

# Chapter 1

## Conclusions

In this thesis we have studied the connection between the classical phase space structures and the spectra of the corresponding quantum system. The basic question was to explore how the relative partition between tori and chaotic trajectories in the classical phase space is reflected in the statistics of the energy levels of the quantum analogue.

Starting with a Hamiltonian that describes the dynamics of the nuclear surface in terms of quadrupole vibrations, we obtained the energy spectra through numerical diagonalisation. The Hamiltonian depends on parameters, which at classical level induce the transition from an integrable system to a non-integrable one. As a consequence, in agreement with the K.A.M. theorem, the phase space starts to be populated with chaotic trajectories which escape from the tori, filling a 3 dimensional volume. We observed that the relative weights of the tori volumes in the phase space depends in a non-trivial way with the energy and the control parameter for non-integrability. Starting from this observation we investigated systematically the properties of the energy spectra as a function of the same control parameter and in different energy ranges. Specifically, we focused on the nearest neighbour distribution and compared it with the predictions of the Poisson and Wigner distributions.

We observed that for all values of the control parameter, the distribution manifests deviations from the expected Wigner distribution characteristic to chaotic systems. We assumed that this effects are related to the significant influence of the tori at the classical level. Therefore, we proposed a distribution which is a linear superposition of the Poisson and Wigner distributions. Then, the coefficient factorising the Poisson distribution will be a measure of closeness to an integrable behaviour. Indeed, if this coefficient is equal to unity, the distribution becomes Poissonian, while when it goes to zero, it transforms Wigner one.

We noticed that when we rise the energy range for the analysis of the level distribution, this coefficient increases, reflecting the observed behaviour in classical

phase space. In other words, the global structure of the classical phase space for an energy interval is reflected by the superposition fitting coefficient. More precisely, we observed that the value of this coefficient increases with the considered energy interval, while in the classical system, as we increase the energy, the volume of the regular trajectories confined on tori increases.

Furthermore, we obtained a non-trivial dependence of the superposition coefficient on the non-integrability parameter. While a greater value of this parameter is expected to make the system more chaotic, and consequently the level distribution gets closer to the Wigner one, from our analysis resulted a more intricate behaviour. The superposition coefficient decreases and increases periodically when the non-integrability parameter changes. This periodic shape shows that for specific values of the control parameter, the system will manifest a more robust integrable-like behaviour. Further work is required to better understand this aspect.