which is a measure of the impact of the change of the BOF between the base scenario and the stressed one.
The SCR_{Mkt} is given by:

$$SCR_{Mkt} = \sqrt{\sum_{i,j} Corr_{i,j} SCR_i SCR_j}$$

where the SCR vector is (Mkt_{IR}, Mkt_{Equity}) and the correlation matrix is:

$Corr_{up}$	IR	Equity
IR	1	0
Equity	0	1

$Corr_{dw}$	IR	Equity
IR	1	0.5
Equity	0.5	1

depending on weather the maximum increase in ΔBOF_{IR} is associated to the scenario IR_{up} or IR_{dw}.

Mkt_{IR} is the ΔBOF computed for the Market Interest Risk, Mkt_{Equity} is the ΔBOF computed for the Market Equity Risk.

The SCR_{Life} is given by:

$$SCR_{Life} = \sqrt{\sum_{i,j} Corr_{i,j} SCR_i SCR_j}$$

where the SCR vector is $(Life_{Mortality}, Life_{Lapse}, Life_{CAT}, Life_{Expenses})$ and the correlation matrix is:

$Corr$	Mortality	Lapse	CAT	Expenses
Mortality	1	0	0.25	0.25
Lapse	0	1	0.25	0.5
CAT	0.25	0.25	1	0.25
Expenses	0.25	0.5	0.25	1

$Life_{Mortality}$ is the ΔBOF computed for the Life Mortality Risk, $Life_{Lapse}$ is the ΔBOF computed for the Life Lapse Risk, $Life_{CAT}$ is the ΔBOF computed for the Life CAT Risk, $Life_{Expenses}$ is the ΔBOF computed for the Life Expenses Risk.

Then once we have computed the SCR_{Mkt} and the SCR_{Life}, the Basic Solvency Capital Requirement is given by:

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} SCR_i SCR_j}$$

where the SCR vector is $(SCR_{Mkt},\ SCR_{Life})$ and the correlation matrix is:

$Corr_{up}$	Market	Life
Market	1	0.25
Life	0.25	1

The Basic Solvency Capital Requirement indicates the amount of additional capital that the insurance company has to reserve in order to cover any possible and unexpected future loss with a 99.5% probability within a year.
Now we will analyze the six stressed scenarios we considered with their respective results.

3.a Market Interest Risk

We created the stressed scenario imposing a change on the EIOPA yield curve: we used the stressed up and down EIOPA yield curve and we recompute all the interested quantities.

$$Mkt_{IR_{up}} = max(\Delta BOF|_{IRup};\ 0)$$

$$Mkt_{IR_{dw}} = max(\Delta BOF|_{IRdw};\ 0)$$

Then since we want to assess the worst scenario, we took the maximum value between the two:

$$Mkt_{IR} = max(Mkt_{IR_{up}};\ Mkt_{IR_{dw}})$$

In our case, both the discount factor and the equity are sensitive to the yield curve change.
We observe that these effects balance each other for the most part for the death and lapse benefits, and for the commissions.
This is because even though an average increase (resp. decrease) of the value of the fund over the years with respect to the base scenario means that the future benefits will increase (resp. decrease), at the same time the financial discount rate is higher (resp. lower) and therefore the present value does not change significantly.
The annual expenses on the other hand are not influenced by the value of the fund, but depend from the IR through the discount rate only, meaning that an increase of the latter will make the future expenses less costly when discounted to the present day.
Thus we find that the SCR_{IR} is 0 for an increase of the rates, while it is positive yet still close to zero for a decrease in the rates.

The duration of the liabilities decreseases in the scenario $Mkt_{IR_{up}}$ and increases in the scenario $Mkt_{IR_{dw}}$.
Indeed an increase of the interest rates makes the financial discount factor increase and consequently future cash flows will count less than those in proximity.
On the contrary a decrease makes the financial discount factor less impacting and stretches the duration of the liabilities.

3.b Market Equity Risk

In order to analyze the equity risk we created the stressed scenario imposing a decrease on the initial value of the equity.

