Fingerprints of classical phase space structure in quantum chaos

Bachelor Thesis

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Outline

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- The aim of this thesis is to study the chaotic behaviour of nuclei considering a liquid-drop-like model.

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- Hamilton's equations

$$\dot{q}_k = \frac{\partial \mathcal{H}}{\partial p_k}, \qquad \dot{p}_k = -\frac{\partial \mathcal{H}}{\partial q_k}$$

- Action-angle variables

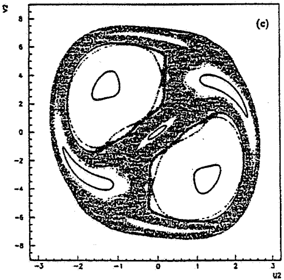
$$I_k = \frac{1}{2\pi} \oint p_k \, \mathrm{d}q_k \,, \qquad \theta_k = \omega_k(I_k)\Delta t$$

- Integrability: A system is *integrable* if the number of degrees of freedom is equal to the number of constants of the motion (in involution which each other).
- For integrable systems the trajectory in phase space is constrained to a n-torus.

K.A.M. Theorem

- If the motion of a system described by an integrable Hamiltonian is perturbed with a small non-integrable term and the frequencies ($\omega_k = \dot{\theta}_k$) of the unperturbed motion are such that for $k = 1, \ldots, n$ we have $m_k \, \omega_k \neq 0, \forall m_k \in \mathbb{Z}$, then the motion will remain on the n-torus, except for a small set of initial conditions, for which the trajectories escape on the energy hyper-surface.

- Poincaré sections: Slices of the phase space which reduce the dimensionality by one.
- Liapunov exponent
  $\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \frac{\mathrm{d}x(t)}{\mathrm{d}x(0)}$
- Chaos is characterised by sensitivity to the initial conditions.



- In Quantum Mechanics we can no longer speak of trajectories or sensitivity to the initial conditions.
- We need a way to compare spectra from different systems in order to search for common patterns or features.
- We can use the level spacings ($E_{n+1} - E_n$) to build histograms (nearest neighbour level spacing distributions)

Giving meaning to quantum chaos

- Berry-Tabor conjecture: The quantum counterpart of a classically integrable system has a Poissonian nearest neighbour distribution.

$$P_P(s) = \mathrm{e}^{-s}$$

- Bohigas-Gianoni-Schmit conjecture: The nearest neighbour distribution of a quantum system with a classically chaotic counterpart is given by the Wigner distribution.

$$P_W(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right)$$

Other measures

- Cumulative spacing distribution

$$I(s) = \int_0^s P(x)\,\mathrm{d}x$$

Outline

- The radius of the nuclear surface

$$R(\theta, \varphi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha^*_{\lambda,\mu} Y_{\lambda,\mu}(\theta, \varphi) \right)$$

- For $\lambda = 2$ we describe the surface in terms of quadrupole vibrations.
- $\alpha_{\lambda,\mu}$ become dynamical degrees of freedom

- We want to study only the vibrations of the surface, so we will use the intrinsic reference frame of the nucleus. Thus we will only have 2 degrees of freedom.
- The Hamiltonian of the system will be given by

$$\mathcal{H}(q_0, p_0, q_2, p_2) = \frac{A}{2}(p_0^2 + p_2^2 + q_0^2 + q_2^2) + \frac{B}{\sqrt{2}} q_0(3q_2^2 - q_0^2) + \frac{D}{4}(q_0^2 + q_2^2)^2$$

