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Diffusion Model Distillation

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Abstract

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List of Abbreviations

ADD Adversarial Diffusion Distillation

AI Artificial Intelligence

ALD Annealed Langevin Dynamic

CLS Classification

CNN Convolutional Neural Network

DDIM Denoising Diffusion Implicit Model

DDPM Denoising Diffusion Probablistic Model

DM Diffusion Model

ELBO Evidence Lower Bound

GAN Generative Adversarial Network

iid independent and identically distributed

KL Kullback-Leibler

LADD Latent Adversarial Diffusion Distillation

LLM Large Language Model

LPIPS Learned Perceptual Image Patch Similarity

MC Markov Chain

MCMC Markov Chain Monte Carlo

MLE Maximum Likelihood Estimation

MLP Multi Layer Perceptron

MSE Mean Squared Error

NCSN Noise-Conditional Score Network

NF Normalizing Flow

NNL Negative Log-Likelihood

ODE Ordinary Differential Equation

RNN Recurrent Neural Network

RoPE Rotary Positional Embedding

SDE Stochastic Differential Equation

SGM Score-Based Generative Model

SOTA State-Of-The-Art

VAE Variational Autoencoder

VI Variational Inference

ViT Vision Transformer

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Chapter 1

Introduction

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1.1 Generative Models

Generative models are a class of machine learning models that are trained to learn the true data distribution to generate new synthetic data elements. They are of high relevance for text and image generation tasks. In particular, famous models for text generation, also known as large language models (LLMs) include ChatGPT [1], Claude [4], Gemini [67] or Lama [70] whereas Stable Diffusion [? ?], Flux [34] and Pixart [11] are widely used for image generation tasks. Both are being used more and more in the professional world and in daily life. Several different generator methods have been developed, such as variational autoencoders (VAEs) [30], generative adversarial networks (GANs) [20, 47], normalizing flows (NFs) [51], and diffusion models (DMs) [56].

The principal idea of a generative model f_θ is to learn a mapping from a simple distribution, e.g., a normal distribution $p_{\text{latent}}(\mathbf{z}) = \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$, often called the latent distribution, to a highly complex data distribution $p_{\text{data}}(\mathbf{x})$. A new data element \mathbf{x} is obtained by sampling from the latent distribution $\mathbf{z} \sim p(\mathbf{z})$ and applying the model

$$\mathbf{x} = f_\theta(\mathbf{z}) \sim p_{\text{model}}(\mathbf{x}) \approx p_{\text{data}}(\mathbf{x}) \quad (1.1)$$

where the model's output distribution $p_{\text{model}}(\mathbf{x})$ should approximate the true data distribution as closely as possible.

In the following, the main concepts of VAEs, NFs and GANs are briefly presented to provide a comprehensive overview followed by an extensive discussion of diffusion and flow-based generative models in chapter 1.2 that are the most promising models today.

1.1.1 Variational Autoencoder

Autoencoder

An autoencoder [23] is a neural network commonly used for dimensionality reduction tasks. It aims to compress high-dimensional data to lower-dimensional space by preserving important information. The architecture consists of two main parts, namely the encoder and the decoder. The encoder maps high-dimensional input data to a latent space, effectively compressing the data, and the decoder learns to reconstruct the original data from its latent space representation. Typically, as learning objective, a simple reconstruction loss, e.g. mean-squared error, is applied comparing the original input data and the output of the autoencoder.

Evidence Lower Bound Loss (ELBO)

The VAE belongs to the class of probabilistic models. A common learning objective for probabilistic models is maximum likelihood estimation (MLE). Given a set of observations $\{\mathbf{x}_i\}_{i=1}^N$ the likelihood is given under the assumption of identical and independent distributed samples

iid as

$$\mathcal{L}_{\text{MLP}} = \prod_{i=1}^n p_{\text{model}}(\mathbf{x}_i) \quad . \quad (1.2)$$

However, in practice, minimizing the negative log-likelihood is often preferred over maximizing the likelihood due to possible instabilities caused by the product of likelihoods

$$\mathcal{L}_{\text{NLL}} = - \sum_{i=1}^n \log p_{\text{model}}(\mathbf{x}_i) \quad . \quad (1.3)$$

This approach works well for simple distributions. However, for models that include latent variables $\mathbf{z} = \mathbf{z}_{1:m}$, such as VAEs, the likelihood becomes an intractable marginalization over the latent space

$$p(\mathbf{x}) = \int p(\mathbf{x}|\mathbf{z})p(\mathbf{z})d\mathbf{z} \quad (1.4)$$

leading to the the so-called evidence lower bound (ELBO) loss [8] being used as training objective for VAEs.

In general, untractable probability distributions are a core challenge in modern statistics. This is especially the case for posterior distributions in Bayesian statistics. Assuming the joint probability distribution

$$p(\mathbf{x}, \mathbf{z}) = p(\mathbf{z}) \cdot p(\mathbf{x}|\mathbf{z}) \quad (1.5)$$

is given with data $\{\mathbf{x}_i\}_{i=1}^N$ and latent variables $\{\mathbf{z}_j\}_{j=1}^M$. During inference, the in general untractable posterior $p(\mathbf{z}|\mathbf{x})$ is the quantity that needs to be computed. In the past, the dominant approach to tackle this challenge was using Monte Carlo Markov Chain simulations (MCMC) [69], which approximate the true posterior distribution by sampling. The drawback of these methods is that the sampling procedures are very slow. Therefore, in recent years, a new method has gained popularity, called variational inference (VI) [9]. In contrast to MCMC VI reformulates the problem as an optimization task

$$q^*(\mathbf{z}) = \arg \min_{q(\mathbf{z}) \in \mathcal{Q}} \text{KL} [q(\mathbf{z}) \parallel p(\mathbf{z}|\mathbf{x})] \quad (1.6)$$

where \mathcal{Q} is a family of distributions. The goal is to find the proposal density $q(\mathbf{z}) \in \mathcal{Q}$ that best approximates the true posterior distribution. To compare the proposal density and the posterior the Kullback-Leibler divergence (KL) [33] is used having the general form of

$$\text{KL}[p_0(\mathbf{x}) \parallel p_1(\mathbf{x})] = \int p_0(\mathbf{x}) \log \frac{p_0(\mathbf{x})}{p_1(\mathbf{x})} d\mathbf{x} \quad (1.7)$$

comparing two distributions p_0 and p_1 . The key for a good choice of \mathcal{Q} is to choose a family that is complex enough so that it contains a $q^*(\mathbf{z})$ close to the true posterior but simple enough to enable efficient optimization. Applying VI to the problem of finding the true posterior yields

$$\text{KL} [q(\mathbf{z}) \parallel p(\mathbf{z}|\mathbf{x})] = \int q(\mathbf{z}) \log \frac{q(\mathbf{z})}{p(\mathbf{z}|\mathbf{x})} d\mathbf{z} \quad (1.8)$$

$$= \int q(\mathbf{z}) \log q(\mathbf{z}) d\mathbf{z} - \int q(\mathbf{z}) \log p(\mathbf{z}|\mathbf{x}) d\mathbf{z} \quad (1.9)$$

$$= \int q(\mathbf{z}) \log q(\mathbf{z}) d\mathbf{z} - \int q(\mathbf{z}) \log \frac{p(\mathbf{z}, \mathbf{x})}{p(\mathbf{x})} d\mathbf{z} \quad (1.10)$$

$$= \mathbb{E} [\log q(\mathbf{z})] - \mathbb{E} [\log p(\mathbf{z}, \mathbf{x})] + \log p(\mathbf{x}) \quad (1.11)$$

From here, the ELBO loss is obtained by

$$\log p(\mathbf{x}) \geq \log p(\mathbf{x}) - \text{KL} [q(\mathbf{z}) \parallel p(\mathbf{z}|\mathbf{x})] \quad (1.12)$$

$$= \mathbb{E} [\log p(\mathbf{z}, \mathbf{x})] - \mathbb{E} [\log q(\mathbf{z})] \quad (1.13)$$

$$= \mathbb{E} [\log p(\mathbf{x}|\mathbf{z})] - \text{KL} [q(\mathbf{z}) \parallel p(\mathbf{z})] \quad (1.14)$$

$$= \text{ELBO} \quad . \quad (1.15)$$

Maximizing the ELBO loss is equivalent to minimizing the KL divergence from 1.6 and most importantly, it is tractable.

For training VAEs the ELBO objective is used. The difference compared to the MSE loss usually used for training autoencoders is that it contains an additional term from the KL divergence that enforces a specific structure on the latent space. Let θ be the decoder parameters and ϕ the encoder parameters then the objective of the VAE is given by

$$\mathcal{L}_{\text{VAE}} = \underbrace{\mathbb{E}_{q_{\phi}(\mathbf{z}|\mathbf{x})} [-\log p_{\theta}(\mathbf{x} | \mathbf{z})]}_{\text{Reconstruction Loss}} + \underbrace{\text{KL} [q_{\phi}(\mathbf{z} | \mathbf{x}) \parallel p(\mathbf{z})]}_{\text{Regularization}} \quad . \quad (1.16)$$

In practice, the latent distribution is often set to a standard normal distribution $p_{\text{latent}}(\mathbf{z}) = \mathcal{N}(\mathbf{z}; \mathbf{0}, \mathbf{1})$ and the reparameterization trick [30] enabling gradient computation is applied. After training, a new data point can be generated by sampling from the latent distribution $\mathbf{z} \sim p_{\text{latent}}(\mathbf{z})$ and applying the decoder of the trained model.