Since the equity is listed in the regulated markets in the EEA, the equity must be classified as of type 1 and consequently we applied a negative shock of 39% to its initial value.

$$S_0^{Equity} = S_0(1 - 39\%)$$
$$Mkt_{Equity} = \Delta BOF|_{Equity}$$

With MatLab the result is 0, but with Excel (see later section) we find a positive ΔBOF, which can be mainly attributed to the fact that the death benefits have a minimum guarantee which is not reduced by the equity shock, and therefore weighs relatively more on the shocked BOF.

The duration of the liabilities increases in this scenario.
Indeed a crash in the market at the beginning means lower benefits and commission payed in the first years. We notice that although the death benefit has a minimum guaranteed threshold, this is not sufficient to make the duration decrease.

3.c Life Mortality Risk

The applied shock here is an instantaneous, and uniform on all dates, +15% increase on the death rate.

$$q_{x+t}^{Mortality} = q_{x+t}(1 + 15\%)$$
$$SCR_{Mortality} = \Delta BOF|_{Mortality}$$

The SCR derived from this risk is null.
The effect is to increase the probability of death throughout the years, and therefore the probability of reaching years which are further into the future is lessened.

The duration of the liabilities decreases in this scenario.
Indeed an increase in the mortality rate means higher death benefits in the earlier years and less probability of reaching the later years, shifting the duration towards the beginning.

3.d Life Lapse Risk

To correctly simulate the risk of having a sudden increase in lapse rate, meaning more clients opting to leave the contract before maturity, or viceversa a decrease in lapse rate, we impose the following scenarios:

- an instantaneous and permanent increase of the lapse rate of 50% for each year as long as it does not exceed the value of 100%

- and an instantaneous and permanent decrease of the lapse rate of 50% for each year as long as the absolute variation does not exceed 20%.

- We simulate also a third scenario, MASS Risk, where we consider a change in the lapse rate so that for the first year it has value of 40% while the values of the next years stay the same as the base scenario.

$$SCR_{Lapse_{up}} = \Delta BOF|_{Lapse_{up}}$$
$$SCR_{Lapse_{dw}} = \Delta BOF|_{Lapse_{dw}}$$
$$SCR_{MASS} = \Delta BOF|_{MASS}$$

Then, since we are searching the worst situation in which the insurance company could be, we take the maximum value between the three stressed scenarios

$$SCR_{Lapse} = max(SCR_{Lapse_{up}}, SCR_{Lapse_{dw}}, SCR_{MASS})$$

In our work SCRs from the up shock and the mass case are 0 in both cases under analysis. Indeed, both shocks provoke a decrease of cash flows in the latter years. Conversely, a decrease of the lapse rate in the down case leads to much greater cash flows both for the death benefits and also for the commissions, which leads to a highly positive $SCR_{Lapse_{dw}}$.

The duration of the liabilities decreases a lot in the $SCR_{Lapse_{up}}$ and SCR_{MASS} scenarios while it increase a lot in the $SCR_{Lapse_{dw}}$ scenario.
Indeed an increase in the lapse rate (either constant or just in the first year) means a shift of the expected benefits towards the earlier years, and vice versa for the down scenario we increase the probability of reaching the latest years of the contract.

3.e Life CAT Risk

The CAT risk deals with the possibility of catastrophes that can generate an unexpected spike in the death rate. In order to analyze the CAT risk we create the stressed scenario of an increase in the mortality rates of an additional 0.15% in the first year

$$q_{x+0}^{CAT} = q_{x+0} \; + \; 0.0015$$
$$SCR_{CAT} = \Delta BOF|_{CAT}$$

Our result is that the SCR is close to 0. The shock increases discounted cash flow related to the first year only while the others slightly decrease.
Furthermore, the maturity is quite long so the effects are not very relevant to in our case.

The duration of the liabilities decreases in this scenario.
Similarly to the Mortality or Lapse Mass risk, an increase in the mortality rate for the first year drags the expected benefits towards the earlier years.

3.f Life Expenses Risk

The expenses risk target all the expense which are not benefits to be paid, so in our case the annual maintenance costs and the commission fee.
In particular we simulate a scenario where they both increase by 10% and also the inflation rate increases additionally by 1%.
Since here we just have an increase in the liabilities and not in our assets we find a positive SCR.

The duration of the liabilities slightly increases in this scenario.

It does make sense since the expenses hold more weight and at the same time we are increasing the inflation rate meaning the maintenance costs increase more towards the later years.

