



Fingerprints of classical phase space structure in quantum chaos

Bachelor Thesis

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Outline

1 Introduction

2 Chaotic dynamics

- Classical Chaos
- Quantum Chaos

3 Surface vibrations

4 Procedure

5 Results

6 Conclusions

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- The aim of this thesis is to study the correspondence between the classical chaotic behaviour of nuclear surface vibrations and the quantum features of energy spectra of same system.

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- Hamilton's equations

$$\dot{q}_k = \frac{\partial \mathcal{H}}{\partial p_k}, \quad \dot{p}_k = -\frac{\partial \mathcal{H}}{\partial q_k}$$

- Action-angle variables

$$I_k = \frac{1}{2\pi} \oint p_k \, dq_k, \quad \theta_k = \omega_k(I_k) \Delta t$$

- Integrability: A system is *integrable* if the number of degrees of freedom is equal to the number of constants of the motion (in involution which each other).
- For integrable systems with n degrees of freedom, the trajectory in phase space is constrained to a n -torus.

K.A.M. Theorem

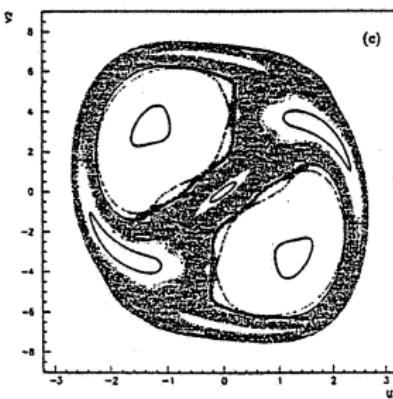
- If the motion of a system described by an integrable Hamiltonian is perturbed with a small non-integrable term and the frequencies ($\omega_k = \dot{\theta}_k$) of the unperturbed motion are such that for $k = 1, \dots, n$, we have $m_k \omega_k \neq 0, \forall m_k \in \mathbb{Z}$, then the motion will remain on the n -torus, except for a small set of initial conditions, for which the trajectories escape on the energy hyper-surface.

Indicators of chaos

- Poincaré sections: Slices of the phase space which reduce the dimensionality by one.
- Liapunov exponent

$$\lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \frac{dx(t)}{dx(0)}$$

- Chaos is characterised by sensitivity to the initial conditions ($\lambda > 0$).



- In Quantum Mechanics we can no longer speak of trajectories or sensitivity to the initial conditions.
- We need a way to compare spectra from different systems in order to search for common patterns or features.
- We can use the level spacings ($E_{n+1} - E_n$) to build the nearest neighbour level spacing distributions.

Giving meaning to quantum chaos

- Berry-Tabor conjecture: The quantum counterpart of a classically integrable system has a Poissonian nearest neighbour distribution.

$$P_P(s) = e^{-s}$$

- Bohigas-Gianoni-Schmit conjecture: The nearest neighbour distribution of a quantum system with a classically chaotic counterpart is given by the Wigner distribution.

$$P_W(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4}s^2\right)$$

Other measures

- Cumulative spacing distribution

$$I(s) = \int_0^s P(x) dx$$

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- The radius of the nuclear surface

$$R(\theta, \varphi) = R_0 \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda, \mu}^* Y_{\lambda, \mu}(\theta, \varphi) \right)$$

- For $\lambda = 2$ we describe the surface in terms of quadrupole vibrations.
- $\alpha_{\lambda, \mu}$ become dynamical degrees of freedom

- We want to study only the vibrations of the surface, so we will use the intrinsic reference frame of the nucleus. Thus we will only have 2 degrees of freedom.
- The Hamiltonian of the system will be given by

$$\begin{aligned}\mathcal{H}(q_0, p_0, q_2, p_2) = & \frac{A}{2}(p_0^2 + p_2^2 + q_0^2 + q_2^2) \\ & + \frac{B}{\sqrt{2}}q_0(3q_2^2 - q_0^2) + \frac{D}{4}(q_0^2 + q_2^2)^2\end{aligned}$$