1.1.2 Normalizing Flow

Normalizing flows (NFs) [51] are another class of probabilistic models, similar to VAEs, which learn a mapping between a latent distribution to the data distribution. However, NFs have some key differences compared to VAEs. First, NFs are fully invertible meaning the network F_{θ} consists of a sequence of (simpler) bijective operations so that an inverse transformation $\overline{F_{\theta}}$ mapping back from data to latent distribution exists. As a consequence, the latent space and the data space need to have the same dimension.

A second decisive advantage is that NFs allow the exact and tractable computation of the

likelihood using the change of variables formula

$$p_{\text{latent}}(\mathbf{z}) = p_{\text{model}}(\mathbf{x}) \left| \frac{\partial F_{\theta}(\mathbf{z})}{\partial \mathbf{z}} \right| = p_{\text{model}}(F_{\theta}(\mathbf{z})) \left| \frac{\partial F_{\theta}(\mathbf{z})}{\partial \mathbf{z}} \right| \quad (1.17)$$

for the latent distribution and

$$p_{\text{model}}(\mathbf{x}) = p_{\text{latent}}(\mathbf{z}) \left| \frac{\partial F_{\theta}(\mathbf{z})}{\partial \mathbf{z}} \right|^{-1} = p_{\text{latent}}(\bar{F}_{\theta}(\mathbf{x})) \left| \frac{\partial \bar{F}_{\theta}(\mathbf{x})}{\partial \mathbf{x}} \right| . \quad (1.18)$$

It is worth noting that computing the likelihood (1.18) includes computing the determinant of a jacobian matrix. Therefore, NFs rely on carefully designed invertible layers (e.g., affine coupling layers [17]) where the determinant of the jacobian can be easily computed. This enables training via exact maximum likelihood estimation and does not rely on the ELBO loss like VAEs.

1.1.3 Generative Adversarial Network

In contrast to VAEs, NFs or DMs, GANs [20] belong to the class of non-probabilistic generators which do not require an explicit likelihood function. The model, also called the generator G , is trained in an adversarial training setup that contains, on the one hand, the generator that should generate new data points being as realistic as possible and, on the other hand, a discriminator that should be able to distinguish if a data point is generated by the generator (“fake”) or from the original dataset (“real”). The generator and the discriminator are alternately trained, where the generator tries to fool the discriminator by generating more realistic data points, and the discriminator D becomes better at distinguishing “fake” from “real”. This is formalized in a MinMax condition

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))] \quad (1.19)$$

where $G(\mathbf{z})$ is a sample generated by the generator and \mathbf{x} is an element from the original dataset. The prediction of the discriminator is close to one if it identifies its input as “real” and close to zero if it is identified as “fake”.

GANs were the dominant class of network architectures for image generating [28, 39, 55] but were overtaken by diffusion models in the last years. The GANs training process turned out to be often instable, and extensive hyperparameter tuning and engineering is required [37, 54]. One challenge is to carefully balance the learning dynamics of the generator and the discriminator. If one overpowers the other, e.g., the discriminator always perfectly distinguishes “real” from “fake” samples, the generator stops learning. Another difficulty is known as mode collapse [68], meaning that the generator just produces samples from a part of the original data distribution but fails to completely capture it. A third challenge is the problem of exploding and vanishing gradients [5] which prevents effective learning of the generator.

1.2 Diffusion Models and Flow Matching

Diffusion models [24, 52, 57, 58, 59, 63] are probabilistic generative models that have achieved remarkable success in image generation tasks [2, 34], demonstrating exceptional performance in terms of image quality and image diversity. While they require significant computational resources and have slower inference compared to some alternatives [20, 30, 51], they have become the dominant generative modeling paradigm in the research community. The main principles of diffusion models are reflected in two processes motivated by non-equilibrium thermodynamics, namely the forward process and the reverse process. In the forward process, the original images are gradually perturbed by systematically adding Gaussian noise until only pure noise is left. The reverse process describes the iterative noise removal by the diffusion model until a clean image is obtained. During image generation with a diffusion model, one starts from randomly sampled noise and applies the diffusion model iteratively, progressively denoising at each step. This repeated application gradually produces a clean, high-quality image.

Over time, several different formulations of diffusion models were proposed. However, there are three dominant formulations recognized in the literature [14]: Denoising diffusion probabilistic models (DDPMs) [24, 57], score-based generative models (SGMs) [59] and stochastic differential equations (SDEs) [63].

Recently, a generalization of DMs was proposed, known as flow matching or Flow-based models [38, 40]. Unlike classical diffusion, flow matching models directly learn a vector field that defines the path between the noise distribution and the data distribution. By enforcing a straight path trajectory, the sampling process becomes computationally more efficient. This formulation was successfully applied in several new models like Flux-dev [34] or Sana [72].

In the following, the different formulations of diffusion models, DDPM, DDIM, SGM and SDE, are presented followed by an introduction to the concept of flow matching.

1.2.1 Denoising Diffusion Probabilistic Models

Unless stated otherwise, the discussion and mathematical formulations of DDPM presented in this chapter are based on the lecture notes by Inga Strümke and Helge Langseth [65].

In DDPMs [24] the forward and reverse process are modeled as Markov Chain (MC), meaning that each step only depends on the immediate previous step. This key concept is illustrated in fig. 1.1. Let $\mathbf{x}_0, \dots, \mathbf{x}_T$ be the states of the MC with \mathbf{x}_0 representing the clean image and \mathbf{x}_T representing pure noise, then one step of the reverse process is given by $p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)$. This shows that the slightly denoised state \mathbf{x}_{t-1} just depends on state \mathbf{x}_t and θ denotes the parameters of the trained diffusion model being used to predict \mathbf{x}_{t-1} . The forward process, which progressively adds noise proceeds from right to left in fig. 1.1 and is given by $q(\mathbf{x}_t|\mathbf{x}_{t-1})$ where no learned parameters are required.

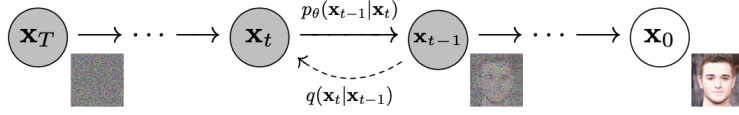


Figure 1.1: The graphical model of DDPM from [24].

Forward Process

Formally, the forward process is defined as a MC that progressively injects noise into the clean image $\mathbf{x}_0 \sim p_{\text{data}}$ which is transformed such that for a large number of steps T the final latent variable \mathbf{x}_T approximately follows a new often simpler prior distribution p_{prior} . This boundary condition is necessary so that in the reverse process, which starts with a sample of p_{prior} , the model is able to transfer it back to the image domain p_{data} . The transformation is governed by a variance schedule $\{\beta_t\}_{t=1}^T$, e.g., a linear [24], a cosine [43] or a learnable [29] schedule, to ensure smooth interpolation between p_{prior} and p_{data} . The schedule dictates the incremental increase of variance β_t .

Concretely, the next state \mathbf{x}_t within the MC is computed by

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}) = K(\mathbf{x}_t; \mathbf{x}_{t-1}, \beta_t) \quad (1.20)$$

with the Markov kernel K , which is chosen as a Gaussian Markov diffusion kernel in DDPM leading to the explicit form

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}) := \mathcal{N}(\mathbf{x}_t; \sqrt{1 - \beta_t}\mathbf{x}_{t-1}, \beta_t\mathbf{I}) \quad (1.21)$$

and the prior distribution $p_{\text{prior}}(\mathbf{x}) = \mathcal{N}(\mathbf{x}; \mathbf{0}, \mathbf{I})$.

A key property of this Gaussian formulation is the existence of a closed-form expression for sampling any \mathbf{x}_t . At every timestep t the intermediate latent variable is computed by

$$\mathbf{x}_t = \sqrt{\alpha_t}\mathbf{x}_{t-1} + \sqrt{1 - \alpha_t}\mathbf{z}_t \quad (1.22)$$

$$= \sqrt{\alpha_t\alpha_{t-1}}\mathbf{x}_{t-2} + \sqrt{\alpha_t(1 - \alpha_{t-1})}\mathbf{z}_{t-1} + \sqrt{1 - \alpha_t}\mathbf{z}_t \quad (1.23)$$

$$= \sqrt{\alpha_t\alpha_{t-1}}\mathbf{x}_{t-2} + \sqrt{1 - \alpha_t\alpha_{t-1}}\mathbf{z}_{t-1:t} \quad (1.24)$$

$$= \dots \quad (1.25)$$

$$= \sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t}\mathbf{z}_{0:t}. \quad (1.26)$$

where $\bar{\alpha}_t = \prod_{s=1}^t \alpha_s$ which shows that $q(\mathbf{x}_t|\mathbf{x}_0)$ follows a normal distribution

$$q(\mathbf{x}_t|\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t; \sqrt{\bar{\alpha}_t}\mathbf{x}_0, (1 - \bar{\alpha}_t)\mathbf{I}) \quad (1.27)$$

This underlines that during the forward process the mean of \mathbf{x}_t is gradually moved towards zero and the variance increases towards one. Therefore, the closed-form solution for \mathbf{x}_t is

$$\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \boldsymbol{\epsilon} \quad (1.28)$$

with $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. This possibility to directly sample every \mathbf{x}_t is crucial for efficient training.

