Chapter 1

Quantum Chaos

1.1 Level repulsion

1.2 Random matrix theory

define random matrices

Nearest neighbour spacing distributions show how the differences between consecutive energy levels fluctuate around the average. In order to understand better this concept we shall begin with the simpler case of real random numbers.

1.2.1 The nearest neighbour spacing distribution for real random numbers

We consider a sequence of uniformly distributed, ordered, real, random numbers. Let E represent a number in the sequence. The probability $P(s)\,\mathrm{d}s$ to have the next number between $E + s$ and $E + s + \mathrm{d}s$ is given by:

$$P(s)\,\mathrm{d}s = P(1 \in \mathrm{d}s \,|\, 0 \in s)P(0 \in s),$$

where $P(n \in s)$ represents the probability for s to contain n numbers and $P(n \in \mathrm{d}s \,|\, m \in s)$ is the *conditioned* probability for the interval of length $\mathrm{d}s$ to contain n numbers when the interval of length s contains m numbers.

Since random numbers are not correlated, the probability of a random number to be found in the interval $\mathrm{d}s$ does not depend on the number of random numbers in s, so

$$P(1 \in \mathrm{d}s \,|\, 0 \in s) = P(1 \in \mathrm{d}s).$$

Since the random numbers are uniformly distributed, the probability (density?) of finding a number in the interval $\mathrm{d}s$ is constant. We denote this constant with

a. Thus

$$P(s)\,\mathrm{d}s = a\,\mathrm{d}s P(0 \in s).$$

$P(0 \in s)$ can be expressed using the complementary probability as $1 - \int_s^\infty P(s')\,\mathrm{d}s'$. Now we can express $P(s)\,\mathrm{d}s$ as follows:

$$P(s)\,\mathrm{d}s = a\,\mathrm{d}s\left(1 - \int_s^\infty P(s')\,\mathrm{d}s'\right).$$

Using the Leibniz rule for differentiating integrals,

$$\frac{\mathrm{d}}{\mathrm{d}x}\int\limits_{G(x)}^{H(x)} F(x,t)\,\mathrm{d}t = \int\limits_{G(x)}^{H(x)} \frac{\partial F}{\partial x}\,\mathrm{d}t + F(x, H(x))\,\frac{\mathrm{d}H}{\mathrm{d}x} - F(x, G(x))\,\frac{\mathrm{d}G}{\mathrm{d}x}$$

we obtain

$$\frac{\mathrm{d}}{\mathrm{d}s}P(s) = -aP(s)$$

when differentiating with respect to s.