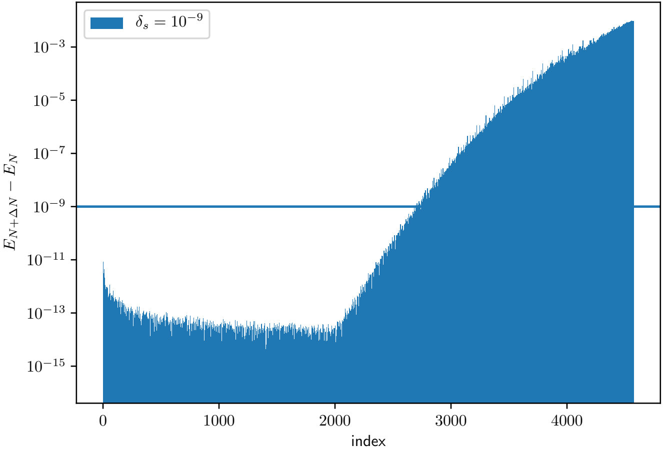
- Quantised Hamiltonian

$$
\begin{aligned}
H = {} & A\left(a_1^\dagger a_1 + a_2^\dagger a_2\right) + \frac{B}{4}\Bigg[\left(3a_1^\dagger {a_2^\dagger}^2 + 3a_1 a_2^2 - {a_1^\dagger}^3 - a_1^3\right) \\
& + 3\left(a_1 {a_2^\dagger}^2 + a_1^\dagger a_2^2 - a_1^\dagger a_1^2 - {a_1^\dagger}^2 a_1 + 2a_1 a_2^\dagger a_2 + 2a_1^\dagger a_2^\dagger a_2\right)\Bigg] \\
& + \frac{D}{16}\Bigg[6\left({a_1^\dagger}^2 a_1^2 + {a_2^\dagger}^2 a_2^2\right) + 2\left(a_1^2 {a_2^\dagger}^2 + {a_1^\dagger}^2 a_2^2\right) + 8a_1^\dagger a_1 a_2^\dagger a_2 \\
& + 4\left(a_1^\dagger a_1^3 + {a_1^\dagger}^3 a_1 + a_2^\dagger a_2^3 + {a_2^\dagger}^3 a_2 + a_1^2 a_2^\dagger a_2 + {a_1^\dagger}^2 a_2^\dagger a_2 + a_1^\dagger a_1 a_2^2 + a_1^\dagger a_1 {a_2^\dagger}^2\right) \\
& + \left({a_1^\dagger}^4 + a_1^4 + {a_2^\dagger}^4 + a_2^4 + 2{a_1^\dagger}^2 {a_2^\dagger}^2 + 2a_1^2 a_2^2\right)\Bigg]
\end{aligned}
$$

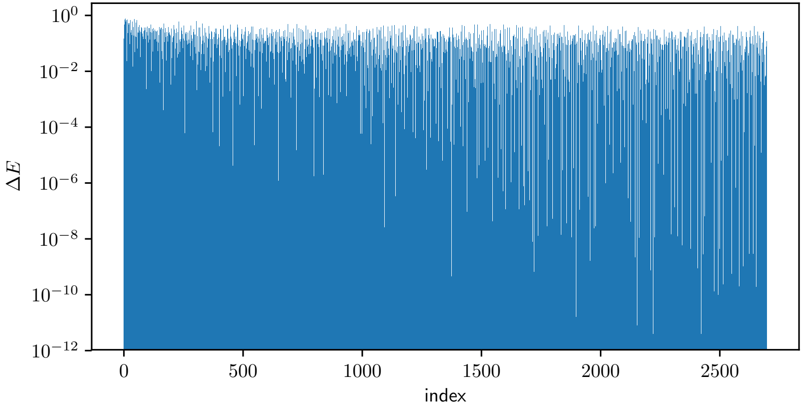
Outline

- Computing the Hamiltonian
- Diagonalisation
- Stability
- Separating the irreducible representations
- Statistics

Stability

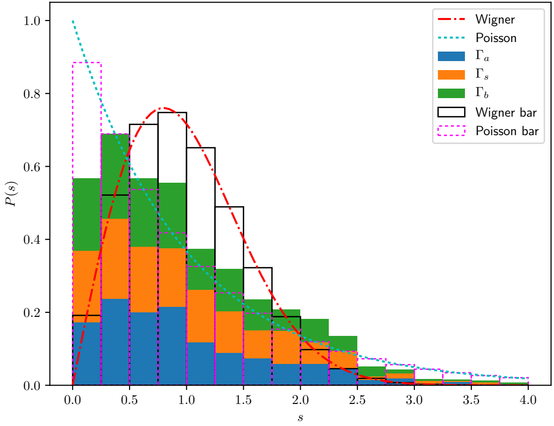

$\delta_s = 10^{-9}$
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
$E_{N+\Delta N} - E_N$
0
1000
2000
3000
4000
index