Reverse Process

The reverse process recovers the clear image by iteratively removing noise. The authors of [19] showed that in the limit of infinitesimal small steps the reverse process has the same distributional form as the forward process, a Gaussian. Particularly, in the reverse process $q(\mathbf{x}_{t-1}|\mathbf{x}_t)$ is the quantity of interest. However, it is not tractable due to

$$q(\mathbf{x}_{t-1}|\mathbf{x}_t) = q(\mathbf{x}_t|\mathbf{x}_{t-1}) \frac{q(\mathbf{x}_{t-1})}{q(\mathbf{x}_t)} \quad (1.29)$$

using Bayes law where $q(\mathbf{x}_{t-1})$ and $q(\mathbf{x}_t)$ are marginal distributions requiring the integration over the whole distribution $q(\mathbf{x}_0)$. Therefore, it is modeled by a neural network parameterized by θ that should learn the Gaussian distribution for each step. Although $q(\mathbf{x}_{t-1}|\mathbf{x}_t)$ is not tractable $q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)$ is which is leveraged during training. The joint distribution learned by the model is given by

$$p_\theta(\mathbf{x}_{1:T}|\mathbf{x}_0) = p(\mathbf{x}_T) \prod_{t=1}^T p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) \quad (1.30)$$

where $p(\mathbf{x}_T)$ is pure noise and $p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)$ is a Gaussian with learned mean and covariance

$$p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t)) \quad (1.31)$$

As seen in the forward process, the variance of the noise follows a predefined schedule which means that the neural network just has to learn the mean $\boldsymbol{\mu}_\theta(\mathbf{x}_t, t)$ and the variance is fixed by $\boldsymbol{\Sigma}_\theta(\mathbf{x}_t, t) = \sigma_t^2 \mathbf{I}$..

Objective Function

Diffusion models belong to the class of probabilistic generative models. A natural choice for the objective of these models is the negative log-likelihood $-\log p_\theta(\mathbf{x}_0)$. As discussed in sec. 1.1.1 the likelihood is not always tractable. Therefore, the ELBO loss is used as learning objective. In the following the ELBO loss is derived for diffusion models. This derivation is mainly based on [46] and [24]. Starting with the expectation of the negative log-likelihood

$$- \mathbb{E}_{q(\mathbf{x}_0)} [\log p_\theta(\mathbf{x}_0)] \quad (1.32)$$

$$= -\mathbb{E}_{q(\mathbf{x}_0)} \left[\log \left(\int d\mathbf{x}_1 \dots d\mathbf{x}_T p(\mathbf{x}_T) \prod_{t=1}^T p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t) \right) \right] \quad (1.33)$$

$$= -\mathbb{E}_{q(\mathbf{x}_0)} \left[\log \left(\int d\mathbf{x}_1 \dots d\mathbf{x}_T p(\mathbf{x}_T) q(\mathbf{x}_1, \dots, \mathbf{x}_T | \mathbf{x}_0) \prod_{t=1}^T \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_t | \mathbf{x}_{t-1})} \right) \right] \quad (1.34)$$

$$= -\mathbb{E}_{q(\mathbf{x}_0)} \left[\log \mathbb{E}_{q(\mathbf{x}_1, \dots, \mathbf{x}_T | \mathbf{x}_0)} \left[p(\mathbf{x}_T) \prod_{t=1}^T \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_t | \mathbf{x}_{t-1})} \right] \right] \quad (1.35)$$

using Jensen's inequality

$$- \mathbb{E}_{q(\mathbf{x}_0)} [\log p_\theta(\mathbf{x}_0)] \leq -\mathbb{E}_{q(\mathbf{x}_0)} \left[\mathbb{E}_{q(\mathbf{x}_1, \dots, \mathbf{x}_T | \mathbf{x}_0)} \left[\log \left(p(\mathbf{x}_T) \prod_{t=1}^T \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_t | \mathbf{x}_{t-1})} \right) \right] \right] \quad (1.36)$$

$$= -\mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} \left[\log \left(p(\mathbf{x}_T) \prod_{t=1}^T \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_t | \mathbf{x}_{t-1})} \right) \right] \quad (1.37)$$

$$= \mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} \left[-\log p(\mathbf{x}_T) - \sum_{t=1}^T \log \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_t | \mathbf{x}_{t-1})} \right] \quad (1.38)$$

$$=: \mathcal{L}_{\text{DDPM}} \quad (1.39)$$

one obtains the loss for training a diffusion model. This can be further transformed to

$$\mathcal{L}_{\text{DDPM}} = \mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} \left[-\log p(\mathbf{x}_T) - \sum_{t=2}^T \log \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_t | \mathbf{x}_{t-1})} - \log \frac{p_\theta(\mathbf{x}_0 | \mathbf{x}_1)}{q(\mathbf{x}_1 | \mathbf{x}_0)} \right] \quad (1.40)$$

$$= \mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} \left[-\log p(\mathbf{x}_T) - \sum_{t=2}^T \log \left(\frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0)} \cdot \frac{q(\mathbf{x}_{t-1} | \mathbf{x}_0)}{q(\mathbf{x}_t | \mathbf{x}_0)} \right) - \log \frac{p_\theta(\mathbf{x}_0 | \mathbf{x}_1)}{q(\mathbf{x}_1 | \mathbf{x}_0)} \right] \quad (1.41)$$

$$= \mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} \left[-\log \frac{p(\mathbf{x}_T)}{q(\mathbf{x}_T | \mathbf{x}_0)} - \sum_{t=2}^T \log \frac{p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)}{q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0)} - \log p_\theta(\mathbf{x}_0 | \mathbf{x}_1) \right] \quad (1.42)$$

$$= \mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} \left[\text{KL}(q(\mathbf{x}_T | \mathbf{x}_0) \| p(\mathbf{x}_T)) + \sum_{t=2}^T \text{KL}(q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) \| p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)) - \log p_\theta(\mathbf{x}_0 | \mathbf{x}_1) \right] \quad (1.43)$$

For training, only non-constant terms are relevant. That is why the loss further simplifies to

$$\begin{aligned} \mathcal{L}_{\text{DDPM}} = & \sum_{t=2}^T \mathbb{E}_{q(\mathbf{x}_0, \mathbf{x}_t)} [\text{KL}(q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) \| p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t))] \\ & - \mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} [\log p_\theta(\mathbf{x}_0 | \mathbf{x}_1)] + \text{const} \end{aligned} \quad (1.44)$$

$$\approx \sum_{t=2}^T \mathbb{E}_{q(\mathbf{x}_0, \mathbf{x}_t)} [\text{KL}(q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) \| p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t))] + \text{const} \quad (1.45)$$

assuming that the term $\mathbb{E}_{q(\mathbf{x}_0, \dots, \mathbf{x}_T)} [\log p_\theta(\mathbf{x}_0 | \mathbf{x}_1)]$ is negligible. Note that in eq. 1.45 only Gaussians, $p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \mu_\theta(\mathbf{x}_t, t), \Sigma_\theta(\mathbf{x}_t, t))$ with $\Sigma_\theta(\mathbf{x}_t, t) = \sigma_t^2 \mathbf{I}$ and $q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_{t-1}; \hat{\mu}(\mathbf{x}_t, \mathbf{x}_0), \hat{\beta}_t \mathbf{I})$ are compared in the KL-divergence meaning there is a closed-form solution. The loss for the diffusion model boils down to

$$\mathcal{L}_{\text{DDPM}} = \sum_{t=2}^T \mathbb{E}_{q(\mathbf{x}_0, \mathbf{x}_t)} \left[\frac{1}{2\sigma^2} \|\hat{\mu}(\mathbf{x}_t, \mathbf{x}_0) - \mu_\theta(\mathbf{x}_t, t)\|^2 \right] . \quad (1.46)$$

The mean and the variance of the forward process $\hat{\mu}$ is obtained by

$$\hat{\mu}(\mathbf{x}_t, \mathbf{x}_0) := \frac{\sqrt{\bar{\alpha}_{t-1}}\beta_t}{1 - \bar{\alpha}_t} \mathbf{x}_0 + \frac{\sqrt{\alpha_t}(1 - \bar{\alpha}_{t-1})}{1 - \bar{\alpha}_t} \mathbf{x}_t \quad (1.47)$$

$$\hat{\beta}_t := \frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t} \beta_t . \quad (1.48)$$

