



Jupiter's Turkish Taco

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1. PROBLEM DEFINITION

Classical optimization problem



Hamiltonian



2. ANALOG HAMILTONIAN

$$H = \sum_{i \in S} b_i w_i + \left(\sum_{i \in S} b_i d_i + \sum_{j=0}^{\text{nglim}} p_j (2^j) - d_{\max} \right)^2$$

$$H = \sum_{i \in S} -\left(\frac{1 - Z(i)}{2}\right) w_i + \left(\sum_{i \in S} \left(\frac{1 - Z(i)}{2}\right) d_i + \sum_{j=0}^{\text{nglim}} \left(\frac{1 - Z(j)}{2}\right) (2^j) - d_{\max} \right)^2$$

- hours needed : many
- errors made : lots of
- number of tears shed : yes
- slept ? lol



EXACT EIGENVALUES = BRUTEFORCE



Goal : find that the ground state is the
correct solution for several easy cases

ngoa



$[((-46+0j), '1110100000'), ((-45+0j),$
 $'1111000000'), ((-45+0j), '11010111000').....$



QOAO VS ADIABATIC

Step 1

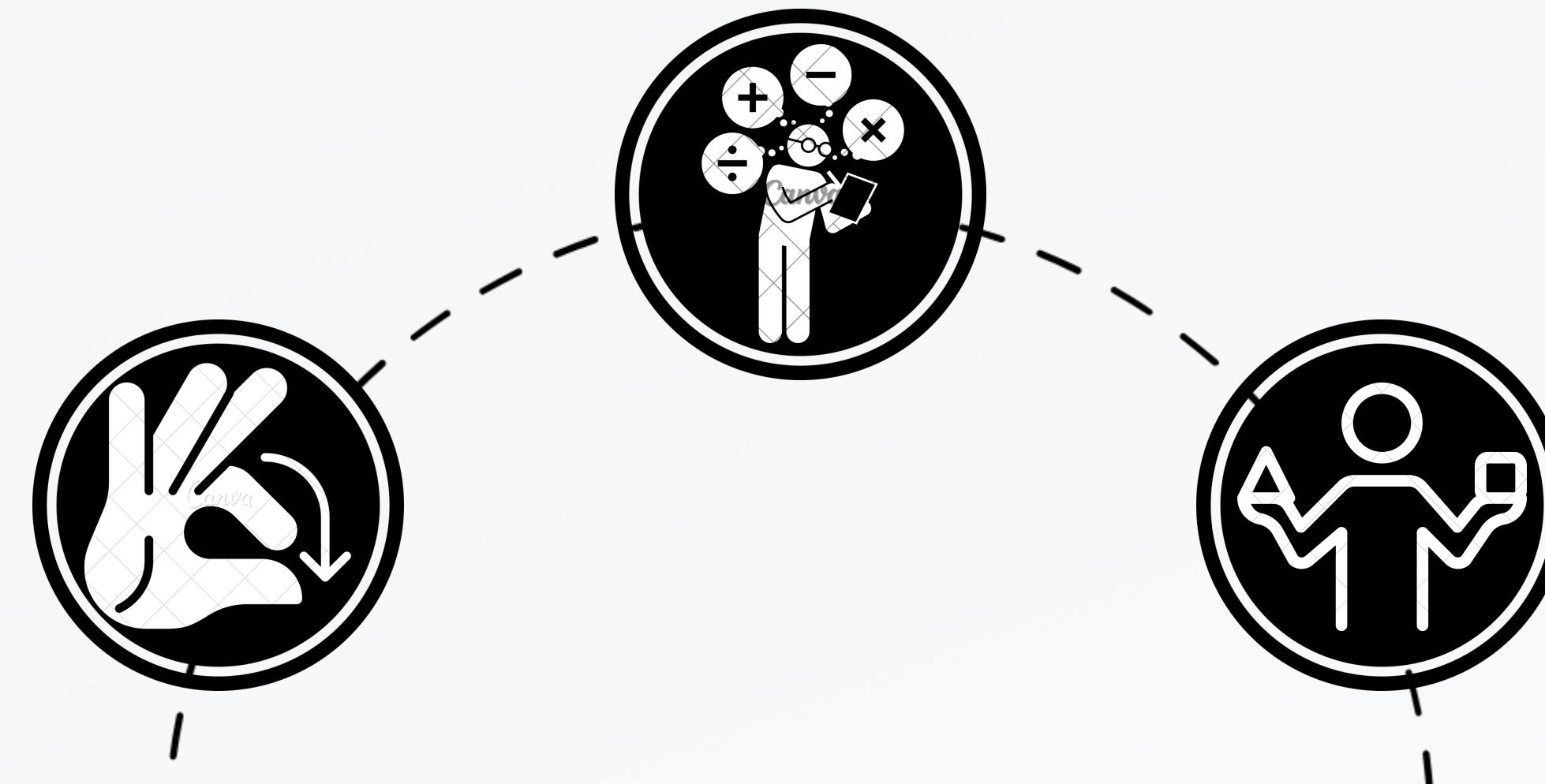
Reduce the hamiltonian
11 qubits -> 6 qubits