4 Deterministic Calculations using Excel

The only difference here with respect to the MatLab computations is that rather than simulating the path of the stock through MC we use a deterministic projection of the value of the fund:

$$S_t = S_0 \, e^{r_t \, t}$$

4.a Result tables

	ASSETS	LIABILITIES	BOF	ΔBOF	dur$_L$
BASE	100.000	107.273,11	-7.273,11	//	6,1477
IR$_{up}$	100.000	107.020,94	-7.020,94	0	6,1489
IR$_{dw}$	100.000	107.416,17	-7.412,17	143,06	6,1490
EQUITY	61.000	68.661,84	-7.661,84	388,73	6,2434
MORTALITY	100.000	107.245,02	-7.245,02	0	6,0867
LAPSE$_{up}$	100.000	104.477,60	-4.477,60	0	4,2785
LAPSE$_{dw}$	100.000	113.532,75	-13.532,75	6.259,64	10,23637
MASS	100.000	95.414,97	4.585,03	0	4,4725
CAT	100.000	107.416,33	-7.416,33	143,21	6,1408
EXPENSES	100.000	108.180,37	-8.180,37	907,25	6,1464

	IR	Equity	Market
SCR	143,06	388,73	476,64

	Mortality	Lapse	CAT	Expenses	Life
SCR	0	6.259,64	143,21	907,25	6.798,45

BSCR $= 6.932,99$

4.b Comments

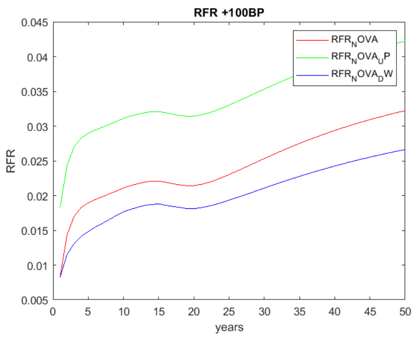
Results obtained in Excel via deterministic projection have in all scenarios similar behaviors as those found through MC simulation via MatLab.
The differences in the values can be attributed mainly to the fact that the death benefit has a minimum guaranteed threshold: what we do in the MC simulation is we compute the 10^6 possible scenarios for each future year, then take the maximum between the value of the fund and the guaranteed threshold, and finally take the mean. The result we find is different with respect to the one we would obtain by taking the mean and only after applying the maximum. This second approach is

basically what we are doing when we simulate a deterministic growth and then take the maximum to compute the benefit.
A way to simply understanding this is to think about a call option: the real price is not the same price one would obtain by projecting in a deterministic way the underlying and then applying the payoff formula.
This is why we obtain a higher BSCR through the MC simulation.
A part from this, all the comments made about the SCR of each different risk scenario and the changes in duration of the contract still hold for the deterministic calculations.

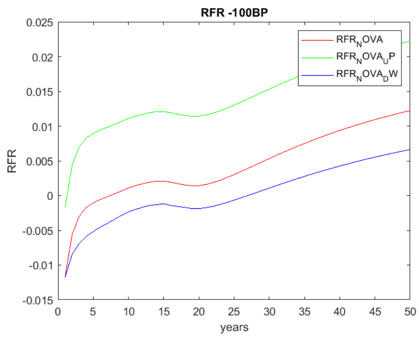
5 Open Questions

5.a IR parallel shift

We consider an upper and lower parallel shift of 100 bps in the EIOPA interest rate curve; in figure 1 we can see how the RFR changes in the two scenarios.



(a) RFR +100bp



(b) RFR -100bp

Figure 1: RISK FREE RATE

Then we compute the value of the asset; in figure 2 we can see how the value has a more consistent difference at high maturity and in the case of +100bp.

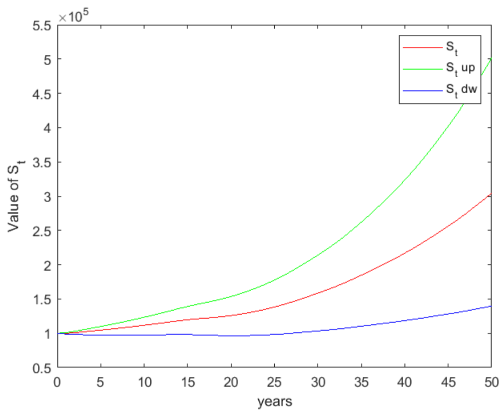


Figure 2: Value of the asset

For what concerns the value of the liabilities, they are affected by the shift of the interest rates: in the first case (+100bp) their values decrease; on the other hand, in the second case (-100bp) their values increase. The reason of this behaviour is that higher rates imply lower discount factors that are used to compute the NPV. The difference is not very significant since it is in the order of one thousand over the base value of 100000.