However, instead of predicting the mean, diffusion models are more often trained on predicting the noise ϵ_θ because [24] empirically found that this lead to more stable training

$$\mathcal{L}_{\text{DDPM}} = \sum_{t=2}^T \frac{1}{2\tilde{\beta}_t} \frac{(1 - \alpha_t)^2}{\alpha_t(1 - \bar{\alpha}_t)} \cdot \mathbb{E}_{q(\mathbf{x}_t | \mathbf{x}_0)} [\|\epsilon_\theta(\mathbf{x}_t, t) - \epsilon_t\|_2^2] \quad (1.49)$$

which is achieved by reparameterization of eq. 1.46 via

$$\hat{\mu}(\mathbf{x}_t, \mathbf{x}_0) = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1 - \alpha_t}{\sqrt{1 - \hat{\alpha}_t}} \epsilon_t \right) . \quad (1.50)$$

In addition, they found empirically that ignoring the weighting term of eq. 1.49 worked better

$$\mathcal{L}_{\text{DDPM}} = \sum_{t=2}^T \mathbb{E}_{q(\mathbf{x}_t | \mathbf{x}_0)} [\|\epsilon_\theta(\mathbf{x}_t, t) - \epsilon_t\|_2^2] \quad (1.51)$$

DDPM Sampler

Algorithm 1 and fig. 1.2 depict the training process of the DDPM approach. In each training iteration, an image $\mathbf{x}_0 \sim p_{\text{data}}$ is sampled from the data distribution. Next, a timestep t is uniformly sampled from a predefined interval $[0, \dots, T]$ and Gaussian noise is drawn according

to $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. Using eq. 1.28 the noisy image \mathbf{x}_t at timestep t is computed. The diffusion model is applied to predict the noise from this noisy input, and the loss function (eq. 1.51) measures the difference between the predicted and true noise. Finally, the model parameters are updated using gradient descent or a more advanced iterative optimization algorithms.

Algorithm 1 Training of DDPM using learning rate η [65]

```

repeat
     $\mathbf{x}_0 \sim p_{\text{data}}$ 
     $t \sim \text{Uniform}(\{1, \dots, T\})$ 
     $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
     $\mathbf{x}_t \leftarrow \sqrt{\bar{\alpha}_t} \cdot \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \cdot \epsilon$ 
     $\theta \leftarrow \theta - \eta \cdot \nabla_{\theta} \|\epsilon_{\theta}(\mathbf{x}_t, t) - \epsilon\|_2^2$ 
until converged
    
```

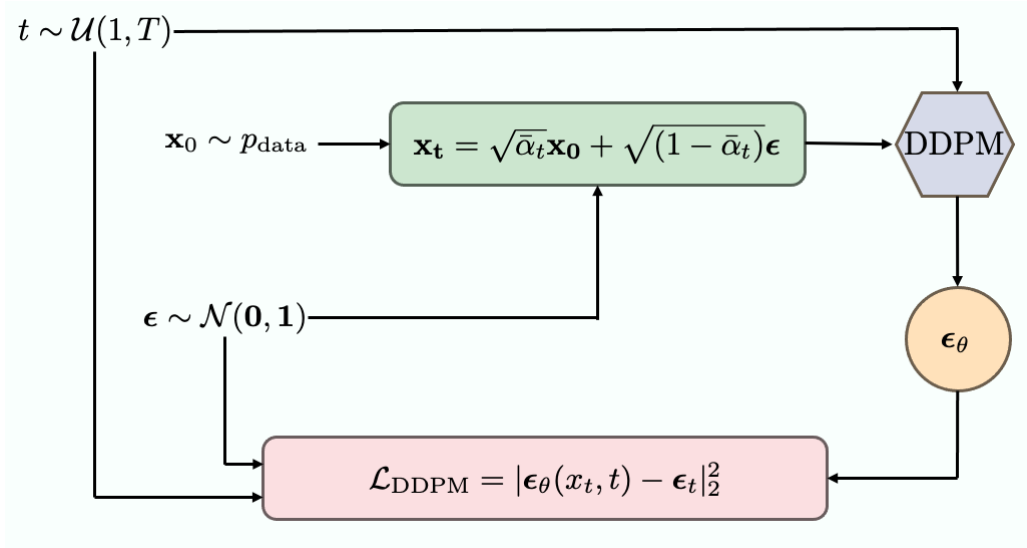


Figure 1.2: DDPM training [10].

Algorithm 2 and fig. 1.3 show schematically the sampling process which reverses the diffusion process to generate images from noise. Starting with pure Gaussian noise $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, the process iteratively denoises the image. At each timestep t , the slightly denoised image \mathbf{x}_{t-1} is computed using

$$\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1 - \alpha_t}{\sqrt{1 - \bar{\alpha}_t}} \epsilon_{\theta}(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z} \quad (1.52)$$

until $t = 2$. In the last step, the final clear images is obtained by

$$\mathbf{x}_0 = \frac{1}{\sqrt{\alpha_1}} \left(\mathbf{x}_1 - \frac{1 - \alpha_1}{\sqrt{1 - \bar{\alpha}_1}} \epsilon_{\theta}(\mathbf{x}_1, 1) \right) + \sigma_1 \mathbf{z} \quad (1.53)$$

Here, it becomes clear that the model has to predict the noise T times which makes inference of such a diffusion model slow.

Algorithm 2 Sampling of DDPM [65]

```

 $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
for  $t = T, \dots, 1$  do
    if  $t > 1$  then
         $\lambda \leftarrow 1$ 
    else
         $\lambda \leftarrow 0$ 
    end if
     $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
     $\mathbf{x}_{t-1} \leftarrow \frac{1}{\sqrt{\alpha_t}} \left( \mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\alpha_t}} \cdot \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \lambda \cdot \sigma_t \cdot \mathbf{z}$ 
end for
return  $\mathbf{x}_0$ 
    
```

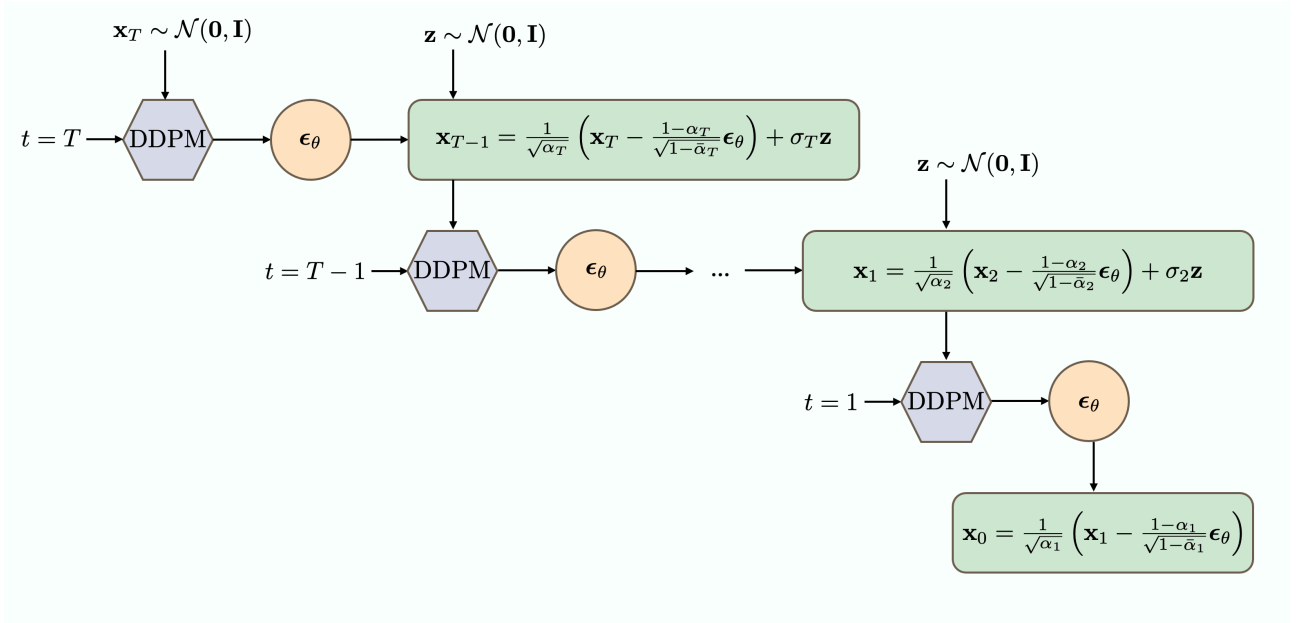


Figure 1.3: DDPM sampling [10].