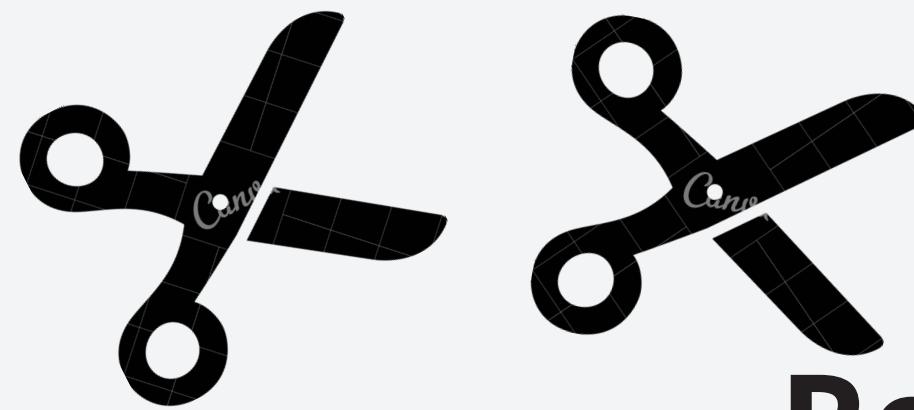
Step 2

Solve the new hamiltonian using the QAOA algorithm.
and using Quantum Adiabatic Evolution

Step 3

Compare the
different solutions





Reduced hamiltonian example

$$H = - \sum_{i=0}^{L-1} \frac{1}{2} v[i] \cdot \left(\frac{1-Z(i)}{2} \right) + \left(\sum_{i=0}^{L-1} \frac{1}{2} d[i] \cdot \left(\frac{1-Z(i)}{2} \right) - d_{max} \right)^2$$

QAOA algorithm

Quantum Adiabatic Evolution

VALUES

QAOA

```
from qibo.symbols import Z, X
from qibo.hamiltonians import SymbolicHamiltonian

def build_cost_hamiltonian(values: list[int], duration: list[int], max_duration: int) -> SymbolicHamiltonian:
    cost_hamiltonian = 0
    L = len(values)
    d_max = max_duration

    term_1 = sum(values[i] * (1 - Z(i)) / 2 for i in range(L))
    a = sum(duration[i] * (1 - Z(i)) / 2 for i in range(L))

    cost_hamiltonian += term_1 + a
```

- [('9.95 %', '010111'), ('9.49 %', '011110'), ...]
- ...The best solution is: 010111

- the solution is: 011101

ADIABATIC

```
from qibo.hamiltonians import Hamiltonian

# build initial ( $H_0$ ) and target ( $H_1$ ) hamiltonians
H0 = build_initial_hamiltonian(nqubits=nqubits)
H1 = build_cost_hamiltonian(values=values, duration=duration, max_duration=max_duration)

# calculate the dense hamiltonian from the symbolic hamiltonian
H1_dense = Hamiltonian(nqubits, H1.calculate_dense().matrix)
H0_dense = Hamiltonian(nqubits, H0.calculate_dense().matrix)

dt = 0.1
T = 20

def s(t): return t

# construct the adiabatic model
adiabatic_model = AdiabaticEvolution(H0_dense, H1_dense, s, dt)
```