- Quantised Hamiltonian

$$\begin{aligned}
 H = & A \left(a_1^\dagger a_1 + a_2^\dagger a_2 \right) + \frac{B}{4} \left[\left(3a_1^\dagger a_2^{\dagger 2} + 3a_1 a_2^2 - a_1^{\dagger 3} - a_1^3 \right) \right. \\
 & + 3 \left(a_1 a_2^{\dagger 2} + a_1^\dagger a_2^2 - a_1^\dagger a_1^2 - a_1^{\dagger 2} a_1 + 2a_1 a_2^\dagger a_2 + 2a_1^\dagger a_2^\dagger a_2 \right) \Big] \\
 & + \frac{D}{16} \left[6 \left(a_1^{\dagger 2} a_1^2 + a_2^{\dagger 2} a_2^2 \right) + 2 \left(a_1^2 a_2^{\dagger 2} + a_1^{\dagger 2} a_2^2 \right) + 8a_1^\dagger a_1 a_2^\dagger a_2 \right. \\
 & + 4 \left(a_1^\dagger a_1^3 + a_1^{\dagger 3} a_1 + a_2^\dagger a_2^3 + a_2^{\dagger 3} a_2 + a_1^2 a_2^\dagger a_2 + a_1^{\dagger 2} a_2^\dagger a_2 + a_1^\dagger a_1 a_2^2 + a_1^\dagger a_1 a_2^{\dagger 2} \right) \\
 & \left. + \left(a_1^{\dagger 4} + a_1^4 + a_2^{\dagger 4} + a_2^4 + 2a_1^{\dagger 2} a_2^{\dagger 2} + 2a_1^2 a_2^2 \right) \right]
 \end{aligned}$$

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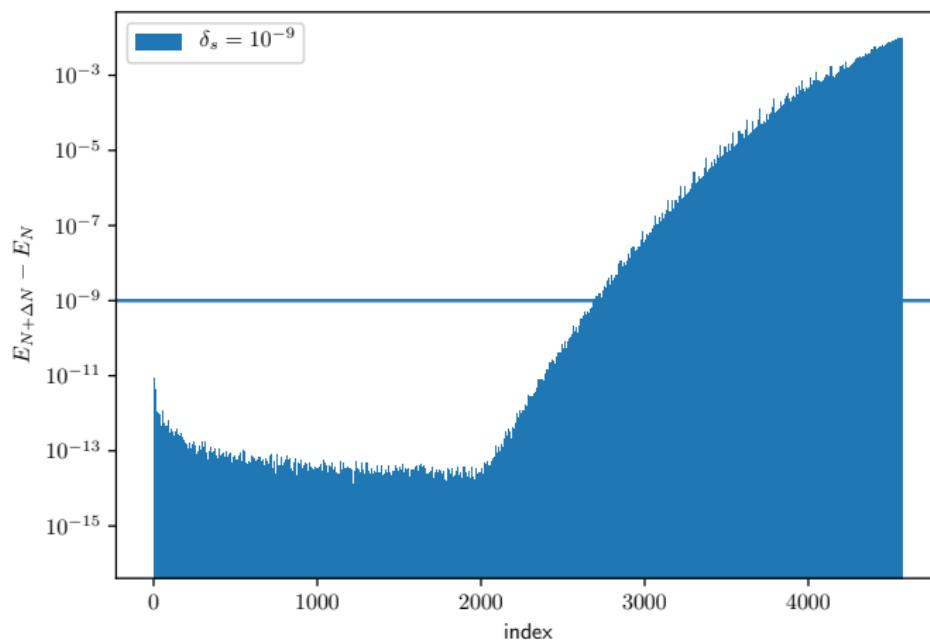
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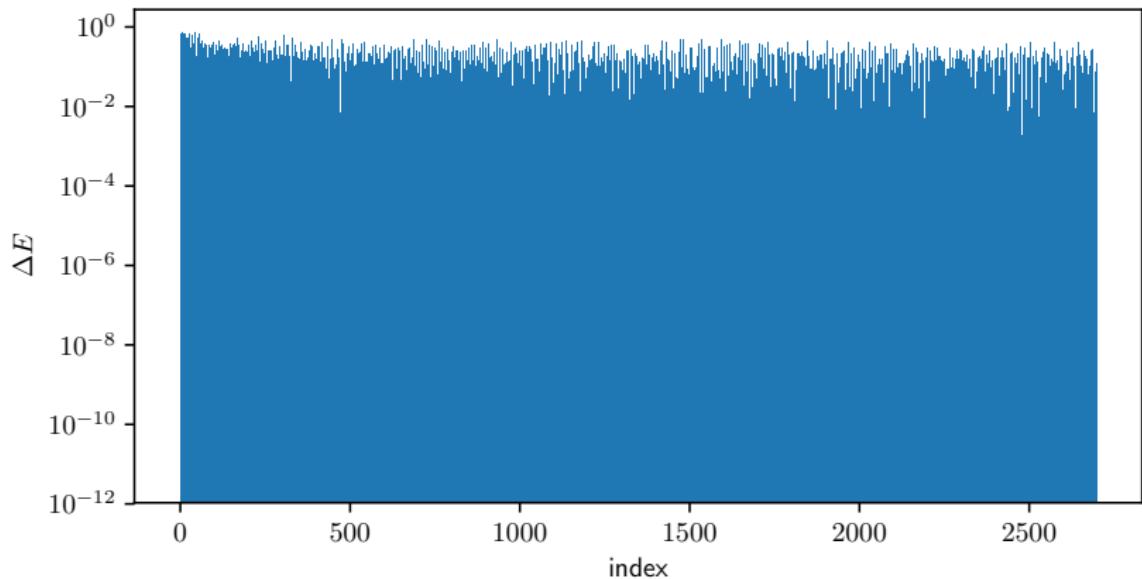
- Computing the Hamiltonian
- Diagonalisation
- Stability
- Separating the irreducible representations
- Statistics



