

Chapter 1

Quantum Chaos

1.1 Random matrix theory

1.1.1 Nearest neighbour spacing distributions

Nearest neighbour spacing distributions show how the differences between consecutive energy levels fluctuate around the average.

We consider a sequence of uniformly distributed, ordered, real, random numbers. Let E represent a number in the sequence. The probability $P(s)ds$ to have the next number between $E + s$ and $E + s + ds$ is given by:

$$P(s) ds = P(1 \in ds | 0 \in s)P(0 \in s),$$

where $P(n \in s)$ represents the probability for s to contain n numbers and $P(n \in ds | m \in s)$ is the *conditioned* probability for the interval of length ds to contain n numbers when the interval of length s contains m numbers.

Since random numbers are not correlated, the probability of a random number to be found in the interval ds does not depend on the number of random numbers in s , so

$$P(1 \in ds | 0 \in s) = P(1 \in ds).$$

Since the random numbers are uniformly distributed, the probability (density?) of finding a number in the interval ds is constant. We denote this constant with a . Thus

$$P(s) ds = a ds P(0 \in s).$$

$P(0 \in s)$ can be expressed using the complementary probability as $1 - \int_s^\infty P(s') ds'$. Now we can express $P(s) ds$ as follows:

$$P(s) ds = a ds \left(1 - \int_s^\infty P(s') ds' \right).$$

Using the Leibniz rule for differentiating integrals,

$$\frac{d}{dx} \int_{G(x)}^{H(x)} F(x, t) dt = \int_{G(x)}^{H(x)} \frac{\partial F}{\partial x} dt + F(x, H(x)) \frac{dH}{dx} - F(x, G(x)) \frac{dG}{dx}$$

we obtain

$$\frac{d}{ds}P(s) = -aP(s)$$

when differentiating with respect to s .