CSFQ - ISING SCHEDULES SETUP

$$\hat{H}(t) = \sum_i (h_i^x(t)\hat{X} + h_i^z(t)\hat{Z}) + \frac{1}{2} \sum_{i,j} J_{ij}(t)\hat{Z}_i\hat{Z}_j$$

$$\hat{H}(t_0) = \sum_i h_i^x(0) \hat{X}_i = \sum_i \hat{X}_i$$

$$\hat{O}_i := \left(\frac{1 - \hat{Z}_i}{2} \right)$$

$$\hat{H}(t_f) = -\sum_i v_i \hat{O}_i + \left(\sum_i \hat{O}_i d_i + \sum_j 2^j \hat{O}_j - d_{max}\right)^2$$

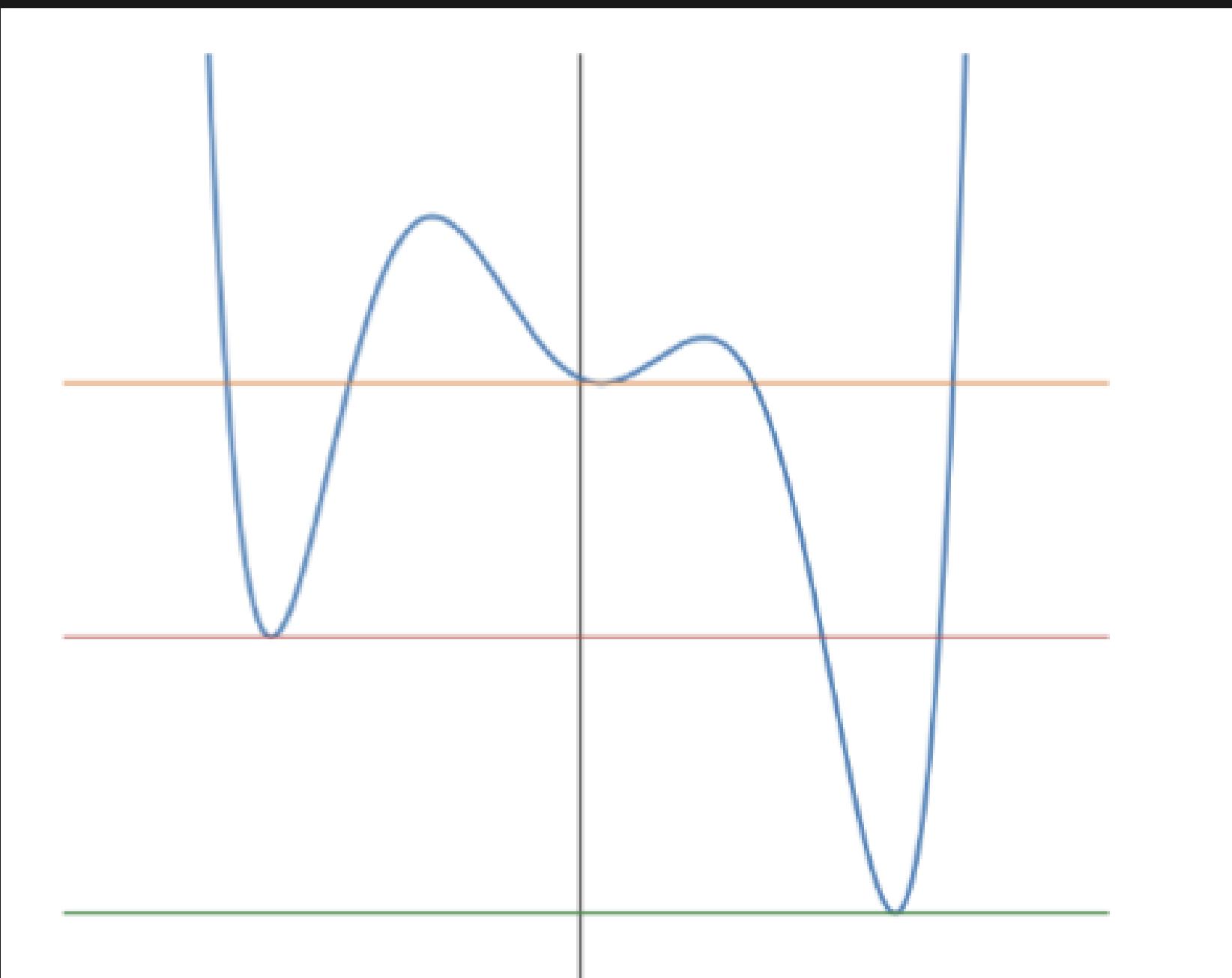
$$\hat{H}(t_f) \approx -\sum_i (v_i + 2d_{max}d_i)\hat{Z}_i + \sum_{i,j} d_id_j\hat{Z}_i\hat{Z}_j$$