Separating the irreducible representations



The nearest neighbour distribution

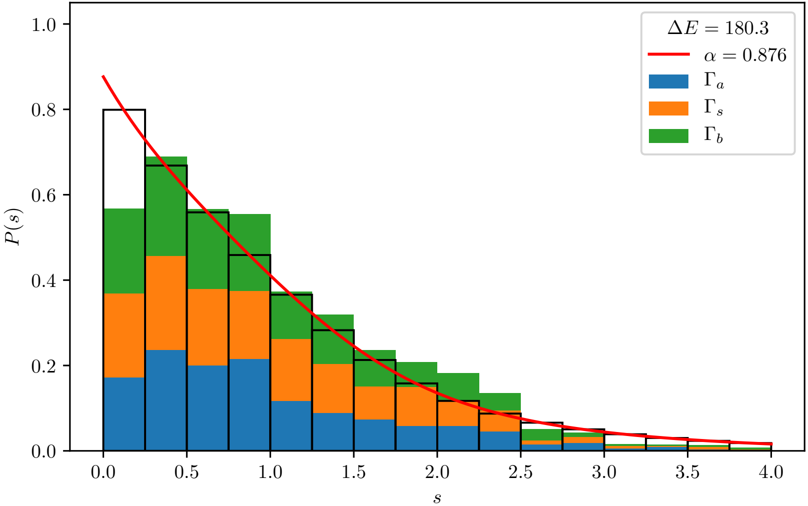
$P(s)$ for $B = 0.55$, $D = 0.4$, $N = 120$ compared with the Poisson and Wigner distributions both in continuous and discrete forms.

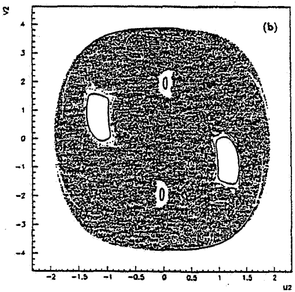


Outline

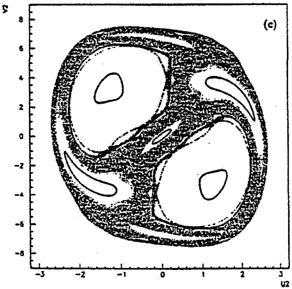
- The nearest neighbour distribution as a linear superposition of the Poisson and Wigner distributions

$$P(s) = \alpha P_P(s) + (1 - \alpha) P_W(s)$$

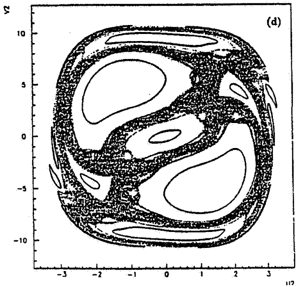

$\Delta E = 180.3$
$\alpha = 0.876$
Γ_a
Γ_s
Γ_b
$P(s)$
s
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0