1.2.2 Denoising Diffusion Implicit Models

The DDPM sampling process can be accelerated by replacing the Markovian diffusion process with a non-Markovian alternative resulting in denoising diffusion implicit models (DDIMs) [58]. Unlike DDPM, DDIM defines the joint distribution as

$$q(\mathbf{x}_{1:T} \mid \mathbf{x}_0) = q(\mathbf{x}_T \mid \mathbf{x}_0) \prod_{t=2}^T q(\mathbf{x}_{t-1} \mid \mathbf{x}_t, \mathbf{x}_0) \quad . \quad (1.54)$$

It becomes clear that the forward process $q(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{x}_0)$ is not Markovian anymore due to its explicit dependence on \mathbf{x}_0 . The authors of [58] parameterize the reverse process as

$$q(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N} \left(\mathbf{x}_{t-1}; \sqrt{\bar{\alpha}_{t-1}} \cdot \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1} - \tilde{\sigma}_t^2} \cdot \frac{\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0}{\sqrt{1 - \bar{\alpha}_t}}, \tilde{\sigma}_t^2 \cdot \mathbf{I} \right) \quad (1.55)$$

The corresponding sampling equation becomes

$$\mathbf{x}_{t-1} = \underbrace{\frac{1}{\sqrt{\bar{\alpha}_t}} (\mathbf{x}_t - \sqrt{1 - \bar{\alpha}_t} \cdot \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t))}_{\text{Predicted } \mathbf{x}_0} + \underbrace{\sqrt{1 - \bar{\alpha}_{t-1} - \tilde{\sigma}_t^2} \cdot \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t)}_{\text{Direction of } \mathbf{x}_t} + \underbrace{\tilde{\sigma}_t \cdot \mathbf{z}}_{\text{Noise}} \quad (1.56)$$

where $\tilde{\sigma}_t$ is a parameter to choose. \mathbf{x}_{t-1} consists of three parts. The first part is a prediction of \mathbf{x}_0 based on $\boldsymbol{\epsilon}_\theta$. The second part is a directional term describing the transition between \mathbf{x}_t and \mathbf{x}_0 . And the last part adds Gaussian noise scaled by $\tilde{\sigma}_t$. The key advantage of DDIM is the flexibility in choosing $\tilde{\sigma}_t$, which does not need to follow a predefined variance schedule. Different values of $\tilde{\sigma}_t$ yield different diffusion processes while using the same trained model. In particular, for the choice

$$\sigma_t = \sqrt{\frac{1 - \bar{\alpha}_{t-1}}{1 - \bar{\alpha}_t}} \sqrt{1 - \frac{\bar{\alpha}_t}{\bar{\alpha}_{t-1}}} \quad (1.57)$$

the DDIM approach becomes Markovian again and here, the deep connection between DDPM and DDIM can be seen. Another aspect to note is that if setting $\tilde{\sigma} = 0$ the whole process becomes deterministic.

1.2.3 Score-Based Generative Models

This discussion is mainly based on [14, 73]. Score-Based Generative Models (SGMs) [60, 61] do not learn directly the data distribution $p_{\text{data}}(\mathbf{x})$ but the score of the probability function $\nabla_x \log p_{\text{data}}(\mathbf{x})$. The score represents a vector field that points to the steepest increase in log probability density, thereby, guiding the data generation process to regions of higher data density. There are several approaches on how the score function can be learned [21, 62, 64]. Here, the one closest to the DDPM approach is presented [60]. During training, the data is perturbed by Gaussian noise at different noise levels and a neural network is trained to estimate the score function based on the noise level $s_\theta(\mathbf{x}_t, t) \approx \nabla_{\mathbf{x}_t} \log q(\mathbf{x}_t)$. These networks are called Noise-Conditional Score Networks (NCSNs). The learning objective boils down to

$$\mathbb{E}_{T \sim \mathcal{U}[1, T], \mathbf{x}_0 \sim q(\mathbf{x}_0), \mathbf{x}_t \sim q(\mathbf{x}_t | \mathbf{x}_0)} \left[\lambda(t) \sigma_t^2 \|\nabla_{\mathbf{x}_t} \log q(\mathbf{x}_t) - s_\theta(\mathbf{x}_t, t)\|^2 \right] \quad (1.58)$$

$$= \mathbb{E}_{T \sim \mathcal{U}[1, T], \mathbf{x}_0 \sim q(\mathbf{x}_0), \mathbf{x}_t \sim q(\mathbf{x}_t | \mathbf{x}_0)} \left[\lambda(t) \sigma_t^2 \|\nabla_{\mathbf{x}_t} \log q(\mathbf{x}_t | \mathbf{x}_0) - s_\theta(\mathbf{x}_t, t)\|^2 \right] + \text{const} \quad (1.59)$$

$$= \mathbb{E}_{T \sim \mathcal{U}[1, T], \mathbf{x}_0 \sim q(\mathbf{x}_0), \mathbf{x}_t \sim q(\mathbf{x}_t | \mathbf{x}_0)} \left[\lambda(t) \left\| -\frac{\mathbf{x}_t - \mathbf{x}_0}{\sigma_t} - \sigma_t s_\theta(\mathbf{x}_t, t) \right\|^2 \right] + \text{const} \quad (1.60)$$

$$= \mathbb{E}_{T \sim \mathcal{U}[1, T], \mathbf{x}_0 \sim q(\mathbf{x}_0), \boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} \left[\lambda(t) \|\boldsymbol{\epsilon} + \sigma_t s_\theta(\mathbf{x}_t, t)\|^2 \right] + \text{const} \quad (1.61)$$

In the final line in eq. 1.61 the relation to DDPM loss becomes visible by identifying $\epsilon(\mathbf{x}_t, t) = -\sigma_t s_\theta(\mathbf{x}_t, t)$ [73], showing that learning to predict noise is equivalent to learning the score function.

For sampling a new element from the data distribution [60] proposed Annealed Langevin Dynamics (ALD) which produces samples by only using the score function being approximated by the learned neural network

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \alpha \nabla_{\mathbf{x}} \log p(\mathbf{x}_{t-1}) + \sqrt{2\alpha} \mathbf{z}_t, \quad 1 \leq t \leq T \quad (1.62)$$

with $\mathbf{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ and α regulates the step size of the update in the direction of the score. By applying eq. 1.62 iteratively to initial Gaussian noise, a new element of the data distribution is generated.

1.2.4 Stochastic Differential Equations

This discussion is mainly based on [14, 73]. Stochastic Differential Equations (SDEs) [64] are a generalization of SGMs and DDPMs to continuous time where the forward and the reverse process are modeled by the solution of stochastic differential equations. The forward process is described by the SDE

$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, t)dt + g(t)d\mathbf{w} \quad (1.63)$$

where $\mathbf{f}(\mathbf{x}, t)$ is the drift coefficient, $g(t)$ the diffusion coefficient and \mathbf{w} a standard Wiener process (aka Brownian Motion). [3] proved that the diffusion process in eq. 1.63 can be reversed leading to the reversed-time SDE given by

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g(t)^2 \nabla_x \log q(\mathbf{x}_t) \right] dt + g(t)d\tilde{\mathbf{w}} \quad (1.64)$$

with a standard Wiener process $\tilde{\mathbf{w}}$ where the time flies backward. Both, forward and reverse SDE, share the same marginals. A score neural network s_θ is trained similarly to SGMs by generalizing the objective function (see eq. 1.58) to continuous time

$$\mathbb{E}_{t \sim \mathcal{U}[0, T], \mathbf{x}_0 \sim q(\mathbf{x}_0), \mathbf{x}_t \sim q(\mathbf{x}_t | \mathbf{x}_0)} \left[\lambda(t) \| s_\theta(\mathbf{x}_t, t) - \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t) \|^2 \right] \quad (1.65)$$

such as $s_\theta(\mathbf{x}, t) \approx \nabla_{\mathbf{x}_t} \log q_t(\mathbf{x}_t)$. For converting noise to data, a solution of eq. 1.64 has to be computed using the score neural network and a SDE solver. Another finding of [64] is that for eq. 1.64 an ODE exists having the same marginal as the SDE being known as probability flow ODE in the literature. It's given by the deterministic version of eq. 1.64

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g(t)^2 \nabla_x \log q(x_t) \right] dt \quad (1.66)$$

Both reversed-SDE or probability flow ODE can be used to generate new samples using SDE solvers [26, 64] or respectively ODE solvers [27, 42, 64, 74].

1.2.5 Flow Matching

The concept of flow matching was introduced for normalizing flows [38] to find transport trajectories between two probability distributions. In contrast to diffusion models, which follows a noisy trajectory flow matching directly learns a probability path $p(t, x)$ connecting two distributions π_1 and π_2 . In particular, the corresponding vector field $u(t, x)$, often referred to as velocity, that generates the probability path is learned via the flow matching loss function

$$\mathcal{L}_{\text{FM}} = \mathbb{E}_{t \sim \mathcal{U}(0,1), x \sim p(t,x)} [\|v_\theta(t, x) - u(t, x)\|^2] \quad (1.67)$$

by a neural network $v_\theta(t, x)$. The physical analogy is that particles are transport from one location to another and the vector field specifies at every time and location in which direction and how fast the particles move, in order to reach a final state. However, in flow matching neither the probability path $p(t, x)$ nor the vector field $u(t, x)$ is directly accessible. Therefore, the marginal probability path is constructed by a mixture of simpler conditional probability paths $p(t, x | x_1)$ such as $p(0, x | x_1) = p(0, x) \approx q(x)$ and $p(1, x | x_1) = \mathcal{N}(x; \mathbf{0}, \mathbf{I})$ with x_1 being sampled from the data distribution

$$p(x, t) = \int p(t, x | x_1) q(x_1) dx_1 \quad (1.68)$$

where $q(x)$ denotes the data distribution. Furthermore, a marginal vector field can be formulated

$$u(t, x) = \int u(t, x | x_1) \frac{p(t, x | x_1) q(x_1)}{p(t, x)} dx_1 \quad (1.69)$$

based on the conditional vector field $u(t, x | x_1)$ which generates the marginal probability path. The trick is not to use the marginal distributions, because the integrals are still intractable, instead reformulate the loss function (eq. 1.67) using the conditional vector field.

$$\mathcal{L}_{\text{CFM}}(\theta) = \mathbb{E}_{t \sim \mathcal{U}(0,1), x \sim p(t,x)} [\|v_\theta(t, x) - u(t, x | x_1)\|^2] \quad (1.70)$$

The key insight is that \mathcal{L}_{CFM} leads to the same gradients w.r.t θ as \mathcal{L}_{FM} and can actually be computed due to the tractable conditional vector field.