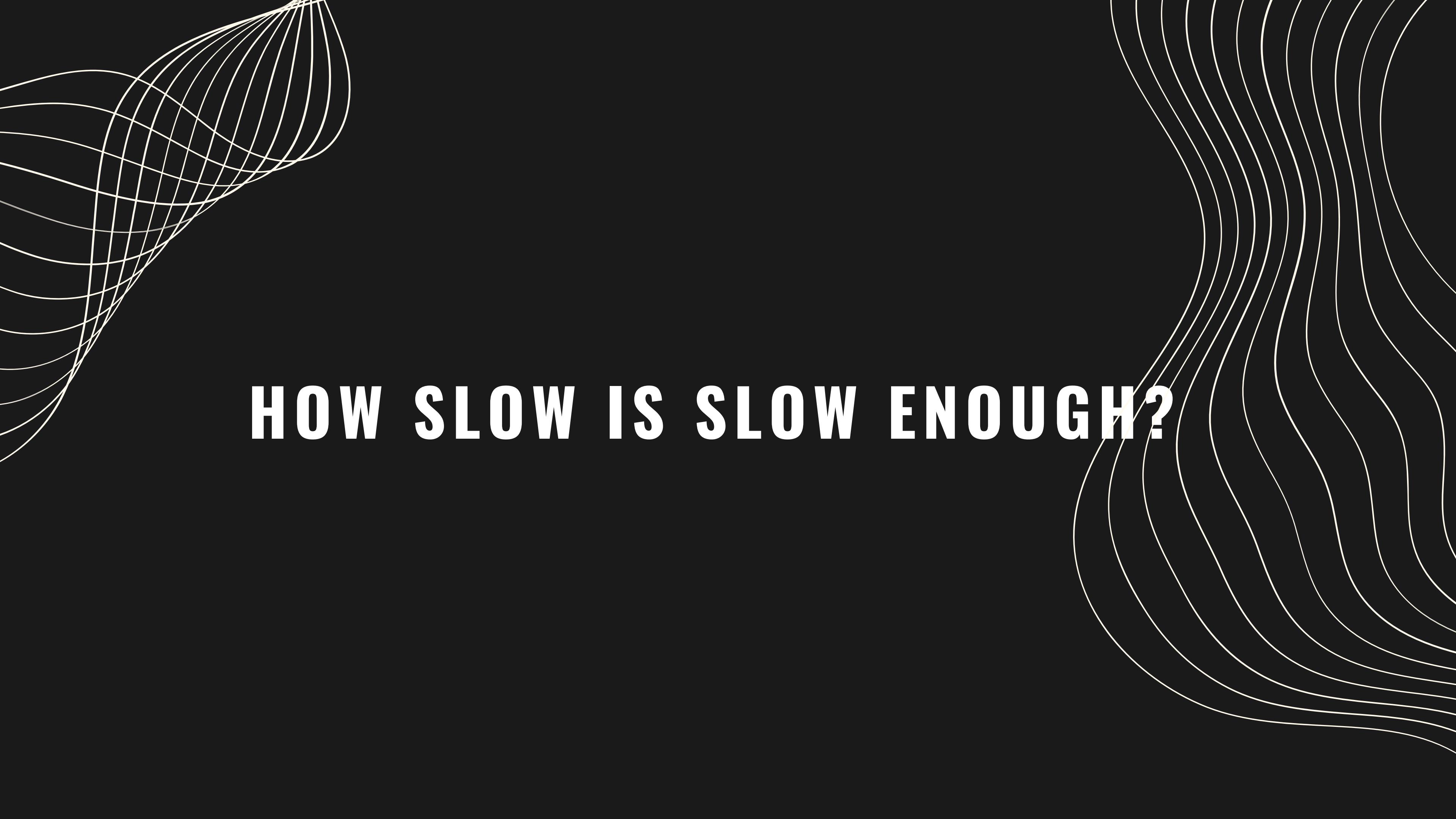
$$\hat{H}(t_f) \approx -\sum_i h_i^z(t_f)\hat{Z}_i + \sum_{i,j} J_{ij}(t_f)\hat{Z}_i\hat{Z}_j$$

