Chapter 1

Quantum Chaos

1.1 Random matrix theory

1.1.1 Nearest neighbour spacing distributions

Nearest neighbour spacing distributions show how the differences between consecutive energy levels fluctuate around the average.

We consider a sequence of uniformly distributed, ordered, real, random numbers. Let E represent a number in the sequence. The probability $P(s)\,\mathrm{d}s$ to have the next number between $E+s$ and $E+s+\mathrm{d}s$ is given by:

$$P(s)\,\mathrm{d}s = P(1 \in \mathrm{d}s \,|\, 0 \in s)P(0 \in s),$$

where $P(n \in s)$ represents the probability for s to contain n numbers and $P(n \in \mathrm{d}s \,|\, m \in s)$ is the *conditioned* probability for the interval of length $\mathrm{d}s$ to contain n numbers when the interval of length s contains m numbers.

Since random numbers are not correlated, the probability of a random number to be found in the interval $\mathrm{d}s$ does not depend on the number of random numbers in s, so

$$P(1 \in \mathrm{d}s \,|\, 0 \in s) = P(1 \in \mathrm{d}s).$$

Since the random numbers are uniformly distributed, the probability (density?) of finding a number in the interval $\mathrm{d}s$ is constant. We denote this constant with a. Thus

$$P(s)\,\mathrm{d}s = a\,\mathrm{d}s P(0 \in s).$$

$P(0 \in s)$ can be expressed using the complementary probability as $1 - \int_s^\infty P(s')\,\mathrm{d}s'$. Now we can express $P(s)\,\mathrm{d}s$ as follows:

$$P(s)\,\mathrm{d}s = a\,\mathrm{d}s\left(1 - \int_s^\infty P(s')\,\mathrm{d}s'\right).$$

Using the Leibniz rule for differentiating integrals,

$$\frac{\mathrm{d}}{\mathrm{d}x}\int\limits_{G(x)}^{H(x)} F(x,t)\,\mathrm{d}t = \int\limits_{G(x)}^{H(x)} \frac{\partial F}{\partial x}\,\mathrm{d}t + F(x,H(x))\,\frac{\mathrm{d}H}{\mathrm{d}x} - F(x,G(x))\,\frac{\mathrm{d}G}{\mathrm{d}x}$$

we obtain

$$\frac{\mathrm{d}}{\mathrm{d}s}P(s) = -aP(s)$$

when differentiating with respect to s.