Stability

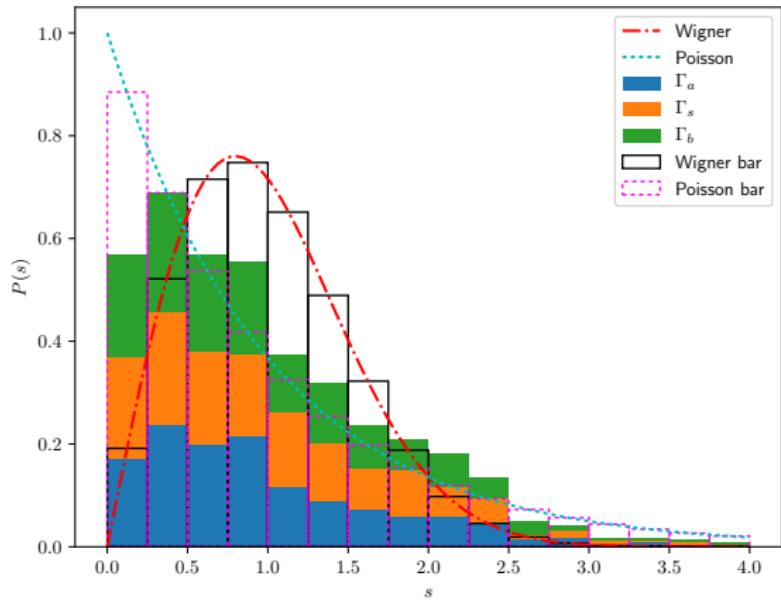


Separating the irreducible representations



The nearest neighbour distribution

$P(s)$ for $B = 0.55$,
 $D = 0.4$, $N = 120$
compared with the
Poisson and
Wigner
distributions both
in continuous and
discrete forms.



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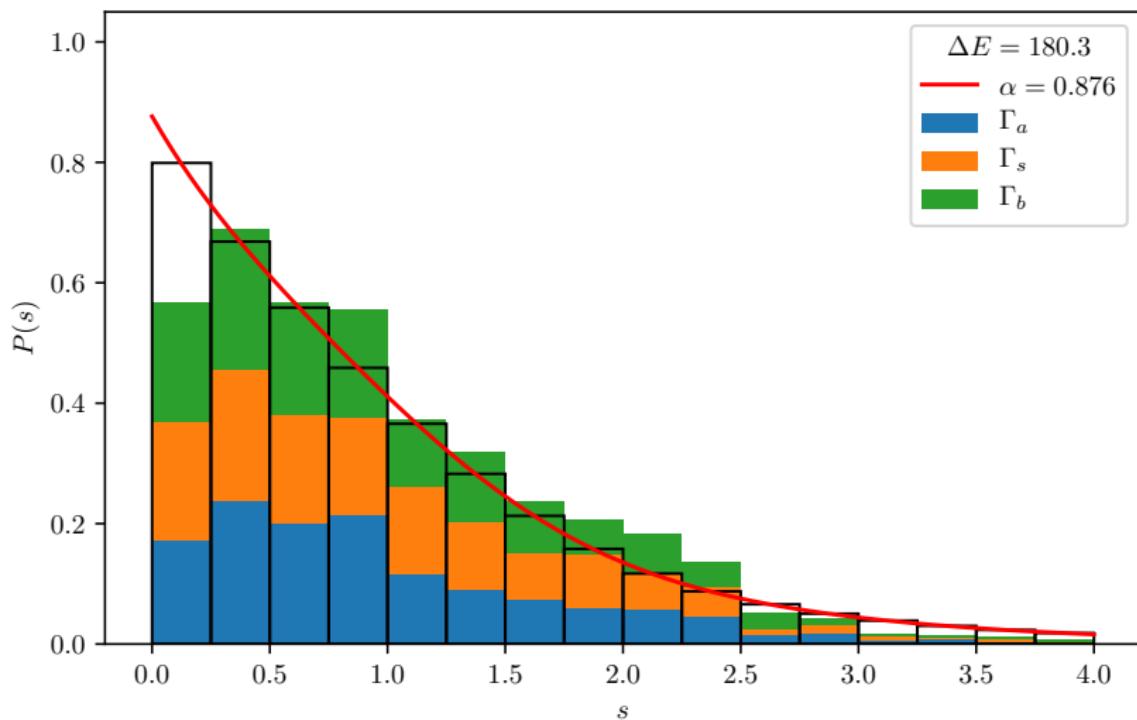
4 Procedure

