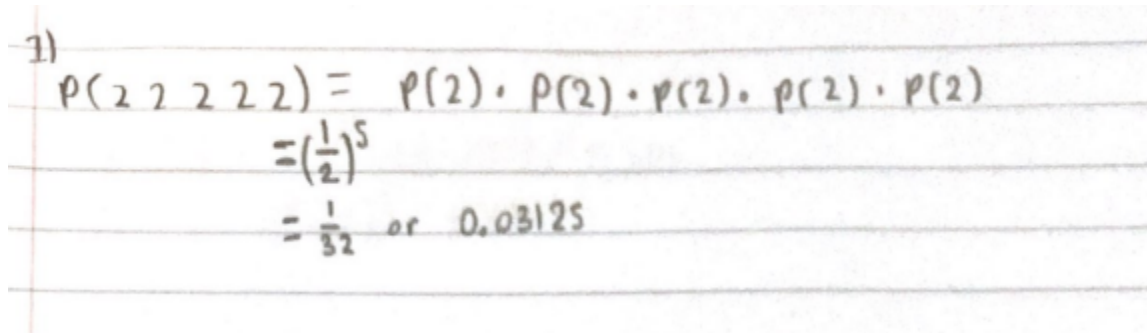


CS3753 HW4 Theory (20 points)

Submit your solution as a PDF or word file via blackboard.

**Probability (20 points)** Show your work.

1. Toss a fair coin five times, what is the probability of seeing five heads in a row?

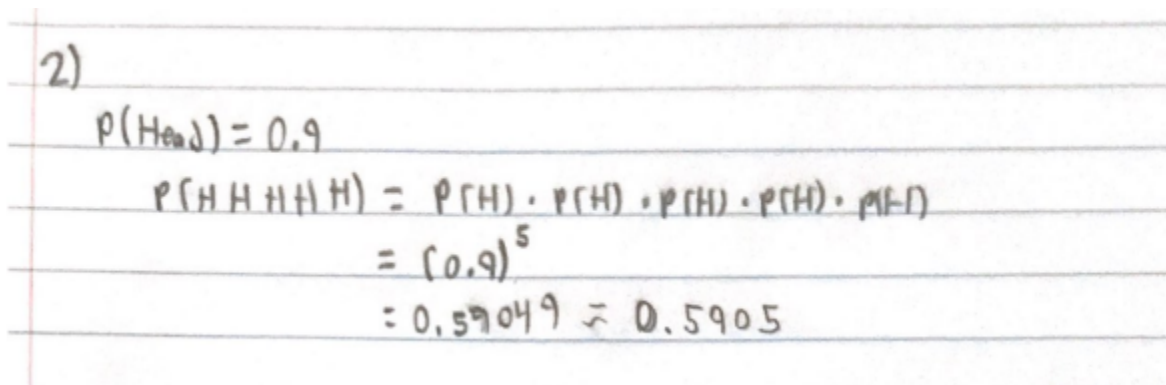


Handwritten solution for problem 1:

$$\begin{aligned} 1) \quad P(2 \ 2 \ 2 \ 2 \ 2) &= P(2) \cdot P(2) \cdot P(2) \cdot P(2) \cdot P(2) \\ &= \left(\frac{1}{2}\right)^5 \\ &= \frac{1}{32} \text{ or } 0.03125 \end{aligned}$$

0.03125

2. Toss a loaded coin ( $p_{\text{head}} = 0.9$ ) five times, what is the probability of seeing five heads in a row?



Handwritten solution for problem 2:

$$\begin{aligned} 2) \quad P(\text{Head}) &= 0.9 \\ P(H \ H \ H \ H \ H) &= P(H) \cdot P(H) \cdot P(H) \cdot P(H) \cdot P(H) \\ &= (0.9)^5 \\ &= 0.59049 \approx 0.5905 \end{aligned}$$

0.5905

3. Toss a fair coin 5 times, what is the probability of seeing a total of 3 heads and 2 tails?

3)

$$P(\text{Head}) = \frac{1}{2} \quad E = \{HHHTT, HH\bar{T}HT, HTHTH, H\bar{T}HHT, \dots\}$$
$$P(\text{Tail}) = \frac{1}{2} \quad 10 \text{ possible ways to order}$$
$$P(HHHHT) = \frac{1}{32} \quad \therefore P(3H 2T) = \frac{10}{32} = \frac{5}{16} = 0.3125$$

0.3125

4. Given a box of coins where exactly half of the coins are fair coins and the other half are loaded coins ( $p_{\text{head}} = 0.9$ ), if you pick one coin from the box and toss it five times, what is the probability to see five heads in a row?

4)

$$P(F) = \frac{1}{2}, P(L) = \frac{1}{2}$$
$$P(5H|F) = \left(\frac{1}{2}\right)^5 = 0.03125$$
$$P(5H|L) = (0.9)^5 = 0.5905$$
$$P(5H) = P(5H|F) P(F) + P(5H|L) P(L)$$
$$= (0.03125)(0.5) + (0.5905)(0.5)$$
$$= 0.310875 \approx 0.31$$

0.31

5. If you randomly pick a coin from the box mentioned above (i.e., half of coins were loaded with  $p_{\text{head}} = 0.9$ ), toss it **five** times and get **five** heads. What is the probability that this is a fair coin?

5)

$$\begin{aligned} P(F|5H) &= P(5H|F) P(F) / P(5H) \\ &= \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) / 0.31 \\ &= 0.05 \end{aligned}$$

0.05