Rectified Flow

[40] proposed rectified flow, a specialized variant of flow matching [38] that aims to straighten the trajectories between source and target distributions π_0 and π_1 by iteratively refining the learned ODE. This approach offers significant computational advantages, specifically, the straight-

ened paths create a more direct mapping between distributions, making the velocity field easier to learn and enabling efficient sampling with as few as 1-2 Euler integration steps, compared to the 50-1000 steps typically required by diffusion models.

The ODE of the rectified flow for $x_0 \sim \pi_0$ and $x_1 \sim \pi_1$ is given by

$$dx_t = v_\theta(x_t, t)dt \quad (1.71)$$

where v_θ is the vector field approximated by a neural network driving the flow such that it follows the direction $x_1 - x_0$. This is enforced by the learning objective

$$\min_v \int_0^1 \mathbb{E} [\|(x_1 - x_0) - v_\theta(x_t, t)\|^2] dt \quad (1.72)$$

where $x_t = tx_1 + (1-t)x_0$ is the linear interpolation between x_0 and x_1 . After training, the ODE in eq. 1.71 can be solved by sampling x_0 , thus obtaining a sample from the data distribution π_1 . One main property of the flow obtained through this learning procedure is that trajectories of the rectified flow do not cross. This is depicted in fig. 1.4. (a) shows the linear interpolations from pairs of $(x_0, x_1) \sim \pi_0 \times \pi_1$. In this case, trajectories intersect. Next, in (b), the vector field is learned. The rectified flow induced by (x_0, x_1) results in rewired trajectories such that they do not cross. This procedure, obtaining the linear interpolation between the elements from the previously learned rectified flow x_0^{k-1}, x_1^{k-1} and learning the rectified flow x_t^k , can be iteratively applied and is called Reflow. The second iteration is depicted in (c) and (d). With every iteration the trajectory becomes more straightened (see algorithm 8). An optional step to enable fast inference is to learn the k -rectified flow by a neural network \hat{T} which directly maps x_0^k to x_1^k . Note, the difference to rectified flow learning is that here, the network approximates the mapping between (x_0^k, x_1^k) whereas rectification yields a different mapping with lower transport cost and straightened trajectory (x_0^{k+1}, x_1^{k+1}) .

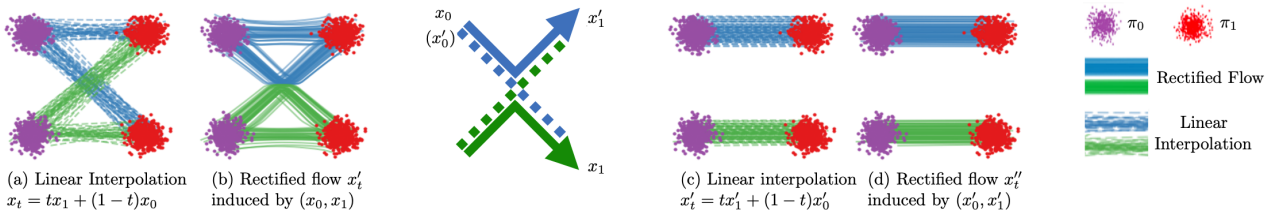


Figure 1.4: Rectified Flow [40].

Algorithm 3 Rectified Flow: Main Algorithm

Procedure: $x' = \text{RectFlow}((x_0, x_1))$

Inputs: Draws from a coupling (x_0, x_1) of π_0 and π_1 ; velocity model $v_\theta : \mathbb{R}^d \rightarrow \mathbb{R}^d$ with parameter θ .

Training: $\hat{\theta} = \arg \min_{\theta} \mathbb{E} [\|x_1 - x_0 - v(t x_1 + (1-t)x_0, t)\|^2]$, with $t \sim \text{Uniform}([0, 1])$.

Sampling: Draw (x'_0, x'_1) following $dx_t = v_{\hat{\theta}}(x_t, t)dt$ starting from $x'_0 \sim \pi_0$ (or backwardly $x'_1 \sim \pi_1$).

Return: $Z = \{Z_t : t \in [0, 1]\}$.

Reflow (optional): $x^{k+1} = \text{RectFlow}((x_0^k, x_1^k))$, starting from $(x_0^0, x_1^0) = (x_0, x_1)$.

Distill (optional): Learn a neural network \hat{T} to distill the k -rectified flow, such that $x_1^k \approx \hat{T}(x_0^k)$.

1.3 Fundamentals of Neural Network Architectures

In this section, the focus lies on the key fundamental building blocks of the neural networks which are used in the experiments in section ??.

1.3.1 Transformer

The introduction of transformer architecture [71] was a decisive moment for the whole AI community. It significantly pushed the development in natural language and image processing forward. Therefore, it is no surprise that the latest models are based on this architecture type [2, 4, 34, 67].

The transformer was introduced as an alternative to recurrent neural networks [53] for processing sequential data like text. In the original paper, the transformer architecture leverages an encoder-decoder structure, however, there are many variants like decoder-only [1, 70] or encoder-only transformer [16].

Transformers, as introduced in the original paper (Fig. 1.5), take text as input, which must first be converted into numerical form through a process called tokenization. This produces a sequence of tokens that are a numerical representation of words or subwords of the text. To provide the model with information about the position of each token in the sequence, positional encodings are added. These preprocessing steps occur before the input is fed into the transformer architecture. Each encoder block consists of two sub-layers: (1) a multi-head self-attention mechanism followed by a residual connection [22] and layer normalization [6]. The decoder blocks contain three sub-layers: (1) a masked multi-head self-attention where the masking prevents that future tokens are seen during the training process, (2) a cross-attention mechanism with the output of the encoder being and (3) an additional feedforward network. All three sub-layers are followed by a residual connection and layer normalization. The final output is projected to vocabulary-sized logits representing a probability distribution over possible next tokens.

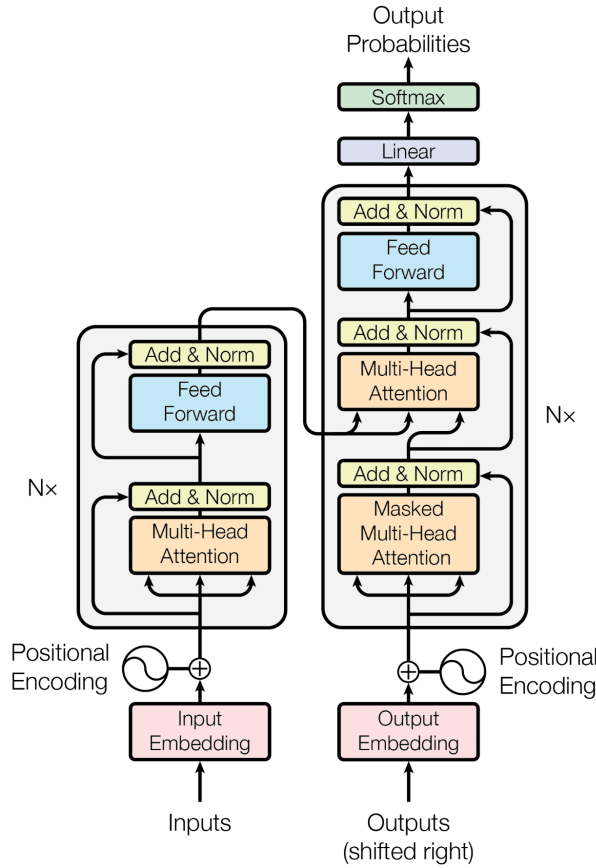


Figure 1.5: Transformer architecture from [71].

1.3.2 Attention Mechanism

The attention mechanism is an integral part of the transformer architecture. It enables the model to focus on the input elements that are most important for the task at hand and ignore the less important ones. This mechanism was first introduced for RNNs [7] to overcome the information bottleneck, as traditionally in RNNs only the last hidden state is given as input for the next prediction. In [7], all previous hidden states were presented to the model as additional input, and the task of the attention mechanism was to filter out those that were relevant for the prediction of the next state. The authors of [71] leveraged the formalism and reformulated it for the transformer architecture that will be discussed in the following.

In the attention mechanism, so-called queries, keys and values are computed. The attention mechanism can be understood as a soft lookup table where queries search for relevant keys, and the corresponding values are retrieved and weighted by similarity. A distinction is made between self-attention (fig. 1.6) and cross-attention (fig. 1.7). In self-attention, the interaction of the tokens within a sequence is calculated. In contrast, in cross-attention you have two input sequences with the goal that the model aligns and relates information between the two different sequences.