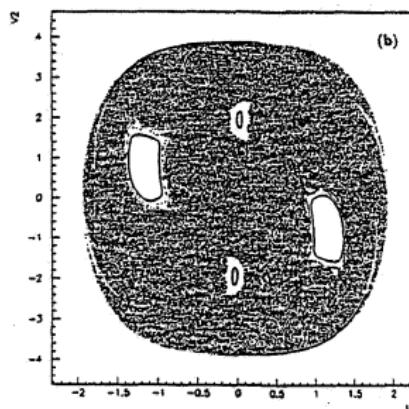
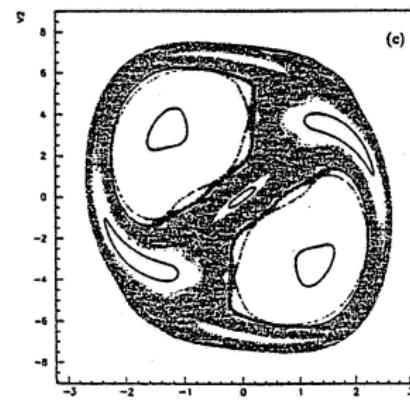
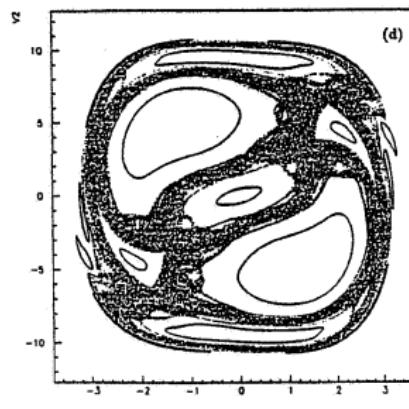
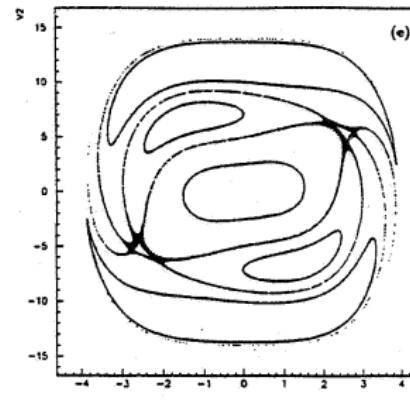
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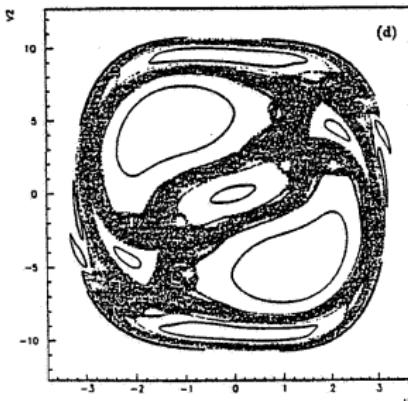
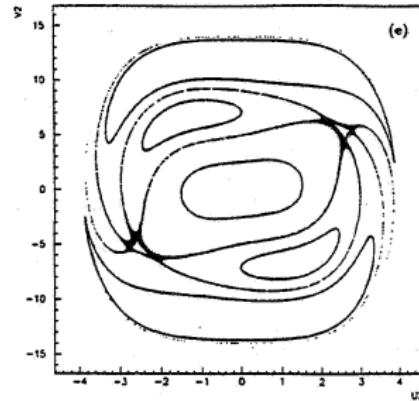
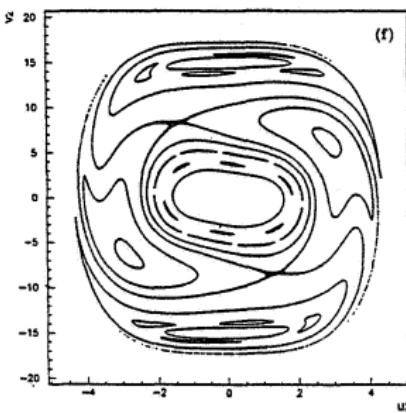
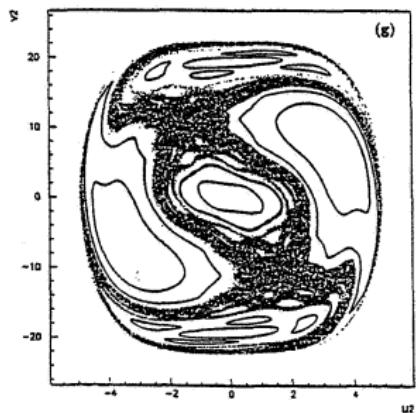
6 Conclusions