5.b Demographic change

In this scenario, the age of the insurer increases and as consequence the mortality rate will increase too. For example if we considered as age of the insurer 80 instead of 60, we cannot take in consideration the same maturity since it is unfeasible that a person lives till 130 years, so we need to decrease the time horizon from 50 to 30. The risk free rates do not change since they depend only on the market and not on the insured.
The effect on the liabilities is an increase in the death benefits, since for each year the probability of dying is higher, while all other liabilities (lapse, expenses, commissions) slightly decrease since the probability of reaching the later years of the contract diminishes.
Moreover the duration of the contract moves towards the beginning so the insurer must be prepared to pay higher benefits early on.

Finally, we consider the case in which we have two models points, one male and one female.
As we know the life tables are different for males and females, women's probabilities of death are lower than the men's ones, so we expect less liabilities early on in the female case.

6 Annex: Matlab Code

Main

```
%% Insurance Project

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%    Group 8:
%    Sebastian Castellano
%    Giulia Mulattieri
%    Virginia Muscionico
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Load data

clear; close all; clc;

addpath(genpath('.'))

% load ISTAT 2021 life tables
load('Life_Tables_2021.mat');

% load risk free rates
load('RiskFreeRate_IT_NOVA310322.mat');

% get RFR vectors
RFR_NOVA = RFR_NOVA(:,2);
RFR_NOVA_UP = RFR_NOVA_UP(:,2);
RFR_NOVA_DW = RFR_NOVA_DW(:,2);


%% Parameters

C0 = 100000;             % insured capital
T = 50;                  % maturity of policy

S0 = 100000;             % initial value of Equity
sigma = 0.25;            % volatility for GBM

x = 60;                  % policy holder (man) initial age
lx = 0.15;               % flat annual lapse rate
Nsim = 1e+6;             % number of simulation for GBM
steps_per_year = 5;      % number of time steps per year for MC
    simulation
fee_rate = 0.02;         % RD
perc_prem = 1.1;         % percentual increase in death benefit
    w.r.t. premium
inflation = 0.02;        % annual inflation rate
```

```
expenses = 50;         % annual expense per policy
comm_rate = 0.014;          % annual commissions
fix_cost = 20;         % fixed cost every time a benefit is
    paid


%% "What if?" cases

% % upward parallel shift of 100bps
% RFR_NOVA = RFR_NOVA + 0.01;
% RFR_NOVA_UP = RFR_NOVA_UP + 0.01;
% RFR_NOVA_DW = RFR_NOVA_DW + 0.01;
%
% % downward parallel shift of 100bps
% RFR_NOVA = RFR_NOVA - 0.01;
% RFR_NOVA_UP = RFR_NOVA_UP - 0.01;
% RFR_NOVA_DW = RFR_NOVA_DW - 0.01;
%
% % increased insured age
% x = 80;
%
% % female model point
% LT_M = LT_F;


%% Mortality rate and Survival probability

% index of the insured person age
i = x+1;

% mortality rate from age x to age x+T-1
qx = 1 - (LT_M{i+1:x+T+1,2}./LT_M{i:x+T,2});

% survival probability from x
t_px = LT_M{i:x+T,2}./LT_M{i,2};

%% Assets

% equity
S = equity(S0, T, RFR_NOVA(1:T), sigma, Nsim, steps_per_year
    );


%% Liabilities

% Capitals to be payed
[C_lapse, C_death, C_expenses, C_comm] = capital(C0, S, T,