Self-Attention

Let $X \in \mathbb{R}^{n \times d}$ be the embedding vector of the tokens. Then the queries Q , keys K and values V are computed by linearly projection of the embedding vector

$$Q = X \cdot W_q^T \quad (1.73)$$

$$K = X \cdot W_k^T \quad (1.74)$$

$$V = X \cdot W_v^T \quad (1.75)$$

with $W_q^T \in \mathbb{R}^{d \times d_k}$, $W_k^T \in \mathbb{R}^{d \times d_k}$ and $W_v^T \in \mathbb{R}^{d \times d_v}$. Now, so-called scaled-dot product attention is used where the attention matrix A that assigns how much focus is put on the different parts of the token sequence is computed by the softmax of the scaled dot product between the queries and the keys

$$A = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) \quad (1.76)$$

The softmax function is defined as the following for a vector $\mathbf{z} = (z_1, z_2, \dots, z_N)$

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}} \quad (1.77)$$

The scaling of the dot product QK^T with the factor $\frac{1}{\sqrt{d_k}}$ is done in order to prevent vanishing gradients in the softmax function which occurs if the values become too small or too big. Finally, the attention matrix is applied with the values. This can be summarized in the following formulation

$$Z = \text{Attention}(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) V \quad (1.78)$$

One main challenge of the discussed attention mechanism is that it has quadratic complexity $\mathcal{O}(n^2)$ in computation and memory consumption which is tackled by several approaches [12, 13, 15, 31].

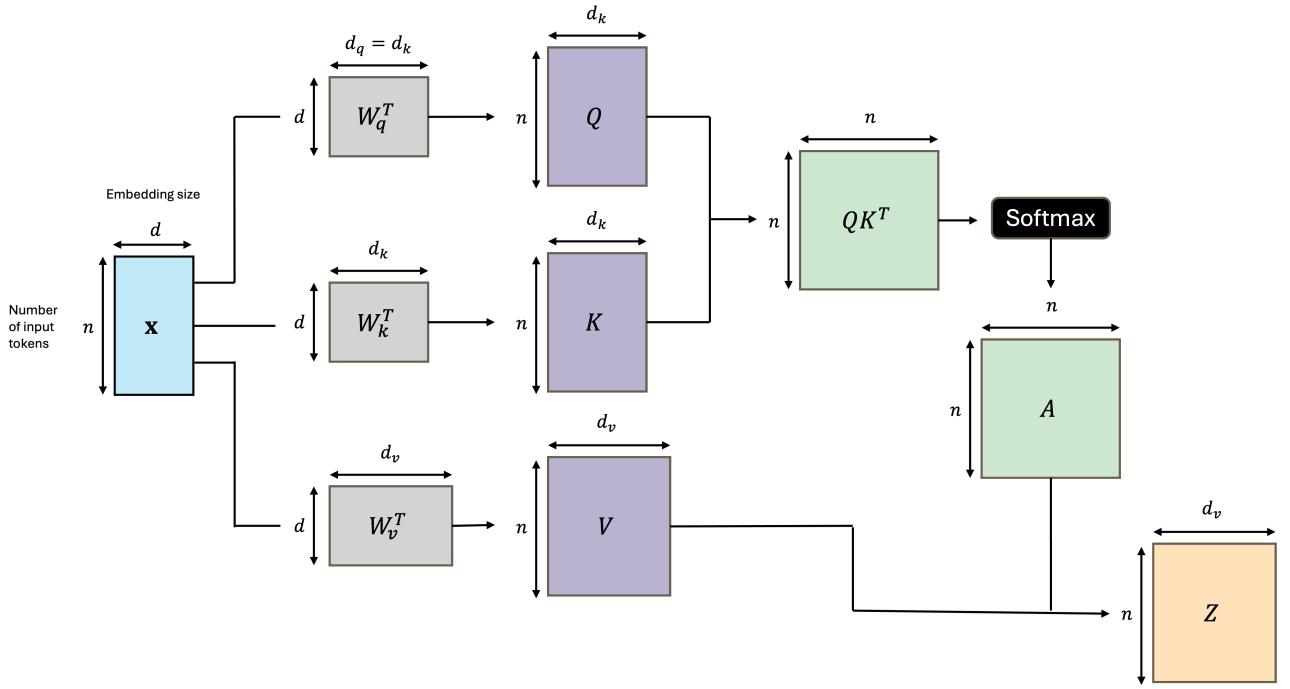


Figure 1.6: Self-attention mechanism inspired from [50].

Cross-Attention

The goal of cross-attention is to relate two different sequences of tokens \mathbf{x}_1 and \mathbf{x}_2 . Commonly, the queries come from the target sequence, e.g., decoder of the transformer, while the keys and values come from the source sequence, e.g., encoder output of transformer. Besides that the computation is similar to the computation of self-attention.

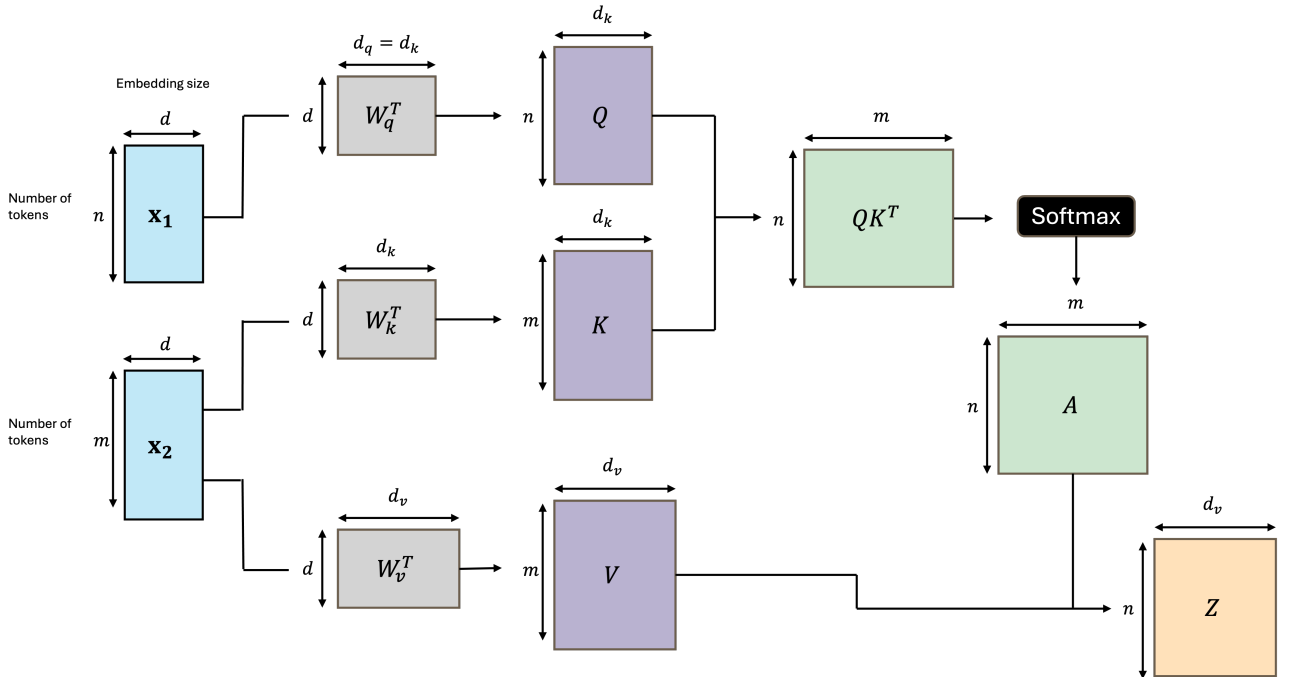


Figure 1.7: Cross-attention mechanism inspired from [50].

Multi-Head Attention

Multi-head attention is another trick used in transformers to make the results of the attention operations even more expressive. Instead of only performing individual self- or cross-attentions in a transformer block, so-called multi-head attentions are used. Here, based on the input embeddings, not just one but N times keys, queries and values are calculated, each with different weight matrices $W_{q_i}^T \in \mathbb{R}^{d \times d_k}$, $W_{k_i}^T \in \mathbb{R}^{d \times d_k}$ and $W_{v_i}^T \in \mathbb{R}^{d \times d_v}$ with $i = 1, \dots, N$, and the attention mechanism is executed. Each attention that is carried out is designated as a head_i . The results of the N attentions are concatenated and combined with another linear projection $W^O \in \mathbb{R}^{Nd_v \times d}$.

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \dots, \text{head}_N) W^O \quad (1.79)$$

$$\text{head}_i = \text{Attention}(XW_{q_i}^T, XW_{k_i}^T, XW_{v_i}^T) \quad (1.80)$$

This is advantageous because the individual heads can concentrate on different tasks, thus reducing the complexity for each head compared to the case where only a single attention has to capture everything.