- The nearest neighbour distribution as a linear superposition of the Poisson and Wigner distributions

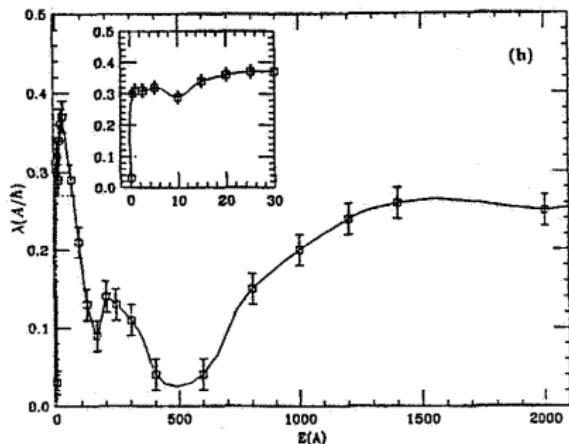
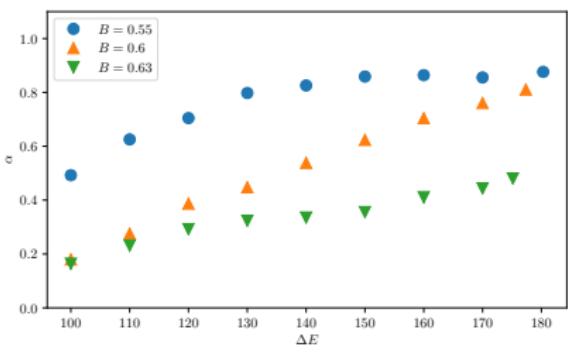
$$P(s) = \alpha P_P(s) + (1 - \alpha) P_W(s)$$



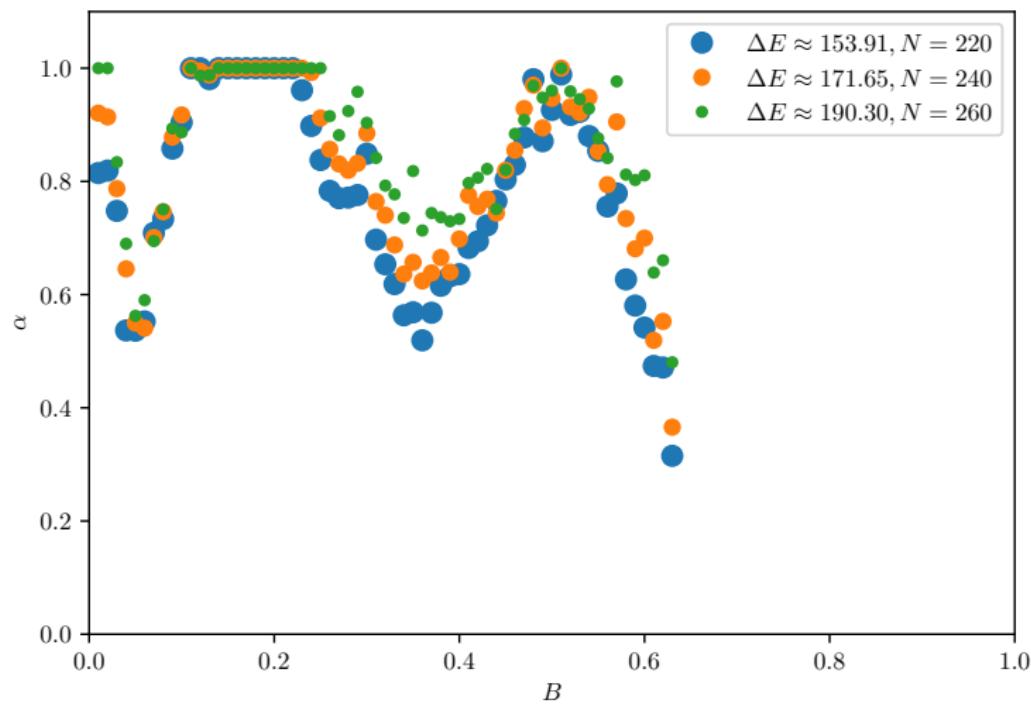
 $E = 30A$  $E = 120A$  $E = 240A$  $E = 400A$