    Nsim, fee_rate, ...
```

```
    perc_prem, fix_cost, inflation, expenses, comm_rate);

Capitals = [C_lapse; C_death; C_expenses; C_comm];

% Liabilities and their duration
[V_t, dur] = liabilities(Capitals, T, qx, t_px, RFR_NOVA(1:T
    ), lx);

F = S0;
BOF_base = F - V_t;

table_Base_scenario = table(BOF_base,dur,...
                            'VariableNames',{'BOF','Duration'});


%% IR_UP and IR_DOWN

[table_IR_UP] = shock(T,RFR_NOVA_UP(1:T),S0,sigma,...
    Nsim,steps_per_year,C0,fee_rate,qx,t_px,lx,BOF_base,
        fix_cost, ...
    inflation, expenses, comm_rate, perc_prem);

[table_IR_DW] = shock(T,RFR_NOVA_DW(1:T),S0,sigma,...
    Nsim,steps_per_year,C0,fee_rate,qx,t_px,lx,BOF_base,
        fix_cost, ...
    inflation, expenses, comm_rate, perc_prem);

%% Stock

S0_stock = S0 * (1-0.39);
[table_Stock] = shock(T,RFR_NOVA(1:T),S0_stock,sigma,Nsim,
    steps_per_year,C0,fee_rate,...
    qx,t_px,lx,BOF_base, fix_cost, ...
    inflation, expenses, comm_rate, perc_prem);

%% Life Mortality Risk

% constant 15% shock in mortality rate
qx_mort = qx*1.15;
t_px_mort = zeros(T,1);
t_px_mort(1) = 1;
for i=2:T
    t_px_mort(i) = t_px_mort(i-1)*(1-qx_mort(i-1));
end
[table_Mortality] = shock(T,RFR_NOVA(1:T),S0,sigma,...
    Nsim,steps_per_year,C0,fee_rate,qx_mort,t_px_mort,lx,
        BOF_base, fix_cost, ...
    inflation, expenses, comm_rate, perc_prem);
```

```
%% Life Lapse Risk

% case upward shock
lx_up = min( 1.5*lx , 1);

[table_Lapse_UP] = shock(T,RFR_NOVA(1:T),S0,sigma,Nsim,
    steps_per_year,C0,fee_rate,...
      qx,t_px,lx_up,BOF_base, fix_cost, ...
      inflation, expenses, comm_rate, perc_prem);

% case downward shock
lx_dw = max( 0.5*lx , lx-0.2);
[table_Lapse_DW] = shock(T,RFR_NOVA(1:T),S0,sigma,Nsim,
    steps_per_year,C0,fee_rate,...
      qx,t_px,lx_dw,BOF_base, fix_cost, ...
      inflation, expenses, comm_rate, perc_prem);

%% MASS

MASS = 1; %control variable for use liabilities_MASS in
    shock function
[table_MASS] = shock(T,RFR_NOVA(1:T),S0,sigma,Nsim,
    steps_per_year,C0,fee_rate,...
      qx,t_px,lx,BOF_base, fix_cost, ...
      inflation, expenses, comm_rate, perc_prem, MASS);

%% Life Catastrophy Risk

% catastrophy shock
qx_cat = qx + 0.0015*[1; zeros(length(qx)-1,1)];
t_px_cat = zeros(T,1);
t_px_cat(1) = 1;
for i=2:T
     t_px_cat(i) = t_px_cat(i-1)*(1-qx_cat(i-1));
end
[table_CAT] = shock(T,RFR_NOVA(1:T),S0,sigma,Nsim,
    steps_per_year,C0,fee_rate,...
      qx_cat,t_px_cat,lx,BOF_base, fix_cost, ...
      inflation, expenses, comm_rate, perc_prem);

%% Expenses risk

expenses = expenses *1.1;
inflation = inflation + 0.01;
[table_Expenses] = shock(T,RFR_NOVA(1:T),S0,sigma,Nsim,
    steps_per_year,C0,fee_rate,...
```

```
    qx,t_px,lx,BOF_base, fix_cost, inflation, expenses,
        comm_rate, perc_prem);

%% Market SCR

SCR_IR = max(table_IR_UP.dBOF(1),table_IR_DW.dBOF(1));
updw = (table_IR_UP.dBOF(1) > table_IR_DW.dBOF(1));

SCR_equity = table_Stock.dBOF(1);

%% LIFE SCR

SCR_mort = table_Mortality.dBOF(1);

SCR_Lapse = max([table_Lapse_UP.dBOF(1), table_Lapse_DW.dBOF
    (1), table_MASS.dBOF(1)]);

SCR_CAT = table_CAT.dBOF(1);

SCR_exp = table_Expenses.dBOF(1);

%% Basic SCR

[BSCR, table_SCR] = bscr(SCR_IR, updw, SCR_equity, SCR_mort,
     SCR_Lapse, SCR_CAT, SCR_exp);
```