$$\begin{aligned}t &: t_0 \longrightarrow t_f \\h_i^x(t) &: 1 \longrightarrow 0 \\h_i^z(t) &: 0 \longrightarrow v_i + 2d_id_{max} \\J_{ij}(t) &: 0 \longrightarrow d_id_j\end{aligned}$$

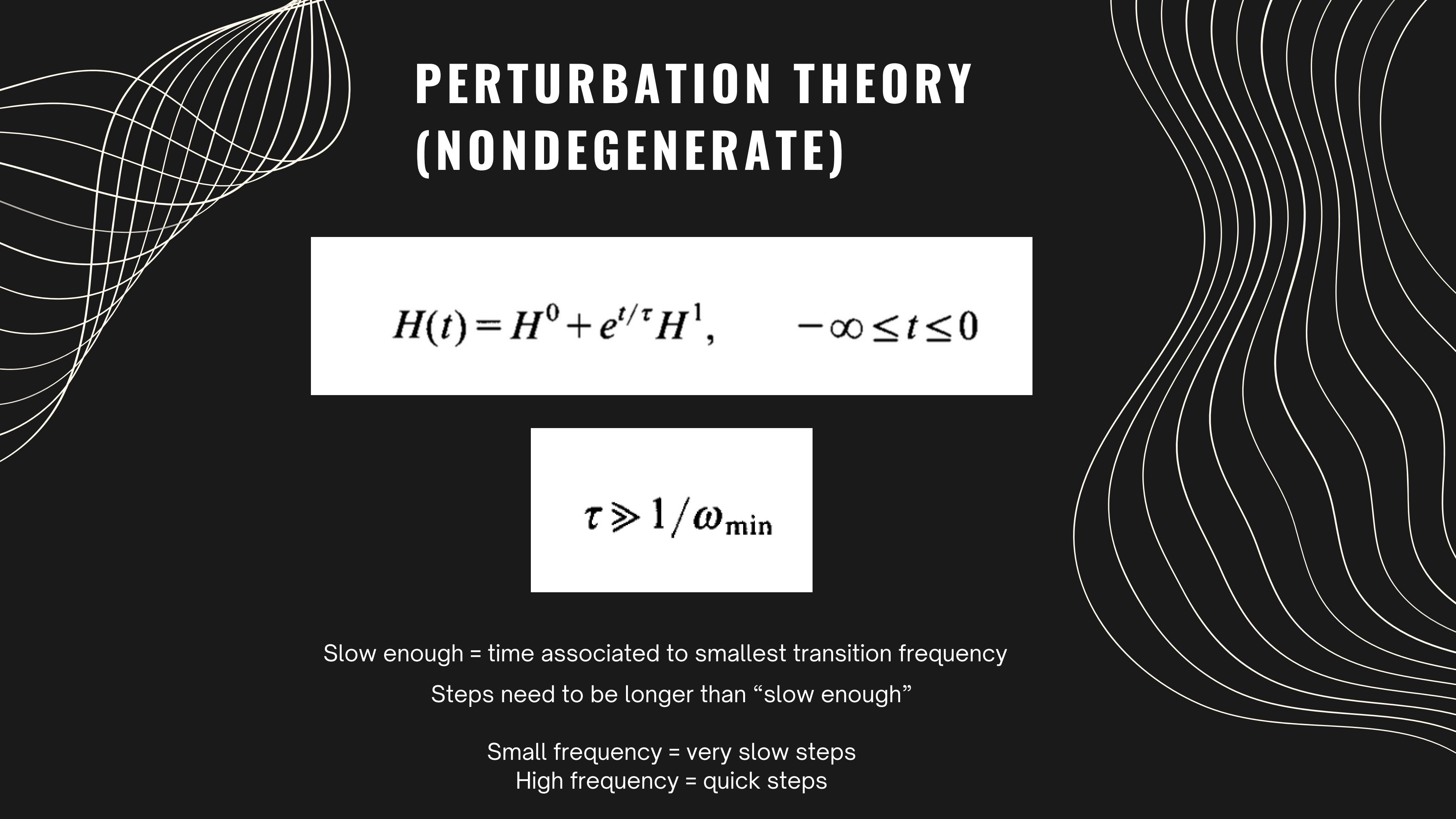
Intuition:

Change potential to tilted washboard slowly,
so previous ground state fits new potential
ground state





HOW SLOW IS SLOW ENOUGH?



PERTURBATION THEORY (NONDEGENERATE)

$$H(t) = H^0 + e^{t/\tau} H^1, \quad -\infty \leq t \leq 0$$

$$\tau \gg 1/\omega_{\min}$$

Slow enough = time associated to smallest transition frequency

Steps need to be longer than “slow enough”

Small frequency = very slow steps

High frequency = quick steps

Energy Plots

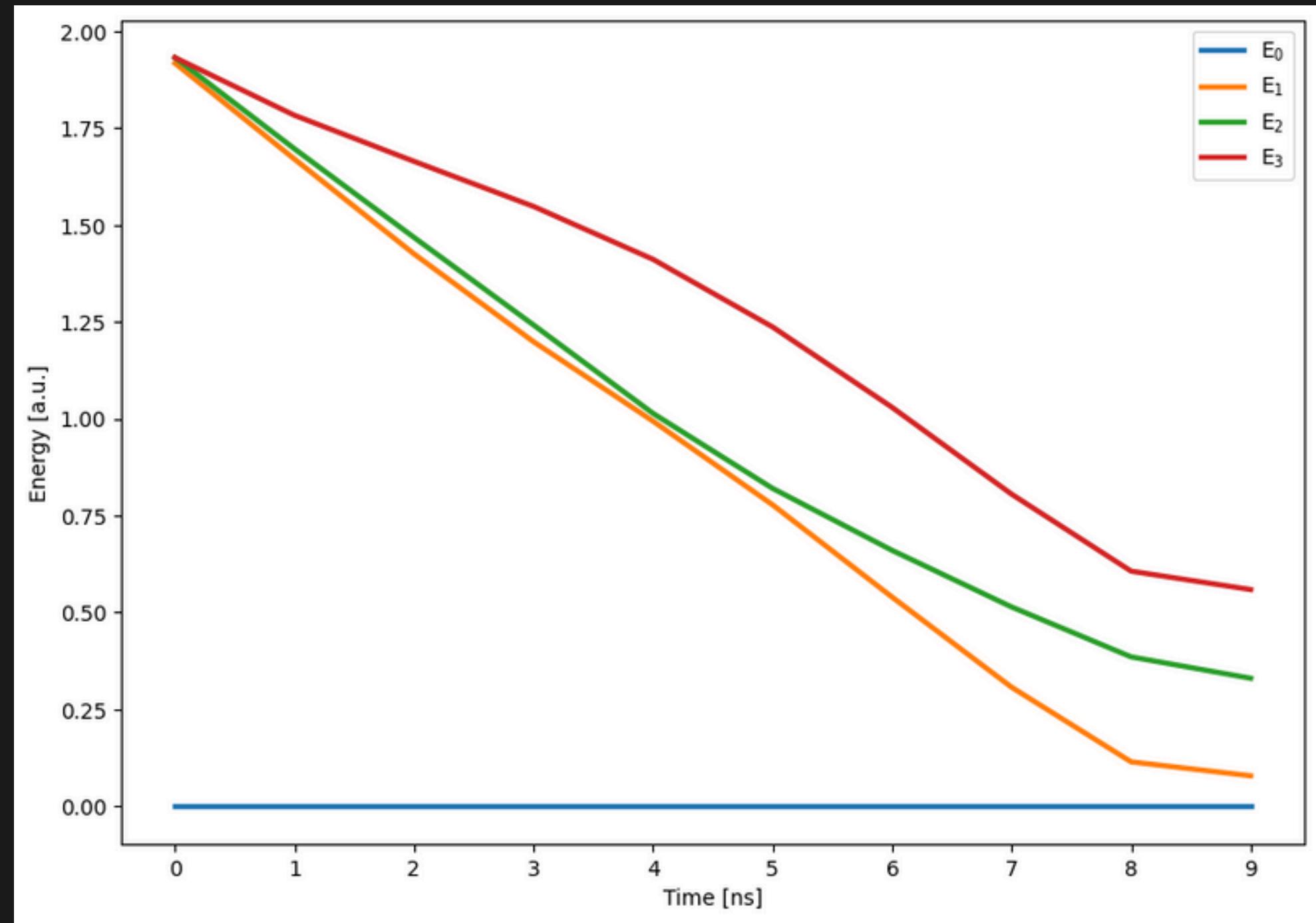


Fig 1: Energy levels of the states with a linear Schedule

Energy Plots

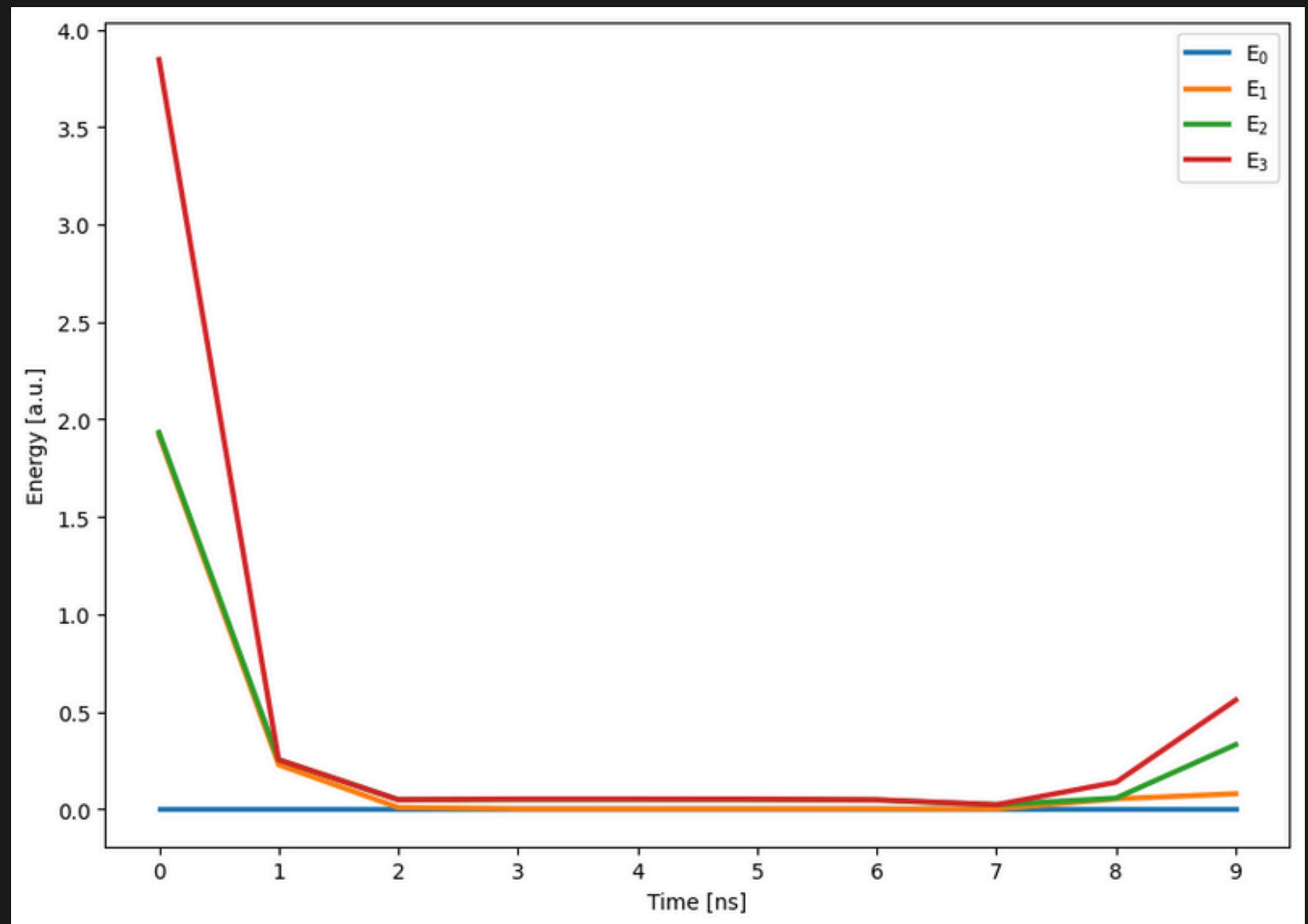