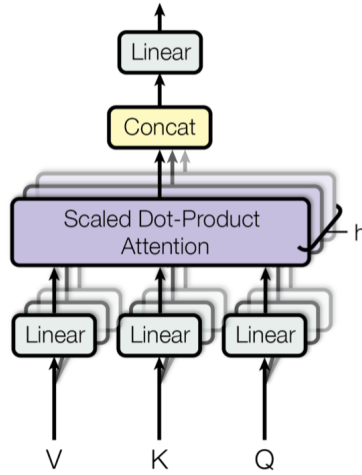


Figure 1.8: Multi-head attention from [71].

1.3.3 Positional Encoding

Before the embedded token sequence is fed as input to the transformer, a positional information is added to each token to provide the network with information about the order of the tokens. There are different methods, which can be divided into the categories relative [35, 44, 66] vs absolute [32, 36, 41, 71] positional embedding and deterministic [32, 36, 44, 66, 71] versus learnable [35, 41] positional embedding.

Sinusoidal Positional Encoding

Sinusoidal positional encoding is used in the original transformer paper [71] which leverages the sin and cosine function to inject positional information into the model. It belongs to the category of absolute and deterministic positional encoding.

$$PE(pos, 2i) = \sin\left(\frac{pos}{10000^{\frac{2i}{d}}}\right) \quad (1.81)$$

$$PE(pos, 2i + 1) = \cos\left(\frac{pos}{10000^{\frac{2i}{d}}}\right) \quad (1.82)$$

where d is the embedding dimension, pos is the position of the token in the sequence and i is the dimension index which is used to compute different frequencies for different parts of the embedding vector. The computed positional embedding has the same dimension like the token embedding vector and is be added to it.

Rotary Position Embedding

In recent years, the focus shifted more and more from absolute positional embedding to relative positional embedding because one hypothesized that relative order of tokens matter and that these methods are better in extrapolating compared to absolute positional embedding methods [75]. Rotary Positiona Embeeding (RoPE) [66] is a a relative positional embedding method used in FLUX-dev [34]. The idea is to encode the position via rotation in the attention mechanism, to be more explicit in the dot product between the queries Q and the keys K . Starting with the 2d case, they propose that queries Q_m which are soley based on the mth embedding vector x_m and its position m via

$$Q_m(x_m, m) = \begin{bmatrix} \cos(m\theta) & -\sin(m\theta) \\ \sin(m\theta) & \cos(m\theta) \end{bmatrix} \begin{bmatrix} W_q^{11} & W_q^{12} \\ W_q^{21} & W_q^{22} \end{bmatrix} \begin{bmatrix} x_m^1 \\ x_m^2 \end{bmatrix} \quad (1.83)$$

$$= R_{\theta, m}^d W_q \mathbf{x}_m \quad (1.84)$$

with a preset non-zero constant θ . **Hier wurde die andere Konvention zum Schreiben des Attention Mechanismus verwendet als oben -> vereinheitlichen???** The keys K_m are computed in exactly the same way. A visualization can be found in fig. 1.9. In the multi-dimensional case the rotation matrix becomes

$$R_{\theta,m}^d = \begin{bmatrix} \cos(m\theta_1) & -\sin(m\theta_1) & \cdots & 0 & 0 \\ \sin(m\theta_1) & \cos(m\theta_1) & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cos(m\theta_{d/2}) & -\sin(m\theta_{d/2}) \\ 0 & 0 & \cdots & \sin(m\theta_{d/2}) & \cos(m\theta_{d/2}) \end{bmatrix} \quad (1.85)$$

By computing the dot product between the queries and the keys one obtain a single rotation matrix just depending on the relative positions

$$Q_m^T K_n = (R_{\theta,m}^d W_q \mathbf{x}_m)^T R_{\theta,n}^d W_k \mathbf{x}_n = \mathbf{x}_m^T W_q^T R_{\theta,n-m}^d W_k \mathbf{x}_n \quad (1.86)$$

because $R_{\theta,n-m}^d = (R_{\theta,m}^d)^T R_{\theta,n}^d$. In contrast to sinusoidal positional encoding being additive, rotary positional encoding is multiplicative. Moreover, it has the property of long-term decay that means the further away two embeddings are the smaller the dot product of the keys and queries will be.

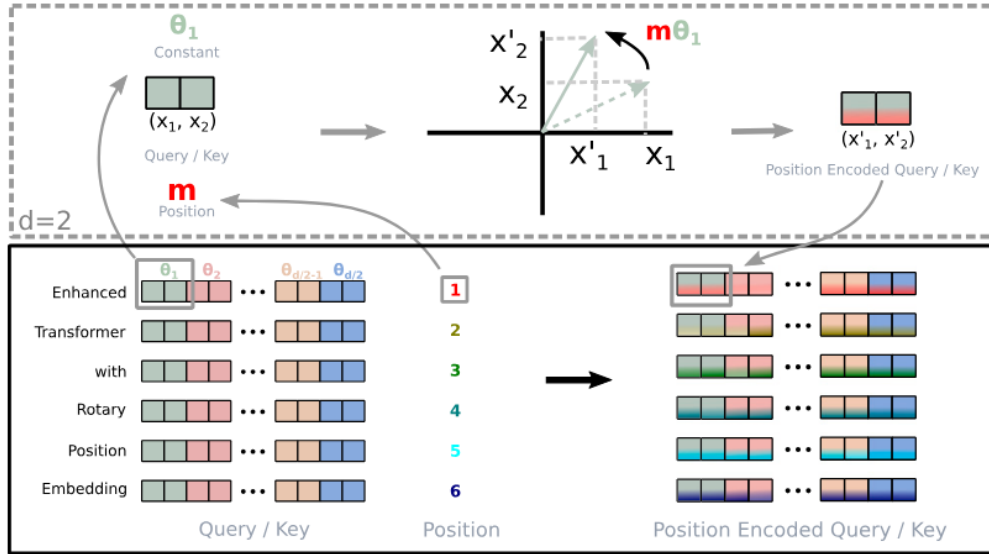


Figure 1.9: Scheme of implementation of rotational position encoding from [66].

1.3.4 Vision Transformer

Original transformers were introduced for language processing tasks. There were some attempts to transfer the methodology of the transformer architecture to computer vision problems [25, 45, 48], but for a long time, convolutional neural networks (CNNs) [49] remained the first choice. CNNs have an inherent advantage over transformers because they possess an image-related inductive bias. On the one hand, CNNs are translationally equivariant by con-

struction and, on the other hand, the concept of locality is embedded in the way they are built. In contrast, these two concepts must be learned from scratch by a transformer, as they are not predefined by the architecture, which makes a sufficiently large dataset essential to achieve competitive performance.

The introduction of vision transformer (ViT) [18] showed that given a sufficiently large dataset transformers can match or even surpass CNNs for vision tasks. The first key aspect in ViT is that not every single pixel is related to all other pixels because this would be computationally infeasible due to the quadratic complexity of the attention mechanism. Instead, the image is split $\mathbf{x} \in R^{N \times W \times C}$ into a sequence of smaller patches $\mathbf{x}_p \in R^{N \times (P^2 \cdot C)}$ where C is the number of channels, H the height of the image, W the width of the image, (P, P) the resolution of a patch and $N = \frac{HW}{P^2}$ the number of patches. At the beginning of the token sequence an additional classification (CLS) token is prepended. The patches are flattened and linearly projected to the encoder dimension. After that a learned positional encoding is added before they are fed into the encoder-only transformer architecture. On top of the transformer encoder a MLP head processes the embedded version of the classification token and provides probability for the different classes from the classification problem.

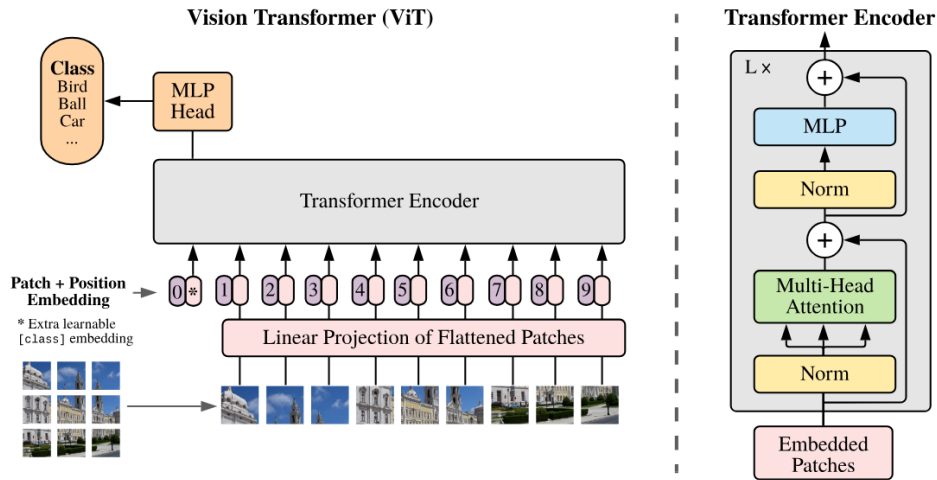


Figure 1.10: Vision transformer from [18].

Chapter 2

Results

sdfsdfs

2.1 Result Distillation

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