Function equity

```
function S = equity(S0, T, spot, sigma, Nsim, steps_per_year
    )
% Compute the equity value for T years using GBM and MC
    method
%
% INPUTS
% S0 = initial value of Equity
% T = time to maturity
% spot = column vector of Spot Rates from time 1 to T
% sigma = volatility for GBM
% Nsim = number of simulation for MonteCarlo
% steps_per_year = number of time steps per year for MC
    simulation
%
% OUTPUT
% S = matrix of Equity prices from time 0 to T (Nsim rows, T
    +1 columns)

subyear = 1/steps_per_year;     % fraction of year for
    simulation
```

```
r = interp1(0:T,[0;spot],0:subyear:T);
f = fwd(r, subyear, T);

% Equity simulation
T_periods = T*steps_per_year + 1;
S_period = zeros(Nsim,T_periods);     % vector stock prices
S_period(:,1) = S0;
rng('default'); % for reproducibility
Z = randn(Nsim, T_periods); % matrix normal random variable
for i = 1:(T_periods-1)
    S_period(:,i+1) = S_period(:,i).*exp( (f(i+1)-sigma^2/2)*subyear + sigma*sqrt(subyear)*Z(:,i));
end
idx_year = 1:steps_per_year:T_periods;
S = S_period(:,idx_year);
end
```

Function fwd

```
function [Fwd] = fwd (r, subyear, T)
% Compute the Forward Rates starting from the EIOPA yield curve
%
% INPUTS
% r = column vector of Spot Rates from t=0 to T
% subyear = fraction of year
% T = time to maturity
%
% OUTPUT
% Fwd = column vector of Forward Rates from t=0 to T
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

t = (0:subyear:T);
n = length(t);
Fwd = zeros(1,n);
Fwd(1) = 0;
for i = 2:n
    Fwd(i) = (r(i)*t(i)-r(i-1)*t(i-1))/(t(i)-t(i-1));
end
end
```

Function capital

```
function [C_lapse_MC,C_death_MC,C_expenses,C_comm_MC] = capital(C0, S, T, Nsim, fee_rate, perc_prem, fix_cost, ...
```

```
    inflation, expenses, comm_rate)
% Compute the value of the capital to be paid in the two cases (for T years)
%
% INPUTS
% C0 = insure capital
% S = matrix of equity values from time 0 to T ((T+1) x Nsim)
% Nsim = number of simulation for GBM
% fee_rate = rate of fee tax payed each year
% perc_prem = value to multiply the premium given in case of death
% fix_cost = cost per benefit payment
% inflation = annual inflation rate
% expenses = annual expenses
% comm_rate = rate of commissions
%
% OUTPUTS
% Clapse_MC = Capital to be paid case of lapse with MonteCarlo method
% Cdeath_MC = Capital to be paid case of death with MonteCarlo method

% fund value (no fees)
F = S;

% fund value (with fees)
F_prime_t = zeros(Nsim,T+1);
F_prime_t(:,1) = F(:,1);
F_prime_t(:,2:end) = F(:,2:end) - fee_rate*F(:,1:end-1);

% capital to be paid, case of lapse
C_lapse_sim = F_prime_t(:,2:end) - fix_cost;
C_lapse_MC = mean(C_lapse_sim,1);
% capital to be paid, case of death
C_death_sim = max(F_prime_t(:,2:end), C0*perc_prem) - fix_cost;
C_death_MC = mean(C_death_sim,1);
% capital to be paid, expenses
C_expenses = expenses*(1+inflation).^(0:(T-1));
% capital to be paid, commission
C_comm_sim = comm_rate*F(:,1:end-1);
C_comm_MC = mean(C_comm_sim,1);
end
```

