Quantum algorithm for calculating a local topological order parameter of a dimerized spin chain

Aalto University
School of Science

Sebastian Dominguez-Calderon*, Marcel Niedermeier, Pascal Vecsei, Jose Lado

1. Introduction

The Heisenberg model describes magnetic interactions between spins fixed on a lattice. This model provides information on how macroscopic properties like magnetism emerge from quantum interactions. On the other hand, quantum computing leverages quantum mechanics to perform calculations, making it an adequate platform to simulate and analyze quantum matter. The present work focuses on developing a quantum algorithm, which calculates quantized Berry phases on a spin chain, which serve as a local topological order parameter. Intuitively, the aim is to characterize different states of a spin chain.

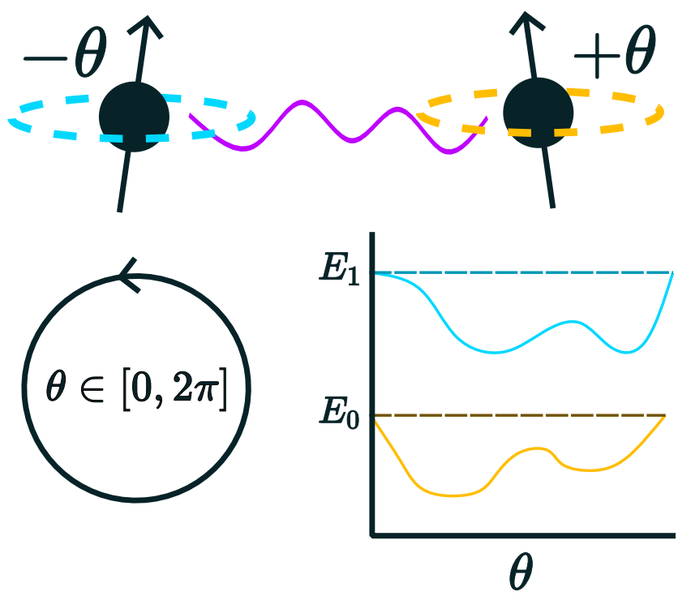
2. Model: Dimerized Heisenberg spin chain

In the dimerized model, one considers an alternating pattern of **interaction strengths** between spins on a 1D chain:

Each link can be characterized by the **Berry phase**, a local topological order parameter, quantized to two values: $\varphi_B = \{0, \pi\}$. Thus, different patterns (states) can be distinguished.

3. Methodology

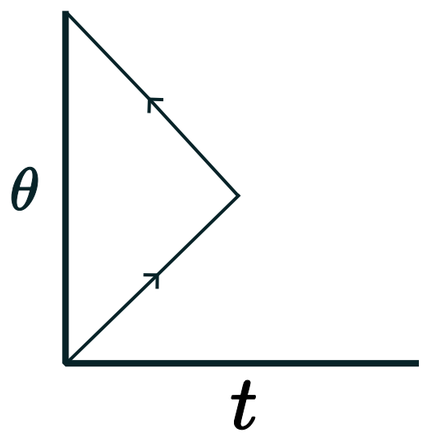
In order to calculate the Berry phase on a link, the link is "twisted" slowly in a closed loop, such that the energy structure does not collapse:



When evolving a system in time in a closed loop, the system picks up two phases, the **dynamical** phase and a **geometrical** (Berry) phase:

$$|\psi\rangle \rightarrow e^{i\,\varphi_D}\, e^{i\,\varphi_B} |\psi\rangle$$

To remove the dynamical phase, the system can be (in a sense) evolved **forward** in time, and then **backwards**:



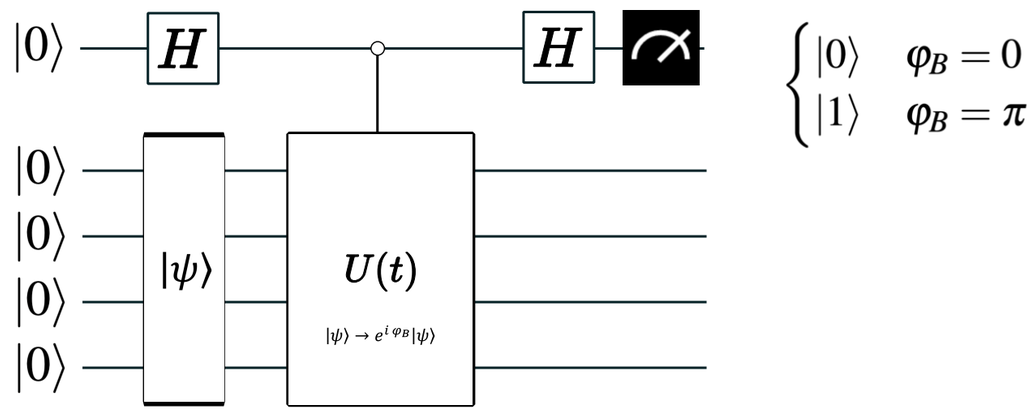
The final state only includes the Berry phase:

$$|\psi\rangle \rightarrow e^{i\,\varphi_B} |\psi\rangle$$

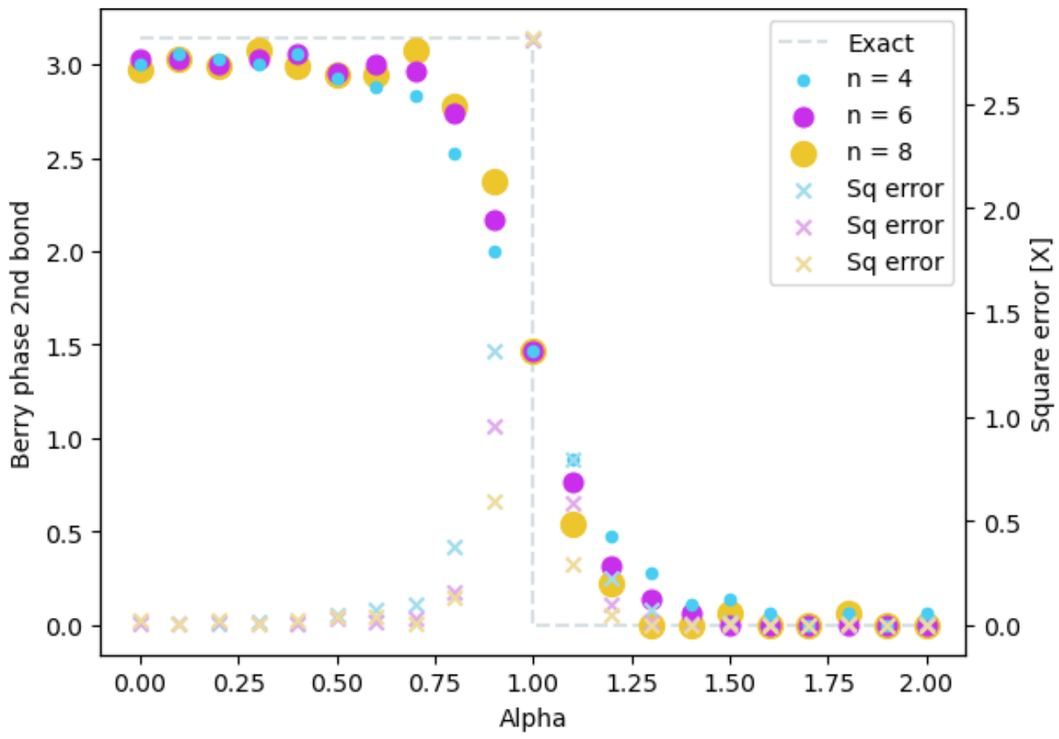
To goal is to measure the Berry phase.

4. Algorithm

Initial spin states can be encoded as **qubits**. The evolution can be encoded into **qubit operations** via approximation schemes. An auxiliary qubit is prepared in a **superposition** between 0 and 1. The evolution of the chain is conditioned on the auxiliary. The Berry phase ends up **modulating** the probability of measuring either 0 or 1 on the auxiliary qubit.



5. Results & conclusion



The algorithm was tested for several dimer strengths (alpha) and system sizes (n), setting J = 1. Errors increase when the alternating link strengths approach (alpha = J). A phase transition can be seen at a link, where the topological order changes from π to 0. This work shows a quantum algorithm which can distinguish different states in dimerized spin chains and can characterize phase transitions, highlighting the use of quantum computers to understand quantum matter.

Poster download, in-depth explanations, & references in the QR code!

SCAN ME

* sebastiandom@protonmail.com