 $E = 240A$

 $E = 400A$

 $E = 600A$

 $E = 1000A$

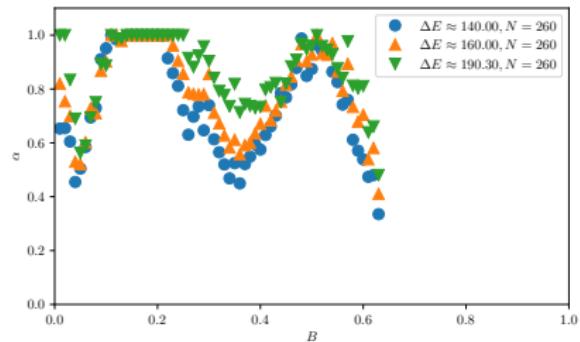
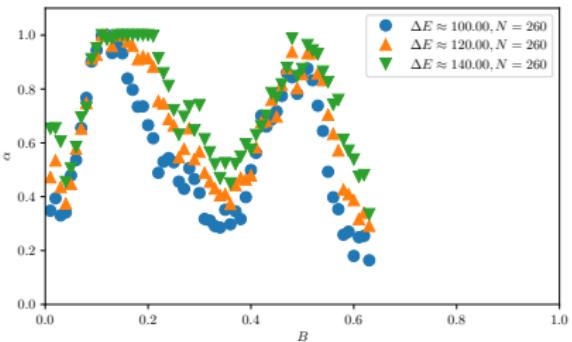
- The global phase space structure is reflected in the plot of α as a function of ΔE



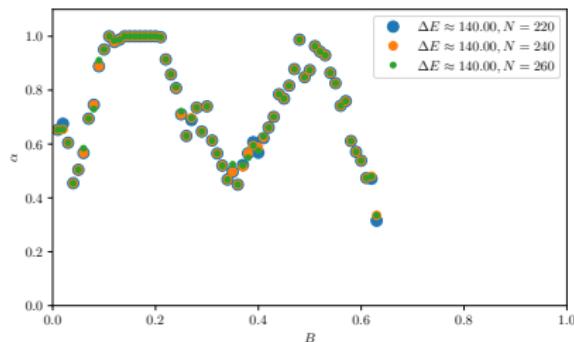
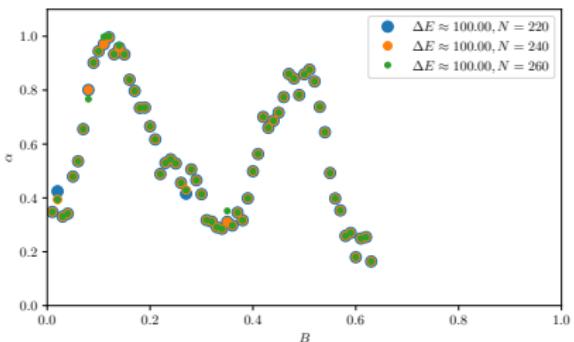
- The nontrivial dependence of α on the non-integrability parameter B for $N = 220, 240, 260$



- The qualitative stability of the shape of $\alpha(B)$ when considering $N = 260$ and different values for the energy interval



- The qualitative stability of the shape of $\alpha(B)$ when considering $N = 220, 240, 260$ and different values for the energy interval



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- We emphasised a correlation between the global phase space structure of the classical system and its quantum correspondent.
- We proposed a description of the nearest neighbour distribution of our spectra through a *superposition of Poisson and Wigner distributions*.
- A non-trivial dependence of the superposition coefficient on the non-integrability parameter was revealed.
- Through our analysis we showed a correlation between the tori volume in classical phase space and the deviation from the Wigner distribution in the quantum system.