Function liabilities

```
function [liabilities, duration] = liabilities(C, T, qx, px,
```

```
    spot, lx)
% Compute the liabilities value for T years
%
% INPUTS
% C = matrix of capitals to be paid from time 1 to T
% T = time to maturity
% qx = column vector of mortality rate from age x to age x+T-1
% px = column vector of survival probability from age x (to reach each age)
% spot = column vector of Spot Rates from time 1 to T
% lx = flat annual lapse rate
%
% OUTPUT
% liabilities = liabilities valued in 0, column vector
% duration = duration of the liabilities

% Capitals
C_lapse = C(1,:);
C_death = C(2,:);
C_expenses = C(3,:);
C_comm = C(4,:);

% discounted (financial and demographic) cash flows
discount_factor = exp(-spot'.*(1:T));
% base_discount = discount * p_survival * p_not_lapse_tillnow (same for every capital)
base_discount = discount_factor.*px'.*(1-lx).^(0:T-1);

% Disc_CashFlows_lapse = sum * base_discount * (p_survival*lapse_rate)
Disc_CashFlows_lapse = zeros(1,T);
Disc_CashFlows_lapse(1:end-1) = C_lapse(1:end-1).*base_discount(1:end-1) ...
                    .*(1-qx(1:end-1)')*lx;
Disc_CashFlows_lapse(end) = C_lapse(end)*base_discount(end)*(1-qx(end)'); % 100% lapse rate at end
% Disc_CashFlows_death = sum * base_discount * (death_rate)
Disc_CashFlows_death = C_death.*base_discount.*qx';
% Disc_CashFlows_exp = sum * base_discount (automatically paid if we reach the year)
Disc_CashFlows_exp = C_expenses.*base_discount;
% Disc_CashFlows_comm = sum * base_discount (automatically paid if we reach the year)
Disc_CashFlows_comm = C_comm.*base_discount;

Disc_CashFlows = Disc_CashFlows_lapse + Disc_CashFlows_death + Disc_CashFlows_exp + ...
```

```
    Disc_CashFlows_comm;

% liabilities
liabilities = sum(Disc_CashFlows);

% durations
duration = sum(Disc_CashFlows.*(1:T))./liabilities;

end
```

Function liabilities_MASS

```
function [liabilities, duration] = liabilities_MASS(C, T, qx
    , px, spot, lx)
% Compute the liabilities value for T years in the case of
    MASS shock
%
% INPUTS
% C = matrix of capital to be paid from time 1 to T (Nsim
    rows)
% T = time to maturity
% qx = column vector of mortality rate from age x to age x+T
    -1
% px = column vector of survival probability from age x (to
    reach each age)
% spot = column vector of Spot Rates from time 1 to T
% lx = flat annual lapse rate
%
% OUTPUT
% liabilities = liabilities valuated in 0, column vector
% duration = duration of the liabilities, column vector

% Capitals
C_lapse = C(1,:);
C_death = C(2,:);
C_expenses = C(3,:);
C_comm = C(4,:);

% discounted (financial and demographic) cash flows
discount_factor = exp(-spot'.*(1:T));
% base_discount = discount * p_survival *
    p_not_lapse_tillnow (same for every capital)
base_discount = discount_factor.*px'.*(1-lx).^(0:T-1);
base_discount(2:end) = base_discount(2:end)/(1-lx)*(1-0.4);

% Disc_CashFlows_lapse = sum * base_discount * (p_survival*
    lapse_rate)
Disc_CashFlows_lapse = zeros(1,T);
```

```
Disc_CashFlows_lapse(1) = C_lapse(1)*discount_factor(1)*(1-qx(1))*0.4;
Disc_CashFlows_lapse(1:end-1) = C_lapse(1:end-1).*base_discount(1:end-1) ...
                        .*(1-qx(1:end-1)')*lx;
Disc_CashFlows_lapse(end) = C_lapse(end)*base_discount(end)*(1-qx(end)'); % 100% lapse rate at end
% Disc_CashFlows_death = sum * base_discount * (death_rate)
Disc_CashFlows_death = C_death.*base_discount.*qx';
% Disc_CashFlows_exp = sum * base_discount (automatically paid if we reach the year)
Disc_CashFlows_exp = C_expenses.*base_discount;
% Disc_CashFlows_comm = sum * base_discount (automatically paid if we reach the year)
Disc_CashFlows_comm = C_comm.*base_discount;

Disc_CashFlows = Disc_CashFlows_lapse + Disc_CashFlows_death + Disc_CashFlows_exp + ...
    Disc_CashFlows_comm;

% liabilities
liabilities = sum(Disc_CashFlows);

% durations
duration = sum(Disc_CashFlows.*(1:T))./liabilities;
end
```