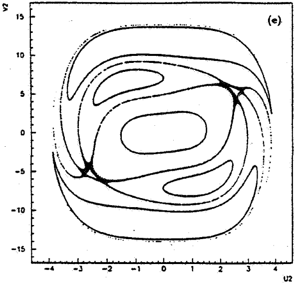
$E = 30$



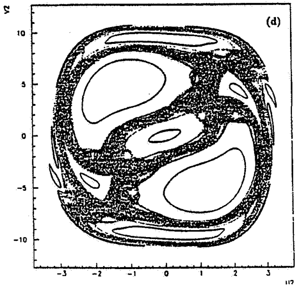
$E = 120$



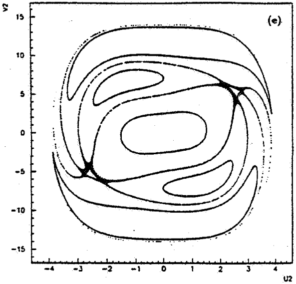
$E = 240$



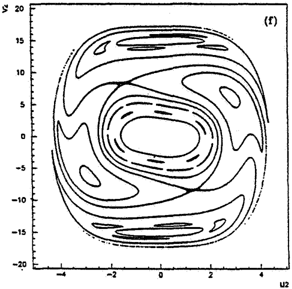
$E = 400$



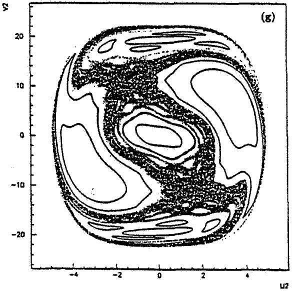
$E = 240$



$E = 400$

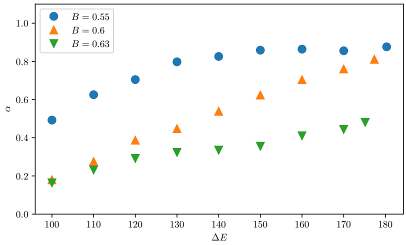


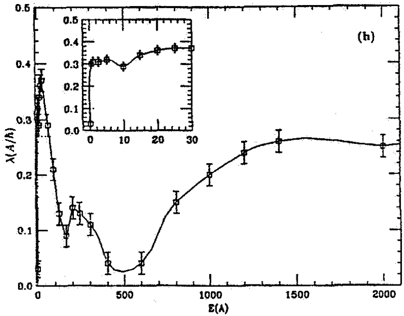
$E = 600$



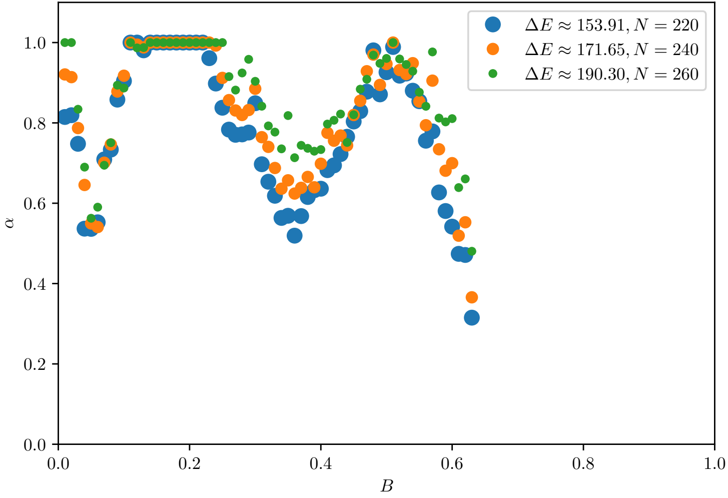
$E = 1000$

- The global phase space structure is reflected in the plot of α as a function of ΔE

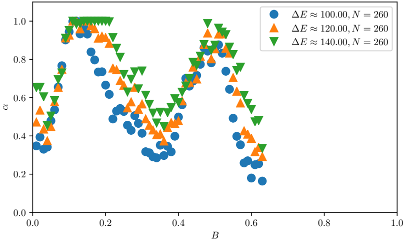


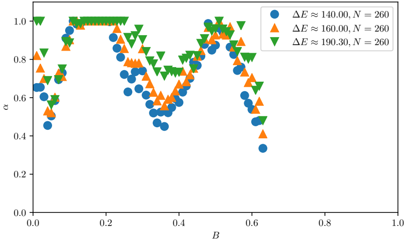


- The nontrivial dependence of α on the non-integrability parameter B for $N = 220, 240, 260$



- The qualitative stability of the shape of $\alpha(B)$ when considering $N = 260$ and different values for the energy interval





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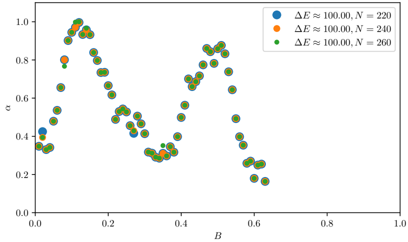

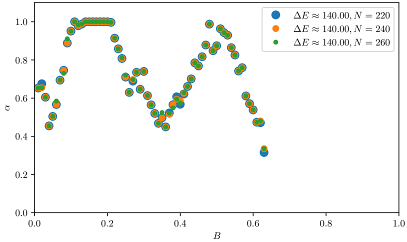

Outline

Conclusions

- We emphasised a correlation between the global phase space structure of the classical system and its quantum corespondent.
- We proposed a description of the nearest neighbour distribution of our spectra through a superposition of Poisson and Wigner distributions.
- A non-trivial dependence of the superposition coefficient on the non-integrability parameter was revealed.
- Through our analysis we showed a correlation between the tori volume in classical phase space and the deviation from the Wigner distribution in the quantum system.