Fig 2: Energy levels of the states with an exponential Schedule

Fluxes

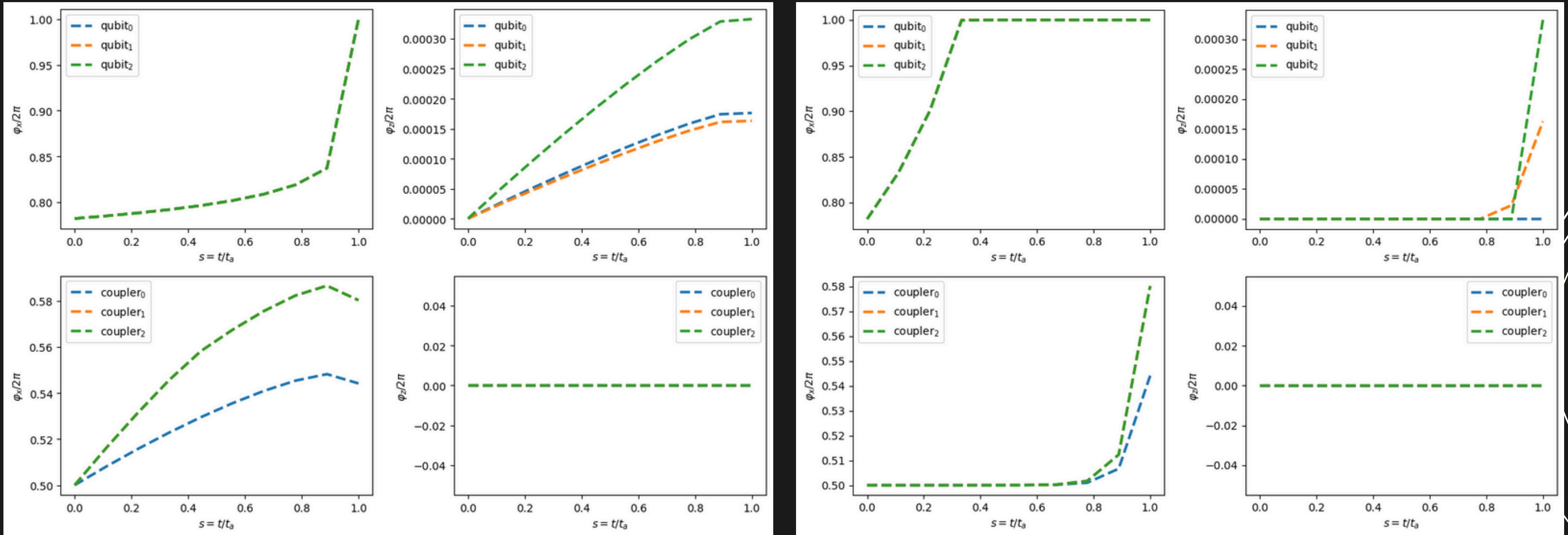


Fig 3-4: Time evolution of calculated fluxes. Left: linear, Right:exponential schedules

FINAL REMARKS

*Let's assume a homogeneous
isotropically constant fun time
throughout this hackathon*

...

Everyone was having fun (we hope...)