Function shock

```
function [table_results] = shock(T,RFR,S0,sigma,Nsim,steps_per_year,C0,fee_rate,...
    qx,t_px,lx,BOF_base,fix_cost, inflation, expenses, comm_rate, perc_prem, MASS)
% Evaluate BOF, delta_BOF and duration of the shock
%
% INPUT
% T = maturity of policy
% RFR = column vector of Spot Rates from time 1 to T
% S0 = initial value of Equity
% sigma = volatility for GBM
% Nsim = number of simulation for MonteCarlo
% steps_per_year = number of time steps per year for MC simulation
% C0 = insured capital
% fee_rate = rate of fee tax payed each year
% qx = column vector of mortality rate from age x to age x+T-1
% t_px = column vector of survival probability from age x (
```

```
    to reach each age)
% lx = flat annual lapse rate
% BOF_base = Base scenario BOF
% fix_cost = cost per benefit payment
% inflation = annual inflation rate
% expenses = annual expenses
% comm_rate = rate of commissions
% perc_prem = value to multiply the premium given in case of
    death
% MASS = control variable for MASS risk
%
% OUTPUT
% table_results = table with BOF, deltaBOF and duration for
    the shock

% equity
S = equity(S0, T, RFR, sigma, Nsim, steps_per_year);

%Capitals
[C_lapse, C_death,  C_expenses, C_comm] = capital(C0, S, T,
    Nsim, fee_rate, perc_prem, ...
    fix_cost, inflation, expenses, comm_rate);
Capitals = [C_lapse; C_death; C_expenses; C_comm];

% Liabilities and their duration
if (nargin == 17)
    [V_t_shock, dur_shock] = liabilities(Capitals, T, qx,
        t_px, RFR, lx);
elseif (nargin == 18)
    [V_t_shock, dur_shock] = liabilities_MASS(Capitals, T,
        qx, t_px, RFR, lx);
end

% Fund value
F = S0;
% BOF shock and deltaBOF
BOF_shock = F - V_t_shock;
delta_BOF = max(0,BOF_base-BOF_shock);

table_results = table(BOF_shock,delta_BOF,dur_shock,...
                       'VariableNames',{'BOF','dBOF','
                           Duration'});

end
```

Function bscr

```
function [bscr,table_scr] = bscr(scr_ir, updw, scr_equity,
```

```
    scr_mort, scr_lapse, scr_cat,  scr_expenses)
% Compute BSCR starting from single SCRs from various risk
    areas,
%   using the Standard Formula approach
%
% INPUT:
% scr_ir: SCR for interest risk
% upwd: boolean variable (1 for IR shock UP, 0 for IR shock
    DW)
% scr_equity: SCR for equity risk
% scr_mort: SCR for life mortality risk
% scr_lapse: SCR for life lapse risk
% scr_cat: SCR for life catastrophy risk
% scr_expenses: SCR for expenses risk
%
% OUTPUT:
% bscr: BSCR computed starting from single SCRs

% define correlation matrices
risks = [1 0.25; 0.25 1];
market_ir_up = [1 0; 0 1];
market_ir_dw = [1 0.5; 0.5 1];
life = [1 0 0.25 0.25; 0 1 0.25 0.5; 0.25 0.25 1 0.25; 0.25
    0.5 0.25 1];

% SCR for market risks
scr = [scr_ir, scr_equity];
if updw == 1
    scr_market = sqrt(scr*market_ir_up*scr');
elseif updw == 0
    scr_market = sqrt(scr*market_ir_dw*scr');
end

% SCR for life risks
scr = [scr_mort, scr_lapse, scr_cat, scr_expenses];
scr_life = sqrt(scr*life*scr');

% BSCR
scr = [scr_market, scr_life];
bscr = sqrt(scr*risks*scr');
table_scr = table(scr_market,scr_life,bscr,'VariableNames',{
    'SCR Market','SCR Life','BSCR'});

end
```